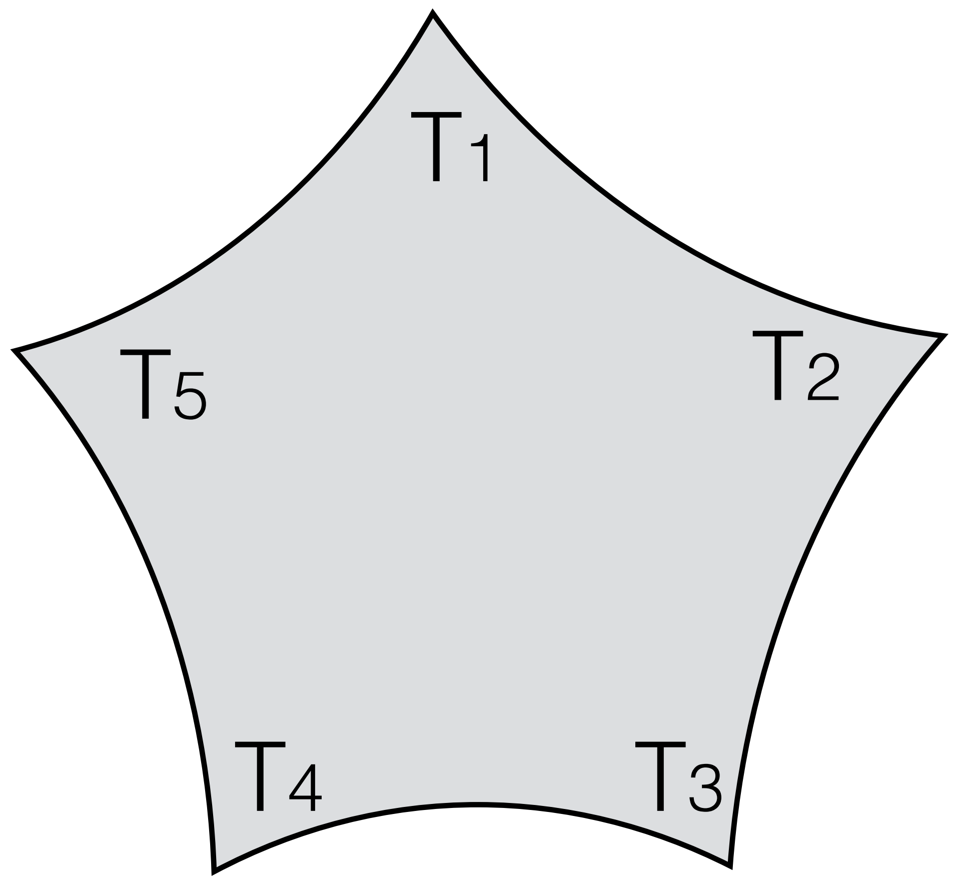
# Boundaries, Interfaces and Dualities

# Dualities I

Complementary weakly coupled descriptions
in a space of exactly marginal couplings



# Dualities II

IR free effective description of
asymptotically free UV theory

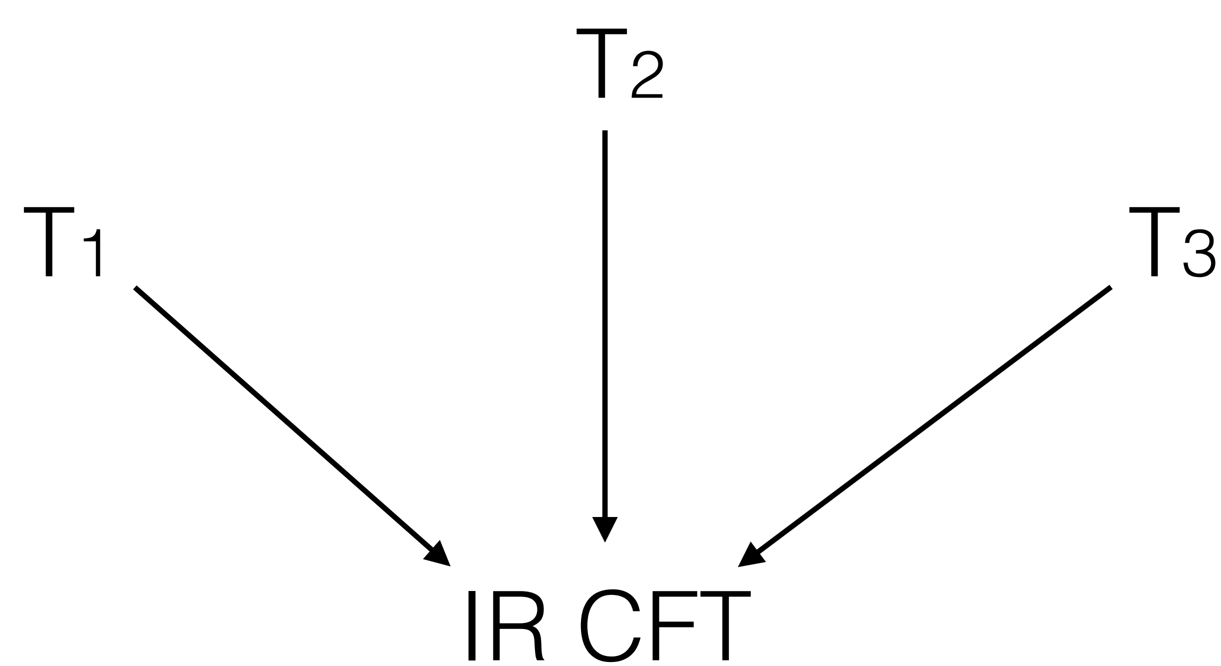
$T_{UV}$

$\downarrow$

$T_{IR}$

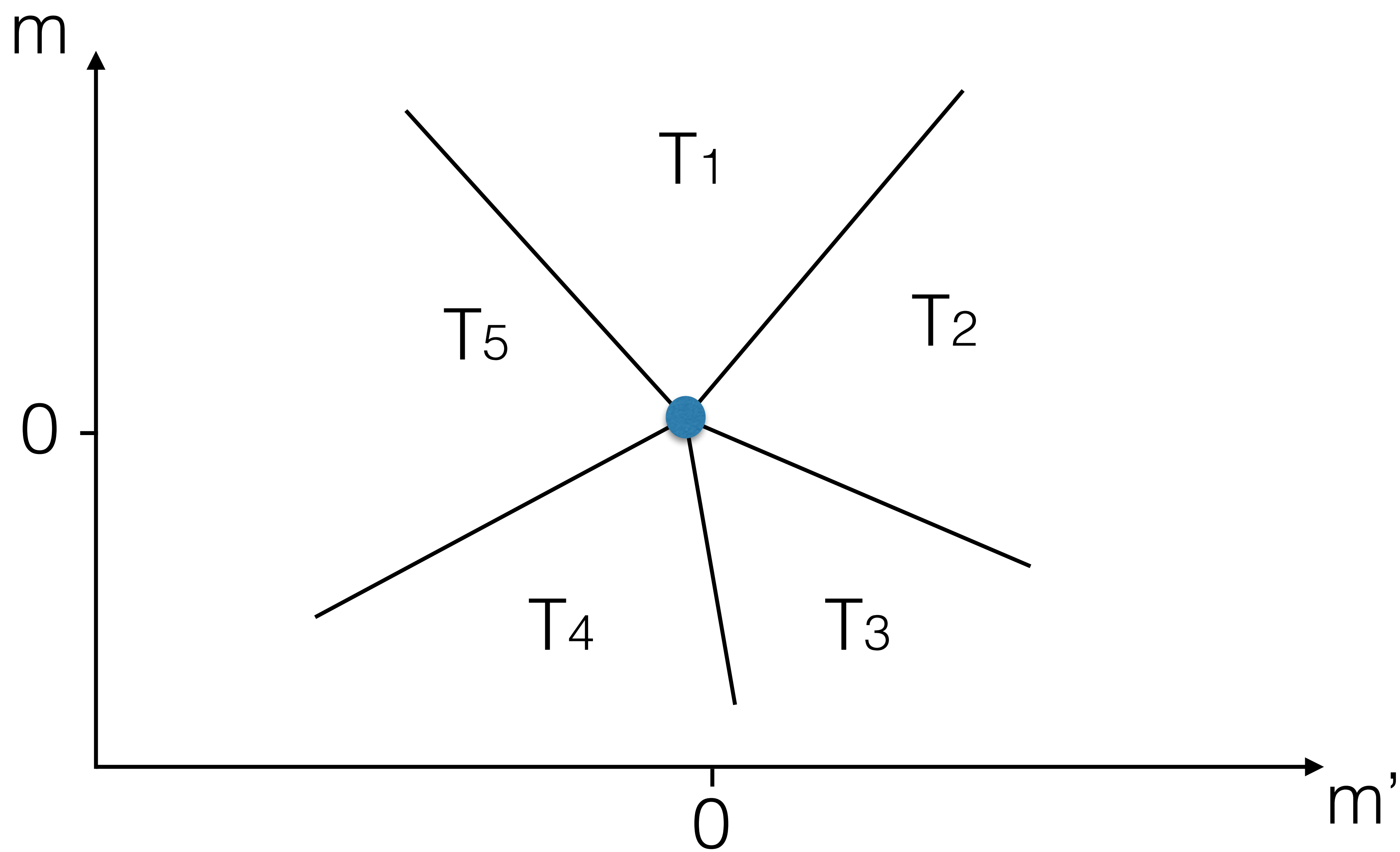
# Dualities III

Alternative UV definitions of a single CFT



# Dualities IV

IR free mass deformations of UV CFT



# A tale of many dimensions and supercharges

- IR gauge theory descriptions of 5d and 6d SCFTs
- 4d: N=4 S-duality, N=2 S-dualities and Seiberg-Witten theories, N=1 Seiberg dualities, ….
- 3d: N=4 mirror symmetries, N=2 mirror symmetries, N=2 Seiberg-like and level-rank dualities, N=0 level-rank dualities, …..
- 2d: (2,2) mirror symmetries, (2,2) Seiberg-like dualities, (0,2) dualities, level-rank dualities,…..

# Exact tests

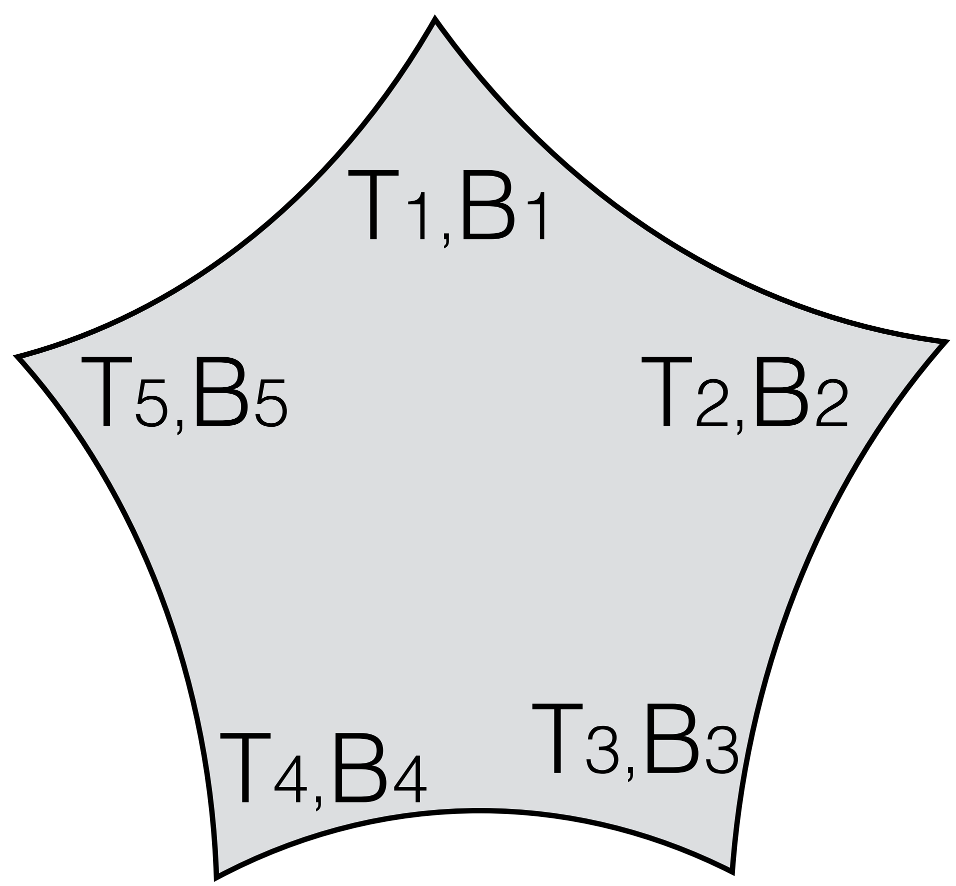
- Anomaly matching, relevant deformations, etc.
- Many supersymmetric dualities can be tested by moduli space of vacua, localization for sphere partition functions, supersymmetric indices, etc.
- The match between dual quantum field theories is often a very non-trivial property of intricate mathematical objects

# Matching defects

- Quantum field theories can be enriched by local extended defects of various dimensions.
- Can we match them across dualities? More precisely, can we extend dualities of the bulk theory to include defects?
- Today focus on boundary conditions

# Dualities I

Transport boundary conditions along moduli space



# Dualities II

IR effective description of UV boundary condition

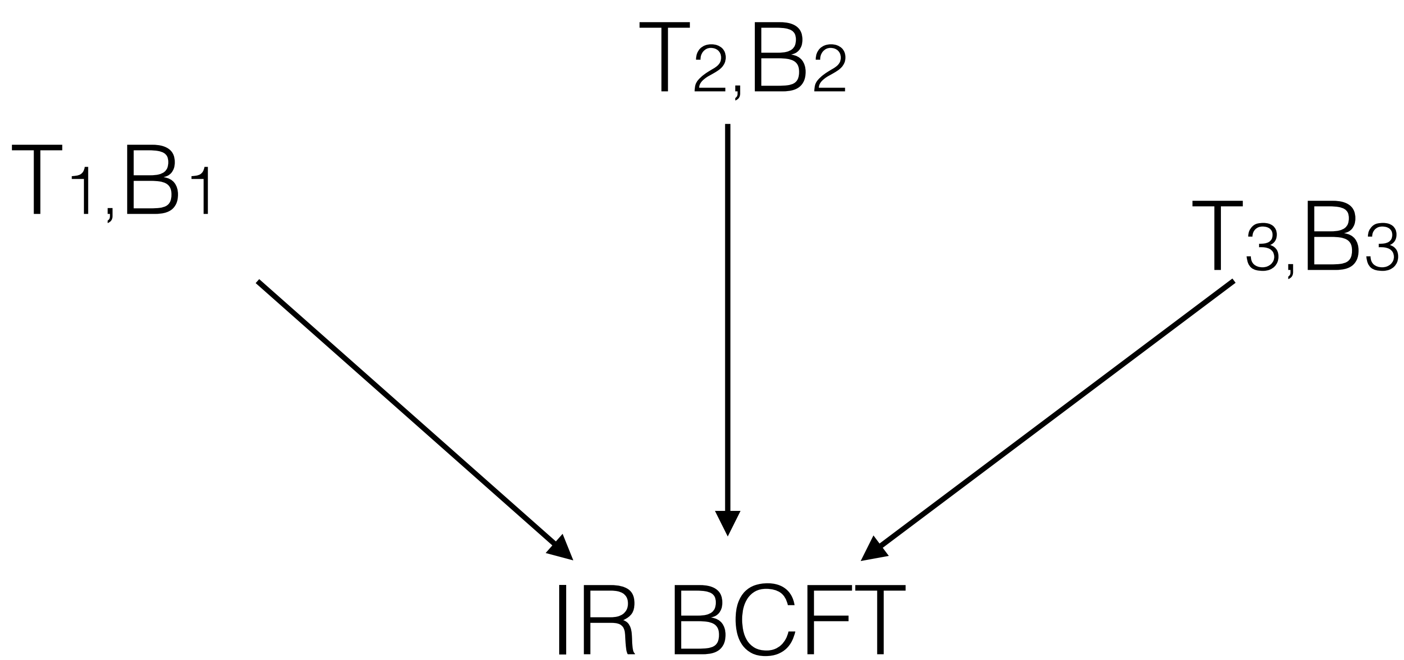
$T_{UV}, B_{UV}$

$\downarrow$

$T_{IR}, B_{IR}$

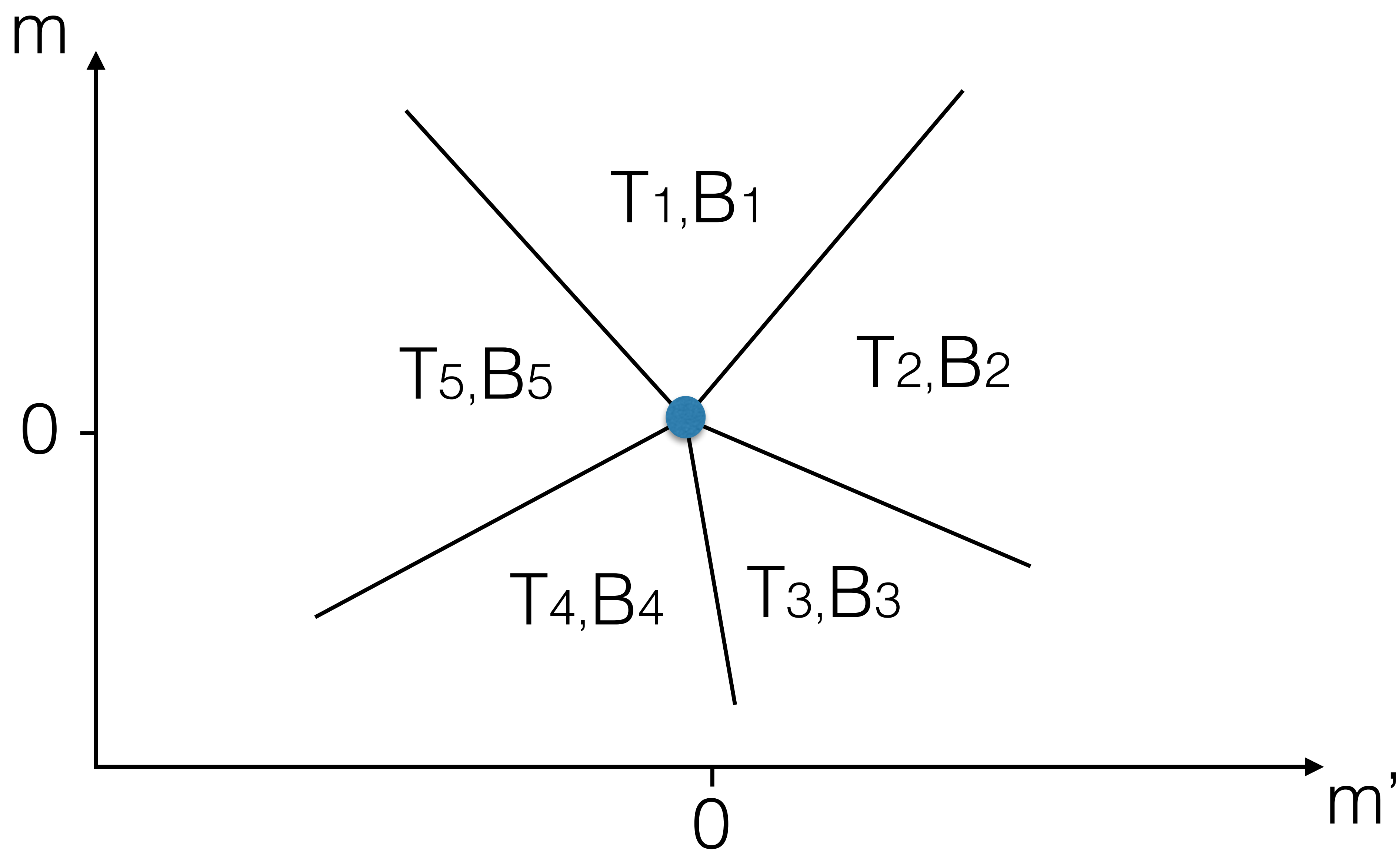
# Dualities III

Alternative UV definitions of a single
Boundary Conformal Field Theory



# Dualities IV

IR free mass deformations of UV BCFT



# Duality interfaces

- Often dualities can be “implemented” by duality interfaces
- Interfaces between dual theories which are dual to trivial (or better “Janus”) interfaces in either theory
- Duality interfaces often “compose” according to duality group law. Relations follow from lower dimensional dualities.

# A tale of many dimensions and supercharges

- 4d N=1 boundaries of 5d SCFTs
- S-duality of 3d N=4, N=2 boundary conditions, N=2 boundary conditions in Seiberg-Witten theory, …..
- Mirror symmetry of 2d (2,2) and (0,2) boundary conditions. Level rank duality of (0,2) boundary conditions. Level-rank duality of N=0 boundary conditions.
- Brane dualities.

# A simple duality

A 3d fermion coupled to $U(1)_{-\frac{1}{2}}$ Chern-Simons

The O(2) Wilson-Fisher fixed point

A 3d complex scalar with quartic potential

# Boundary conditions for fermions

- Set to zero a chiral half of the fermion at the boundary:

  - $B_\psi^+ : \psi_+|_\partial = 0$
  - $B_\psi^- : \psi_-|_\partial = 0$
  - $B_\psi^- \equiv B^+ + \lambda_+^{2d} + \left( \int_\partial \psi_- \lambda_+^{2d} \right)$
  - Boundary conditions have half of anomalies of a 2d fermion

# Neumann b.c.

- Neumann b.c for gauge field: gauge transformation non-trivial at the boundary, $F_{\perp ||} = J_{||}$
- Require anomaly cancellation at the boundary
- Classically breaks Topological U(1) symmetry:

$$J_{\perp}^{\rm top} = F_{|| ||}$$

# Bare Neumann b.c.

- $B^+_\psi$ cancels bulk Chern-Simons anomaly inflow
- Breaks U(1) topological!
- Monopole charge has opposite image.
- Flows to “exceptional transition” boundary condition.

$$\phi \sim \frac{c e^{i\theta_{2d}}}{x_\perp^{\Delta_\phi}} + \cdots$$

# Enriched Neumann

- $B_\psi^- + \lambda_+^{\rm 2d}$ Cancels anomaly from bulk CS term
- Topological U(1) restored by acting on 2d fermion!

$$J_\perp^{\rm top} = F_{\| \|} = \partial_\| J_\lambda^\|$$

- Flows to generic O(2) preserving b.c. : “ordinary transition”

$$\psi_-|_\partial \lambda_+^{\rm 2d} \to \partial_\perp \phi|_\partial$$

# Dirichlet b.c.

- Dirichlet b.c. for gauge fields: gauge tranformations trivial at boundary, $A_{\|} = 0$
- Gauge symmetry becomes global symmetry at the boundary.
- Interesting for dualities! Dual boundary condition remembers original gauge group.
- Boundary monopoles! Flow to “special transition”?

# General level-rank duality

$$U(k)_{\frac{N_f}{2}-N} + N_f \, \text{fermions} \leftrightarrow SU(N)_k + N_f \, \text{WF} \, \text{scalars}$$

- Observation: $B_\psi^- + N \times \lambda_+^{2d}$ cancels anomaly for Neumann b.c. on the left hand side
- Global symmetry and anomalies match Dirichlet b.c. on the right hand side
- Observation: $k \times \eta_-^{\text{2d}}$ cancels anomaly for Neumann b.c. on the right hand side
- Global symmetry and anomalies match Dirichlet b.c. on the left hand side

# Level rank duality in pure Chern-Simons

- Relation between boundary conditions already non-trivial:

  - $SU(N)_k$ Chern-Simons + Dirichlet b.c

  - $U(k)_N$ Chern-Simons + N x k 2d fermions + Neumann b.c.

- Follows from 2d coset formulation of level rank duality: 2d bifundamental fermions are secretly the product of two WZW models.

# Tests

- Large N,k calculations (Radicevic)
- Supersymmetrize: level rank duality has a 3d N=2 ancestor.
- Bifundamental fermions -> bifundamental (0,2) Fermi multiplets
- Test with supersymmetric index of boundary operators (Dimofte)

Happy Birthday!