Problems

PCMI GSS 2024 - $\mathbb{A}^1$-algebraic topology (following F. Morel)

1 Show that $B\mathbb{Z} = S^1$ is 1-truncated.

2 If X is a pointed nice[1] topological space, and $\tilde{X} \to X$ is the universal cover then $\tilde{X}$ is 2-connective and we have a fibre sequence
$$\tilde{X} \to X \to B\pi_1(X)$$

3 Show that in $\mathcal{H}(k)$:

    (a) $\Sigma(\mathbb{G}_m, 1) \simeq \mathbb{P}^1$.

    (b) Show that $\mathbb{P}^n/\mathbb{P}^{n-1} \simeq \mathbb{A}^n/(\mathbb{A}^n \setminus \{0\})$

    (c) Show that $\Sigma^{n-1}(\mathbb{G}_m, 1)^{\wedge n} \simeq (\mathbb{A}^{n-1} \setminus \{0\}, 1)$

4 Show that SL_n is $\mathbb{A}^1$-1-connective.

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[1] reasonable enough so that the universal cover exists