SIMPLIFIED TOV EQUATIONS FOR RELATIVISTIC MATTER

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BASED ON: S. ADLER & B. DOHERTY arXiv: 2309.13380 V9

- TOLMAN OPPENHEIMER VOLKOFF (TOV)

$p(r) =$ PRESSURE $\rho(r) =$ DENSITY

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r)$$

$$\frac{dp(r)}{dr} = -\frac{\rho(r) + p(r)}{2}\frac{d\nu(r)}{dr} \qquad \nu(r) = \log g_{00}(r)$$

$$\frac{d\nu(r)}{dr} = \frac{N(r)}{D(r)} \qquad N(r) = \frac{2}{r^2}\left[m(r) + 4\pi r^3 p(r)\right]$$

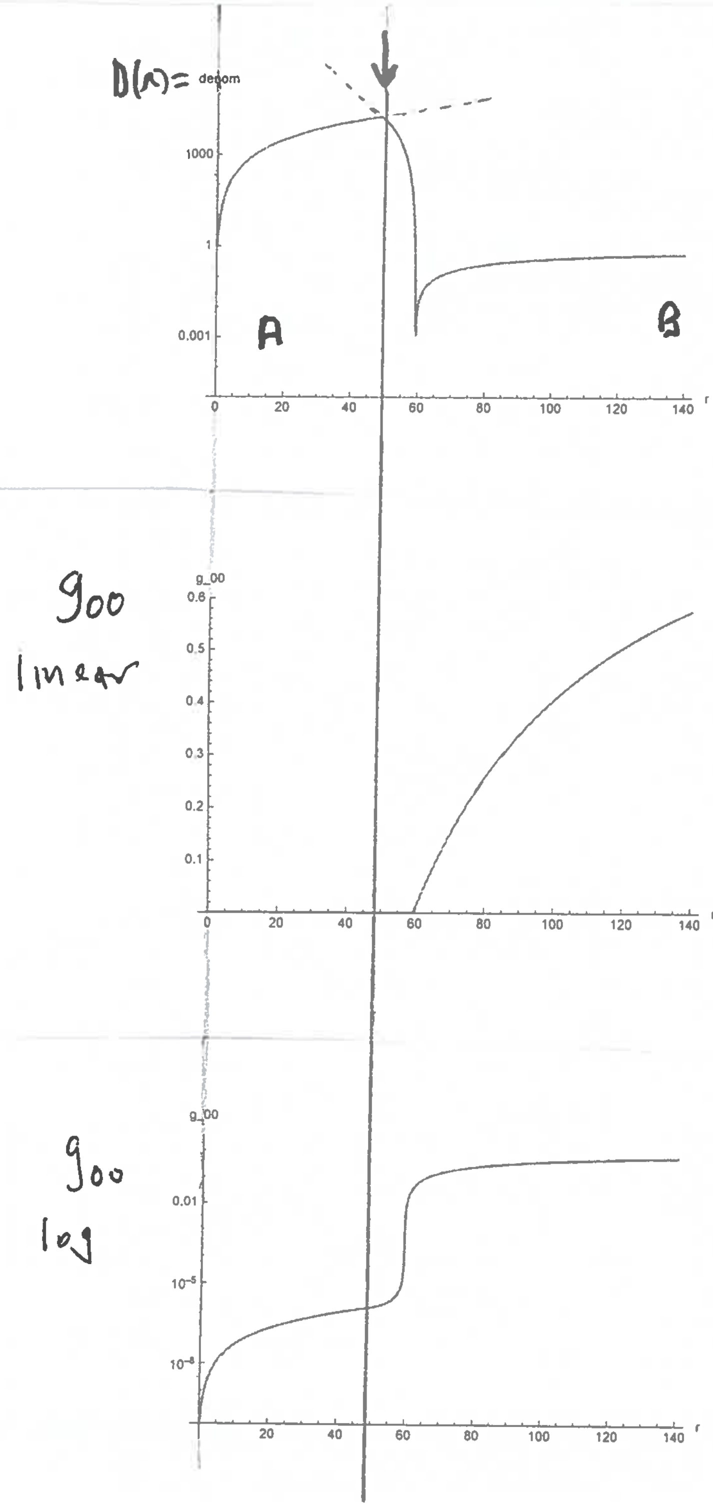
$$D(r) = 1 - \frac{2m(r)}{r}$$

- GIVE EQ. OF STATE $\rho(p)$

MODEL STUDIED: $\rho = 3p$ FOR $p \leq p_{jump}$

$p + \rho = \beta > 0$ FOR $p > p_{jump}$

BOUNDEDNESS $\Rightarrow$ p CONTINUOUS $\Rightarrow$ JUMP IN ρ



↓ $p_{jump} = 0.95$

$\Rightarrow r_{jump} = 48.895$

REGION	EQ. OF STATE
A	$\rho + p = .01$
B	$\rho = 3p$

NEW VARIABLES - RESCALING INVARIANT

$t = \log r \qquad dt = dr/r$

$\alpha(t) = m(r)/r \qquad \delta(t) = 4\pi r^2 p(r) \qquad \rho(r) = 3p(r)$

REWRITTEN TOV

$$\frac{d\alpha(t)}{dt} = 3\delta(t) - \alpha(t)$$

$$\frac{d\delta(t)}{dt} = -4\delta(t)\,\frac{\delta(t) + 2\alpha(t) - 1/2}{1 - 2\alpha(t)} \qquad (\text{SO } D(t) = 1 - 2\alpha(t))$$

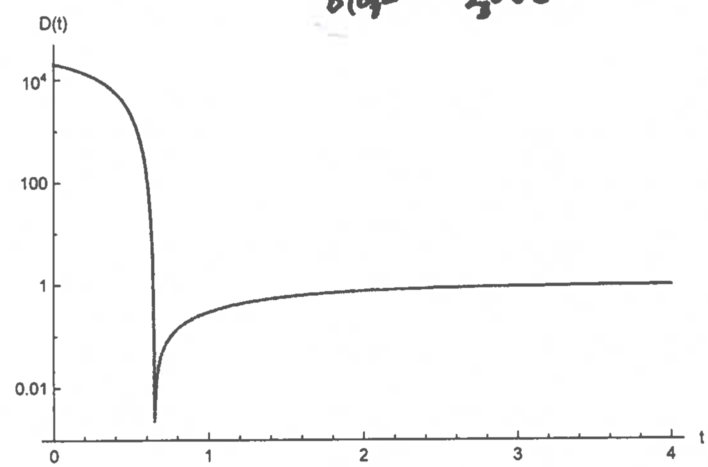
"AUTONOMOUS" - NO EXPLICIT t DEPENDENCE ON RIGHT

$\Rightarrow$ ANALYTICITY OF α, δ IN t FIXED BY POLES ON RIGHT

SO IF $\alpha(t) \neq 1/2$, α AND δ ARE C^∞

EXAMPLE:

$\alpha(0) = -10{,}000$
$\delta(0) = 3000$



↑ LOOKS SHARP, BUT IS C^∞!

CONJECTURE:

$\alpha(0) < 1/2$, $\delta(0) > 0$ AND $3\delta(0) - \alpha(0) > 0 \Rightarrow$

TOV EQUATIONS FOR RELATIVISTIC MATTER YIELD A KINK SOLUTION OR "SIMULATED HORIZON"