

REPORT

The program on the variational methods of geometric functionals has developed the general theory and helped to find connections between themes. The following is a description of some of the advances obtained since the program.

During the program, Xin Zhou proved the Multiplicity One Conjecture in the Almgren-Pitts setting. The isoperimetric problem in the real projective spaces was solved by Celso Viana. The conjecture for smooth generic metrics stated that minimal hypersurfaces that are constructed by the min-max theory have integer multiplicity one. This was a key tool in the development of a Morse theory for the area functional.

Since the program advances were obtained in the construction of minimal surfaces with genus bounds. The Pitts-Rubinstein conjecture on the existence of minimal surfaces in three-manifolds isotopic to strongly irreducible Heegaard splittings was solved by Daniel Ketover, Yevgeny Liokumovich and Antoine Song. A main difficulty in the Pitts-Rubinstein approach to this problem was the issue of multiplicity. Zhichao Wang and Xin Zhou announced the solution of this problem for unstable surfaces. They used this to construct four embedded minimal spheres for generic smooth metrics on three-spheres (for general metrics this is a conjecture of Yau). Some exciting new constructions of embedded minimal tori were announced recently by Adrian Chu and Yangyang Li, and by Xingzhen Li and Zhichao Wang. These works solve conjectures of White in positive Ricci curvature.

During the program there was general interest in variational constructions of minimal surfaces using PDE methods. Otis Chodosh and Christos Mantoulidis have used this approach to determine the constant in the Weyl law for the volume spectrum of surfaces. The solution of the stable Bernstein problem was since then extended to hypersurfaces of dimension up to five, starting with work of Otis Chodosh and Chao Li.

By discussions that started during the program, we have explored some connections of minimal surface theory with the geodesic flow in negative curvature (we wrote a paper with Danny Calegari and Andre Neves). We investigated, with Lucas Ambrozio and Andre Neves, questions related to the rigidity of metrics for the variationally defined invariants of the area functional.