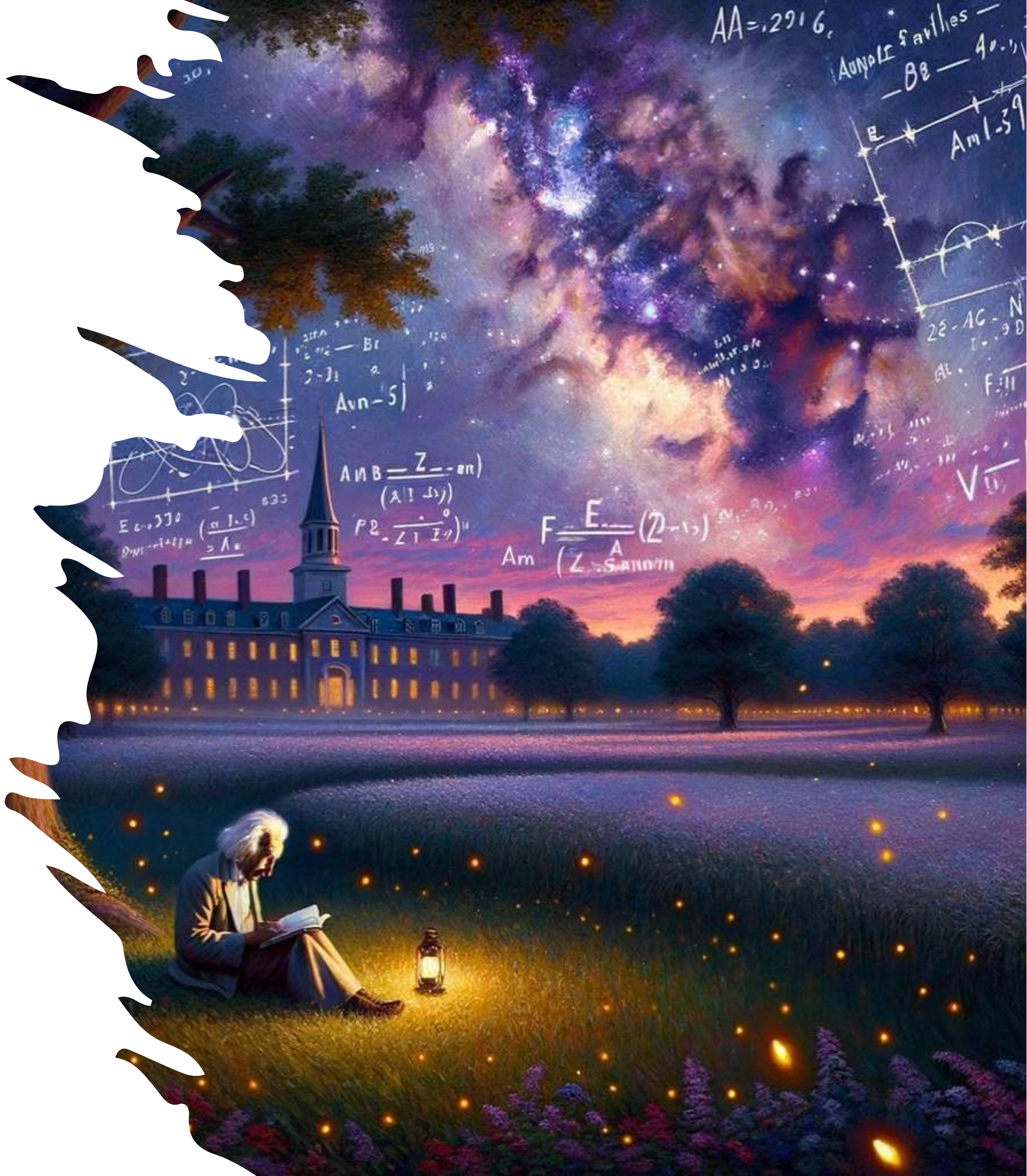
Wednesday

ias.edu/amplitudes2024

Gong Show

starting at 4:10 pm

(note the early start)



Surfaceology for Colored Yukawa Theory

Marcos Skowronek
Based on work with S. De, A. Pokraka, M. Spradlin, A. Volovich

Department of Physics
Brown University

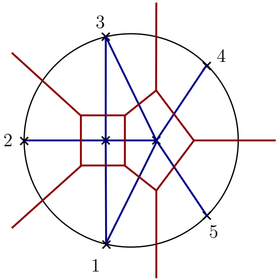
Amplitudes 2024, IAS

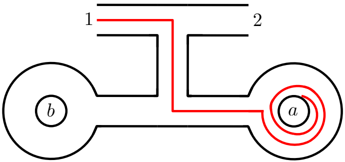
BROWN

The curve integral

Recent works: amplitudes can be written as single integrals over combinatorial objects constructed from surfaces.
Novel results: hidden zeroes, new factorization relations, gluons and pions from scalars, etc.

N. Arkani-Hamed, Q. Cao, J. Dong, C. Figuereido, S. He, H. Frost, P-G. Plamondon, G. Salvatori, H. Thomas



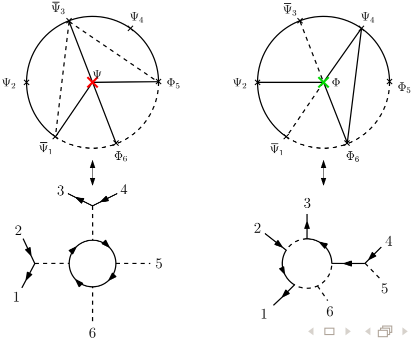


Missing step: fermions?

Real world $\longrightarrow$ we need to include models with fermionic particles.
Yukawa with colored fermions and scalars:

$$\mathcal{L} = i\mathrm{Tr}\big(\bar{\Psi}\partial\!\!\!/\Psi\big) - \frac{1}{2}\mathrm{Tr}\big(\partial_\mu\Phi\partial^\mu\Phi\big) + g\mathrm{Tr}\big(\bar{\Psi}\Psi\Phi\big) + \lambda\mathrm{Tr}\big(\Phi^3\big)\ .$$

Vertex orientation $\Rightarrow$ strict constraints on the amplitude.
Species and placement of curves is completely determined.



Final formula for the amplitude

Tropical numerator $\mathcal{N}(\boldsymbol{t})$: combinatorial object, no need to sum over Feynman diagrams!

$$\mathcal{N}(\boldsymbol{t}) = \underbrace{\left(\prod_{i=\bar{\Psi}} \mathcal{P}F_i(\boldsymbol{t})\right)}_{\text{external lines}} \underbrace{\left(\sum_{\mathcal{O}_\Psi \in \text{conf}(\Psi_0)} \bar{\Theta}^{\mathcal{O}}_{\Psi\Psi}(\boldsymbol{t}) \prod_{a\in\mathcal{O}} \mathcal{P}\mathrm{tr}_a(\boldsymbol{t})\right)}_{\text{internal traces}},$$

$$\mathcal{P}F_i(\boldsymbol{t}) = \bar{v}(p_i)\mathcal{P}\left\{\prod_C \not{P}_C(\boldsymbol{t})\right\} u(p_{i+1}),$$

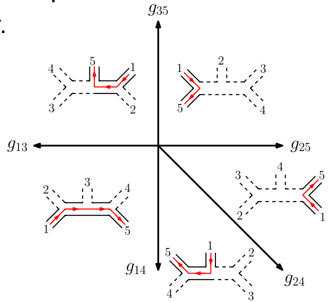
$$\mathcal{P}\mathrm{tr}_a(\boldsymbol{t}) = -\,\mathrm{tr}\left[\left\{\mathcal{P}\prod_C \not{P}_C(\boldsymbol{t})\right\}\right],$$

Loop-integration

Integrate out loop momenta $\longrightarrow$ tensor structure is encoded into tropical determinants.

$$\mathcal{A} = \int \frac{d^E \boldsymbol{t}}{\text{MCG}} \, \mathcal{I}^{\text{scalar}} \sum_{\mathcal{O}_\Psi \in \text{conf}} \overline{\det \Omega}_{\mathcal{O}} \, \mathcal{N}_{\mathcal{O}}(\boldsymbol{t}) \Big|_{\not{P} \to \gamma^\mu}$$

Spanning over the whole parameter space results in the complete set of Feynman diagrams for the theory.



THANK YOU!

Christoph Bartsch, PhD student (3rd year)
w/ Karol Kampf and Jaroslav Trnka at Charles U., Prague

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Amplitudes for pions (NLSM) and related scalar theories (trϕ^3, NLSM+ϕ^3)

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Use soft theorems (e.g. Adler Zero) to efficiently represent and compute amplitudes.

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Reveal structural similarities between trϕ^3 and NLSM$(+\phi^3)$.

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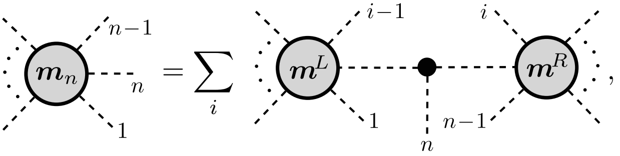
Reveal structural similarities between trϕ^3 and NLSM$(+\phi^3)$.

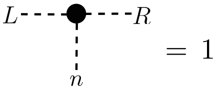
Today: tr$\phi^3 \longrightarrow$ NLSM+$\phi^3 \longrightarrow$ NLSM in under 2 minutes

Cubic recursion relations: $\mathrm{tr}\,\phi^3$

Mafra (1603.09731)



based on *root vertex*



Cubic recursion relations: tr ϕ^3

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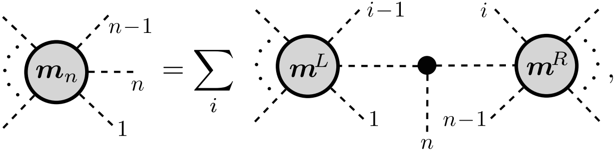

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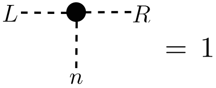


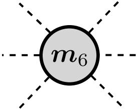
e.g.

$$m_3(123) = 1, \qquad m_4(1234) = \frac{1}{X_{24}} m_3(234) + \frac{1}{X_{13}} m_3(123),$$

$$m_5(12345) = \frac{1}{X_{25}} m_4(2345) + \frac{1}{X_{13}X_{35}} m_3(123) m_3(345) + \frac{1}{X_{14}} m_4(1234),$$

with $X_{ij} = s_{i,j-1} = (p_i + \ldots + p_{j-1})^2$.

Consider two-step modification of trϕ^3:



$$= \frac{1}{X_{26}} m_5(23456) + \frac{1}{X_{15}} m_5(12345) + \frac{1}{X_{13}X_{36}} m_3(123) m_4(3456) + \frac{1}{X_{14}X_{46}} m_4(1234) m_3(456)$$

Consider two-step modification of trϕ^3:

$$= \frac{-X_{26}^2}{X_{26}} m_5(23456) + \frac{-X_{15}^2}{X_{15}} m_5(12345)$$

$$+ \frac{(X_{36} - X_{13}) X_{13}}{X_{13} X_{36}} m_3(123) m_4(3456)$$

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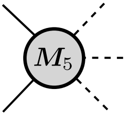
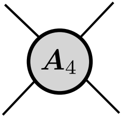
$M_5(1^\phi 2^\pi 3^\pi 4^\phi 5^\phi) =$



$A_4(1^\pi 2^\pi 3^\pi 4^\pi) =$



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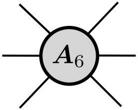

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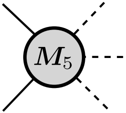
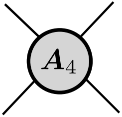
$M_5(1^\phi 2^\pi 3^\pi 4^\phi 5^\phi) =$ 

$A_4(1^\pi 2^\pi 3^\pi 4^\pi) =$ 

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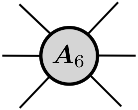

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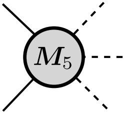
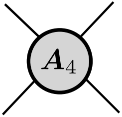
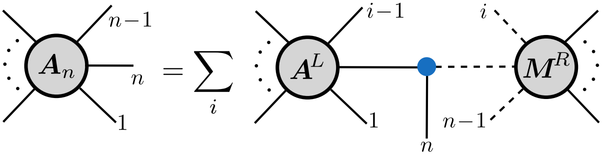
$M_5(1^\phi 2^\pi 3^\pi 4^\phi 5^\phi) =$ 

$A_4(1^\pi 2^\pi 3^\pi 4^\pi) =$ 

Soft Factor Expansion for NLSM Amplitudes

Soft Factor Expansion: NLSM at n points



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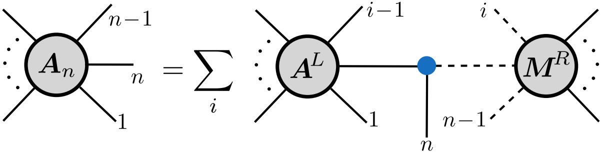
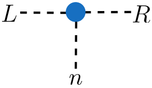


based on *root vertex*



$$= (X_L - X_R)X_R$$

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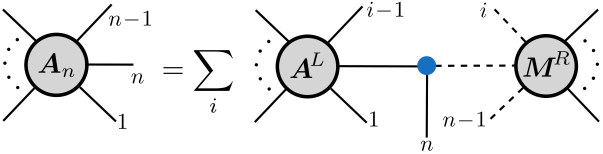

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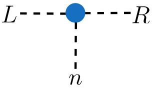

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Efficient representation: $\mathcal{O}(n)$ terms.

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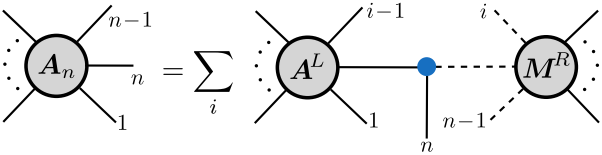

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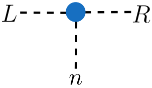

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Adler Zero $p_n \to 0$ manifest term by term.

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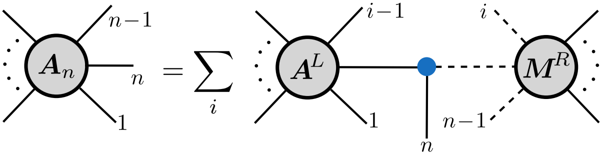

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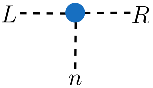


$= (X_L - X_R)X_R$

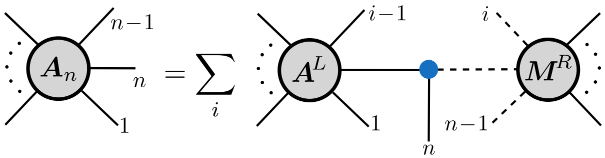
Efficient representation: $\mathcal{O}(n)$ terms.

Adler Zero $p_n \to 0$ manifest term by term.

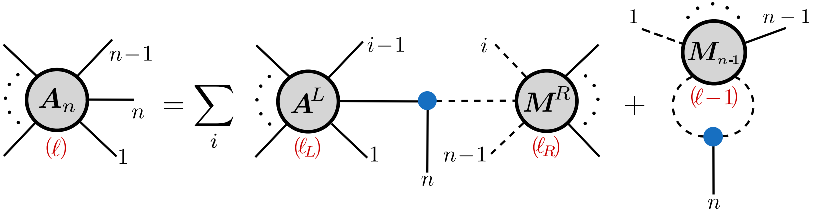
Serves as basis for **cubic recursion relations**.

CB, Kampf, Novotný, Trnka (24xx.xxxx)

Soft Factor Expansion: NLSM at n points



Soft Factor Expansion: NLSM at n points (Loops!)



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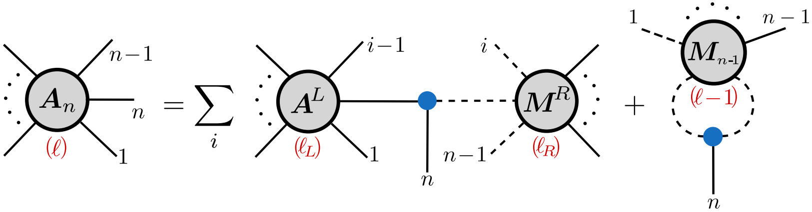
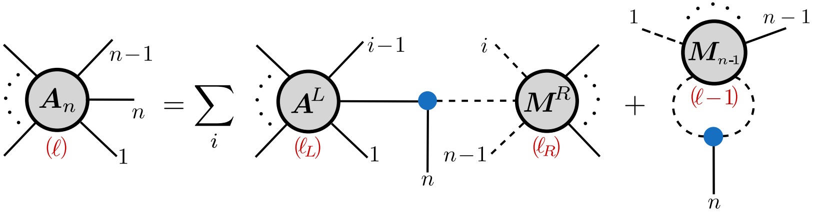


Equivalent to integrand from δ-shift of trϕ^3 surfacehedron.

Arkani-Hamed, Figueiredo (2403.04826)

Soft Factor Expansion: NLSM at n points (Loops!)



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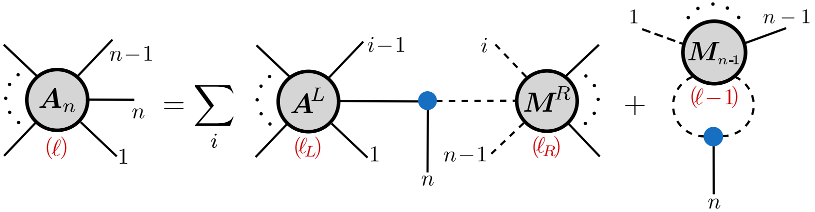
Manifest loop-level generalization of Adler Zero.

CB, Kampf, Novotný, Trnka (2401.04731)

Soft Factor Expansion: NLSM at n points (Loops!)



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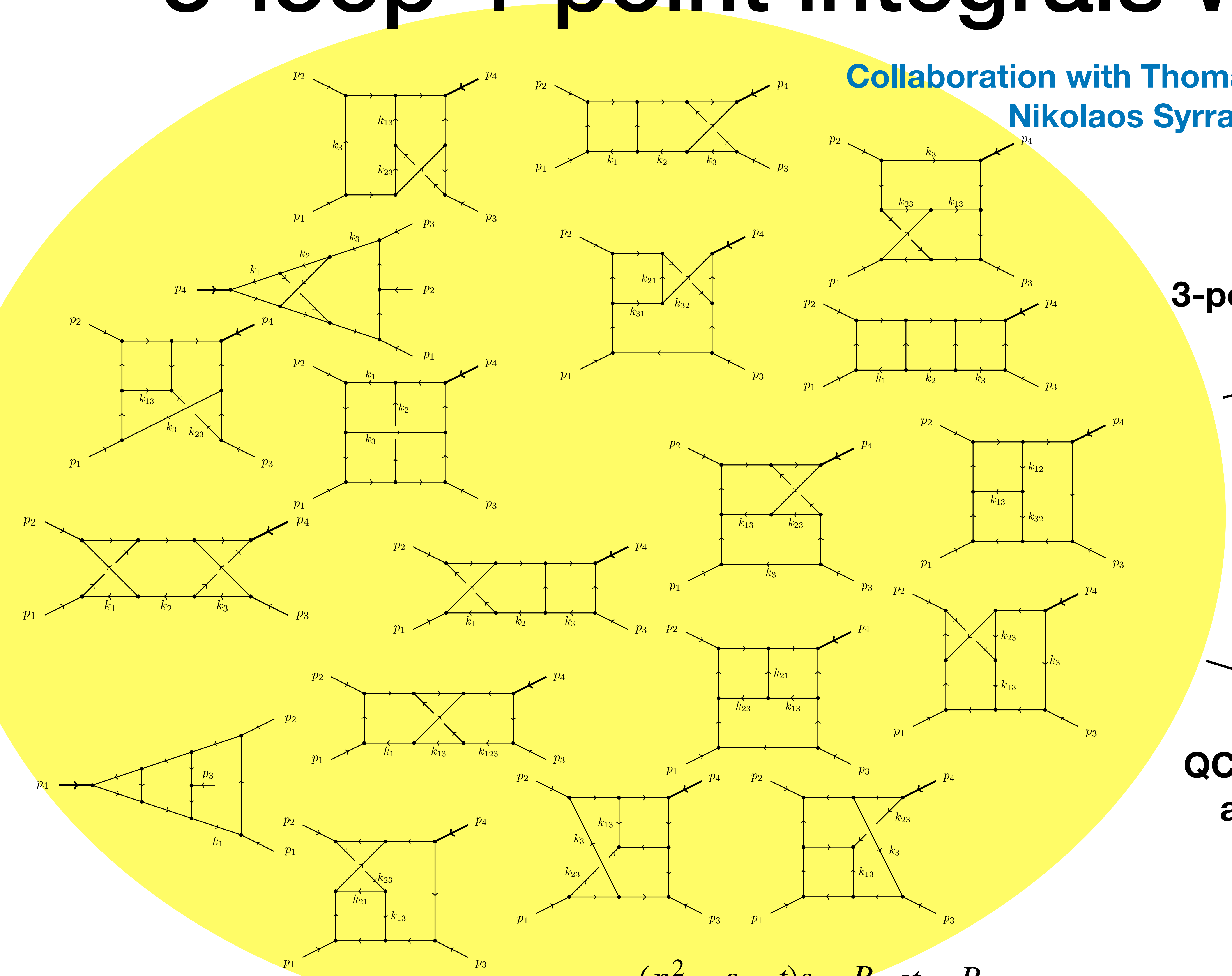
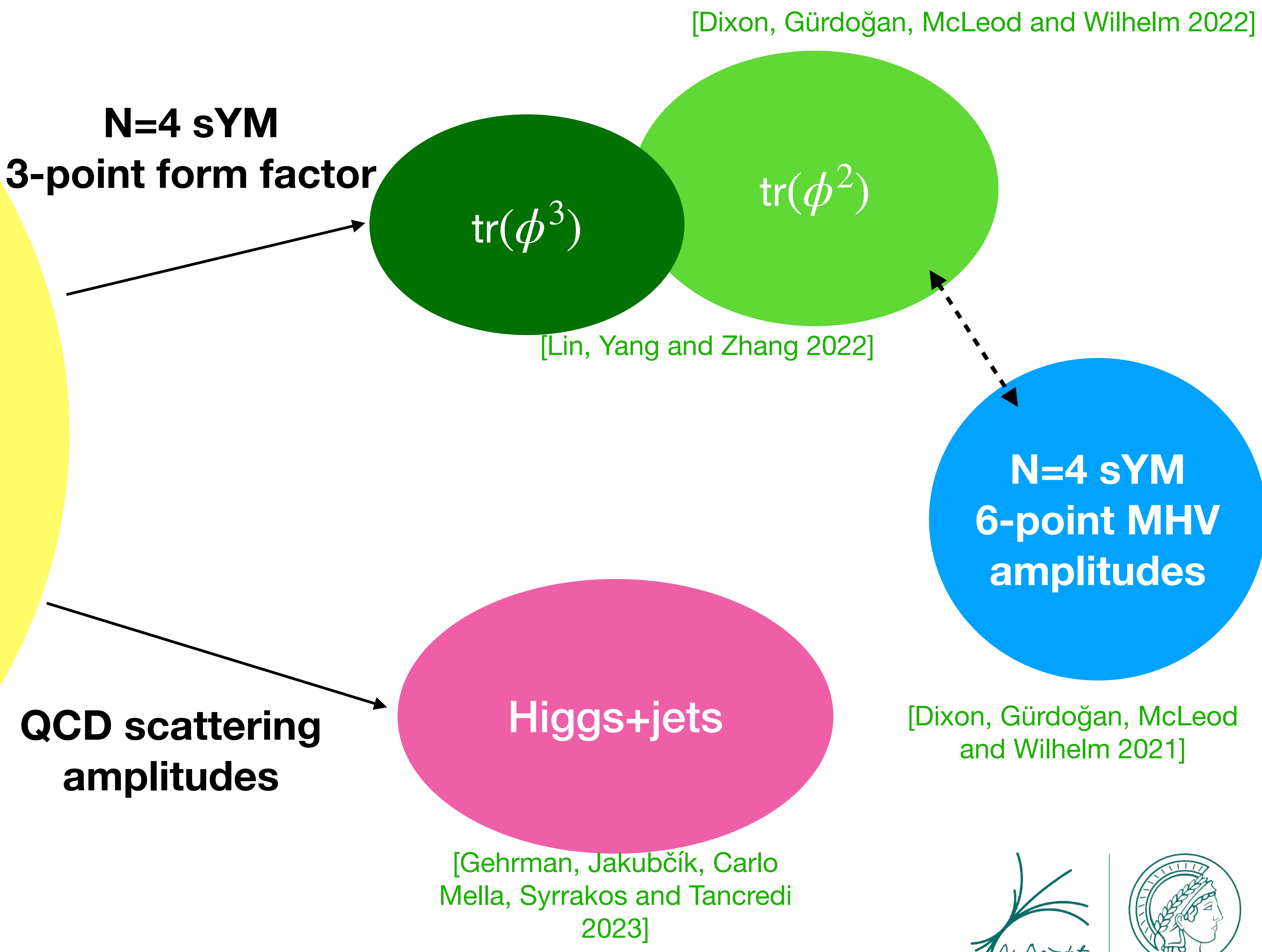
Thank you!

3-loop 4-point integrals with one off-shell leg

Collaboration with Thomas Gehrman Johannes Henn, Petr Jakubčík , Cesare Carlo Mella, Nikolaos Syrrakos , Lorenzo Tancredi and William J. Torres Bobadilla





$$\vec{\alpha} = \{p_4^2, s, t, p_4^2 - s - t, p_4^2 - s, p_4^2 - t, s + t, \frac{(p_4^2 - s - t)s - R}{(p_4^2 - s - t)s + R}, \frac{st - R}{st + R}, p_4^4 - t\left(p_4^2 + s\right), p_4^4 - s\left(p_4^2 + t\right),$$
$$t^2 + p_4^2\left(s - t\right), s^2 - p_4^2\left(s - t\right), -p_4^2 t + \left(p_4^2 - s\right)^2\} \text{ with } R = \sqrt{-p_4^2 s(p_4^2 - s - t)t}$$

Jungwon Lim

MAX-PLANCK-INSTITUT FÜR PHYSIK

ORIGINS Excellence Cluster

universe+

Amplitude Zeros from the Double Copy

Amplitudes 2024 Gong Show

Umut Oktem

Based on 2403.10594 with C. Bartsch, T. V. Brown, K. Kampf, S. Paranjape and J. Trnka

University of California Davis, Center for Quantum Mathematics and Physics

Motivation

- Constraints from amplitude poles well known and effective, what about amplitude zeros?
- Some examples
  - Adler zero of NLSM amplitudes
  - Standard model radiation zeros, $q_1\bar{q}_1 \to W^{\pm}\gamma$ and $q_1\bar{q}_1 \to W^{\pm}Z$ zero[1]
- Recent work[2] on hidden zeros of partial amplitudes in NLSM, Yang-Mills, and $Tr(\phi^3)$

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[1] L. Dixon, Z. Kunszt, and A. Signer. "Vector boson pair production in hadronic collisions at Order α_s: Lepton correlations and anomalous couplings". In: *Physical Review D* 60.11 (Nov. 1999). ISSN: 1089-4918.

[2] Nima Arkani-Hamed et al. *Hidden zeros for particle/string amplitudes and the unity of colored scalars, pions and gluons*. 2024. arXiv: 2312.16282 [hep-th].

BCJ and Zeros

- Hidden zeros $s_{13} = s_{14} = s_{15} = 0$ of 6-pt NLSM from BCJ[3]

$$\mathcal{A}_6[123456] = \frac{1}{s_{12}s_{123}s_{56}}\Big[\mathcal{A}_6[162345]\, s_{15}(s_{12}+s_{23})(s_{14}-s_{56}) - \mathcal{A}_6[162354]\, s_{14}(s_{12}+s_{23})(s_{25}+s_{35}) + \mathcal{A}_6[162435]\, s_{13}s_{15}s_{24} + \mathcal{A}_6[162453]\, s_{13}s_{24}(s_{15}+s_{35}) - \mathcal{A}_6[162534]\, s_{14}s_{25}(s_{12}+s_{23}) + \mathcal{A}_6[162543]\, s_{13}s_{25}(s_{56}-s_{24})\Big].$$

[3] Z. Bern, J. J. M. Carrasco, and H. Johansson. "New relations for gauge-theory amplitudes". In: *Physical Review D* 78.8 (Oct. 2008). ISSN: 1550-2368. DOI: 10.1103/physrevd.78.085011.

The Zeros from Double Copy

- BCJ implies hidden zeros, do zeros double copy as well?

$$(\text{DC amp}) = (\text{BCJ amp}) \otimes (\text{BCJ amp}) \tag{1}$$

- We can write the special Galileon amplitude as a double copy of zero satisfying NLSM

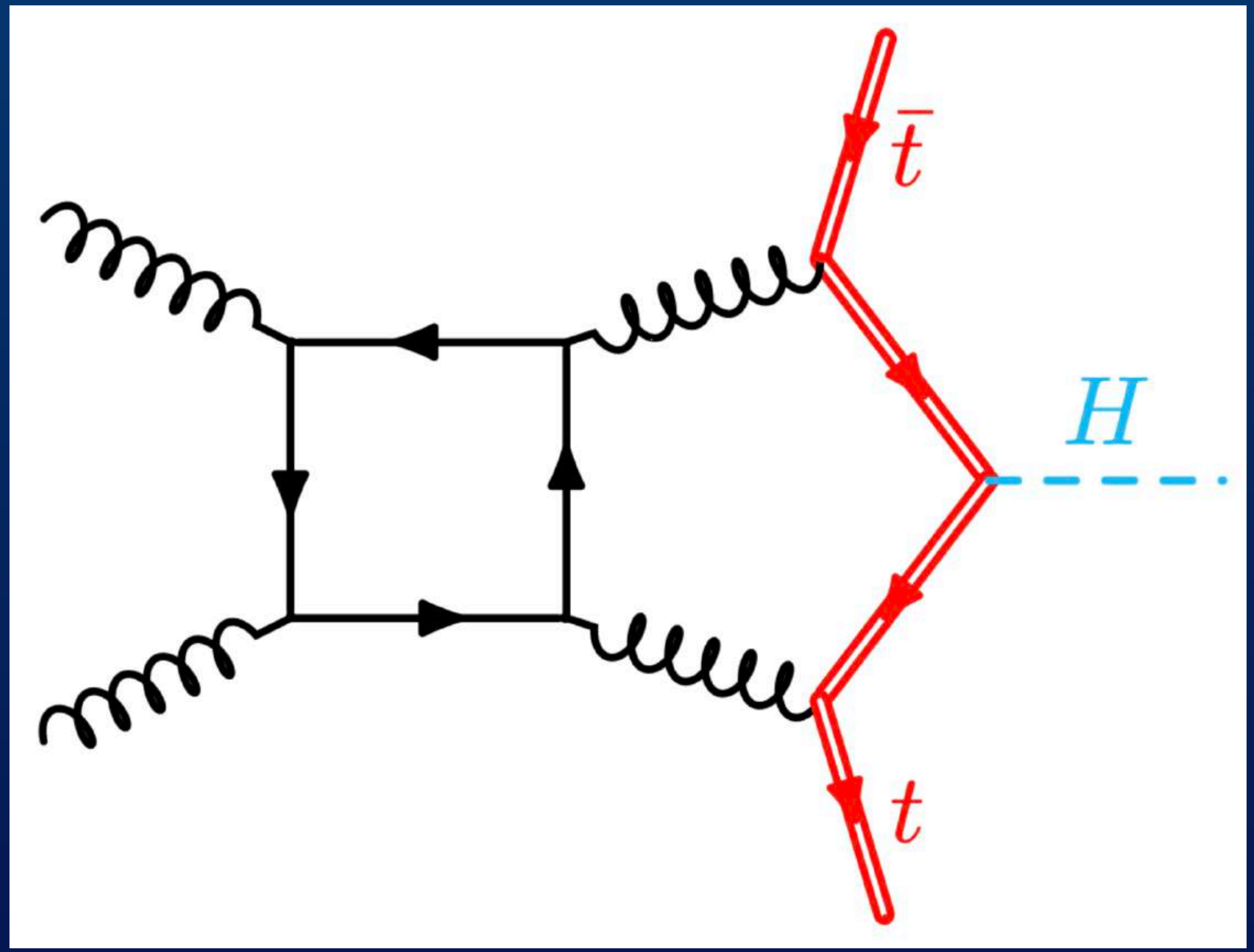
$$M_n = \sum_{\sigma\gamma} S[\sigma|\gamma] A_n(1\gamma m{+}1 n) A_n(1 m{+}1 n\sigma)\,, \tag{2}$$

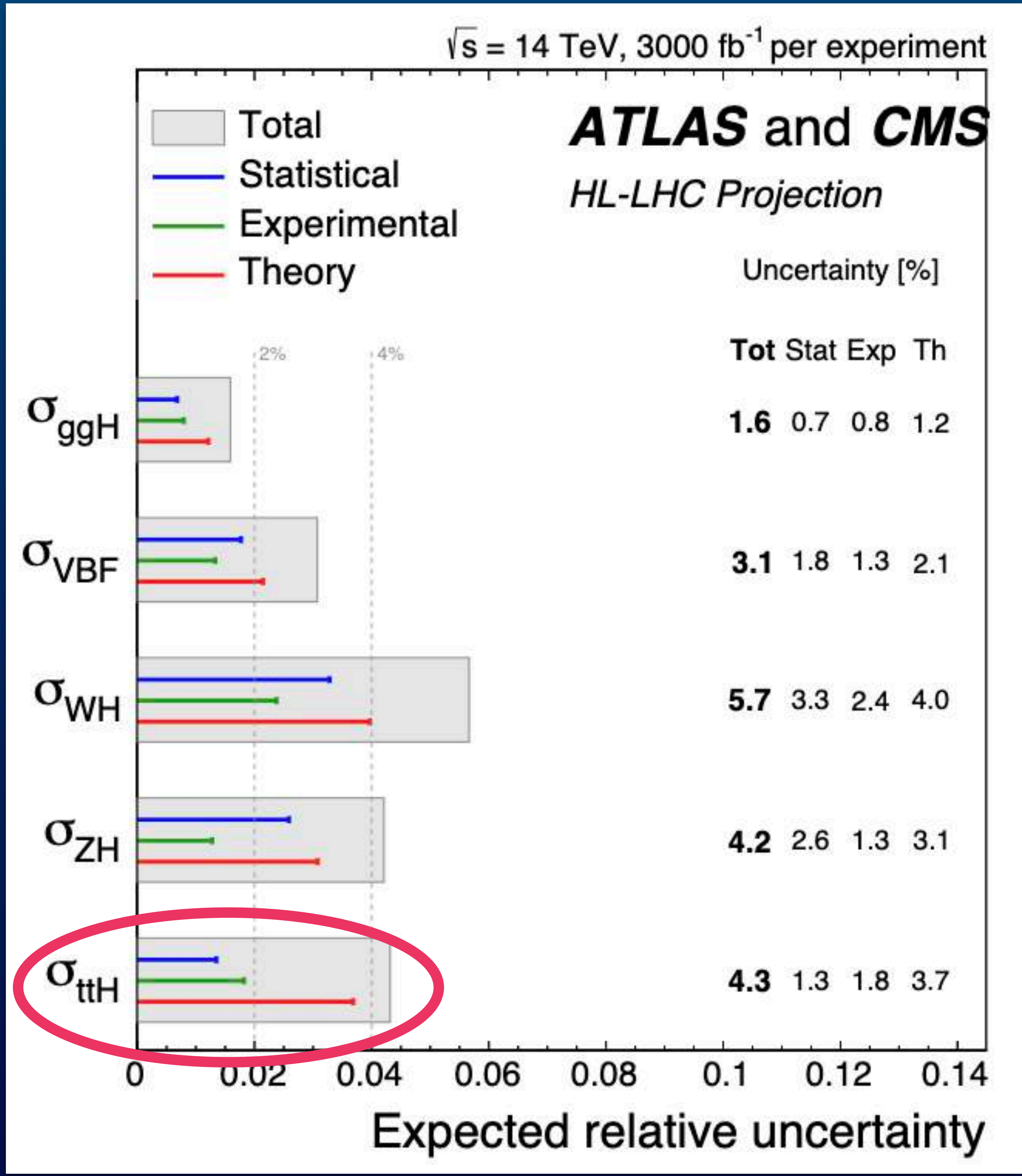
- We can always find a basis where the kernel S vanishes at hidden zeros
- Zeros carry over to special Galileon, what about gravity? Leave it to future work

The End

Thanks for Listening!

Analytic Structure of Multi-Scale Two-Loop Feynman Integrals for $t\bar{t}H$ Production at the LHC





[Yellow Report, CERN-2019-007]

Gustavo Figueiredo | Florida State University

Amplitudes 2024 Gong Show

In collaboration with: Fernando Febres Cordero, Manfred Kraus, Ben Page, Laura Reina

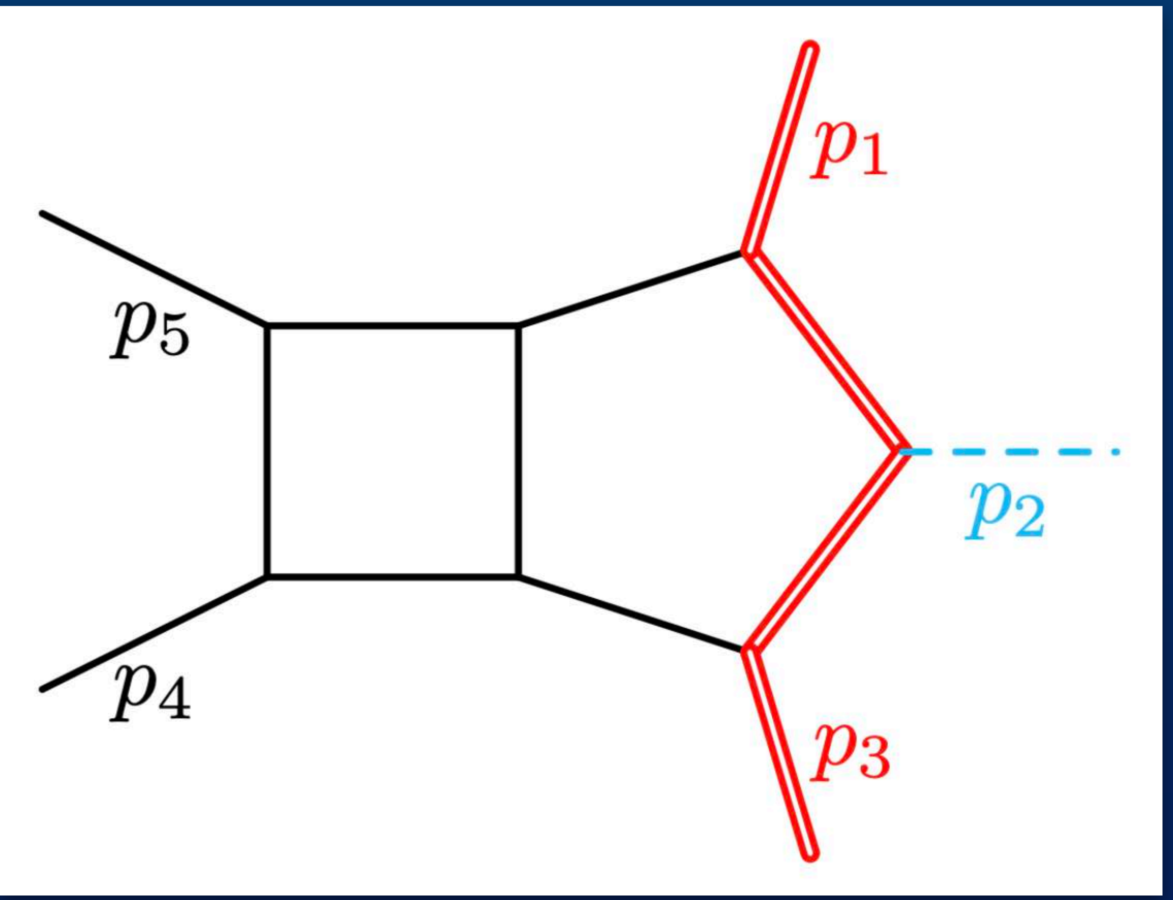
Talk based on: arXiv:2312.08131 [hep-ph]

Companion poster next week

$t\bar{t}H$ Families

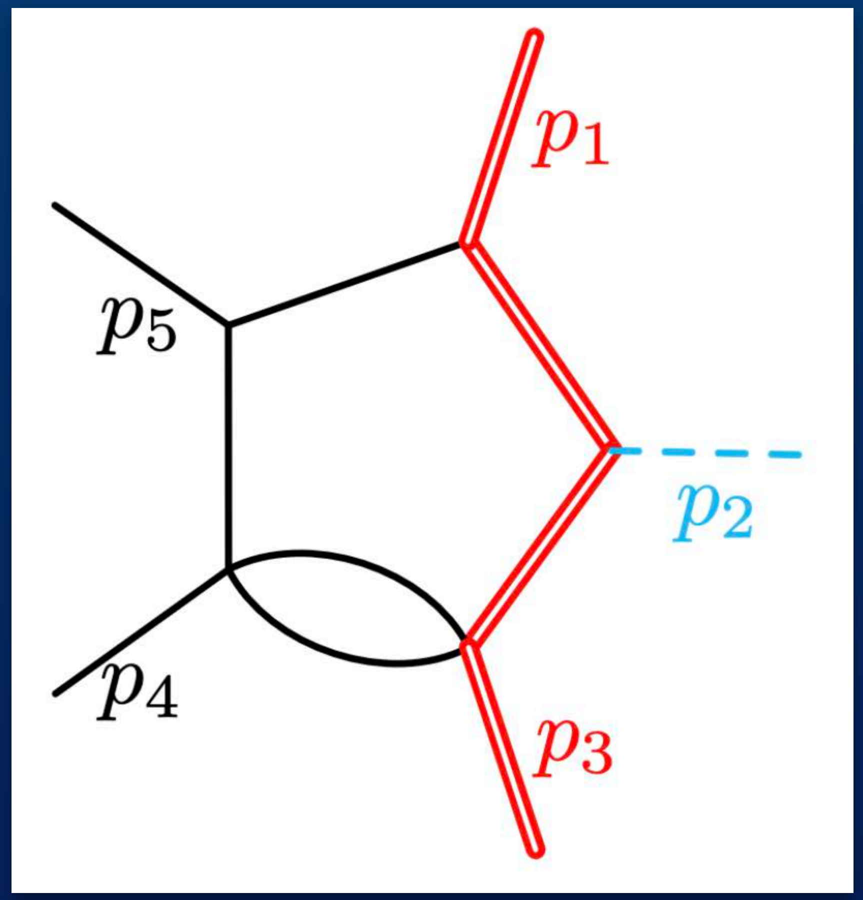
$$q_1(p_4)\ q_2(p_5) \to t(p_1)\ H(p_2)\ \bar{t}(p_3)$$

Kinematic Variables: $x_m = \left\{ v_{12}, v_{23}, v_{34}, v_{45}, v_{15}, m_t^2, q^2 \right\}$



T_1

- 111 Integrals
- 152 differential forms



$\tilde{T}_2$ & $Z(\tilde{T}_2)$

- 19 Integrals (each)
- 73 dlog forms $\subset T_1$

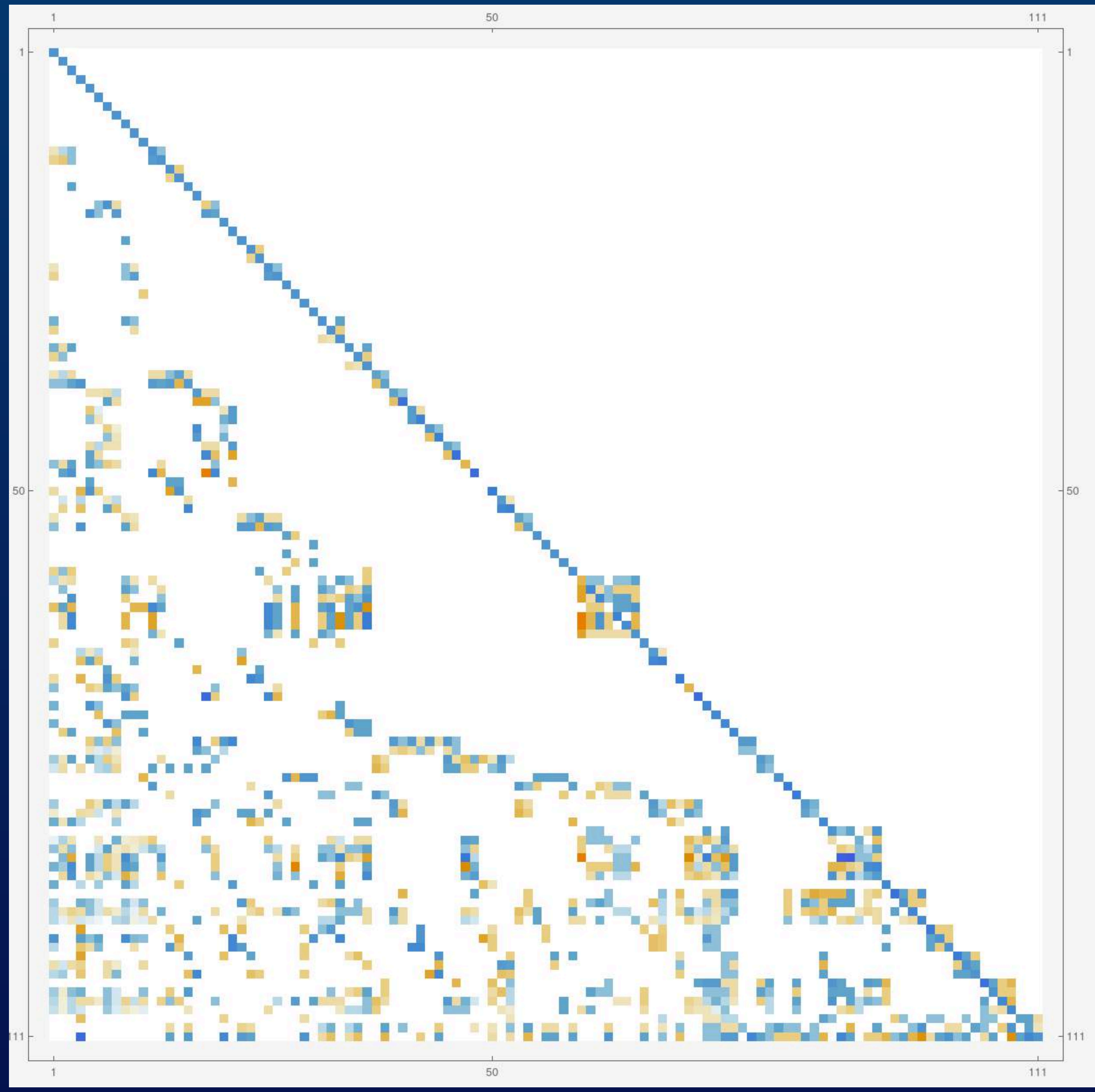
$\tilde{T}_2$ & $Z(\tilde{T}_2)$ related by kinematic map

$$Z : p_1 \leftrightarrow p_3,\ p_4 \leftrightarrow p_5$$

$$p_2^2 = q^2$$

$$v_{ij} = 2p_i \cdot p_j$$

$$\mathrm{d}\mathbb{A}_{T_1}(x_n) =$$

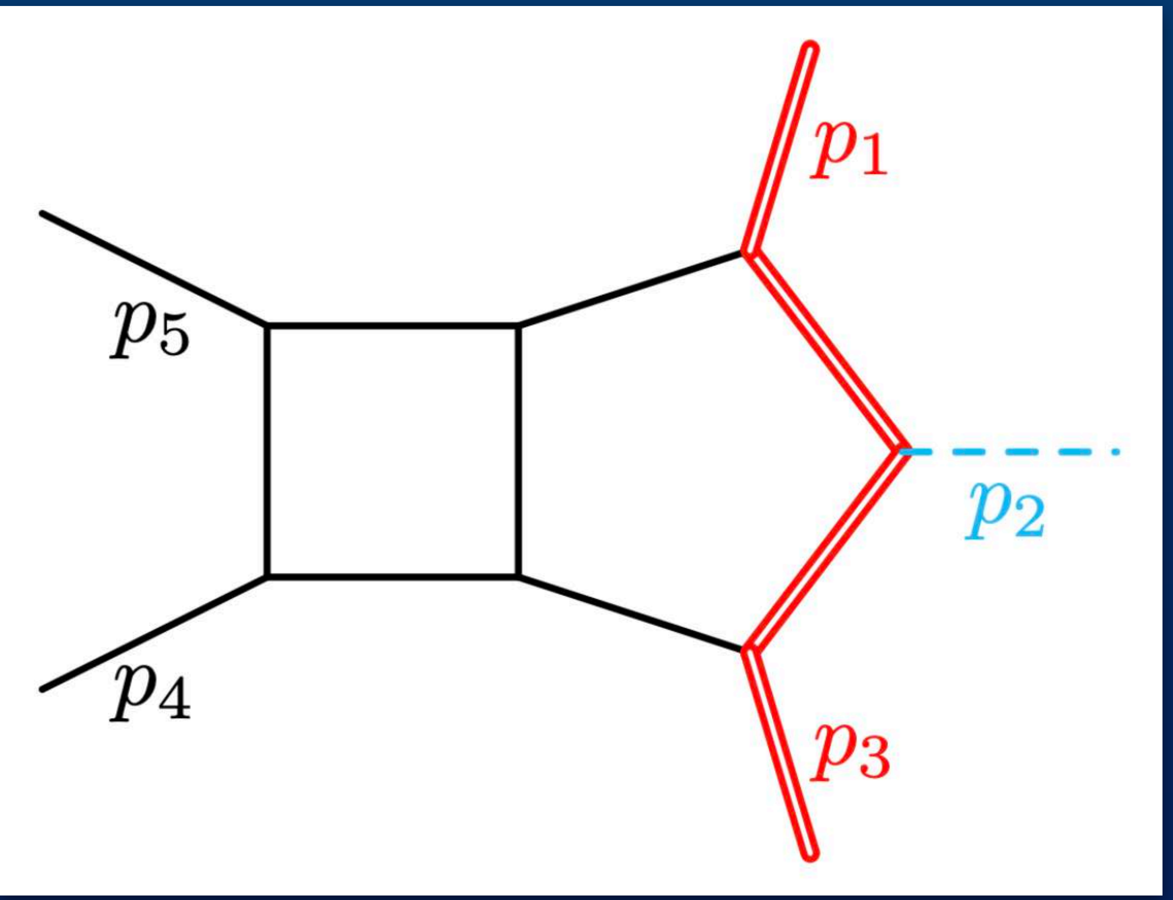


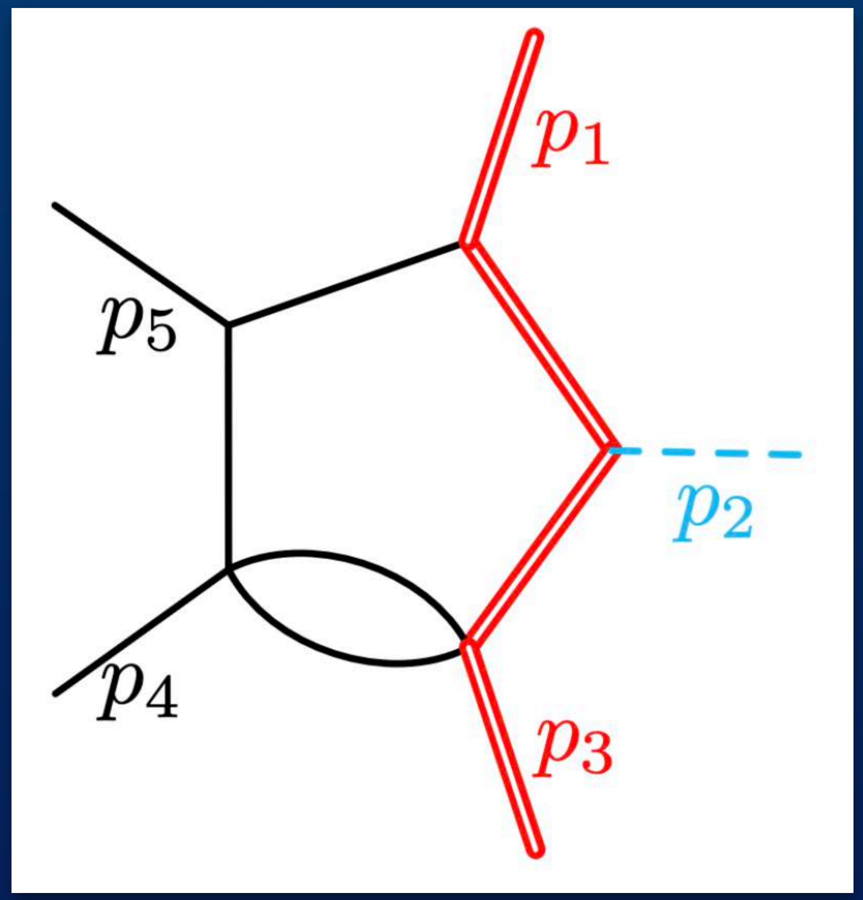
$$\mathrm{d}\mathbb{A} = \sum_{\alpha=1}^{\kappa} \mathrm{M}_\alpha \omega_\alpha$$ where (mostly) $$\omega_\alpha \sim \mathrm{d}log\left(W_\alpha\right)$$

$t\bar{t}H$ Families

$$q_1(p_4)\ q_2(p_5) \to t(p_1)\ H(p_2)\ \bar{t}(p_3)$$

Kinematic Variables: $x_m = \left\{ v_{12}, v_{23}, v_{34}, v_{45}, v_{15}, m_t^2, q^2 \right\}$





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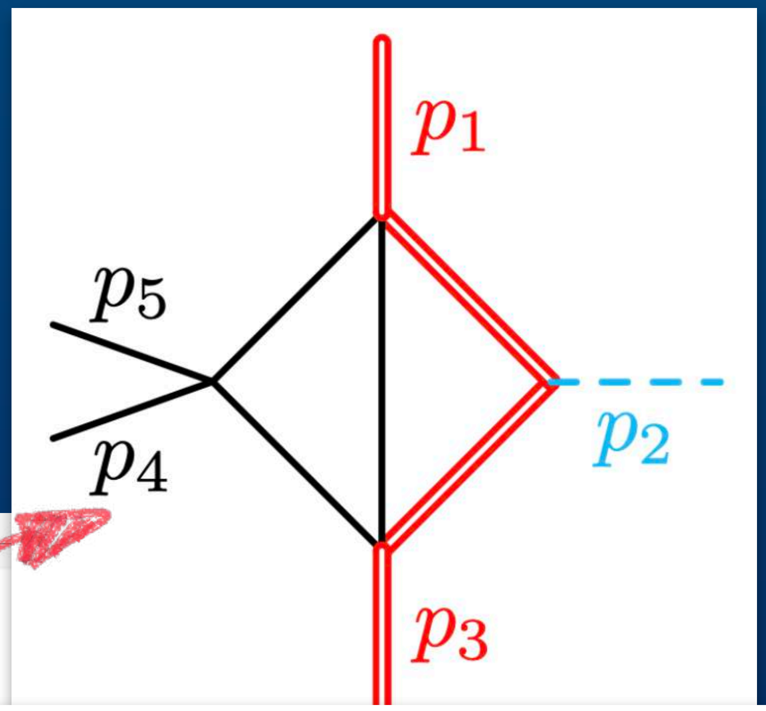
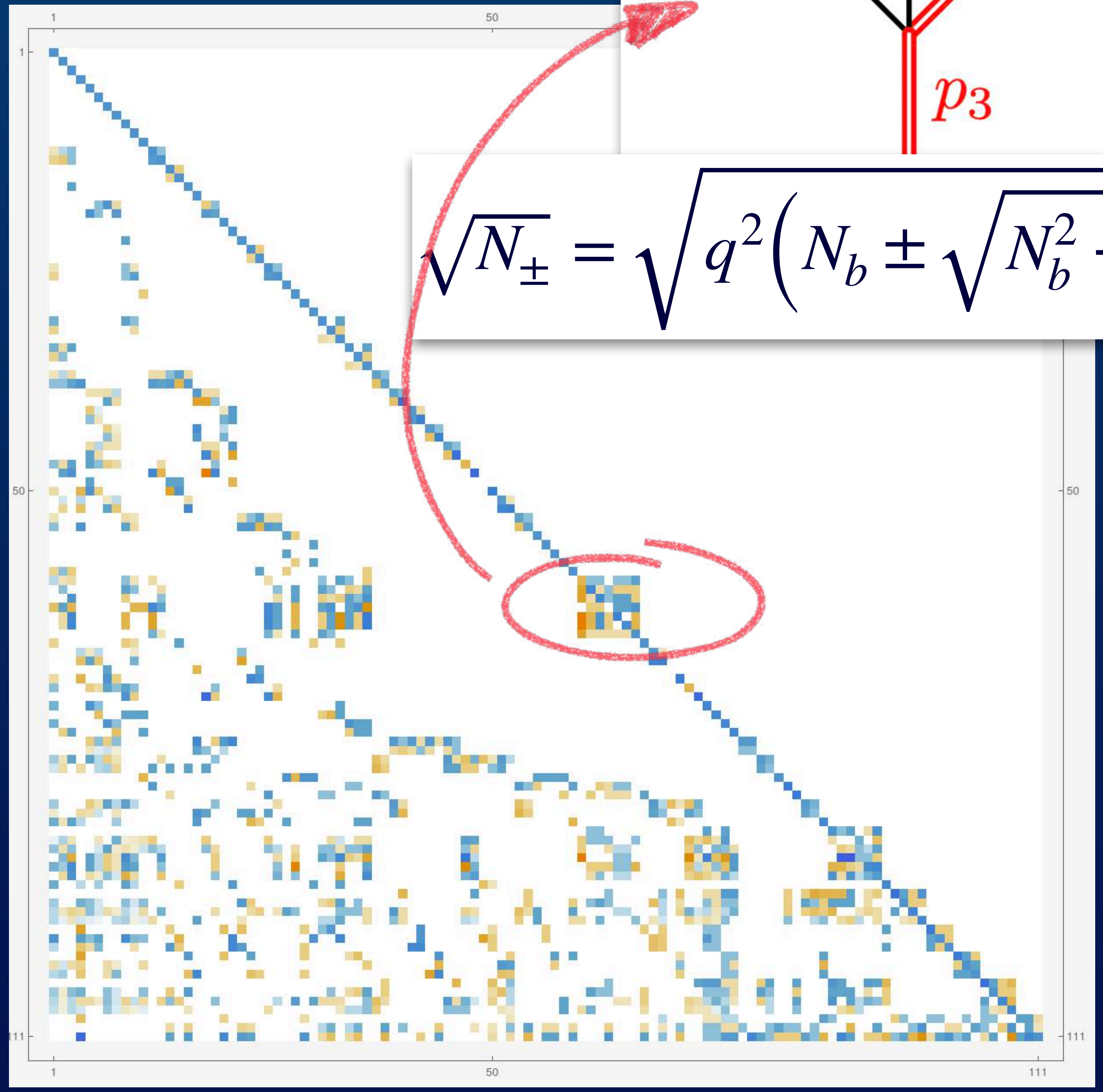


$$\sqrt{N_\pm} = \sqrt{q^2\left(N_b \pm \sqrt{N_b^2 - N_c}\right)}$$

$$\mathrm{d}\mathbb{A}_{T_1}(x_n) =$$



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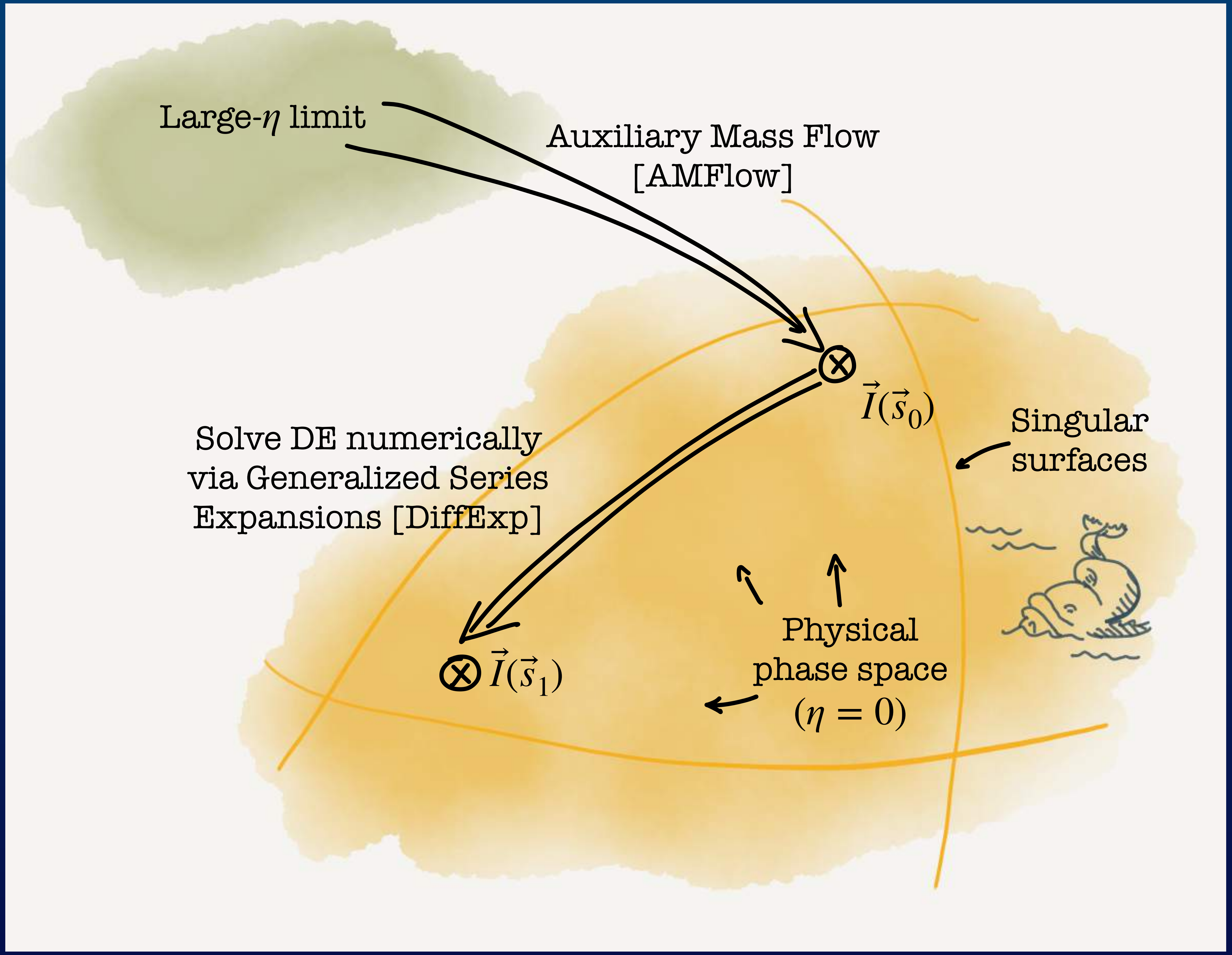
Results

Analytic Solutions

n	Linearly independent	Irreducible
1	7	7
2	31	16
3	85	69
4	121	114

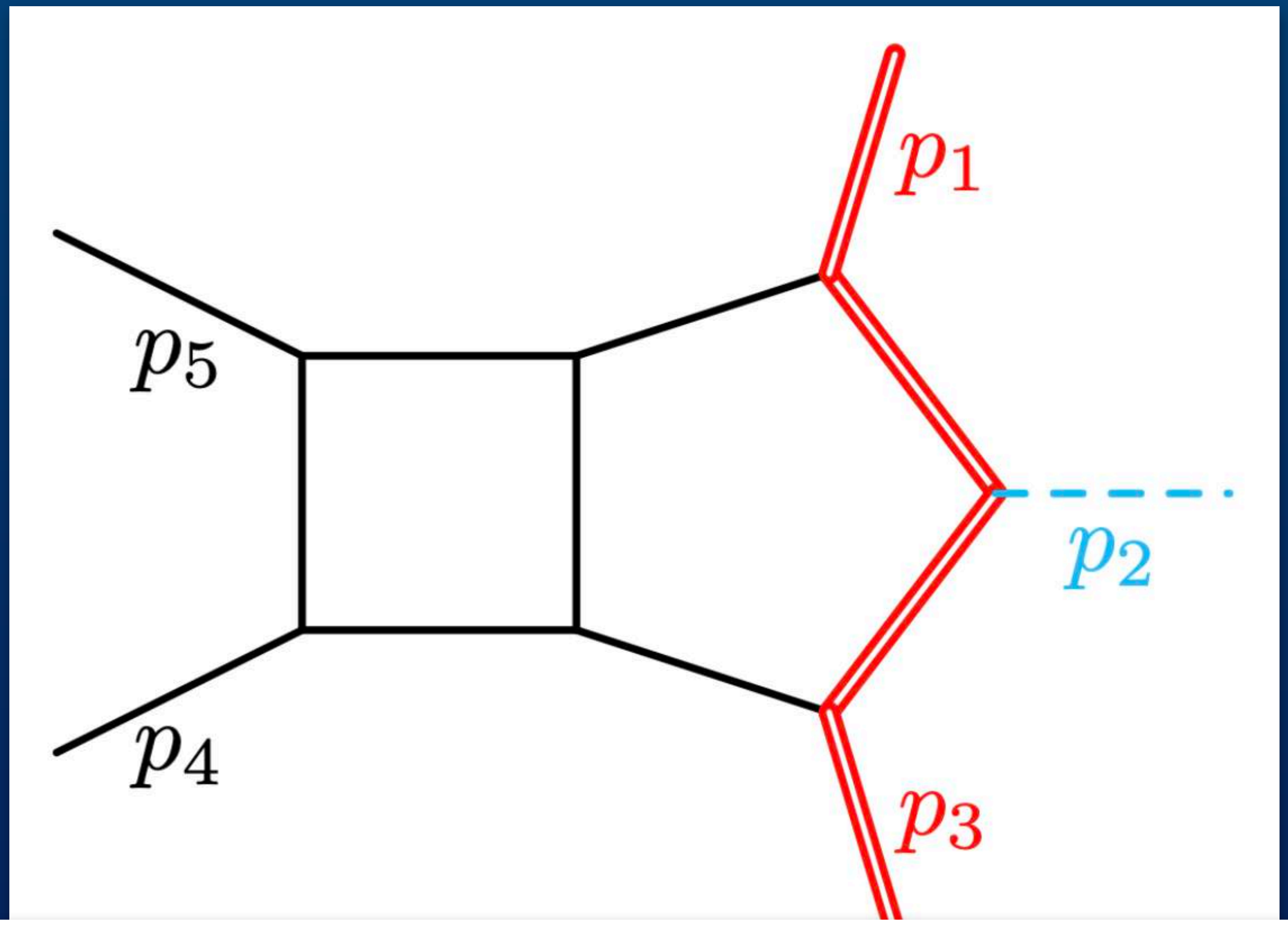
Modulo boundary constants

Numerical Solutions



Machine-readable expressions for all our results & numerical integration scripts available on ArXiv!

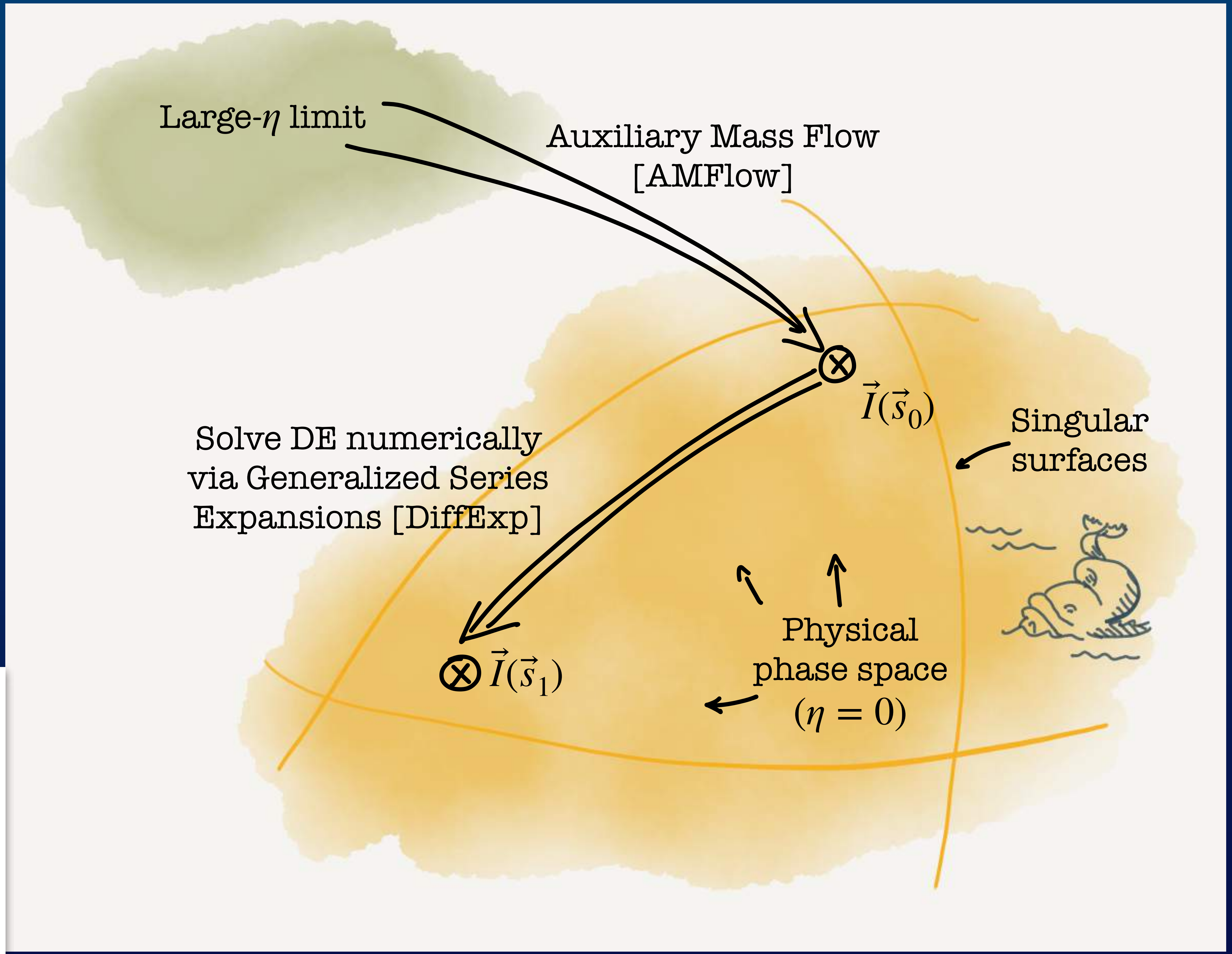
Results



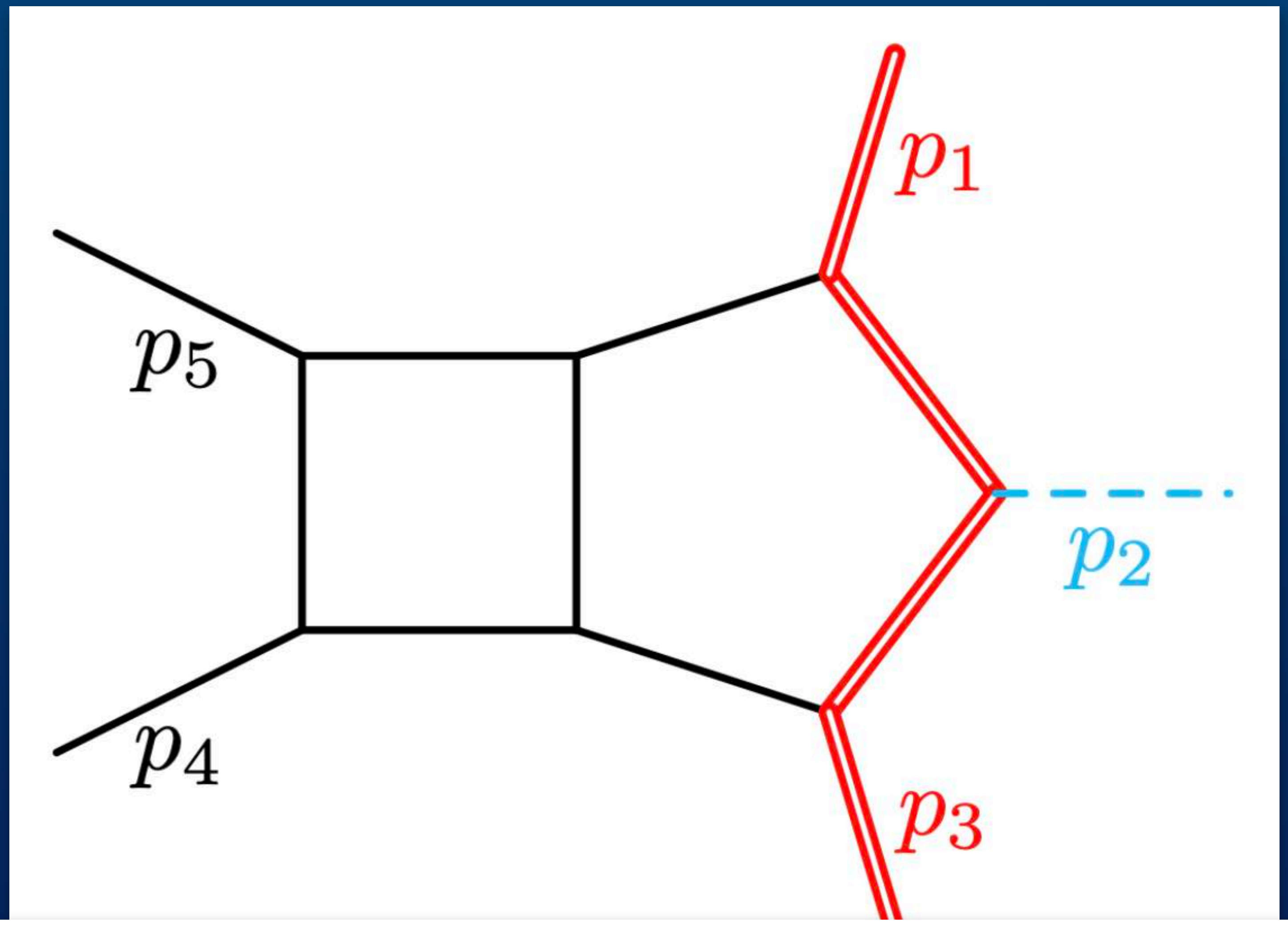
$$\vec{s}_1 = \left\{ \frac{19}{3}, \frac{46}{3}, -\frac{24}{7}, \frac{383}{5}, -\frac{61}{28}, \frac{25}{118}, \frac{97}{896} \right\}$$

	$\mathcal{O}(\epsilon^0)$	$\mathcal{O}(\epsilon^1)$	$\mathcal{O}(\epsilon^2)$	$\mathcal{O}(\epsilon^3)$	$\mathcal{O}(\epsilon^4)$
$(\vec{I}_1)_{109}$	0	0	0	-3.703380133 $+5.885655074\,i$	2.149576969 $-10.432322830\,i$
$(\vec{I}_1)_{110}$	0	0	0	0	0
$(\vec{I}_1)_{111}$	0	0	-1.306045093 $-12.647039669\,i$	2.05552771 $+25.35139955\,i$	-85.55528965 $-75.93834102\,i$

Numerical Solutions



Results



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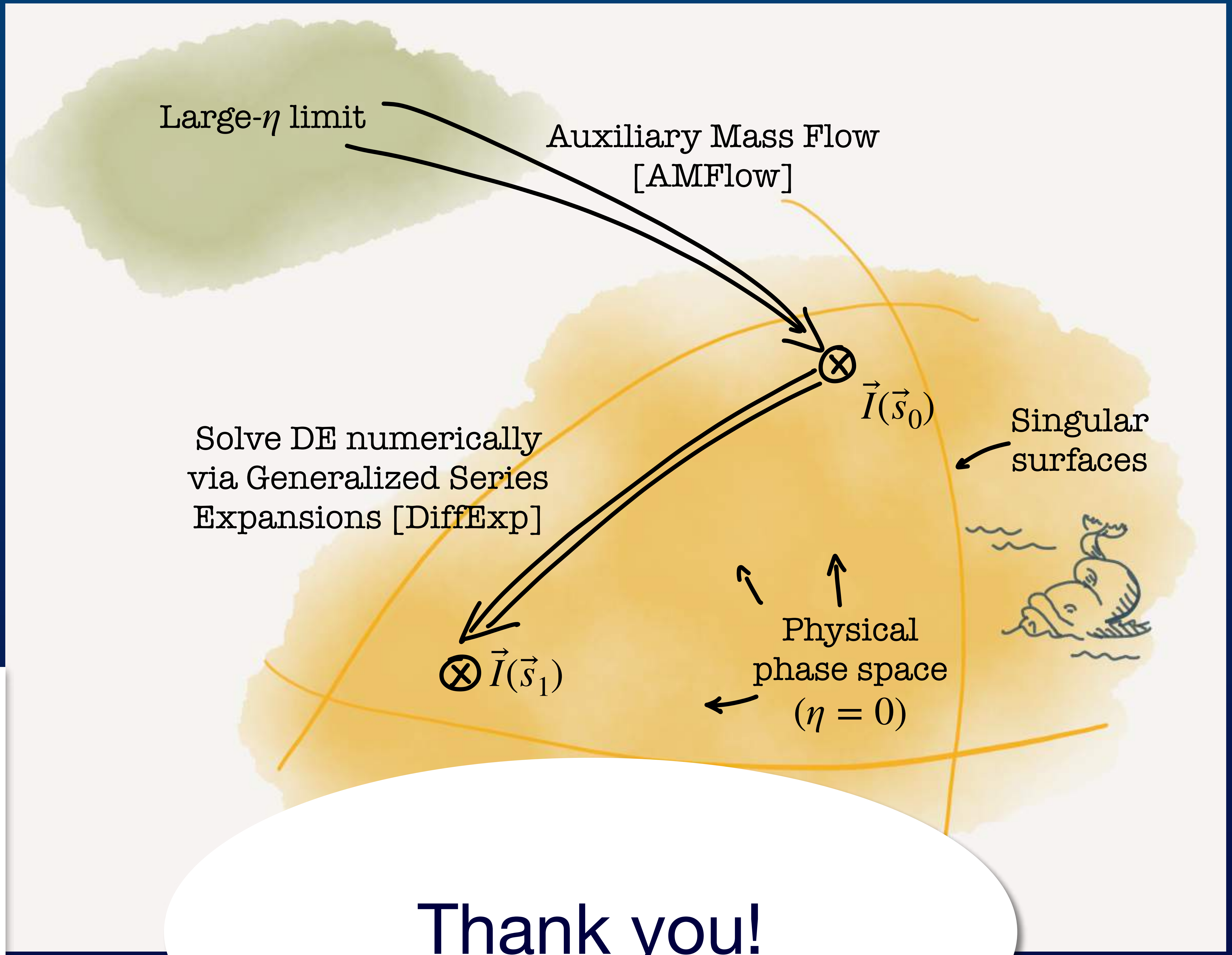


Thank you!

Integrated Unitarity
for Scattering Amplitudes

2403.18047

Piotr Bargieła

University of Zurich

gong show
Amplitudes 2024
IAS, Princeton

Universität Zürich[UZH]

European Research Council
Established by the European Commission

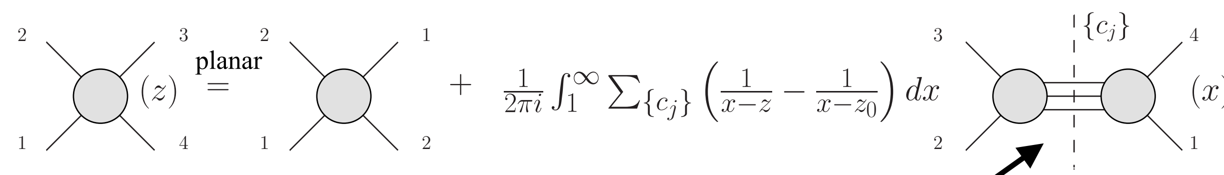
Integrated Unitarity

properties

- **method** for computing Scattering Amplitudes
- ~ Generalized Unitarity @ integrated level :
  constrains both MIs and their coefficients with cuts
- allows **algorithmic** usage of dispersion relation
- ansatz matching as an alternative to explicit integration
- beneficial for IBP computational complexity
- requires understanding of analytic structure

trade-off

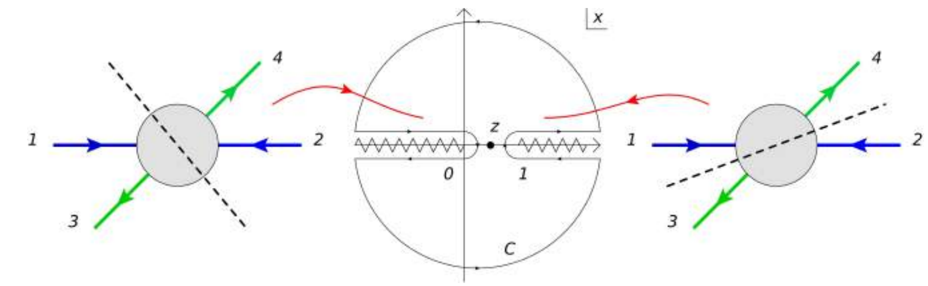
$$\mathscr{A}(z) = \mathscr{A}_0 + \frac{1}{2\pi i}\left(\int_0^\infty \mathrm{Cut}_t + \int_1^\infty \mathrm{Cut}_u\right)\left(\frac{1}{x-z} - \frac{1}{x-z_0}\right)\mathscr{A}(x)dx$$



IBP & DEQ simpler on a cut

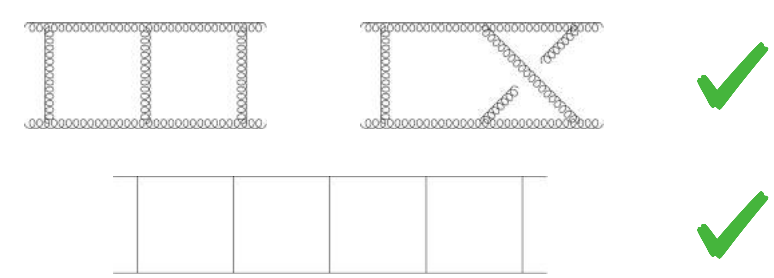
$$\begin{cases} \mathrm{Disc}_0\,\mathscr{A} & = \mathrm{Cut}_t\,\mathscr{A} \\ \mathrm{Disc}_1\,\mathscr{A} & = \mathrm{Cut}_u\,\mathscr{A} \\ \mathrm{Disc}_\infty\,\mathscr{A} & = \mathrm{Cut}_s\,\mathscr{A} \end{cases}$$

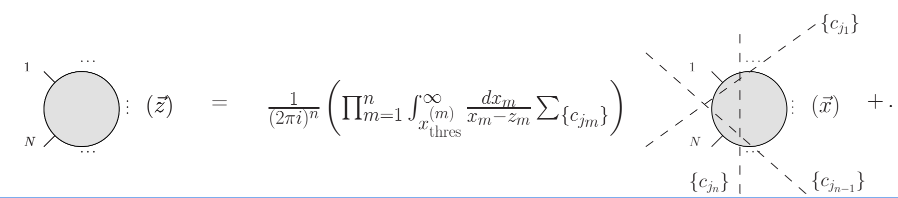
$$\mathscr{A}(z) = \epsilon^{\#}\sum_{n\geq 0}\epsilon^n \sum_{\vec{\alpha}\,:\,|\vec{\alpha}|\leq n} r_{n,\vec{\alpha}}(z)\,G(\vec{\alpha},z)$$



status

- formulated for **4-point massless** kinematics
- validated for 2-loop nonplanar massless QCD amplitude
- provided new **4-loop ladder** integral result
- multivariate generalization ongoing





THANK YOU

Supersymmetric Yang-Mills theories without anti-commuting variables

Saurabh Pant

Based on : 2005.12324 and 24xx.xxxx

Amplitudes 2024

June 12, 2024

IISER PUNE

The Idea

- Supersymmetric Yang-Mills theories can be characterized by a transformation of the bosonic fields such that the Jacobian determinant of the transformation exactly is equal to the product of the fermion and ghost determinants.

$$Z = \int DA\, D\lambda\, DC\, D\bar{C}\; e^{-S[A,\lambda,C,\bar{C}]}$$

↓ Integrate out fermions and ghosts

$$Z = \int DA\; e^{-S_g[A]}\, \Delta_{FD}[A]\, \Delta_{GD}[A]$$

↓ Field transformation

$$Z = \int DA'\; e^{-\int d^4x\, A' \Box A'}$$

Nicolai, 1979; Nicolai, 1980

Flume and Lechtenfeld, 1984;

- Where is the information of interaction and supersymmetry?

$$\langle\langle A_1(x_1)\ldots . A_n(x_n)\rangle\rangle = \left\langle T_g^{-1}[A'_1](x_1)\ldots T_g^{-1}[A'_n](x_n)\right\rangle_0$$

Dietz and Lechtenfeld, 1985; Nicolai and Plefka, 2020

Results

- For super Yang-Mills theory derived the map up to third order in the coupling constant.
- Determinant matching condition imposes the constraint $r = 2(D-2)$, where r is the number of fermionic degrees of freedom, and D is the number of space-time dimensions.

$$D = 3,4,6,10 \iff r = 2,4,8,16$$

Ananth, Lechtenfeld, Malcha, Nicolai, Pandey and **Pant;** 2020

- Is there a simpler version or closed form of Nicolai Map for super Yang-Mills theories?
- We work in the light-cone gauge and find transformation that satisfies all the conditions of Nicolai map.
- Found two inequivalent maps and addressed the uniqueness of this approach.
- Computed amplitudes using the inverse map.

Ananth, Bhave and **Pant;** 2024

Hidden Structures in Intersection Numbers [Crisanti, Smith, 2405.18178]

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1. What is intersection theory? [Matsumoto, 1998]

A mathematical framework that allows us to build inner products between twisted period integrals:

$$I_{\alpha_1 \cdots \alpha_m} \sim \int u \, \varphi_{\alpha_1 \cdots \alpha_m} \qquad \varphi_{\alpha_1 \cdots \alpha_m} = \frac{dz_1 \wedge \cdots \wedge dz_m}{z_1^{\alpha_1} \cdots z_m^{\alpha_m}} \qquad \langle \varphi_L | \varphi_R \rangle_u = \sum_{p \in \mathcal{P}} \mathrm{Res}_{z=p}(\psi \varphi_R)$$

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Through a parametric representation, Feynman integrals can be written as twisted period integrals. Allows us to decompose to MIs using this inner product:

$$I_{\alpha_1\cdots\alpha_m} \sim \int\left(\prod_i d^dk_i\right)\frac{1}{D_1^{\alpha_1}\cdots D_m^{\alpha_m}} \longrightarrow \sim \int\left(\prod_i dz_i\right)\frac{p(z_1,\cdots,z_m)^\gamma}{z_1^{\alpha_1}\cdots z_m^{\alpha_m}}$$

$$J = \sum_{i=1}^n c_i I_i \longrightarrow c_i = \sum_{i=1}^n \langle J|I_j\rangle\left(\mathbf{C}^{-1}\right)_{ji}$$

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For dLog or 1-forms, much is already known:

$$\langle\varphi_L|\varphi_R\rangle \propto \sum_{\{r_i,r_j\}\in\mathcal{P}(\varphi_L)\cap\mathcal{P}(\varphi_R)} \frac{1}{\gamma_i\,\gamma_j}$$

$$\langle\varphi_L,\varphi_R\rangle_\omega = \int dx\,dy\,\delta(\omega_x)\delta(\omega_y)\,\widehat{\varphi}_L\,\widehat{\varphi}_R$$

$$= \sum_{(x^*,y^*)} \det{}^{-1}\begin{bmatrix}\frac{\partial\omega_x}{\partial x} & \frac{\partial\omega_x}{\partial y}\\ \frac{\partial\omega_y}{\partial x} & \frac{\partial\omega_y}{\partial y}\end{bmatrix}\widehat{\varphi}_L\,\widehat{\varphi}_R\Bigg|_{(x,y)=(x^*,y^*)},$$

[Mizera, 2018]

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[Mizera, 2018]

In our work, we find unexpected symmetries in intersection numbers beyond dLog and 1- forms for the first time!

$$\langle b(\mathbf{z})^p | b(\mathbf{z})^q \rangle = f_n(p, q; \gamma) \times \frac{\det \left(\mathbf{H}(b_h) \right)^{n+p+q}}{\det \left(\mathbf{H}(b) \right)^{n+p+q+1}} \qquad u(\mathbf{z}) = b(\mathbf{z})^\gamma \qquad b(\mathbf{z}) \text{ quadratic}$$

Completely trivialises the computation of the **C**-matrix at one loop

Hessian determinants are are multivariable polynomial discriminants!

Thank you for listening!

[arXiv: 2405.18178]

S-matrix thermodynamics reloaded

Emanuele Gendy Abd El Sayed

Based on ongoing work with J. Elias Miró and P. Baratella

The idea:

Express the **thermodynamics** of a system in terms of the **S-matrix** only

$$F(\beta) = F_0(\beta) - \frac{1}{2\pi i}\int_0^\infty \mathrm{d}E\, e^{-\beta E}\,\mathrm{Tr}_c \ln S(E+i\epsilon)$$

[Dashen, Ma, Bernstein ’69-’70]

Connect properties of **(reasonably) hot and dense matter** with **zero temperature** quantities like their scattering amplitudes

Exploit computational power of S-matrix methods **bypassing off-shell approach**

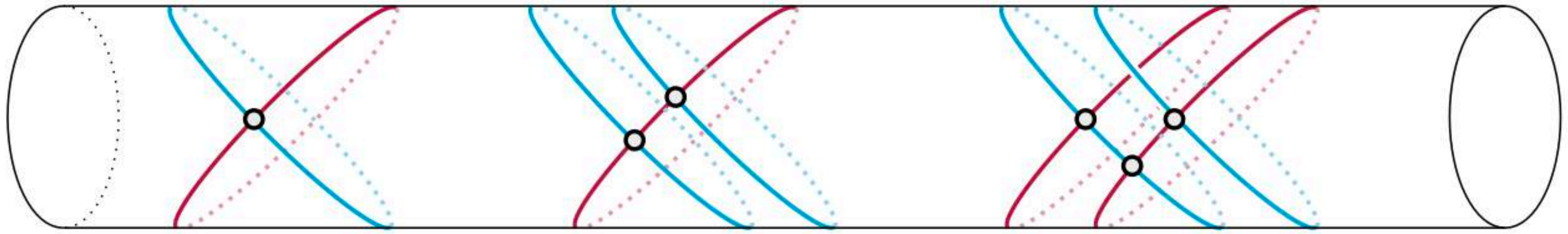
Unresolved IR divergences prevented application to relativistic systems

UNTIL NOW!

The application:

We studied a simple, integrable system where **F is known exactly**

$$F = \frac{L}{\beta}\sqrt{\frac{\beta^2}{\ell_s^4} - \frac{\pi}{3\ell_s^2}} = F_0 - L\frac{\ell_s^2\pi^2}{72\beta^4} - L\frac{\ell_s^4\pi^3}{432\beta^6} - L\frac{5\ell_s^6\pi^4}{10368\beta^8} + \mathcal{O}(\ell_s^8)$$



Reproduced F up to **NNLO**

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Thank you!

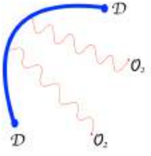
Exact Results for Giant Graviton Scattering

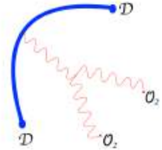
Augustus Brown, Francesco Galvagno, Congkao Wen, QMUL

$\mathcal{N}=4$ SYM: 6 Φ^I, packaged by Y_I. No. of colours $=N$, 't Hooft coupling $=\lambda=g_{YM}^2 N$

2 types of $\frac{1}{2}$-BPS operators of interest:

- $\mathcal{O}_2(x,Y)=Y_{I_1}Y_{I_2}\mathrm{Tr}\big(\Phi^{I_1}(x)\Phi^{I_2}(x)\big)$: Bottom component of stress tensor multiplet
- $\mathcal{D}(x,Y)=\det_N Y_I\,\Phi^I(x)$: Determinant operator, $\Delta=N$





AdS/CFT: $\langle\mathcal{O}_2\mathcal{O}_2\mathcal{D}\mathcal{D}\rangle\leftrightarrow$
Scattering of two gravitons with a D3-brane (giant graviton) that moves along an AdS_5 geodesic

This has been studied at weak 't Hooft coupling λ up to $O(\lambda^2)$ [Jiang, Komatsu, Vescovi]

Supersymmetric localisation: Calculated the *integrated correlator* of $\langle\mathcal{O}_2\mathcal{O}_2\mathcal{D}\mathcal{D}\rangle$ to all orders in λ at leading and subleading order in N

Expanded at large λ, and completed with modular functions

Unitarity for a dS S-matrix

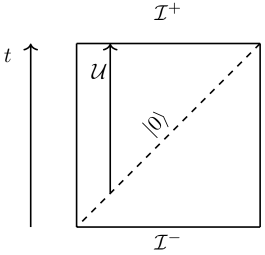
Santiago Agüí Salcedo

Cambridge University DAMTP
Harding Distinguished Postgraduate Scholars Programme

12th June 2024

A S-matrix with particle production

Broken time translations are a key difference between Minkowski and cosmological spacetimes:



- The vacuum is time dependent.
- Particle production at the level of the free theory.
- Asymptotic states need to take particle production into account.

Asymptotic states are built in a different way [2309.07092]:

$$|\mathbf{k}\rangle_{\text{in}} = \hat{\mathcal{U}}\hat{a}^\dagger_{\mathbf{k}}\,|0\rangle \ , \ {}_{\text{out}}\langle\mathbf{k}| = \langle 0|\,\hat{a}_{\mathbf{k}}\hat{\mathcal{U}}^\dagger_0 \tag{1}$$

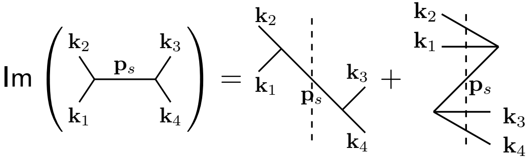
$$S_{n\to n'} = \ {}_{\text{out}}\langle n'|n\rangle_{\text{in}} = \delta_{n,n'} + iA_{n\to n'} \tag{2}$$

New cutting rules and Positivity

The dS S-matrix satisfies a set of unitarity cutting rules [SAS+Melville, in prep]:

$$\hat{\mathcal{U}}^\dagger \hat{\mathcal{U}}_0 \hat{\mathcal{U}}_0^\dagger \hat{\mathcal{U}} = \mathbb{I} \tag{3}$$

The lack of energy conservation means that unitarity cuts need to account for particle production via more intermediate states:



(4)

These new terms do not spoil flat space positivity:

$$\text{Im}\left(A^{\text{exc,s}}_{2\to 2'}\right) = A_{2\to 1} A^*_{2'\to 1} + \underbrace{A_{3\to 0} A^*_{3\to 0}}_{\langle 2'|5\rangle\langle 5|2\rangle} \tag{5}$$

A Leafy Celestial Dual for MHV Amplitudes

Walker Melton

[2312.07820] + [2402.04150] w/ Atul Sharma and Andrew Strominger
[2403.18896] w/ Atul Sharma, Andrew Strominger, and Tianli Wang

Amplitudes 2024 Gong Show

Celestial Leaf Amplitudes

- Celestial holography posits that 4D amplitudes can be computed holographically by a 2D CFT.

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$$\mathcal{L}^{\mathrm{MHV}}_{\Delta_i} = \frac{z_{12}^4}{z_{12}\cdots z_{n1}} \int_{\hat{x}^2=-1} d^3\hat{x} \prod_{j=1}^{n} \frac{\Gamma(2\bar{h}_j)}{(\epsilon - \mathrm{i}\hat{q}_j\hat{x})^{2\bar{h}_j}}$$

Celestial Leaf Amplitudes

- Celestial holography posits that 4D amplitudes can be computed holographically by a 2D CFT.
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- Leaf amplitudes describe scattering that occurs on a single hyperbolic slice of spacetime.

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- Amplitudes are linear combinations of leaf amplitudes

$$\mathcal{A}^{\mathrm{MHV}}_{\Delta_i} = \frac{\delta(\Delta_1+\Delta_2+\Delta_3-3)}{8\pi^3}\left(\mathcal{L}^{\mathrm{MHV}}_{\Delta_i}(z_i,\bar{z}_i) + \mathcal{L}^{\mathrm{MHV}}_{\Delta_i}(z_i,-\bar{z}_i)\right)$$

A Celestial Dual for MHV Leaf Amplitudes

- The dual leaf CFT for MHV amplitudes has action

$$S = \frac{1}{8\pi}\int d^2z \psi^i \bar{\partial}\psi^i + \frac{1}{4\pi}\int d^2z \rho\bar{\partial}\eta + \frac{1}{4\pi}\int d^2z [\partial\phi\bar{\partial}\phi + 4\pi\mu e^{2b\phi}]$$

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- This theory contains the 'gluon' operators

$$\mathcal{O}_{\Delta}^{+a} \propto T^a_{ij} :\psi^i\psi^j: e^{(\Delta-1)b\phi}(z,\bar{z})$$
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- In the semiclassical limit $b \to 0$

$$\langle \mathcal{O}_{\Delta_1}^{-a_1}\mathcal{O}_{\Delta_2}^{-a_2}\cdots\mathcal{O}_{\Delta_n}^{+a_n}\rangle = \frac{\mathrm{Tr}[T^{a_1}\cdots T^{a_n}]z_{12}^4}{z_{12}\cdots z_{n1}}\int_{\hat{x}^2=-1} d^3\hat{x}\prod_{j=1}^{n}\frac{\Gamma(2\bar{h}_j)}{(\epsilon - \mathrm{i}\hat{q}_j\hat{x})^{2\bar{h}_j}}$$
$$= \mathcal{L}^{\mathrm{MHV}}_{\Delta_i}$$

Schubert arrangements in the Grassmannian

Schubert hyperplane H_q in $\mathbb{P}^{\binom{n}{k}-1}$ $\quad \det \begin{bmatrix} P \\ Q \end{bmatrix} = 0$

scattering potential $\quad L(p;s) = \sum_{i=1}^{d} s_i \log(\det M_i(p)) \qquad M_i(p) = \begin{bmatrix} P \\ Q_i \end{bmatrix}$

number of complex critical points $=$ ML-degree of $\mathrm{Gr}(k,n) \setminus \mathcal{H} = \left| \chi(\mathrm{Gr}(k,n) \setminus \mathcal{H}) \right|$

counted by $\quad P_{k,n}(d, \mathbb{K}) = \chi(\mathbb{K}^{k(n-k)}) \sum_{i=0}^{k(n-k)} (-1)^i \chi_{k,n}(i, \mathbb{K}) \binom{d}{i} \in \mathbb{Q}[d]$

$\chi_{k,n}(i, \mathbb{K}) \in \mathbb{Z}$ *denotes the Euler characteristic of the intersection of* i *generic Schubert divisors*

$$\chi_{k,n}(0, \mathbb{C}) = \chi(\mathrm{Gr}_{k,n}(\mathbb{C})) = \binom{n}{k}, \quad \chi_{k,n}(1, \mathbb{C}) = \chi(D_i) = \binom{n}{k} - 1 \qquad \chi_{k,n}(k(n-k), \mathbb{C}) = \frac{(k(n-k))!(k-1)!(k-2)!\dots 2!1!}{(n-1)!(n-2)!\dots(n-k)!}$$

Zaslavsky (1975) $\quad P_{1,n}(d, \mathbb{R}) = d + \binom{d-1}{2} + \binom{d-1}{3} + \cdots + \binom{d-1}{n}$

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ELIA MAZZUCCHELLI

Grassmannian string integrals

MAX PLANCK INSTITUTE FOR PHYSICS

$$\mathcal{I}_{k,n}(\mathbf{X},\{c\}) := (\alpha')^d \int_{\mathrm{G}_+(k,n)/T} \omega_{k,n} R_{k,n} = \int_{\mathbb{R}^d_{>0}} \prod_{i=1}^{d} \frac{dx_i}{x_i}\, x_i^{\alpha' X_i} \prod_{I}^{D-d} \mathcal{F}_I(\mathbf{x})^{-\alpha' c_I}$$

$$\omega_{k,n} = \Omega(\mathrm{G}_+(k,n)/T) \qquad R_{k,n} := \prod_{a_1,a_2,\ldots,a_k} (a_1,a_2,\ldots,a_k)^{\alpha' s_{a_1,a_2,\ldots,a_k}}$$

scattering equations $\quad d\log R_{k,n} = 0$

defines a map $\Phi : \Delta_d \to \mathrm{Int}(\mathcal{P}),\, x \mapsto X$ solving SE $\qquad \mathcal{P}(k,n) := \sum_I c_I \mathbf{N}[\mathcal{F}_I]$

$$\lim_{\alpha' \to 0} \mathcal{I}_{k,n}(\mathbf{X},\{c\}) = \quad \Omega(\mathcal{P}) = \Phi_* \left(\prod_{i=1}^{d} \frac{dx_i}{x_i} \right) = \sum_{x^* \in \mathrm{Crit}(\Phi)} \prod_{i=1}^{d} d\log x_i \Big|_{x^*}$$

$k = 2\ :\ \mathcal{P}(2,n) = \text{Associahedron } A_{n-3}\,,\quad \Omega(A_{n-3}) = n\text{-point amplitude in } \mathrm{Tr}\,\varphi^3$

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A Double Copy from Twisted Cohomology at Genus One

Gong Show: Amplitudes 2024, IAS

Rishabh Bhardwaj

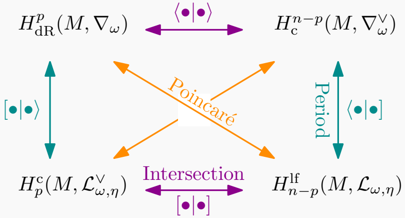
Based on the work [arxiv:2312.02148] with A. Pokraka, L. Ren and C. Rodriguez

Brown University

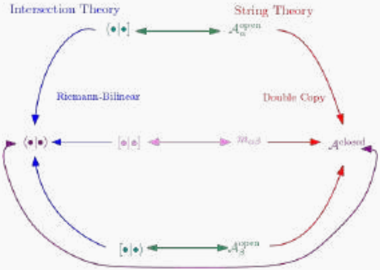
Introduction

- The field theory limit of the tree-level string KLT gave rise to the much celebrated double copy relations.
- In string theory this relates open string amplitudes $\mathcal{A}^{\text{open}}$ to the closed string ones $\mathcal{A}^{\text{closed}}$.
- But often various ingredients of this correspondence are technically challenging to compute.
- The intersection theory of twisted (co)homology groups developed in the 1970s by Aomoto and Gelfand serves as a highly efficient tool to study tree-level KLT! [Mizera '16, '19].

Introduction



(a) Connections between twisted cohomology groups and their duals



(b) Connection between intersection theory and string theory

- In our work we aimed to generalise this construction at genus one (one-loop string amplitudes)!

Toy model: The Riemann Wirtinger integrals

- An appropriate generalisation of tree-level string integrals (hypergeometric functions) at genus-one is the RW integrals, defined as follows:

$$\int_{\gamma_i} u(z_1)\, F(z_1 - z_j, \eta|\tau)\, \mathrm{d}z_1, \quad u(z_1) = e^{2\pi i s_{1A} z_1} \prod_{i=2}^{n} \vartheta_1^{s_{1i}}(z_1 - z_i), \quad (1)$$

  with $n \geq 3$ punctures and an extra condition

$$\boxed{s_{1B} = s_{1A}\tau + \sum_{j=2}^{n} s_{1j} z_j - \eta = \mathrm{const.}\,,} \quad (2)$$

  where $s_{1B} \in \mathbb{C}$ is a generic complex number.
- The above constraint is necessary for the RW integrals to have appropriate monodromies around the two cycles of the torus [Mano '08, Mano and Watanabe '12 & Goto '22].

Twisted (co)homology: Definition

- We can define IBP relations w.r.t the connection $\nabla = \mathrm{d} + \omega\wedge$ on these integrals, where $\omega = d\log u$ is the twist.
- This allows us to define the corresponding twisted cohomology class w.r.t this connection in the usual way:

$$H^p(M, \nabla_\omega) = \frac{\ker\left(\nabla_\omega : \Omega^p(M, \nabla_\omega) \to \Omega^{p+1}(M, \nabla_\omega)\right)}{\mathrm{im}\left(\nabla_\omega : \Omega^{p-1}(M, \nabla_\omega) \to \Omega^p(M, \nabla_\omega)\right)}, \tag{3}$$

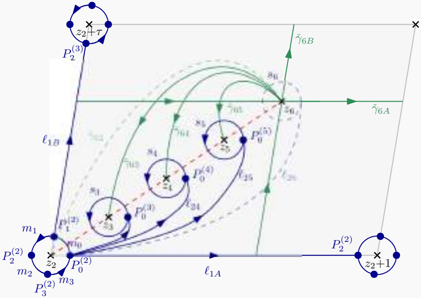
- The twisted homology group is defined w.r.t the Poincaré dual boundary operator ∂_ω as:

$$H_p(M, \check{\mathcal{L}}_\omega) = \frac{\ker\left(\partial_\omega : C_p(M, \check{\mathcal{L}}_\omega) \to C_{p-1}(M, \check{\mathcal{L}}_\omega)\right)}{\mathrm{im}\left(\partial_\omega : C_{p+1}(M, \check{\mathcal{L}}_\omega) \to C_p(M, \check{\mathcal{L}}_\omega)\right)}, \tag{4}$$

- More explicitly, the twisted cohomology basis is given as [Goto '22]:

$$\varphi_a = F(z_1 - z_{a+1}, \eta)\, \mathrm{d}z_1 \quad \text{for} \quad a = 1, 2, \cdots, n-1 \tag{5}$$

  Defined over the local system $\mathcal{L}_\omega := \mathcal{L}_\omega(s_{1A}, s_{12}, \ldots, s_{1n}) = \mathbb{C}u^{-1}$.
- And the corresponding twisted homology basis is:



  Defined over the dual local system
  $\check{\mathcal{L}}_\omega := \mathcal{L}_{-\omega}(-s_{1A}, -s_{12}, \ldots, -s_{1n}) = \mathbb{C}u$

Riemann bilinear relations (double-copy) at genus one

- The RW integrals can be written as period integrals on this basis of twisted (co)homology :

$$P_{ia} := [\gamma_i \otimes u_{\gamma_i}|\varphi_a\rangle = \int_{\gamma_i \otimes u_{\gamma_i}} \varphi_a \tag{6}$$

together with its dual defined as $\overline{P}_{bj} := \langle\overline{\varphi_b}|\overline{\gamma_j}] = \overline{[\gamma_j|\varphi_a\rangle}$.

- The corresponding intersection index is given as:

$$H_{ij} := [\gamma_i|\check{\gamma}_j]\,, \tag{7}$$

- Gluing these pieces together, we arrive at the Riemann bilinear relations [Matsubara-Heo '20, RB-Pokraka-Ren-Rodriguez '23]:

$$\boxed{\langle\bar{\varphi}_a|\varphi_b\rangle := \int_M |u|^2\, \bar{\varphi}_a \wedge \varphi_b = \left(\, \underline{\overline{P}} \cdot \underline{H}^{-1} \cdot \underline{P}\, \right)_{ba}} \tag{8}$$

Numerical double-copy at genus one

- Various numerical checks have helped us support our analysis:

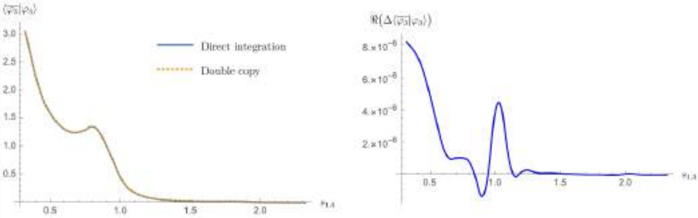


Figure 1: Numerical comparison of the (real part of the) complex Riemann-Wirtinger integral $\langle\overline{\varphi}_3|\varphi_3\rangle$ by numerical integration and by using the double copy formula.

Unfolding the generalized double copy structure

Alan (Shih-Kuan) Chen

Leinweber Center for Theoretical Physics, The University of Michigan

LSA PHYSICS
UNIVERSITY OF MICHIGAN

In collaboration with Henriette Elvang

– Generalized double copy [Chi, Elvang, Herderschee, Jones, Paranjape 2021]

Field-theory $$\text{Gravity}^{+} = \text{YM} \overset{\text{FT}}{\otimes} \text{YM} \qquad \mathbb{1}^{\text{BAS}}$$

String-theory $$\text{Closed} = \text{Open} \overset{\text{string}}{\otimes} \text{Open} \qquad \mathbb{1}^{\text{string}}$$

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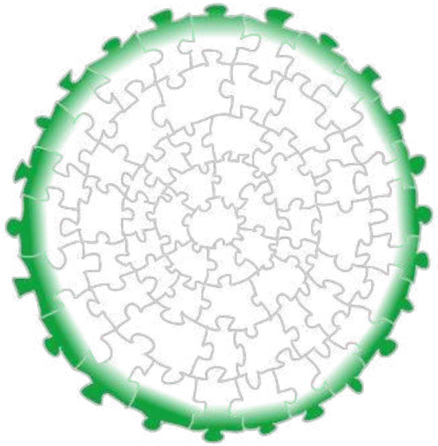

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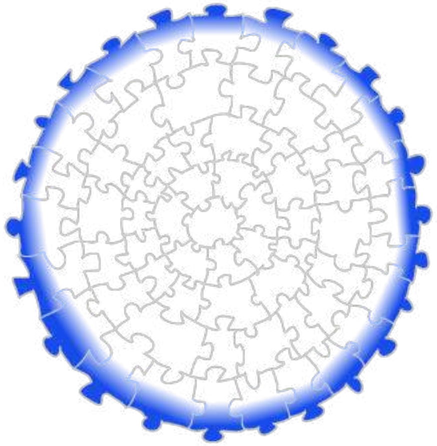

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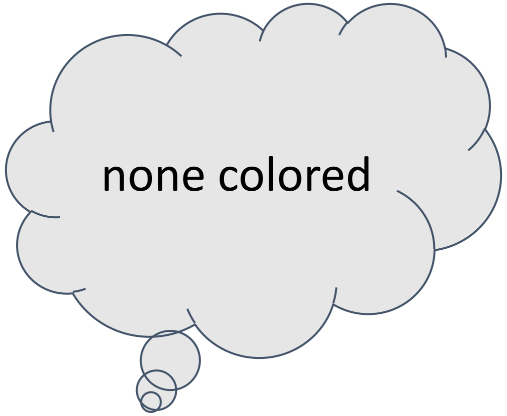

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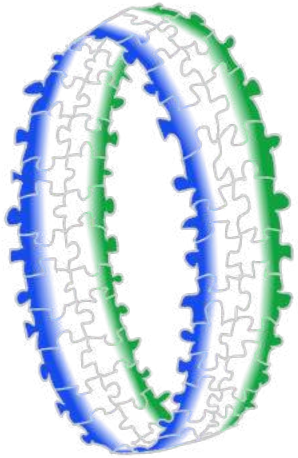
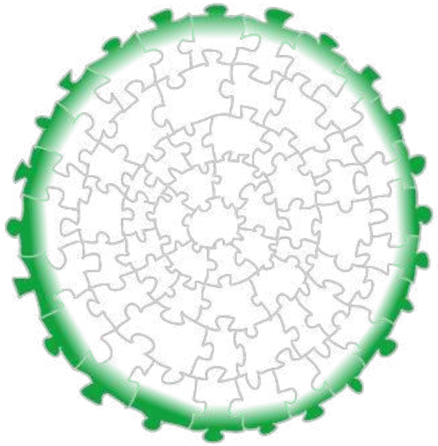


FT KKBCJ
string monodromy



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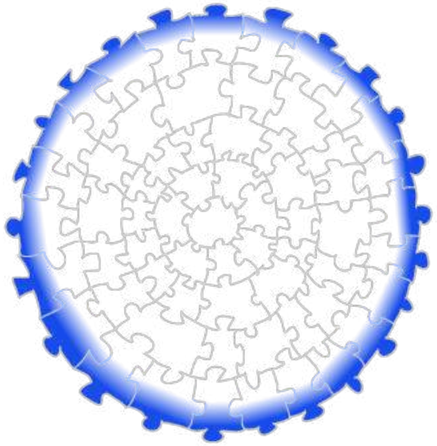

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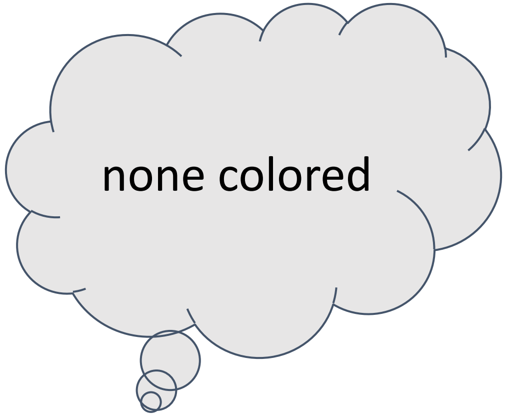

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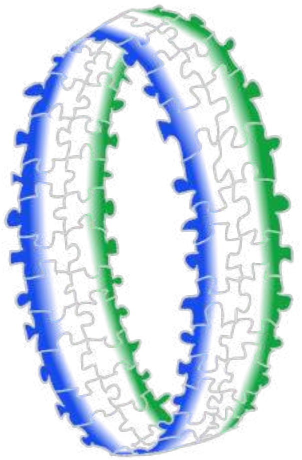

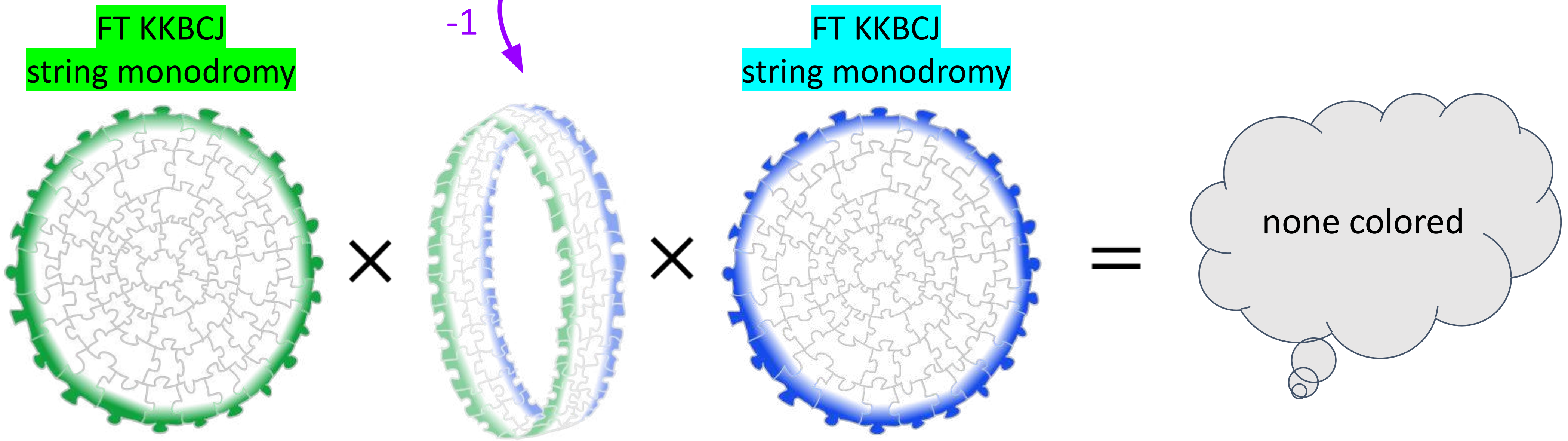

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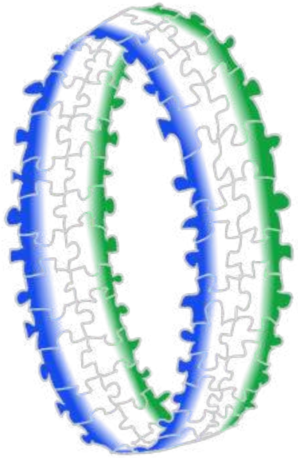

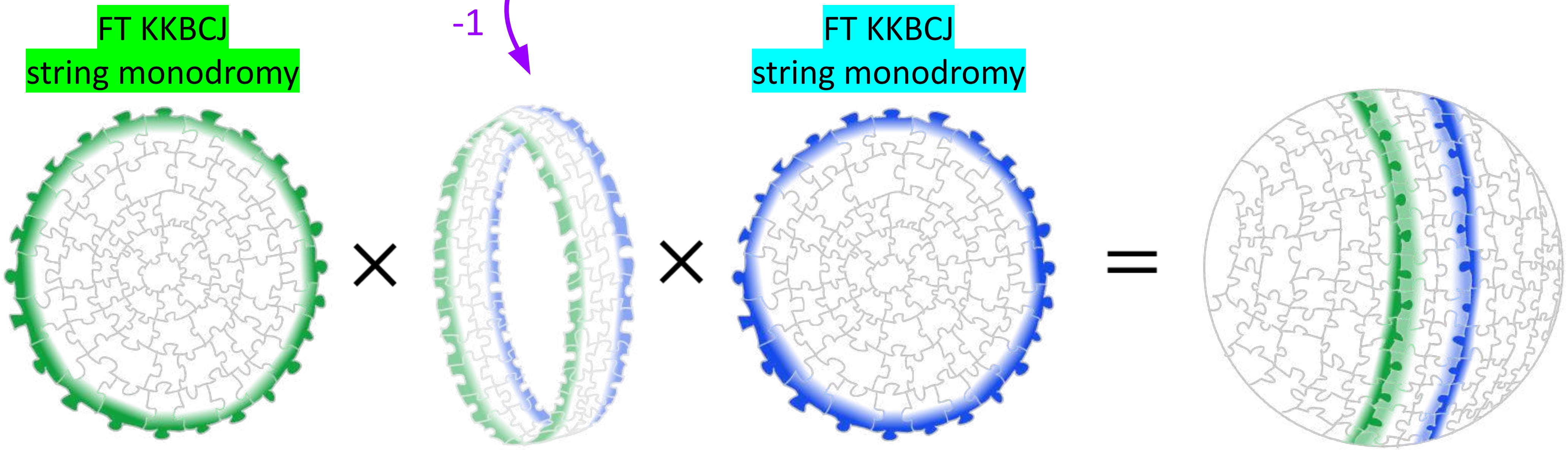

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Generalized

$$\text{EFT} \overset{\text{generalized}}{\otimes} \text{EFT} = \text{EFT}$$

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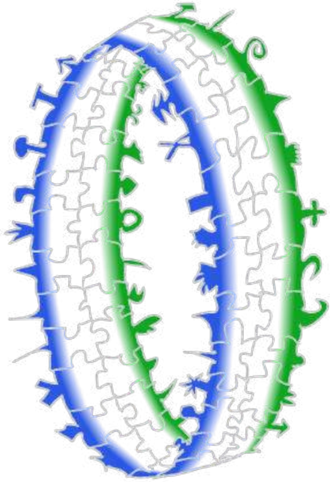

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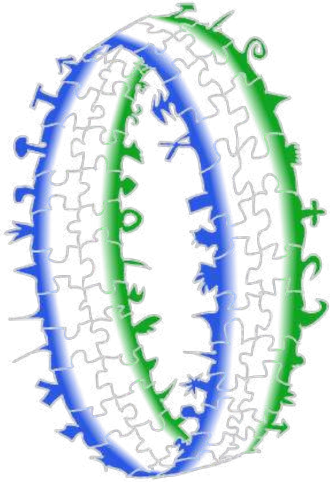

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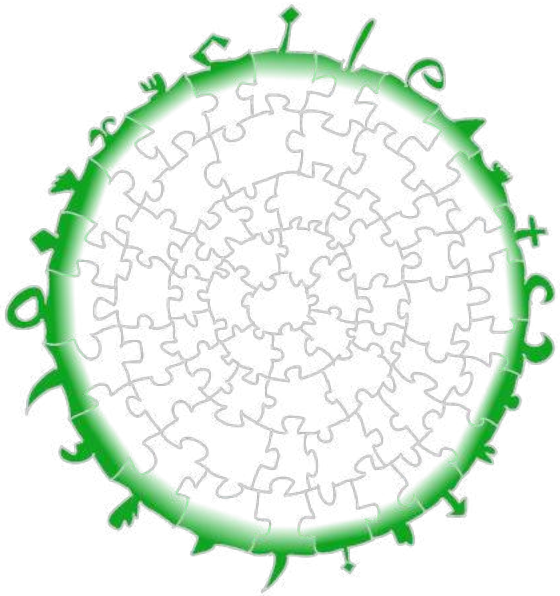

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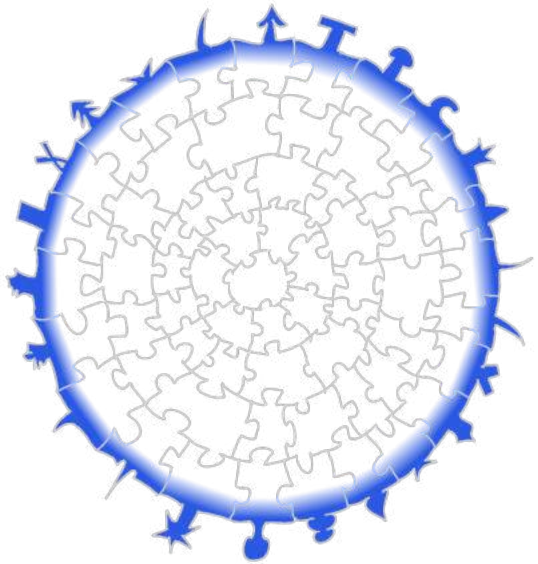
Generalized KKBCJ



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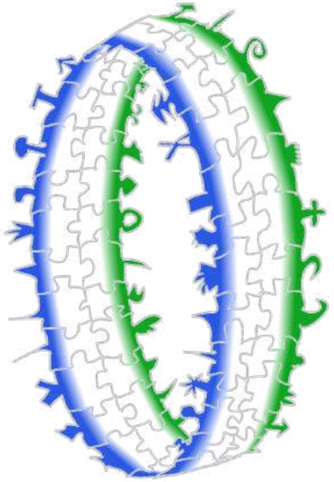

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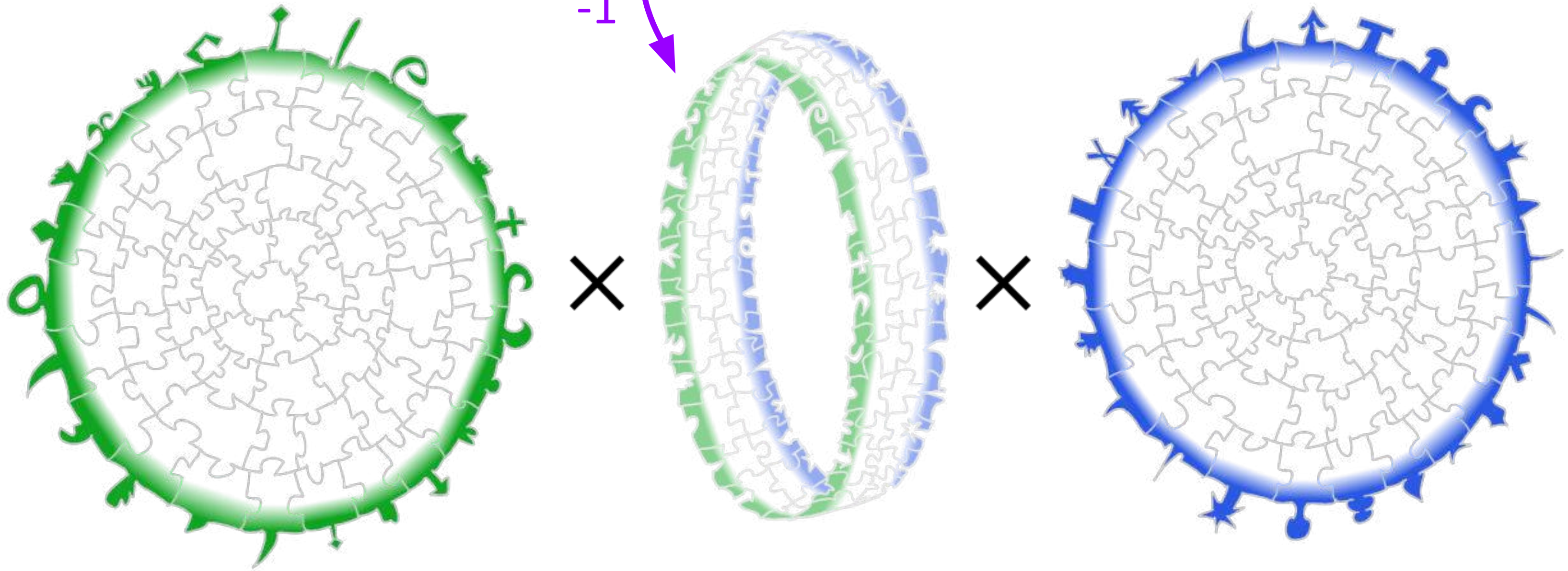
Generalized KKBCJ

-1

Generalized KKBCJ



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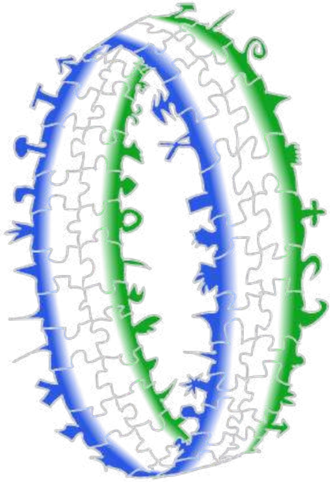

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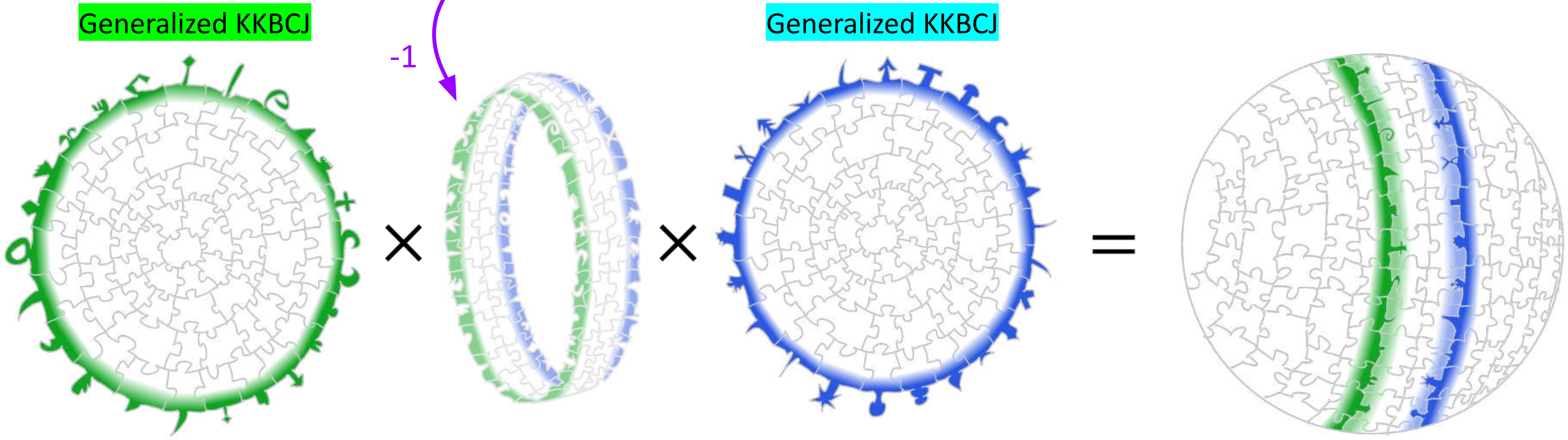

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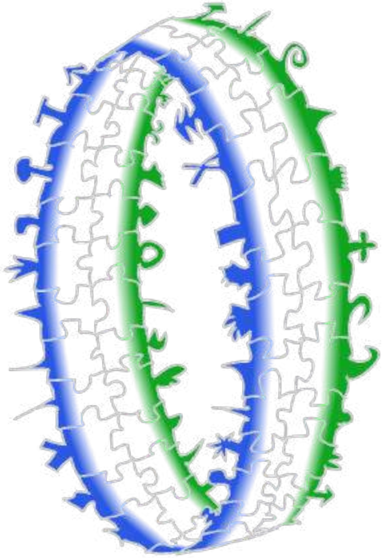
Generalized

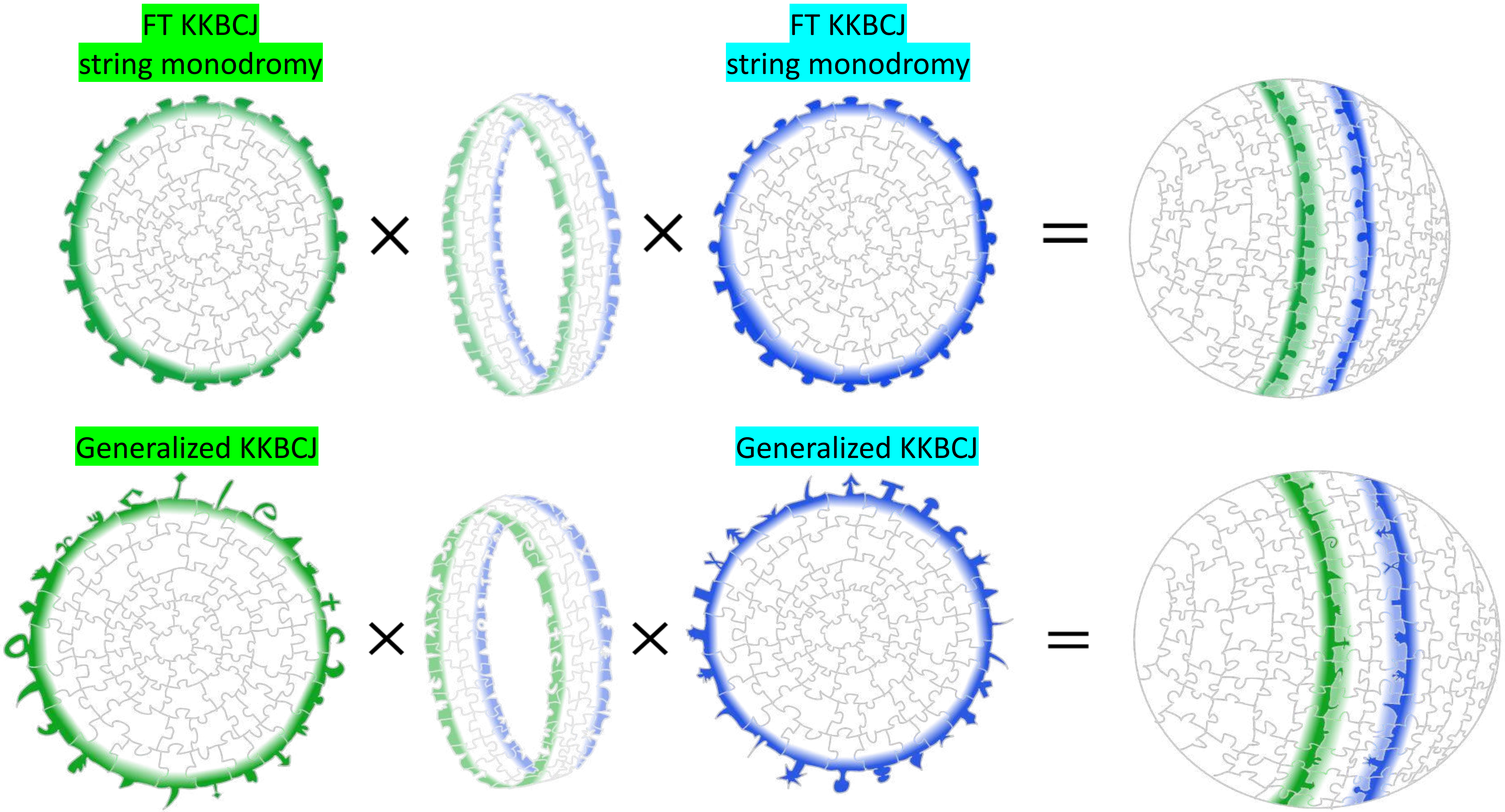
$$\text{EFT} \overset{\text{generalized}}{\otimes} \text{EFT} = \text{EFT}$$

$$\mathbb{1}^{\text{BASEFT}}$$



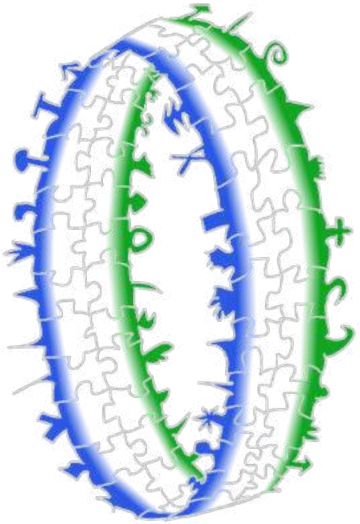
– The only consistency requirement is that the “inverse kernel amplitude” BASEFT must satisfy the minimal rank condition

– The output turns out to be the same (up to a shifting of parameters)

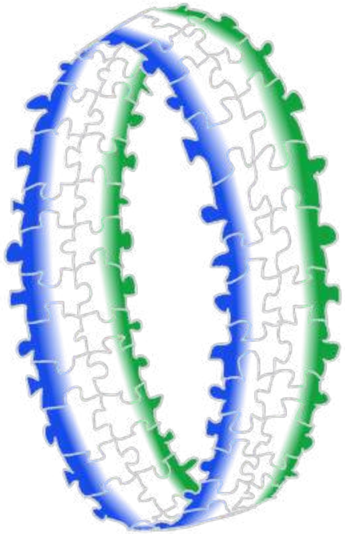


Hybrid Mode Decomposition

$\mathbb{1}^{G\tilde{G}}$



$\mathbb{1}^{FF}$



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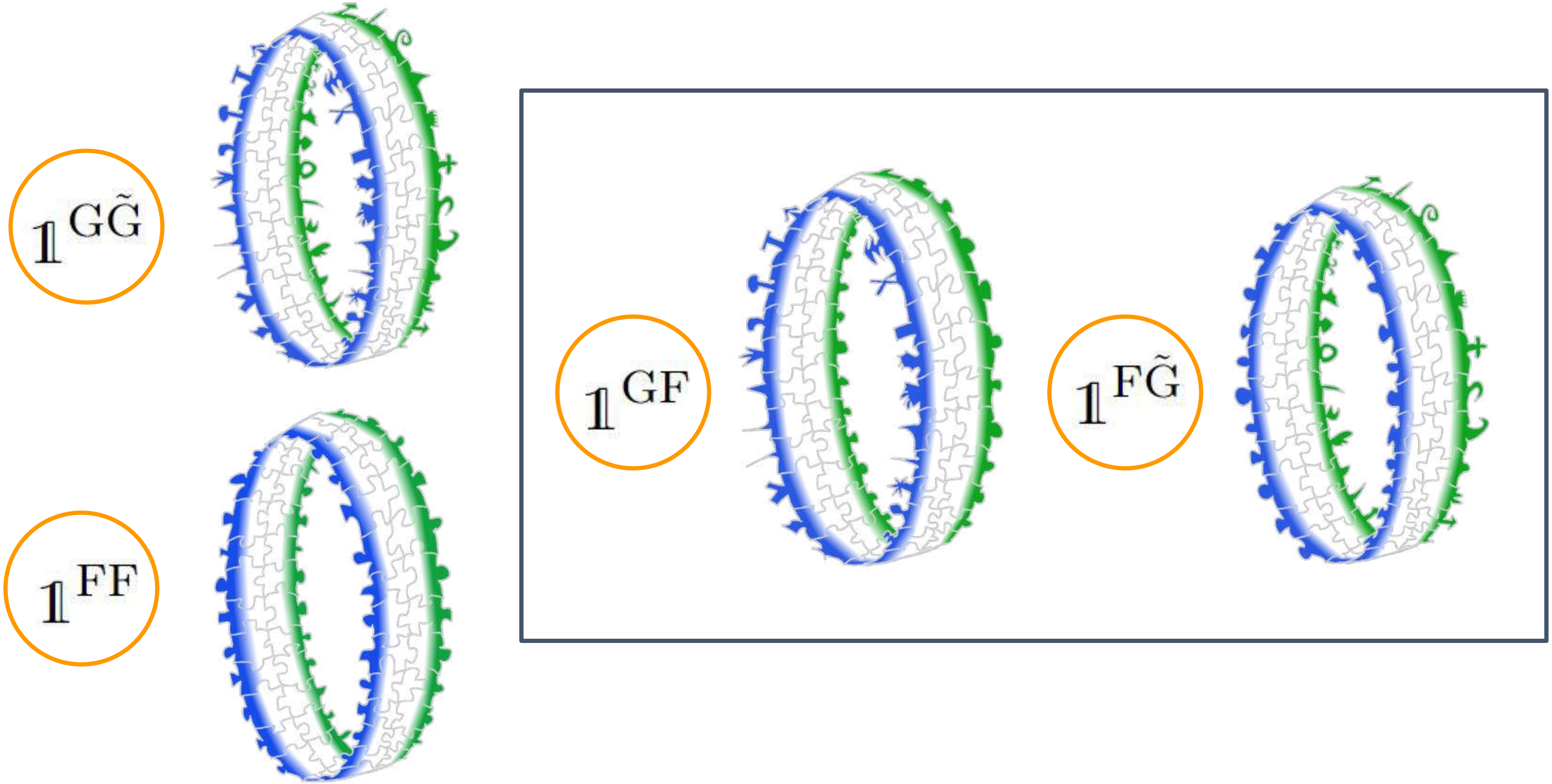

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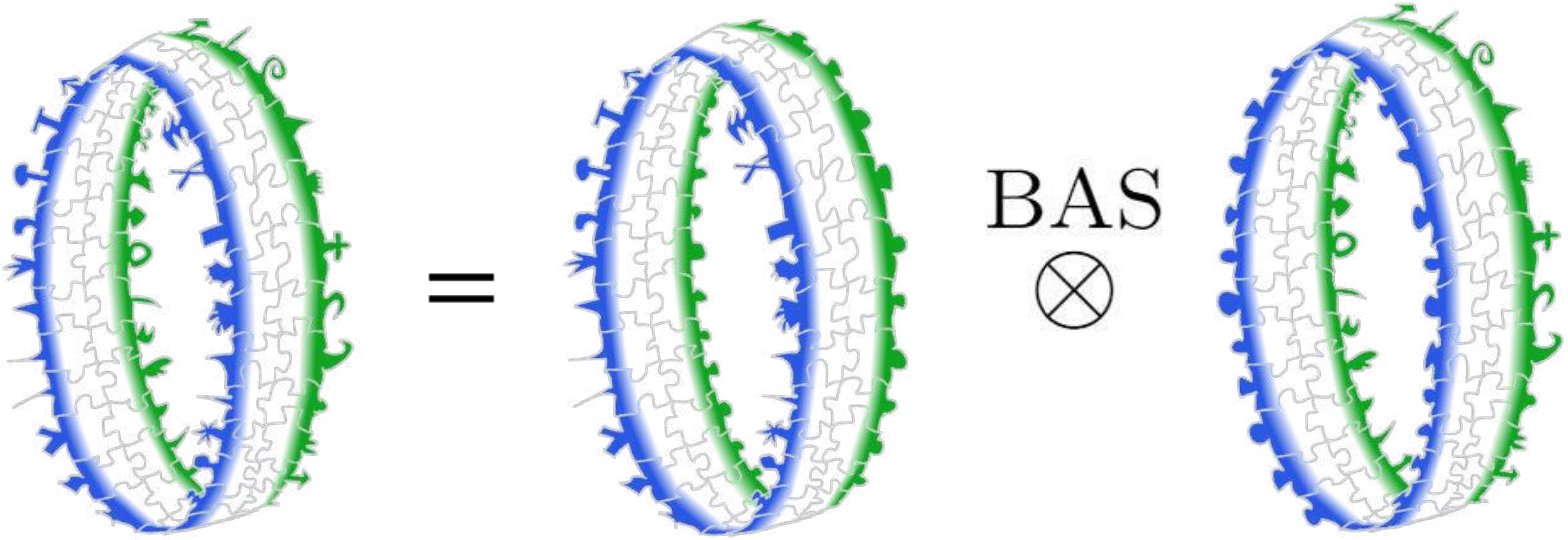

$$\mathbb{1}^{G\tilde{G}} = \mathbb{1}^{GF} \overset{\text{BAS}}{\otimes} \mathbb{1}^{F\tilde{G}}$$

– Application: complete parametrization of inverse kernel amplitude

$$m^{\mathrm{GF}} \sim Z = Z(\zeta_2, \zeta_3, \zeta_5, \cdots, \zeta_{3,5}, \zeta_{3,7}, \cdots)$$

order	independent new MZVs	independent commutators	new parameters
8	$\zeta_{3,5}$	$[M_5, M_3]$	$\tilde{b}_{3,5}$
9	none	$[M_3, [M_3, M_3]] = 0$	none
10	$\zeta_{3,7}$	$[M_7, M_3]$	$\tilde{b}_{3,7}$
11	$\zeta_{3,3,5}$	$[M_3, [M_5, M_3]]$	$\tilde{b}_{3,3,5}$
12	$\zeta_{3,9}$ $\zeta_{1,1,4,6}$	$[M_9, M_3]$ $[M_7, M_5]$	$\tilde{b}_{3,9}$ $\tilde{b}_{5,7}$
13	$\zeta_{3,5,5}$ $\zeta_{3,3,7}$	$[M_5, [M_5, M_3]]$ $[M_3, [M_7, M_3]]$	$\tilde{b}_{3,5,5}$ $\tilde{b}_{3,3,7}$
14	$\zeta_{3,3,3,5}$ $\zeta_{3,11}$ $\zeta_{5,9}$	$[M_3, [M_3, [M_5, M3]]]$ $[M_{11}, M_3]$ $[M_9, M_5]$	$\tilde{b}_{3,3,3,5}$ $\tilde{b}_{3,11}$ $\tilde{b}_{5,9}$
15	$\zeta_{3,3,9}$ $\zeta_{5,3,7}$ $\zeta_{1,1,3,4,6}$	$[M_3, [M_9, M_3]]$ $[M_5, [M_7, M_3]]$ $[M_3, [M_7, M_5]]$	$\tilde{b}_{3,3,9}$ $\tilde{b}_{5,3,7}$ $\tilde{b}_{3,5,7}$
16	$\zeta_{5,11}$ $\zeta_{3,13}$ $\zeta_{3,3,3,7}$ $\zeta_{3,3,5,5}$ $\zeta_{1,1,6,8}$	$[M_{11}, M_5]$ $[M_{13}, M_3]$ $[M_3, [M_3, [M_7, M_3]]]$ $[M_3, [M_5, [M_5, M_3]]]$ $[M_9, M_7]$	$\tilde{b}_{5,11}$ $\tilde{b}_{3,13}$ $\tilde{b}_{3,3,3,7}$ $\tilde{b}_{3,3,5,5}$ $\tilde{b}_{7,9}$

See [Schlotterer, Stieberger 2012] for a parametrization of Z amplitudes and the definition of M matrices

– Application: generalized single-valued projection

$$m^{\mathrm{J}} = \left(m^{\mathrm{Z}}\right)^{\top} \overset{\text{String}}{\otimes} m^{\mathrm{Z}}$$

$$m^{\mathrm{J}} = \mathrm{sv}(m^{\mathrm{Z}})$$

$$\begin{aligned}
\mathrm{sv:}\quad & \zeta(\mathrm{even}) \to 0\,, \quad \zeta(\mathrm{odd}) \to 2\,\zeta(\mathrm{odd})\,, \\
& \zeta_{3,5} \to -10\,\zeta_3\,\zeta_5\,, \\
& \zeta_{3,7} \to -28\,\zeta_3\,\zeta_7 - 12\,\zeta_5^2\,, \\
& \zeta_{3,3,5} \to 2\,\zeta_{3,3,5} - 5\,\zeta_3^2\,\zeta_5 + 90\,\zeta_2\,\zeta_9 + \frac{12}{5}\,\zeta_2^2\,\zeta_7 - \frac{8}{7}\,\zeta_2^3\,\zeta_5^2\,,
\end{aligned}$$

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\end{aligned}$$

$$\begin{aligned}
\tilde{b}_{2k+1} &= \tilde{b}^{(\mathrm{R})}_{2k+1} + \tilde{b}^{(\mathrm{L})}_{2k+1}\\
\tilde{b}_{i_1,i_2} &= -\tilde{b}^{(\mathrm{R})}_{i_1,i_2} + \tilde{b}^{(\mathrm{L})}_{i_1,i_2} + \frac{1}{2}(\tilde{b}^{(\mathrm{R})}_{i_2}\tilde{b}^{(\mathrm{L})}_{i_1} - \tilde{b}^{(\mathrm{L})}_{i_2}\tilde{b}^{(\mathrm{R})}_{i_1})\\
\cdots\ &\text{etc.}
\end{aligned}$$

Thank you

Residue Operations and a Canonical Basis for Cosmological Integrals

SHOUNAK DE
(BROWN UNIVERSITY)

wip w/ A. Pokraka (24xx.xxxxx)

Cosmological Correlators as Twisted Integrals

→ Object of interest: Wavefunction for cosmological fluctuations

Flat-space wavefunction $\left(\psi^{(n)}_{\text{flat}}\right)$ $\xrightarrow{\text{seeds}}$ Wavefunction in power-law FRW cosmology $\left(\psi^{(n)}_{\text{FRW}}\right)$

Cosmological Correlators as Twisted Integrals

→ Object of interest: Wavefunction for cosmological fluctuations

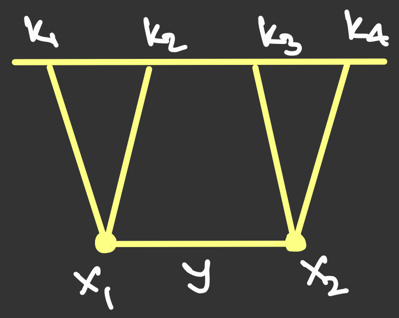
Flat-space wavefunction $(\psi^{(n)}_{\text{flat}})$ $\xrightarrow{\text{seeds}}$ Wavefunction in power-law FRW cosmology $(\psi^{(n)}_{\text{FRW}})$

$\psi^{(n)}_{\text{FRW}}$ of a theory of conformally coupled scalars can be written as a twisted integral over $\psi^{(n)}_{\text{flat}}$:

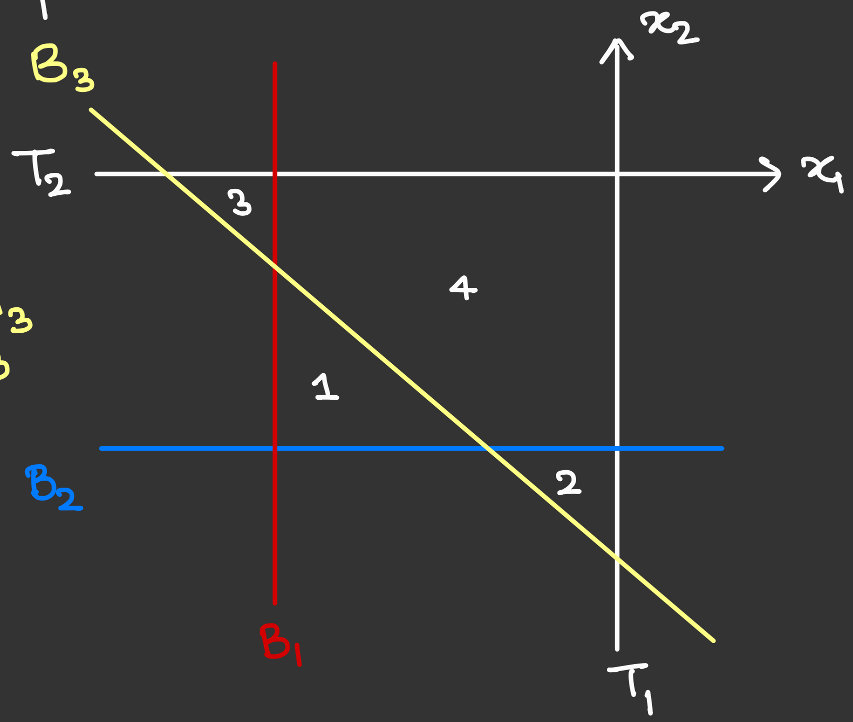
$$\psi^{(n)}_{\text{FRW}}(X_v, Y_e) = \int_0^\infty \underbrace{\prod_{v\in\mathcal{V}} dx_v\, x_v^{\varepsilon}}_{\text{Cosmological twist}}\; \underbrace{\psi^{(n)}_{\text{flat}}(x_v + X_v, Y_e)}_{\text{universal integrand}}$$

where $\psi^{(n)}_{\text{flat}}(X_v, Y_e) = \int_{-\infty}^{0} \prod_{v\in\mathcal{V}} d\eta_v\, e^{iX_v\eta_v} \prod_{e\in\mathcal{E}} \mathcal{G}(Y_e, \eta_v, \eta_v')$.

→ Arrangement of singular lines in the (x_1, x_2)-plane:

$\psi^{(2)}_{FRW} =$ 

$$= \int \frac{(x_1 x_2)^{\varepsilon}\, dx_1 \wedge dx_2}{T_1^{m_1} T_2^{m_2} B_1^{n_1} B_2^{n_2} B_3^{n_3}}$$



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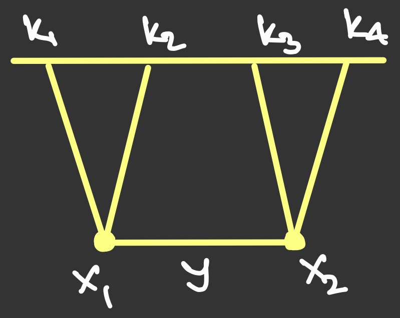
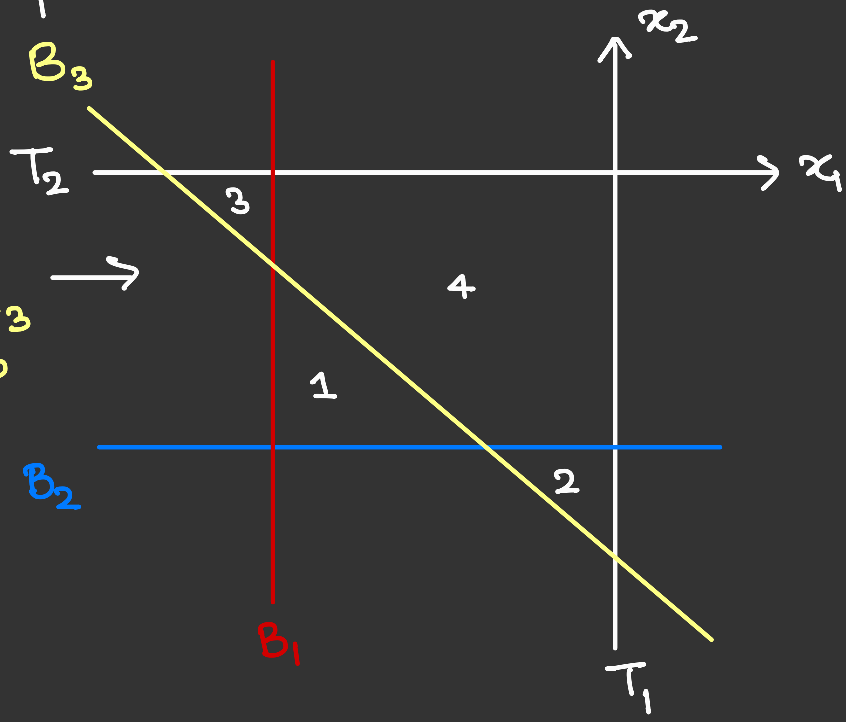
$\psi^{(2)}_{FRW} =$ 

$$= \int \frac{(x_1 x_2)^{\varepsilon}\, dx_1 \wedge dx_2}{T_1^{m_1} T_2^{m_2} B_1^{n_1} B_2^{n_2} B_3^{n_3}} \rightarrow$$



→ Twisted cohomology predicts the size of the vector space:

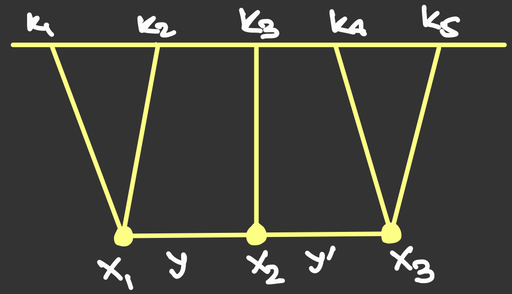
# of independent master integrals = # of bounded regions defined by divisors of the integrand

(Aomoto '75, Mastrolia et al. '19)

DEQ takes the ε-form: $d\vec{I} = \varepsilon \tilde{A} \vec{I}$

(N. Arkani-Hamed et al. '23, SD, A. Pokraka '23, ...)

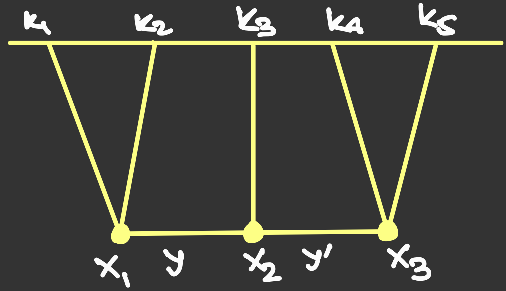
$\longrightarrow \quad \psi^{(3)}_{FRW} =$



$$= \int_0^\infty (x_1 x_2 x_3)^{\varepsilon} \frac{(B_5+B_6)}{B_1 B_2 B_3 B_4 B_5 B_6} \, dx_1 \wedge dx_2 \wedge dx_3$$

25 bounded regions $\Rightarrow$ 25 dim^{n}''al vector space

$\longrightarrow \quad \psi^{(3)}_{FRW} =$



$$= \int_0^\infty (x_1 x_2 x_3)^\varepsilon \frac{(B_5+B_6)}{B_1 B_2 B_3 B_4 B_5 B_6} \, dx_1 \wedge dx_2 \wedge dx_3$$

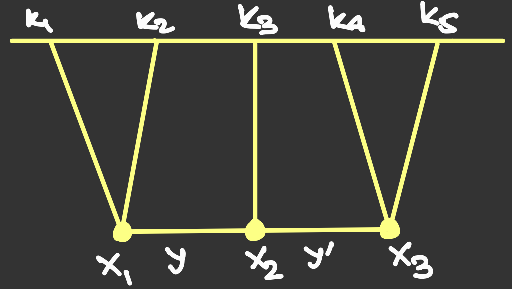
25 bounded regions ⇒ 25 dim$^{n''}$al vector space

Complexity of DEQ governed by a few universal rules
"KINEMATIC FLOW"

(N. Arkani-Hamed et al. '23)

Basis functions = $4^e = 16 \quad \forall e=2$

$\longrightarrow \quad \psi^{(3)}_{FRW} =$



$$= \int_0^\infty (x_1 x_2 x_3)^\varepsilon \frac{(B_5 + B_6)}{B_1 B_2 B_3 B_4 B_5 B_6} dx_1 \wedge dx_2 \wedge dx_3$$

25 bounded regions ⇒ 25 dim^{n}'al vector space

Complexity of DEQ governed by a few universal rules
"KINEMATIC FLOW"

(N. Arkani-Hamed et al. '23)

Basis functions $= 4^{n-3} = 16 \; (\forall n = 5)$

QUESTION: What physics governs the reduction of master integrals?

→ Eliminate unphysical sector using residue operations

→ Steinmann-like relations + Linear rel^{n} among hyperplanes

$$\text{Res}_{E_{g_1}} \cdots \text{Res}_{E_{g_k}} \Psi^{(n)}_{flat} = 0 \quad + \quad B_2 + B_4 = B_5 + B_6$$

Eliminates trivial dual forms

↓

Physical sector of 16 forms on which DEQ system closes

→ Eliminate unphysical sector using residue operations

→ Steinmann-like relations

$$\text{Res}_{E_{g_1}} \dots \text{Res}_{E_{g_k}} \psi^{(n)}_{\text{flat}} = 0$$

(Benincasa et al. '20,...)

\+ Linear rel^n among hyperplanes

$$B_2 + B_4 = B_5 + B_6$$

↓

Eliminates trivial dual forms (unphysical boundaries)

↓

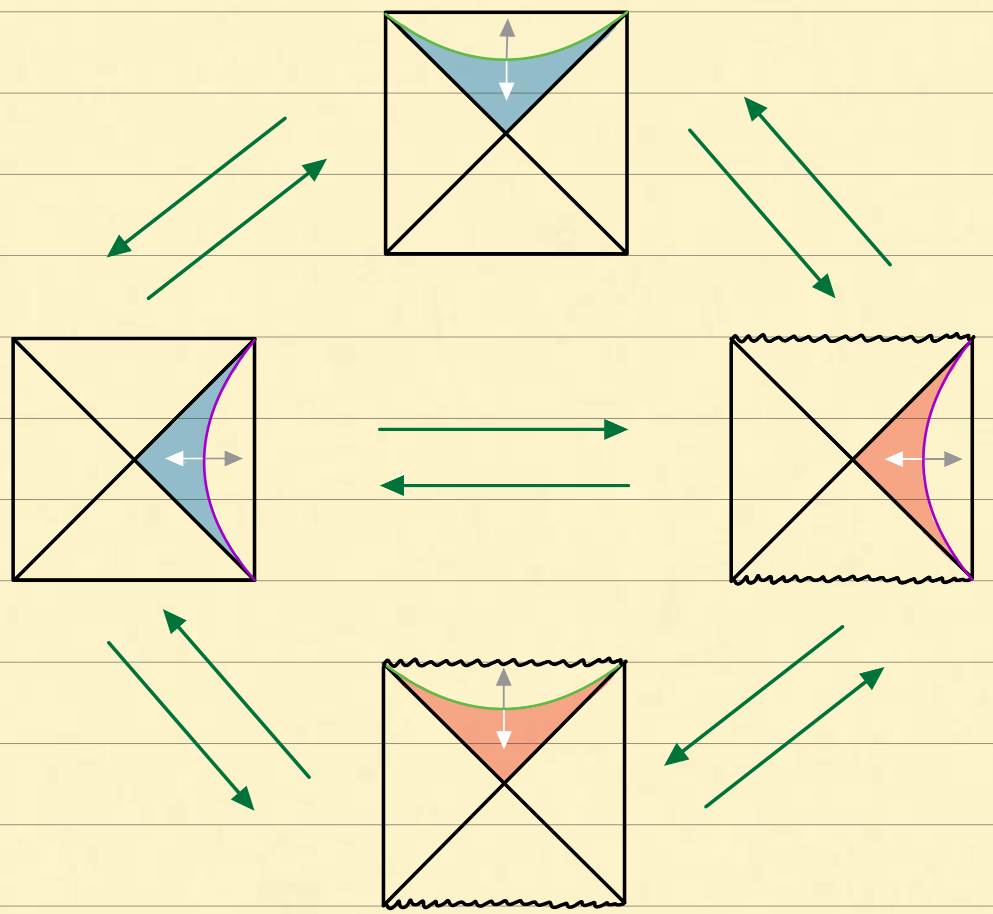
Physical sector of 16 forms on which DEQ system closes

Such rules seem to generalize to higher orders at tree-level!

$$\#(\text{physical integrals}) = \dim H^{n-3} - \#(\text{unphysical boundaries})$$

dS/CFT from $T\overline{T} + \Lambda_d$

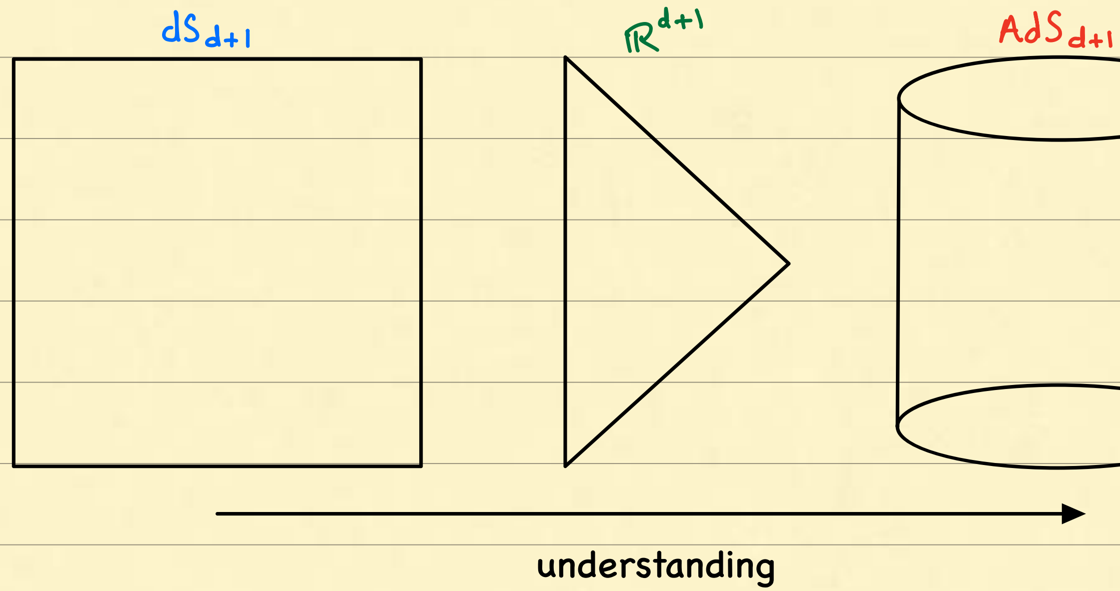
(w/ Vasudev Shyam, Eva Silverstein, Ronak Soni, Gonzalo Torroba; arXiv:24xx.xxxxx)



ALL FLOWS LEAD TO dS/CFT...

dS/CFT - the story so far

- Need to understand QG in real world where $\Lambda > 0$.
- No explicit examples of a holographic duality in dS, i.e. no dS/CFT.



- Previously built dS/CFT from the bottom up: analytic continuations of EAdS, higher-spin dS/CFT, cosmological bootstrap etc. .
- Here we move away from AdS/CFT lamppost to derive novel boundary field theory which describes microscopic theory of global dS, i.e. dS/CFT !

Hamiltonian Constraint = $T\bar{T}$ deformation !

$\Lambda = \pm \frac{d(d-1)}{l^2}$

$$H_{c,\pm\Lambda}\Psi_{WDW} = 0 \qquad H_{c,\pm\Lambda} = \left[\frac{16\pi G_N}{\sqrt{g}}\left(\tilde{\pi}^{ab}\tilde{\pi}_{ab} - \frac{1}{(d-1)}\tilde{\pi}^2\right) - \frac{\sqrt{g}}{16\pi G_N}\left(R - 2\Lambda\right)\right]$$

For D=3, $\Lambda > 0$, shift: $\tilde{\pi}_{ab} \to \pi_{ab} - \frac{1}{16\pi G_N l}\sqrt{g}\, g_{ab}$
(d=2)

$$H_{c,+\Lambda} = \frac{16\pi G_N}{\sqrt{g}}\left[\tilde{\pi}^{ab}\tilde{\pi}_{ab} - \tilde{\pi}^2\right] - \frac{\sqrt{g}}{16\pi G_N}\left[R - \frac{2}{l^2}\right]$$

$$H_{c,+\Lambda} = \underbrace{\frac{16\pi G_N}{\sqrt{g}}\left[\pi^{ab}\pi_{ab} - \pi^2\right]}_{T\bar{T}\text{ operator}} + \frac{2}{l}\pi^a{}_a - \frac{\sqrt{g}}{16\pi G_N}R$$

$$\Longrightarrow \lim_{\sqrt{g}\to\infty} H_{c,+\Lambda} = \frac{1}{\sqrt{g}}\pi^a{}_a - \frac{l}{32\pi G_N}R$$

similarly
for $\Lambda < 0$

$$\lim_{\sqrt{g}\to\infty} H_{c,-\Lambda} = \frac{i}{\sqrt{g}}\pi^a{}_a - \frac{l}{32\pi G_N}R$$

$$\underbrace{\left[T^a{}_a - \frac{(i)l}{32\pi G_N}R\right]Z_{CFT} = 0}_{\text{Weyl anomaly equation!}} \xrightarrow[\text{deformation}]{T\bar{T}} H_{c,\pm\Lambda}\Psi_{WDW} = 0$$

$$T^{ab}(x) \equiv \frac{-2}{\sqrt{g}}\frac{\delta}{\delta g_{ab}(x)} = \frac{-2i}{\sqrt{g}}\pi^{ab}(x) \quad \therefore$$

for $\Lambda > 0$ $\pi^{ab} \in \mathbb{R}$, $T^{ab} \in i\mathbb{R}$ ("non-unitary")

for $\Lambda < 0$ $\pi^{ab} \in i\mathbb{R}$, $T^{ab} \in \mathbb{R}$

Hamiltonian Constraints Galore

- Cauchy/Radial slices in Lorentzian AdS **OR** "Radial" slices in Euclidean AdS

$$H_{C,L,+\Lambda} = \left[\frac{16\pi G_N}{\sqrt{\pm g}}\left(\pi^{ab}\pi_{ab} - \frac{1}{(d-1)}\pi^2\right) \mp \frac{\sqrt{\pm g}}{16\pi G_N}\left(R \mp 2\,\frac{d(d-1)}{l^2}\right)\right]$$

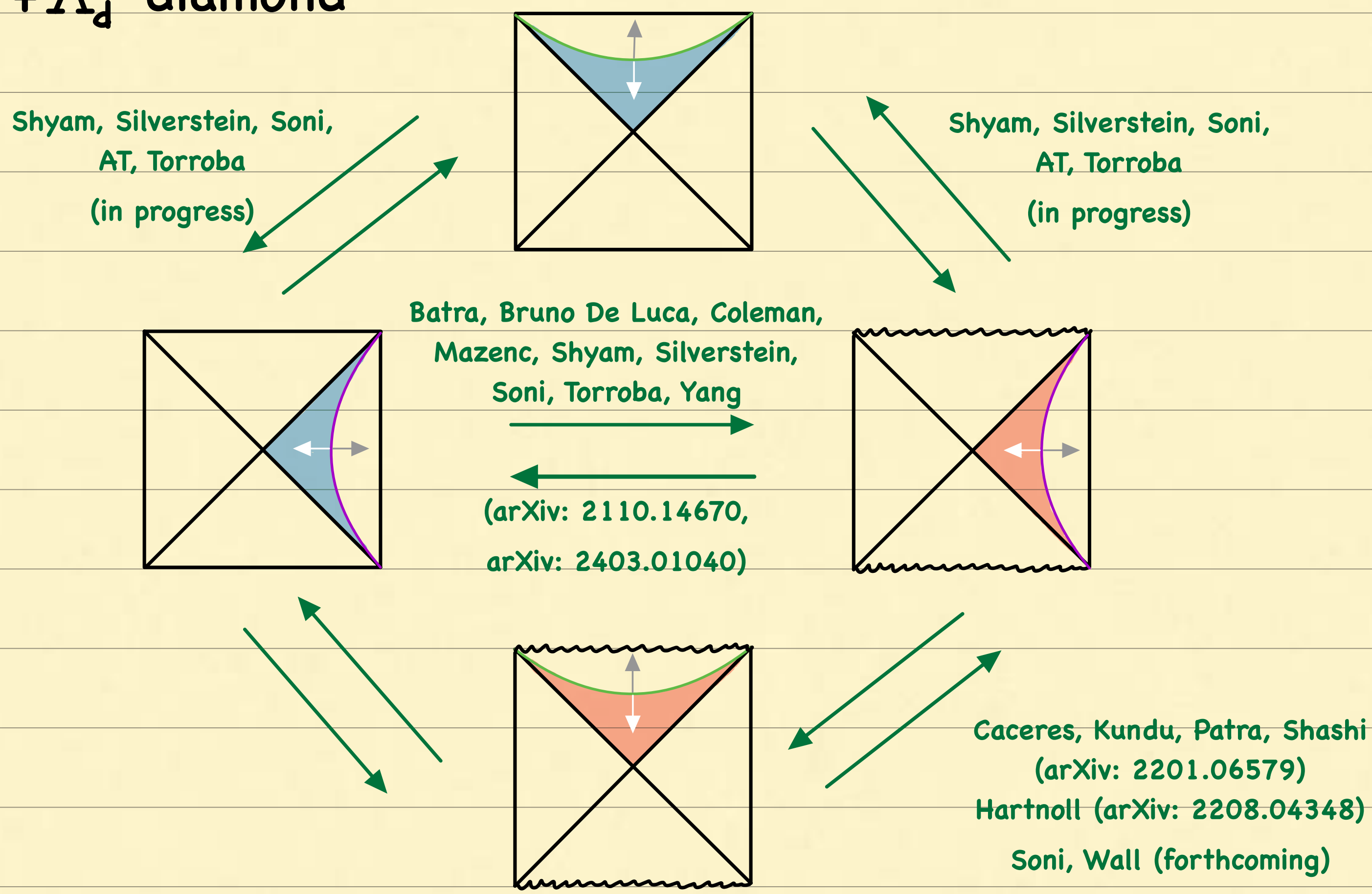
use Λ_d to switch CC at horizon where CC "vanishes"

- In general: $\lim_{l\to\infty} H_{r,E,-\Lambda,R<0\,,R>0} = \lim_{l\to\infty} H_{C,L,+\Lambda,R>0\,,R<0}$

- Use dimensionless deformation parameter $y = \frac{8\pi G_N l}{L_R^2}$ ← circumference of codimension-1 slice

- Two different field theories match at $Z_{\eta=1,\, y=y_0} = Z_{\eta=-1,\, y=y_0}$
  AdS — dS

The $T\bar{T} + \Lambda_d$ diamond



- Microscopic theory of static patch, i.e. GH entropy captured by BTZ states!
- Same density of states for dS/CFT w/ "imaginary" spectrum, i.e. $T\bar{T}+\Lambda_d$ is all we need for dS/CFT!
- <u>BUT</u> what is entropy in boundary theory when there is no boundary?

ALL FLOWS LEAD TO dS/CFT...

Thank you for listening!

Any questions feel free to find me at Amplitudes conference/school or e-mail at735@cantab.ac.uk

Extra slides

Hamiltonian Constraints Galore (cont.)

- Cauchy slices in Lorentzian dS

$$H_{c,L,+\Lambda} = \left[\frac{16\pi G_N}{\sqrt{g}} \left(\pi^{ab}\pi_{ab} - \frac{1}{(d-1)}\pi^2 \right) - \frac{\sqrt{g}}{16\pi G_N} \left(R - 2\frac{d(d-1)}{l^2} \right) \right]$$

- Radial slices in static patch of Lorentzian dS

$$H_{r,L,+\Lambda} = \left[\frac{16\pi G_N}{\sqrt{-g}} \left(\pi^{ab}\pi_{ab} - \frac{1}{(d-1)}\pi^2 \right) + \frac{\sqrt{-g}}{16\pi G_N} \left(R - 2\frac{d(d-1)}{l^2} \right) \right]$$

- Cauchy slices in Lorentzian AdS OR radial slices in EAdS

$$H_{r,E,-\Lambda} = \left[\frac{16\pi G_N}{\sqrt{g}} \left(\pi^{ab}\pi_{ab} - \frac{1}{(d-1)}\pi^2 \right) - \frac{\sqrt{g}}{16\pi G_N} \left(R + 2\frac{d(d-1)}{l^2} \right) \right]$$

- Radial slices in Lorentzian AdS (e.g. BTZ or Vacuum AdS)

$$H_{r,E,-\Lambda} = \left[\frac{16\pi G_N}{\sqrt{-g}} \left(\pi^{ab}\pi_{ab} - \frac{1}{(d-1)}\pi^2 \right) + \frac{\sqrt{-g}}{16\pi G_N} \left(R + 2\frac{d(d-1)}{l^2} \right) \right]$$

Hamiltonian Constraints = $T\bar{T}$

$\Lambda = \pm \frac{d(d-1)}{l^2}$

$$H_{c,\pm\Lambda} = \left[\frac{16\pi G_N}{\sqrt{g}} \left(\tilde{\pi}^{ab}\tilde{\pi}_{ab} - \frac{1}{(d-1)}\tilde{\pi}^2 \right) - \frac{\sqrt{g}}{16\pi G_N}\left(R - 2\Lambda \right) \right]$$

For D=3, $\Lambda > 0$, shift: (d=2) (canonical transformation)

$$\tilde{\pi}_{ab} \longrightarrow \pi_{ab} - \frac{1}{16\pi G_N l}\sqrt{g}\, g_{ab}$$

$$\tilde{\pi}^a{}_a = g^{ab}\tilde{\pi}_{ab} = \pi^a{}_a - \frac{1}{16\pi G_N l}\sqrt{g}\,\delta^a{}_a$$

$$\tilde{\pi}^{ab}\tilde{\pi}_{ab} = \left(\pi^{ab} - \frac{1}{16\pi G_N l}\sqrt{g}\, g^{ab}\right)\left(\pi_{ab} - \frac{1}{16\pi G_N l}\sqrt{g}\, g_{ab}\right) = \pi^{ab}\pi_{ab} - \frac{2}{16\pi G_N l}\sqrt{g}\,\pi^a{}_a + \frac{1}{(16\pi G_N l)^2}(\sqrt{g})^2\delta^a{}_a$$

$$\tilde{\pi}^2 = (\tilde{\pi}^a{}_a)^2 = \left(\pi^a{}_a - \frac{1}{16\pi G_N l}\sqrt{g}\,\delta^a{}_a\right)\left(\pi^a{}_a - \frac{1}{16\pi G_N l}\sqrt{g}\,\delta^a{}_a\right)$$

$$= (\pi^a{}_a)^2 - \frac{2}{16\pi G_N l}\sqrt{g}\,\delta^a{}_a\pi^a{}_a + \frac{1}{(16\pi G_N l)^2}(\sqrt{g})^2(\delta^a{}_a)^2$$

$$H_{c,\pm\Lambda} = \frac{16\pi G_N}{\sqrt{g}}\left[\tilde{\pi}^{ab}\tilde{\pi}_{ab} - \tilde{\pi}^2\right] - \frac{\sqrt{g}}{16\pi G_N}\left[R - \frac{2}{l^2}\right]$$

$$H_{c,\pm\Lambda} = \frac{16\pi G_N}{\sqrt{g}}\left[\pi^{ab}\pi_{ab} - \pi^2\right] - \frac{2}{l}\pi^a{}_a + \frac{2}{l}\delta^a{}_a\pi^a{}_a + \frac{\sqrt{g}}{16\pi G_N l^2}\delta^a{}_a$$
$$- \frac{\sqrt{g}}{16\pi G_N}R + \frac{\sqrt{g}}{16\pi G_N}\frac{2}{l^2} - \frac{\sqrt{g}}{16\pi G_N l^2}(\delta^a{}_a)^2$$

$$= \frac{16\pi G_N}{\sqrt{g}}\left[\pi^{ab}\pi_{ab} - \pi^2\right] + \frac{2}{l}\pi^a{}_a - \frac{\sqrt{g}}{16\pi G_N}R$$

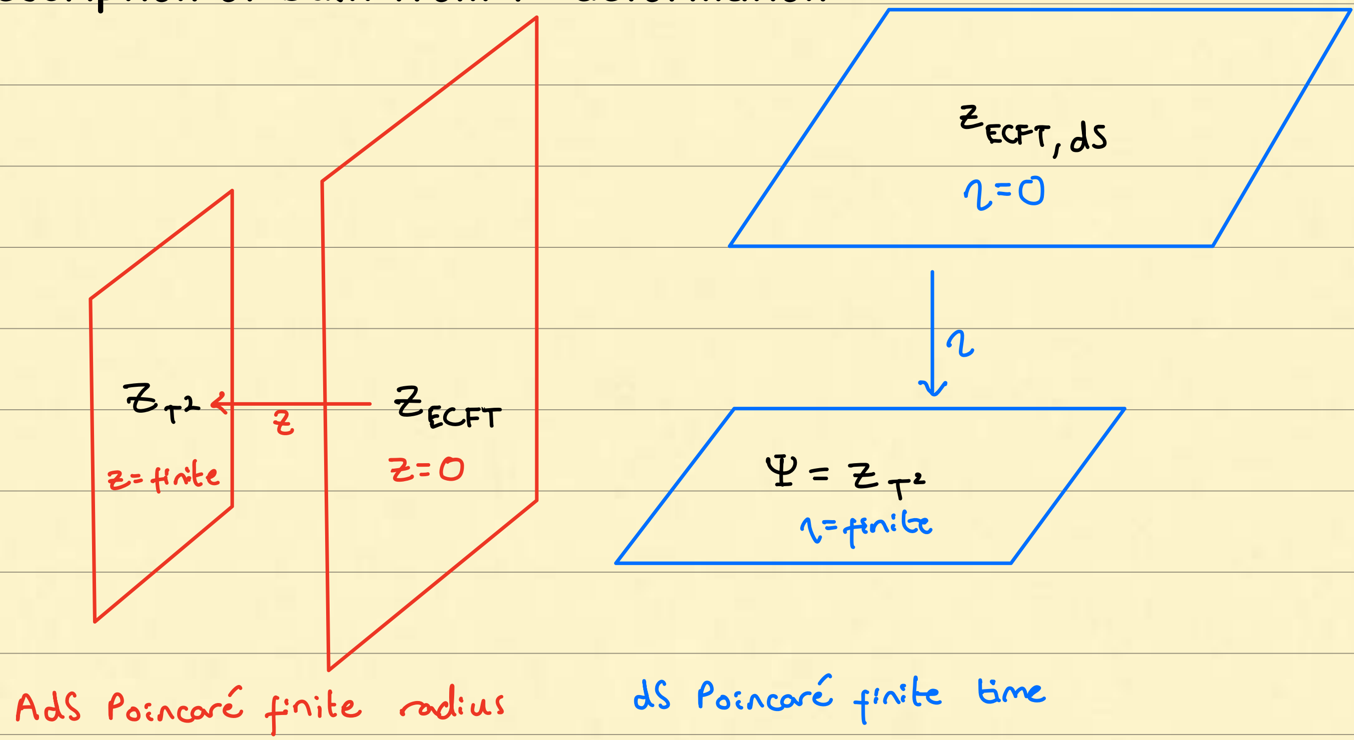
Cauchy Slice Holography Dictionary

Bulk at finite time	T^2-flow of boundary
Conformal time η	Deformation parameter $\mu^{\frac{1}{d}}$
WDW wavefunction $\Psi_{WDW}[g_{ij}, \phi, \ldots; \eta]$	Euclidean partition function $Z^{(\mu)}_{T^2}[\gamma_{ij}, J, \ldots]$
Configurations $g_{ij}(\eta), \phi(\eta), \ldots$	Sources $\gamma_{ij}(\mu), J(\mu), \ldots$
Conj. momenta $\Pi^{ab}(\eta) = -i \frac{\delta}{\delta g_{ab}(\eta)}$, $\Pi_\phi(\eta) = -i \frac{\delta}{\delta \phi(\eta)}$	Expectation values $\langle T^{ij} \rangle_\mu = \frac{2}{\sqrt{\gamma}} \frac{\delta}{\delta \gamma_{ij}} \log Z^{(\mu)}_{T^2}$, $\langle O \rangle_\mu = \frac{1}{\sqrt{\gamma}} \frac{\delta}{\delta J} \log Z^{(\mu)}_{T^2}$
Gauge constraints $H\Psi = 0$, $\nabla_a \Pi^{ab} = 0$	Properties of correlators $\langle H \rangle = 0$, $\langle \nabla_i T^{ij} \rangle = 0$

- We extend all aspects of usual holographic dictionary to provide a boundary description of bulk CQG physics w/ B.C.

$$\lim_{\eta \to 0} \Psi_{WDW} = \lim_{\mu \to 0} Z_{T^2} = Z_{CFT}$$

Holographic description of bulk from T^2-deformation



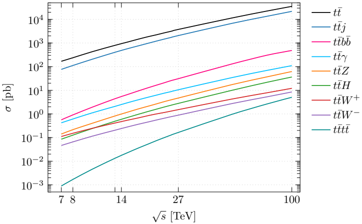
- Hartman, Kruthoff, Shaghoulian, Tajdini (2019) showed that 2-pt correlators obtained from Z_{T^2} matched bulk correlation functions at finite radius in EAdS.

- We assume the existence of a Z_{CFT} in dS, then derive the T^2-flow equation for functional derivatives of $\log Z_{T^2}$ w.r.t. sources.

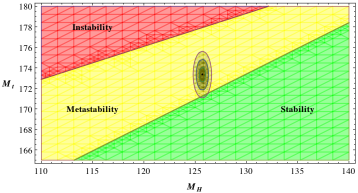
Progress towards two-loop QCD corrections to pp → ttj

Colomba Brancaccio

In collaboration with: Simon Badger, Matteo Becchetti, Heribertus Bayu Hartanto, Simone Zoia



Snowmass report, '21



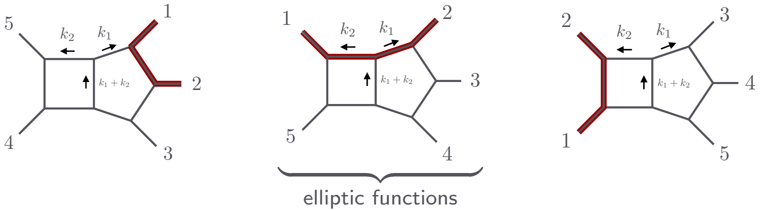
Branchina, Messina, Platania, '14

Cooking up the amplitude: our recipe for the computation

- **Helicity amplitudes**: they encode spin correlations in the narrow width approximation
- Limit of large color number: **leading color approximation**
- The amplitude can be written in terms of master integrals:

$$A = \sum_j C_{ij} I_j$$

- **Master integrals are now available**:



Badger, Becchetti, Giraudo, Zoia, '24

- Ongoing effort: numerical reconstruction of the rational coefficients using **finite field** strategy

Peraro, FiniteFlow, '19

Non-planar integrated correlator in $\mathcal{N} = 4$ SYM

arXiv: 2404.18900

Shun-Qing Zhang (MPP Munich)

Amplitudes 2024 (IAS Princeton)

MAX-PLANCK-INSTITUT FÜR PHYSIK

universe+

European Research Council
Established by the European Commission

Motivation

- 4-point function

dynamic part

$$\langle \mathcal{O}_2\mathcal{O}_2\mathcal{O}_2\mathcal{O}_2\rangle = \text{free} + I_4(x_i, Y_i)\, \boxed{T_N(U,V;g_{\text{YM}})}$$

Pert. Integrands known:
L=10 (planar), L=4 (non-planar)

- *Integrate* over *U,V* [Binder, Chester, Pufu, Wang].

$$\boxed{\mathcal{C}_{SU(N)}} = \int dM_{U,V}\, T_N(U,V;g_{\text{YM}})$$

Exact

At small g_{YM} [Dorigoni, Green, Wen]

non − planar

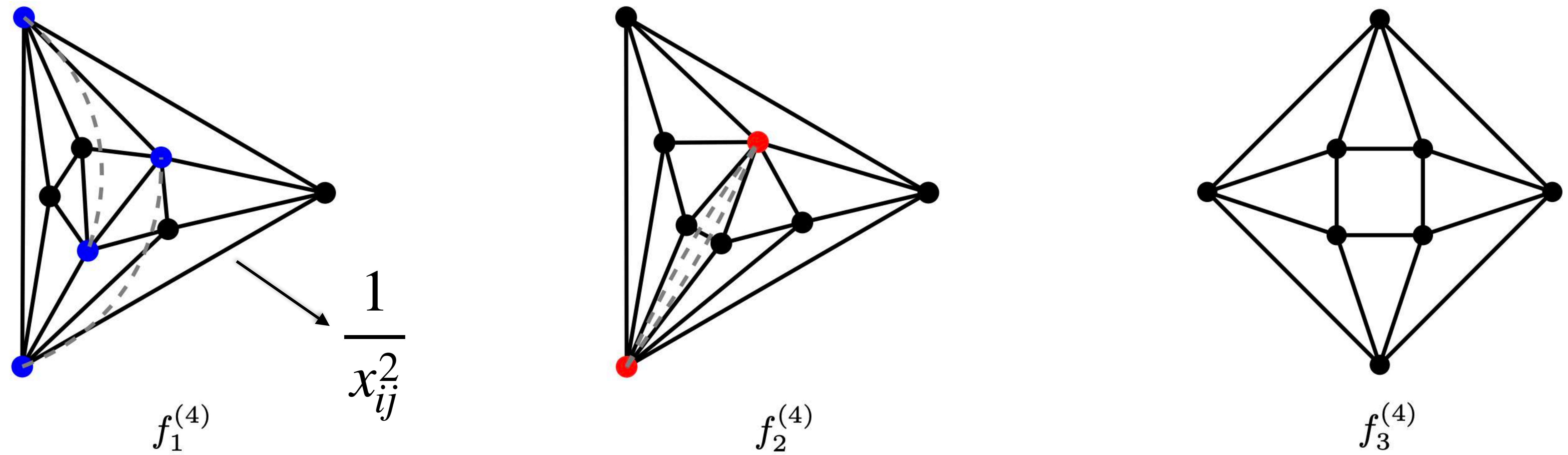
- $$\mathcal{C}^{\text{pert}}_{SU(N)} = 4c\Big[\frac{3\zeta(3)a}{2} - \frac{75\zeta(5)a^2}{8} + \frac{735\,\zeta(7)a^3}{16} - \frac{6615\,\zeta(9)(1+\boxed{\frac{2}{7N^2}})a^4}{32} + \mathcal{O}(a^5)\Big]$$

$$\left(c = \frac{N^2-1}{4}, \quad a = \frac{\lambda}{4\pi^2}\right)$$

$\mathscr{C}^{\text{pert}}_{SU(N)}$: periods of f-graphs, $\mathscr{P}_{f^{(L)}}$ [Wen, SQZ]

[Bourjaily, Eden, Heslop, Korchemsky, Sokatchev, Tran …]

L=4 (planar): 3 topologies



$$\frac{-1}{4!\,(-4)^4} \times \left(\mathscr{P}_{f_1^{(4)}} + \mathscr{P}_{f_2^{(4)}} - \mathscr{P}_{f_3^{(4)}}\right) = \frac{-6615\zeta(9)}{32}.$$

$\frac{8!}{8} \times 252\zeta_9$ $\quad$ $\frac{8!}{24} \times 252\zeta_9$ $\quad$ $\frac{8!}{16} \times 168\zeta_9$

Non-planar sector at $L = 4$

- **Non-planar data** [Fleury, Pereira] and **Gram det.**

$$\frac{-1}{4!\,(-4)^4} \times \frac{1}{N^2} \times \sum_{\alpha=1}^{32} \boxed{c^{(4)}_{1;\alpha}} \mathscr{P}_{f^{(4)}_\alpha} = -\frac{2}{7N^2} \times \frac{6615\zeta(9)}{32} .$$

- **MZV's cancel**

$$\mathscr{P}_{f^{(4)}_4} = \frac{8!}{16} \times \left(\frac{432}{5}\zeta(5,3) + 252\zeta(5)\zeta(3) - \frac{58\pi^8}{2625} \right) ,$$

$$\mathscr{P}_{f^{(4)}_{12}} = \frac{8!}{4} \times \left(\frac{432}{5}\zeta(5,3) - 36\zeta(3)^2 + 360\zeta(5)\zeta(3) + \frac{189\zeta(7)}{2} - \frac{131\zeta(9)}{2} - \frac{58\pi^8}{2625} \right) .$$

Outlook

- **2nd correlator** (*different* measure), which also displays a **simple** pattern of **ζ's**

$$\mathscr{C}^{\text{2nd}}_{SU(N)} = 4c\left[-60\zeta_5\, a + \frac{3\,(36\zeta_3^2 + 175\zeta_7)\, a^2}{2} - \frac{45\,(20\zeta_3\zeta_5 + 49\zeta_9)\, a^3}{2} + \mathcal{O}(a^4)\right].$$

- **Generic weights,** $\langle \mathcal{O}_{p_1}\mathcal{O}_{p_2}\mathcal{O}_{p_3}\mathcal{O}_{p_4}\rangle$ [Brown, Heslop, Wen, Xie] utilising **10D symmetry** [Caron-Huot, Coronado]

- **Wilson line** [Pufu, Rodriguez, Wang; Billo, Frau, Lerda], **determinant operators** $\mathscr{D}$ [Jiang, Wu, Zhang; Brown,Galvagno, Wen].

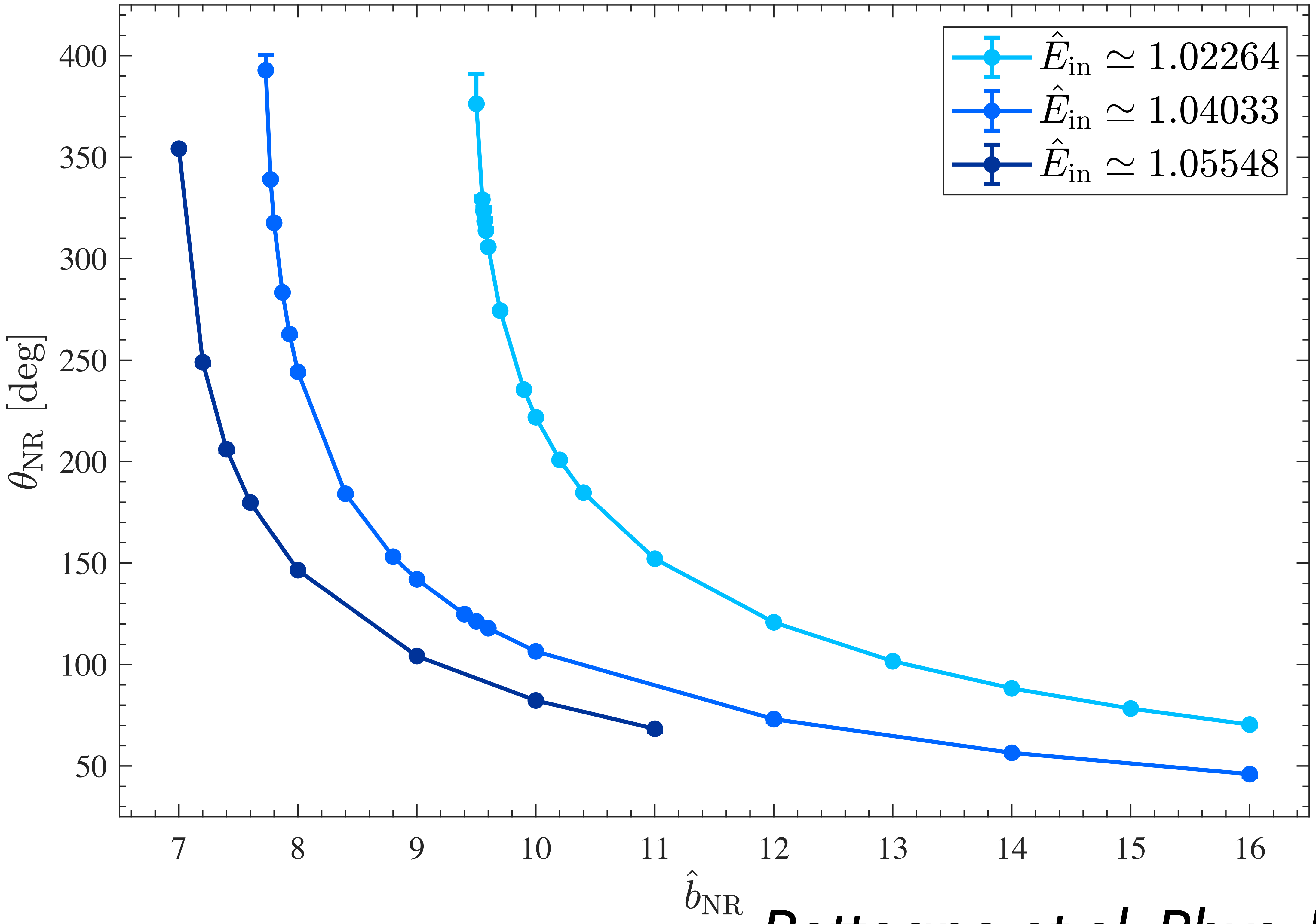
THE ROYAL SOCIETY
Piero Rettegno, Geraint Pratten, Lucy M. Thomas,
Patricia Schmidt, Thibault Damour
UNIVERSITY OF BIRMINGHAM
GRAVITATIONAL WAVE ASTRONOMY

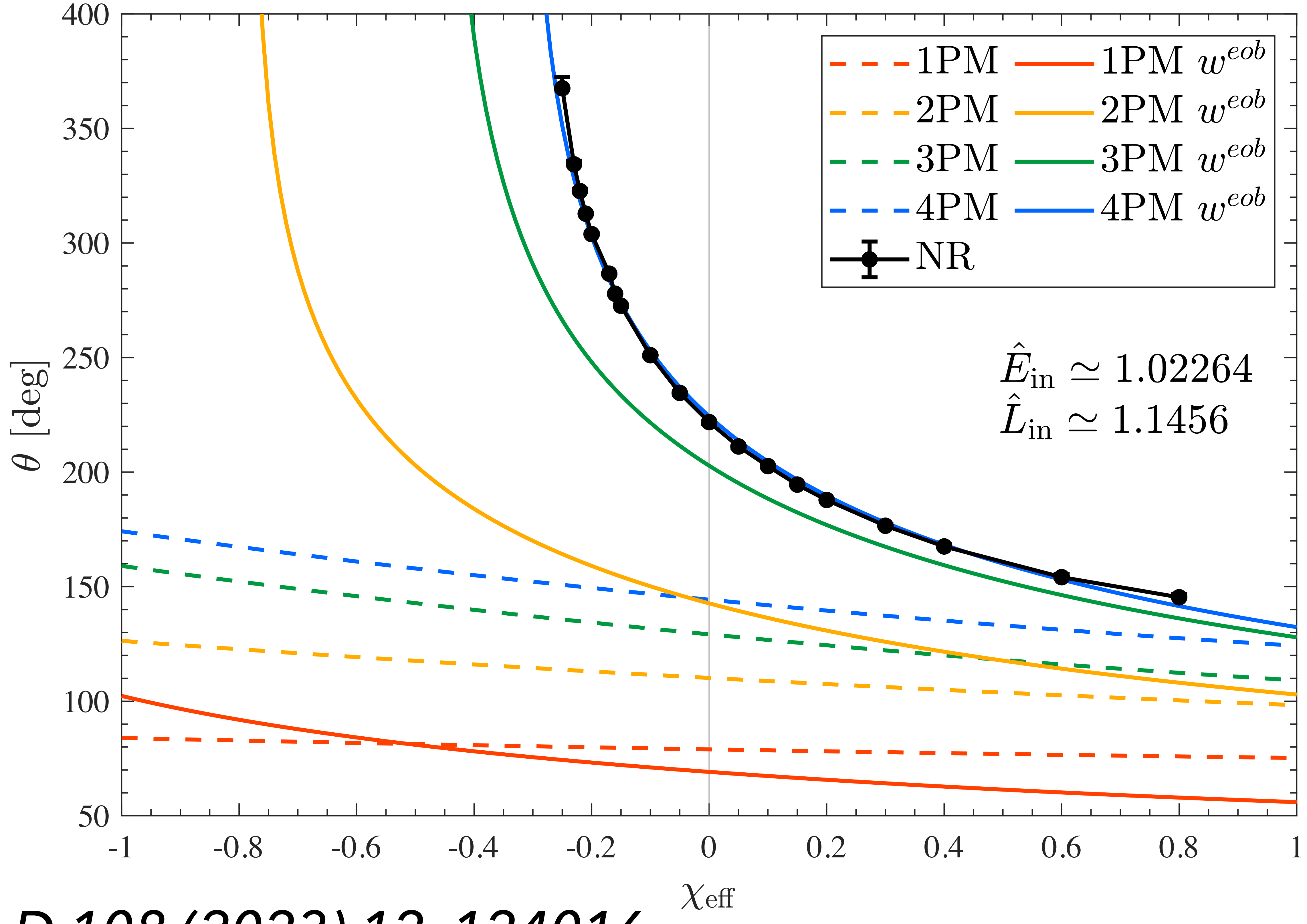
STRONG–FIELD SCATTERING OF BLACK HOLES: NR VS. PM

- Equal-mass scattering simulations:
  - Nonspinning - higher energies
  - Equal spins
  - Unequal spins

EOB mass-shell condition: $p_{\bar{r}}^2 + \frac{\ell^2}{\bar{r}^2} = p_\infty^2 + w^{eob}(\bar{r};\gamma)$

$$w_{n\mathrm{PM}}\left(\bar{r},\gamma,\ell,S\right) = w_{\mathrm{orb}}\left(\bar{r},\gamma\right) + \sum_{k=1,3} w_{\mathrm{k}}\left(\bar{r},\gamma\right)\frac{\ell\, S^{\,k}}{\bar{r}^{\,k+1}} + \sum_{k=2,4} w_{\mathrm{k}}\left(\bar{r},\gamma\right)\frac{S^{\,k}}{\bar{r}^{\,k}}$$





Rettegno et al. Phys. Rev. D 108 (2023) 12, 124016

Purity and the breakdown of perturbative unitarity in cosmology

Ciaran McCulloch

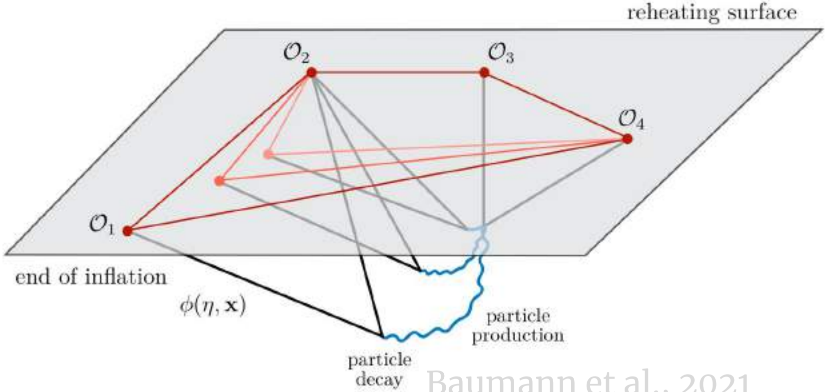
Work with C. Duaso Pueyo, H. Goodhew, and E. Pajer

UNIVERSITY OF CAMBRIDGE

Background

- Inflation: the early universe underwent rapid, quasi-de Sitter expansion
- Adiabatic density perturbations: massless scalar field with perturbative interactions
- Can we sharply diagnose when perturbation theory breaks down?

What couplings + kinematics?



Baumann et al., 2021

A new indicator

- Scalar wavefunction:

$$\Psi[\varphi] = \exp\left(\frac{1}{2}\int_{\mathbf{k}} \psi_2(\mathbf{k})\varphi(\mathbf{k})\varphi(-\mathbf{k}) + \frac{1}{3!}\int_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3} \delta\left(\sum_i \mathbf{k}_i\right)\psi_3(\{\mathbf{k}_i\})\varphi(\mathbf{k}_1)\varphi(\mathbf{k}_2)\varphi(\mathbf{k}_3) + \dots\right)$$

- Density matrix: $\rho = |\Psi\rangle\langle\Psi|$

- Reduced density matrix: trace out all Fourier modes but one

$$\rho_{\mathbf{p}} := \mathrm{Tr}_{\mathbf{k}\neq\mathbf{p}}\,\rho$$

- The purity is bounded! $0 \leq \mathrm{Tr}\left(\rho_{\mathbf{p}}^2\right) \leq 1$

$$\boxed{\mathrm{Tr}\left(\rho_{\mathbf{p}}^2\right) = 1 - \frac{1}{2}\int_{\mathbf{k}} \frac{\left(\mathrm{Re}\,\psi_3(\mathbf{p},\mathbf{k},-\mathbf{p}-\mathbf{k})\right)^2}{\mathrm{Re}\,\psi_2(\mathbf{p})\,\mathrm{Re}\,\psi_2(\mathbf{k})\,\mathrm{Re}\,\psi_2(-\mathbf{p}-\mathbf{k})}}$$

What do we learn?

- Flat space: mostly familiar results, similar to partial wave unitarity
- De Sitter: local-type non-Gaussianity (standard signal)

$$\psi_3(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \propto k_1^3 + k_2^3 + k_3^3$$

$$L^{-1} < \frac{k_i}{k_j} < L$$

Finite purity requires cutoff in *ratio* of momenta, not overall size

- De Sitter: a family of interactions

$$\mathcal{L}_{\rm int} = \frac{g_n}{2\Lambda^{2+2n}} \dot{\varphi}((\partial_i)^n \dot{\varphi})^2$$

$$|g_n| \left(\frac{H}{\Lambda}\right)^{2n+2} \lesssim \frac{C^{2n+2}}{\Gamma(2n+3)}$$

No cutoff this time, but order 1 Wilson coefficients are too large!

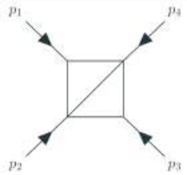
Spectral Analysis of the Feynman Integral:

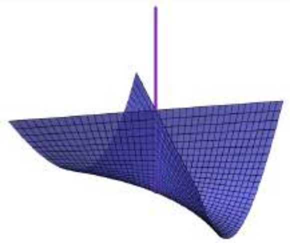
Stratifying Wave Fronts and Landau Singularities

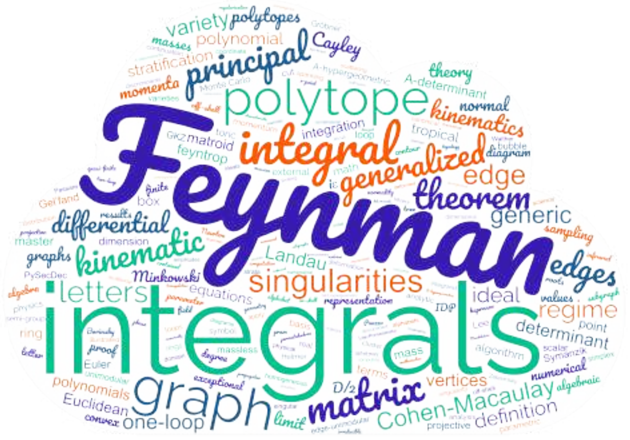
Felix Tellander (University of Oxford)

June 12, 2024

UNIVERSITY OF OXFORD



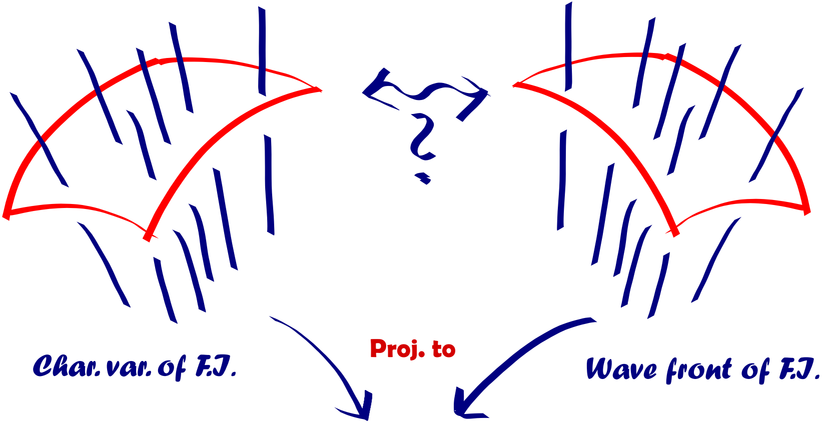




- 350 Feynman
- 337 Integrals
- 146 Graph
- 138 Polytope
- 134 Integral
- 114 Matrix
- 103 Landau

(Theoretical) Physics + Analysis + Algebra

Where and why is a Feynman integral (F.I.) singular?



Landau variety

Landau singularity

Euler discriminant

(Principal A-determinant)

String loops and gravitational positivity bounds

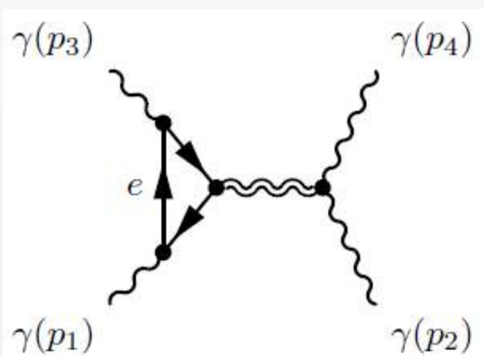
Junsei Tokuda (IBS, Korea) [arXiv: 2406.xxxxx S.Caron-Huot, **JT**]

- Positivity bound: $c_2 > 0.$ [Pham+ ('85), Adams+ ('06)]

$$\mathcal{M}(s,t) \sim (\text{poles}) + \lambda + c_2\left(s^2 + t^2 + u^2\right) + \cdots.$$

- If this were valid with gravity, one obtains interesting bounds!

e.g.) QED + Gravity (D=4) [Cheung-Remmen ('14)] + works by other authors



$$c_2 \supset \frac{-11}{90\pi^2}\frac{e^2}{M_{\rm pl}^2 m_e^2} \Rightarrow$$

WGC-like bound!

$$\frac{e}{m_e} \gtrsim \frac{1}{M_{\rm pl}}$$

- Unitarity-based robust bound: $c_2 > \dfrac{-\mathcal{O}(1)}{M_{\rm pl}^2 m_e^2}$ (e.g. [Caron-Huot+ ('21)])
- Quantum gravity really admits this m_e^{-1}-enhanced negativity??

String loops and gravitational positivity bounds

Junsei Tokuda (IBS, Korea) [arXiv: 2406.xxxxx S.Caron-Huot, **JT**]

- We consider the case where **the grav. exchange is Reggeized:**

$$\mathcal{M}(s,t) \propto f(t) s^{2+\alpha' t+\alpha'' t^2+\cdots}$$

- Finite-energy sum rule: $c_2 \sim (\text{positive term}) + \frac{1}{M_{\rm pl}^2}\underline{[-f'/f + \alpha''/\alpha']_{t=0}}$.
  Source of negativity

- **We identify high-energy (stringy) loop processes** which give m_e^{-1} -enhanced contributions to $f(t)$ and $\alpha(t)$.

- **We evaluate them precisely,** and find that **they exactly cancel** the m_e^{-1} -enhanced term predicted by EFT.

  - Key: Only the scattering at large impact parameters $b \sim m_e^{-1} \gg M_{\rm string}^{-1}$ matters. -> ***t*-channel factorization**!!!

Classical observables using Newman-Janis deformation in electromagnetic scattering

Samim Akhtar

The Institute of Mathematical Sciences

Based on JHEP 05 (2024) 148 [arXiv:2401.15574] with A. Manna and A. Manu

Amplitudes 2024 Gong Show

- The Newman-Janis shift is a complex coordinate transformation ($z \rightarrow z + ia$) which generates rotating solutions from static ones, notably the Kerr solution from Schwarzschild solution. This transformation has an EM counterpart where the Coulomb solution maps to the so-called $\sqrt{\text{Kerr}}$ solution, a rotating charged object and is a solution to the Maxwell's equations.
- The origin of the shift is identified as arising from the exponentiation of spin operators for the minimally coupled three-particle amplitudes of spinning particles coupled to gravity, in the large-spin limit. *Arkani−Hamed, Huang, O′Connell"19*

$$\mathcal{M}_3^{s,\pm} \xrightarrow{s\to\infty,\hbar\to 0} e^{\mp q\cdot a}\mathcal{M}_3^{0,\pm} \,, \tag{1}$$

where $a = s/m$ parametrizes the spin.

- The goal is to investigate the applicability of the NJ shift in deriving various classical observables (both conservative and radiative) for Kerr Black Hole scattering. As a simpler setup, we considered the analogous problem in $\sqrt{\text{Kerr}}$ scattering mediated by EM interaction.

NJ Deformation

- The linear impulse, Δp_1^μ for the scalar particle in the background of a $\sqrt{\text{Kerr}}$ particle can be obtained from a scalar-scalar scattering by complexifying the impact parameter ($b \to b + ia_2$). $^{Arkani-Hamed, Huang, O'Connell"19}$
- We employ the KMOC formalism and apply the shift to the scalar-scalar scattering through a specific deformation of the polarization vectors of the exchange photon.

$$\mathcal{A}_{4,\text{scalar}-\sqrt{\text{Kerr}}}[p_1, p_2 \to p_1', p_2'] = \frac{1}{q^2}\mathcal{A}^\mu_{3,\text{scalar}}[p_2', p_2, q]\widetilde{\mathcal{P}}_{\mu\nu}\ \mathcal{A}^\nu_{3,\text{scalar}}[p_1', p_1, -q]\,, \quad (2)$$

where the deformed photon projector $\widetilde{\mathcal{P}}_{\mu\nu}$ is given by

$$\begin{aligned}\widetilde{\mathcal{P}}^{\mu\nu}(q) &:= e^{q\cdot a_2}\epsilon_+^\mu(q)\epsilon_-^\nu(q) + e^{-q\cdot a_2}\epsilon_+^\nu(q)\epsilon_-^\mu(q)\\ &= \cosh(a_2\cdot q)\eta^{\mu\nu} + \sinh(a_2\cdot q)\Pi^{\mu\nu}(q)\,.\end{aligned} \quad (3)$$

We used an ansatz for the antisymmetric part of the projector, $\Pi^{\mu\nu}$ that can be used in the computation of all the physical observables.

Results and Conclusions

- We used the NJ deformation technique in computing the following observables for scalar-$\sqrt{\text{Kerr}}$ scattering at LO in coupling, and to all order in spin:
  - The radiation kernel, $\mathcal{R}_1^\mu(\bar{k})$ for the scalar particle,
  - The spin kick, Δa_2^μ imparted on the $\sqrt{\text{Kerr}}$ particle.
  - The orbital angular impulse, $\Delta L_i^{\mu\nu}$ of the scalar and the $\sqrt{\text{Kerr}}$ particle.

  These match with the results obtained through solving classical EOMs perturbatively.
- We found that the result for the angular impulse of the $\sqrt{\text{Kerr}}$ particle is consistent with the total angular momentum conservation if we take the spin tensor $S^{\mu\nu}$ as the fundamental spin d.o.f instead of a^μ which are related via the duality relation, $a^\mu = \frac{1}{2m^2}\epsilon^{\mu\nu\rho\sigma}p_\nu S_{\rho\sigma}$.
- We would like to compute the observables to higher orders in coupling (building "loop integrands") for the $\sqrt{\text{Kerr}}$ as well as BH scattering using the on-shell methods along with the NJ deformation.

Thanks! Come see my poster next week for more!

Reverse Unitarity by Quantum Perturbiner Method

Hojin Lee(Seoul National University)
with Kanghoon Lee(APCTP)

Quantum Perturbiner Method is a systematic way of computing loop integrands at arbitrary loop level for N-point amplitudes using *off-shell recursion relation*.

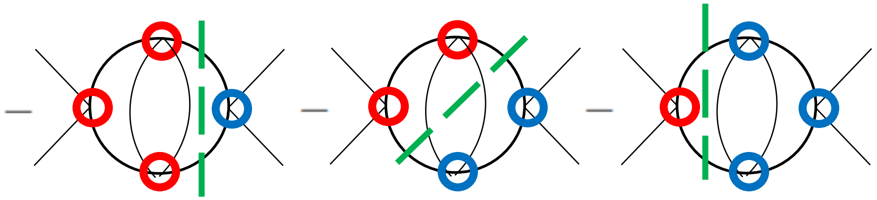
Efficient for computing loop integrands using computer.

Cross sections from optical theorem: *Reverse Unitarity*.

$$\mathrm{Im}\mathcal{M}\,(A \to A) = 2E_{\mathrm{CM}}|\vec{p}_{\mathrm{CM}}| \sum_X \sigma\,(A \to X)$$

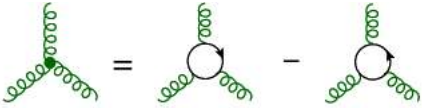
Able to derive l.h.s using *Largest Time Equation* and double field prescription without drawing Feynman diagrams.

$2\mathrm{Im}\mathcal{M} = -$



$$S[\phi^A, j^A] = -\frac{1}{2}\int_{x,y} \phi^A_x K^{AB}_{xy} \phi^B_y - \frac{\lambda^A}{4!}\int_x V^{ABCDE} \phi^B_x \phi^C_x \phi^D_x \phi^E_x + \int_x \eta^{AB} j^A_x \phi^B_x$$
$$= S[\phi^+, j^+] - S^*[\phi^-, j^-]$$

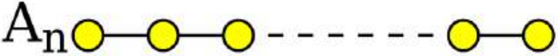
$$\begin{pmatrix} 2 & -1 & 0 & \dots & 0 \\ -1 & 2 & -1 & \dots & 0 \\ 0 & -1 & 2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 2 \end{pmatrix}$$



A Ridiculous Formula for the Number of SU(N) Gluonic Color Factors

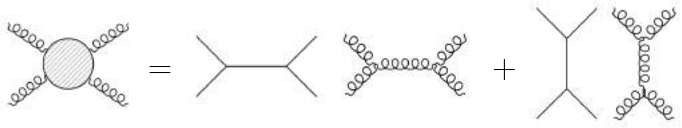
Presented by Michael Plesser

From 24XX.XXXX with J. Bourjaily and C. Vergu



Color Factors in Gauge Theory

In gauge theory we can organize amplitudes into color factors $\mathcal{C}_i$ and partial amplitudes A_i



The $\mathcal{C}_i$ satisfy (group-specific) identities, EG

SU(2): $\mathrm{Tr}[123] + \mathrm{Tr}[132] = 0 \leftrightarrow d^{abc} = 0$

SU(3): $\mathrm{Tr}[123] + \mathrm{Tr}[132] \neq 0 \leftrightarrow d^{abc} \neq 0$

A natural question:

How many <u>independent</u> n-pt color factors are there for a given group?

Counting with Schur's Lemma

The Tl;dr

The multiplicity of $\mathbf{1}$ in $\mathbf{Ad}^{\otimes n}$ gives the number of color factors for n-external gluons, $|\mathcal{C}_n|$, to all loop orders

SU(N) n-pts:

$$|\mathcal{C}_n| = \langle \mathbf{1} | \mathbf{Ad}^{\otimes n} \rangle$$

A bound is given by the *Subfactorial* or *Derangement* numbers

$$|\mathcal{C}_n| \leq \ !n$$

This bound is <u>saturated</u> when $n \leq N$

$$\boxed{|\mathcal{C}_{n\leq N}| = \ !n \ = \left[\frac{n!}{e} \right]}$$

(With $[x]$ meaning 'x rounded to the nearest integer')

Embracing the Derangement!

Derangements are permutations with no 1-cycles
Wrapping each cycle with ‘Tr’ gives the Deranged trace basis

EG $n = 4$:

$$\left[\frac{4!}{e}\right] = [8.829...] \implies |\mathcal{C}_4| \leq 9$$

Tr[1234]	Tr[1342]	Tr[1423]
Tr[1243]	Tr[1324]	Tr[1432]
Tr[12]Tr[34]	Tr[13]Tr[24]	Tr[14]Tr[23]

These satisfy 6 SU(2) identities,
1 SU(3) identity, and
0 SU($N \geq 4$) identities

More to the Story!

- Tree-level "irrep basis" is smaller than DDM
- Cayley-Hamilton Identities
  (Exponential growth for large n, not factorial!)
- Other gauge groups
  (E_8 is "simpler" than SU(3)??? D_n does what???)
- ST duality and graphical reduction
  (Biedenharn-Elliot identity, hexagon relation, etc)
- Motzkin and Riordan numbers for SU(2)
  (Racah W's and Wigner 6j's)
- Adding fundamentals and higher reps, its easy!

If any of this sounds cool, let's talk
(or email mkp5771@psu.edu)

Thanks!

EFT diagrammatic approach to post-Newtonian theory beyond General Relativity

Nicola Bartolo, Pierpaolo Mastrolia, Matteo Pegorin, Angelo Ricciardone

Università degli Studi di Padova

INFN Padova

Funded by the European Union NextGenerationEU

- Accurate gravitational waveform are fundamental to detect and analyze gravitational wave events.
- While the accuracy of General Relativity waveforms is approaching an adequate level, currently the **accuracy of gravitational waveform for beyond GR theories is lacking behind**.
- **Improving their precision is of utmost importance** to fully leverage the capabilities of next-generation gravitational wave observatories and is a fundamental goal of these collaborations.
  [LISA Consortium Waveform WG (2023)]
- The recent progress in gravity calculations made possible by **EFT and scattering amplitudes approaches may be key to close this gap**.

EFT diagrammatic approach to post-Newtonian theory beyond General Relativity

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- Many beyond GR theories introduce higher order operators or additional fields to the Einstein-Hilbert action, for example scalar-tensor theories and generalizations thereof. [Brans, Dicke (1961); Damour, Esposito-Farese (1992); Gleyzes, Langlois, Piazza, Vernizzi (2015); ...]

Example: scalar-tensor theory

Full theory (Beyond GR action + compact objects couplings + ...)

$$S_{tot}[\{x_a^\mu\}, g_{\mu\nu}, \varphi] = -2\Lambda^2 \int \mathrm{d}^4x \sqrt{-g} \left(R - \frac{1}{2}\Gamma^\mu \Gamma_\mu - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \ldots \right) - \sum_{a=1}^{2} m_a^0 \int \mathrm{d}\sigma_a \sqrt{g_{\mu\nu}(x_a)\frac{\mathrm{d}x_a^\mu}{\mathrm{d}\sigma}\frac{\mathrm{d}x_a^\nu}{\mathrm{d}\sigma}} \left(1 + \alpha_a^0 \varphi + \ldots\right) + \ldots$$

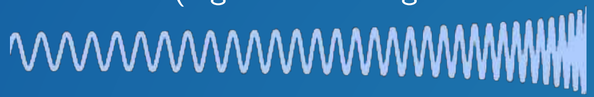
Integrate out field d.o.f.

$$e^{iS_{eff}[\{x_a^\mu\}]} = \int Dg_{\mu\nu} D\varphi \, e^{iS_{tot}[\{x_a^\mu\}, g_{\mu\nu}, \varphi]}$$

Effective action (post-Newtonian corrections)

$$S_{eff}[\{x_a^\mu\}] = \int \mathrm{d}t\, L = \int \mathrm{d}t \left(\frac{1}{2} m_1^0 v_1^2 + \frac{1}{2} m_2^0 v_2^2 + G\frac{m_1^0 m_2^0}{r}(1 + \alpha_1^0\alpha_2^0) + \ldots \right)$$

Physical observables (e.g. scalar and gravitational waveform)



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- Many beyond GR theories introduce higher order operators or additional fields to the Einstein-Hilbert action, for example scalar-tensor theories and generalizations thereof. [Brans, Dicke (1961); Damour, Esposito-Farese (1992); Gleyzes, Langlois, Piazza, Vernizzi (2015); ...]

- These new interaction terms result in new vertices and diagrams, which yield the corrections to the observables due to the **modified theory of gravity** $\Longrightarrow$ **post-Newtonian corrections can be evaluated within NRGR/EFT diagrammatic approach!** [Goldberger, Rothstein (2004); Kuntz, Piazza, Vernizzi (2019); Bernard, Dones, Mougiakakos (2024); ...]

Example: scalar-tensor theory

Full theory (Beyond GR action + compact objects couplings + ...)

New interaction terms

$$S_{tot}[\{x_a^\mu\}, g_{\mu\nu}, \varphi] = -2\Lambda^2 \int d^4x\, \sqrt{-g}\left(R - \frac{1}{2}\Gamma^\mu\Gamma_\mu - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \dots\right) - \sum_{a=1}^{2} m_a^0 \int d\sigma_a \sqrt{g_{\mu\nu}(x_a)\frac{dx_a^\mu}{d\sigma}\frac{dx_a^\nu}{d\sigma}}\,(1+\alpha_a^0\varphi+\dots) + \dots$$

Integrate out field d.o.f.

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Effective action (post-Newtonian corrections)

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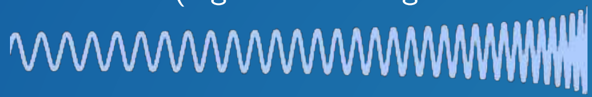
Physical observables (e.g. scalar and gravitational waveform)



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  $\implies$ **post-Newtonian corrections can be evaluated within NRGR/EFT diagrammatic approach!**

  [Goldberger, Rothstein (2004); Kuntz, Piazza, Vernizzi (2019); Bernard, Dones, Mougiakakos (2024); ...]

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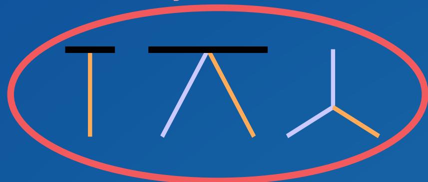
New Feynman rules



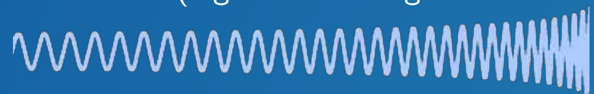
Integrate out field d.o.f.

$$e^{iS_{eff}[\{x_a^\mu\}]} = \int Dg_{\mu\nu}\,D\varphi\; e^{iS_{tot}[\{x_a^\mu\}, g_{\mu\nu}, \varphi]}$$

Effective action (post-Newtonian corrections)

$$S_{eff}[\{x_a^\mu\}] = \int \mathrm{d}t\, L = \int \mathrm{d}t \left(\frac{1}{2}m_1^0 v_1^2 + \frac{1}{2}m_2^0 v_2^2 + G\frac{m_1^0 m_2^0}{r}(1+\alpha_1^0\alpha_2^0) + \dots\right)$$

Physical observables (e.g. scalar and gravitational waveform)



EFT diagrammatic approach to post-Newtonian theory beyond General Relativity

Nicola Bartolo, Pierpaolo Mastrolia, Matteo Pegorin, Angelo Ricciardone

Università degli Studi di Padova
INFN Padova

Funded by the European Union NextGenerationEU

- Many beyond GR theories introduce higher order operators or additional fields to the Einstein-Hilbert action, for example scalar-tensor theories and generalizations thereof. [Brans, Dicke (1961); Damour, Esposito-Farese (1992); Gleyzes, Langlois, Piazza, Vernizzi (2015); ...]

- These new interaction terms result in new vertices and diagrams, which yield the corrections to the observables due to the **modified theory of gravity** $\implies$ **post-Newtonian corrections can be evaluated within NRGR/EFT diagrammatic approach!**

  [Goldberger, Rothstein (2004); Kuntz, Piazza, Vernizzi (2019); Bernard, Dones, Mougiakakos (2024); ...]

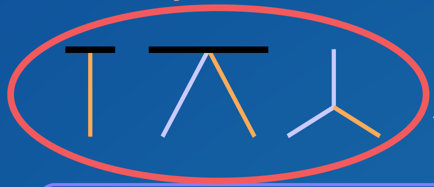
Example: scalar-tensor theory

Full theory (Beyond GR action + compact objects couplings + ...)

New interaction terms

$$S_{tot}[\{x_a^\mu\}, g_{\mu\nu}, \varphi] = -2\Lambda^2 \int d^4x \sqrt{-g}\left(R - \frac{1}{2}\Gamma^\mu\Gamma_\mu - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \dots\right)$$
$$- \sum_{a=1}^{2} m_a^0 \int d\sigma_a \sqrt{g_{\mu\nu}(x_a)\frac{dx_a^\mu}{d\sigma}\frac{dx_a^\nu}{d\sigma}}\,(1+\alpha_a^0\varphi+\dots) + \dots$$

New Feynman rules



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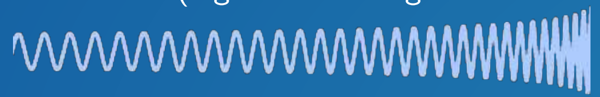
New diagrams

Effective action (post-Newtonian corrections)

$$S_{eff}[\{x_a^\mu\}] = \int dt\, L = \int dt\left(\frac{1}{2} m_1^0 v_1^2 + \frac{1}{2} m_2^0 v_2^2 + G\frac{m_1^0 m_2^0}{r}(1+\alpha_1^0\alpha_2^0) + \dots\right)$$

Physical observables (e.g. scalar and gravitational waveform)



EFT DIAGRAMMATIC APPROACH TO POST-NEWTONIAN THEORY BEYOND GENERAL RELATIVITY

Nicola Bartolo, Pierpaolo Mastrolia, Matteo Pegorin, Angelo Ricciardone

UNIVERSITÀ DEGLI STUDI DI PADOVA

INFN PADOVA

Funded by the European Union NextGenerationEU

- Many beyond GR theories introduce higher order operators or additional fields to the Einstein-Hilbert action, for example scalar-tensor theories and generalizations thereof. [Brans, Dicke (1961); Damour, Esposito-Farese (1992); Gleyzes, Langlois, Piazza, Vernizzi (2015); ...]

- These new interaction terms result in new vertices and diagrams, which yield the corrections to the observables due to the **modified theory of gravity** $\Longrightarrow$ **post-Newtonian corrections can be evaluated within NRGR/EFT diagrammatic approach!** [Goldberger, Rothstein (2004); Kuntz, Piazza, Vernizzi (2019); Bernard, Dones, Mougiakakos (2024); ...]

Example: scalar-tensor theory

Full theory (Beyond GR action + compact objects couplings + ...)

New interaction terms

$$S_{tot}[\{x_a^\mu\}, g_{\mu\nu}, \varphi] = -2\Lambda^2 \int \mathrm{d}^4x \sqrt{-g}\left(R - \frac{1}{2}\Gamma^\mu\Gamma_\mu - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \dots\right) - \sum_{a=1}^{2} m_a^0 \int \mathrm{d}\sigma_a \sqrt{g_{\mu\nu}(x_a)\frac{\mathrm{d}x_a^\mu}{\mathrm{d}\sigma}\frac{\mathrm{d}x_a^\nu}{\mathrm{d}\sigma}}\,(1+\alpha_a^0\varphi+\dots) + \dots$$

New Feynman rules

Integrate out field d.o.f.

$$e^{iS_{eff}[\{x_a^\mu\}]} = \int Dg_{\mu\nu}\, D\varphi\, e^{iS_{tot}[\{x_a^\mu\}, g_{\mu\nu}, \varphi]}$$

$=$  $+ \dots$

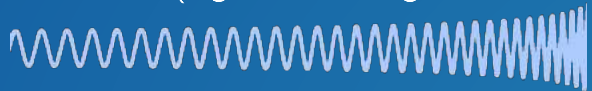
New diagrams

Effective action (post-Newtonian corrections)

$$S_{eff}[\{x_a^\mu\}] = \int \mathrm{d}t\, L = \int \mathrm{d}t \left(\frac{1}{2} m_1^0 v_1^2 + \frac{1}{2} m_2^0 v_2^2 + G\frac{m_1^0 m_2^0}{r}(1+\alpha_1^0\alpha_2^0) + \dots\right)$$

Beyond GR contributions

Physical observables (e.g. scalar and gravitational waveform)



EFT diagrammatic approach to post-Newtonian theory beyond General Relativity

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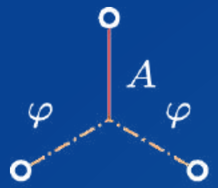


$$= -i\int \mathrm{d}t \left[\frac{G^2 m_1^0 m_2^0}{r^2}\Big(2(\alpha_0^1+\alpha_0^2)(m_1^0\alpha_0^1+m_2^0\alpha_0^2)(\mathbf{v}_1\cdot\mathbf{v}_2) + m_1^0\alpha_0^1\alpha_0^2(\mathbf{v}_1\cdot\hat{\mathbf{r}})^2 - \alpha_0^1\alpha_0^2\left(m_1^0 v_1^2 + m_2^0 v_2^2 - m_2^0(\mathbf{v}_2\cdot\hat{\mathbf{r}})^2\right) - 4(\alpha_0^1+\alpha_0^2)(m_1^0\alpha_0^1+m_2^0\alpha_0^2)(\mathbf{v}_1\cdot\hat{\mathbf{r}})(\mathbf{v}_2\cdot\hat{\mathbf{r}})\Big)\right] + \mathcal{O}(L\, v^6)$$

- Presently working to obtain **post-Newtonian** predictions in scalar-tensor theories.
- Developed an **in-house code** for automated evaluation.
- Results will be important for **phenomenological forecasts**.
- **Challenges**: many beyond GR theories present caveats that should be taken into account.

Thank you for your attention

Regge Amplitudes from Glauber SCET

Amplitudes 2024

Anjie Gao

2401.00931 with Ian Moult, Sanjay Raman, Gregory Ridgway, Iain Stewart

+ ongoing work

Massachusetts Institute of Technology

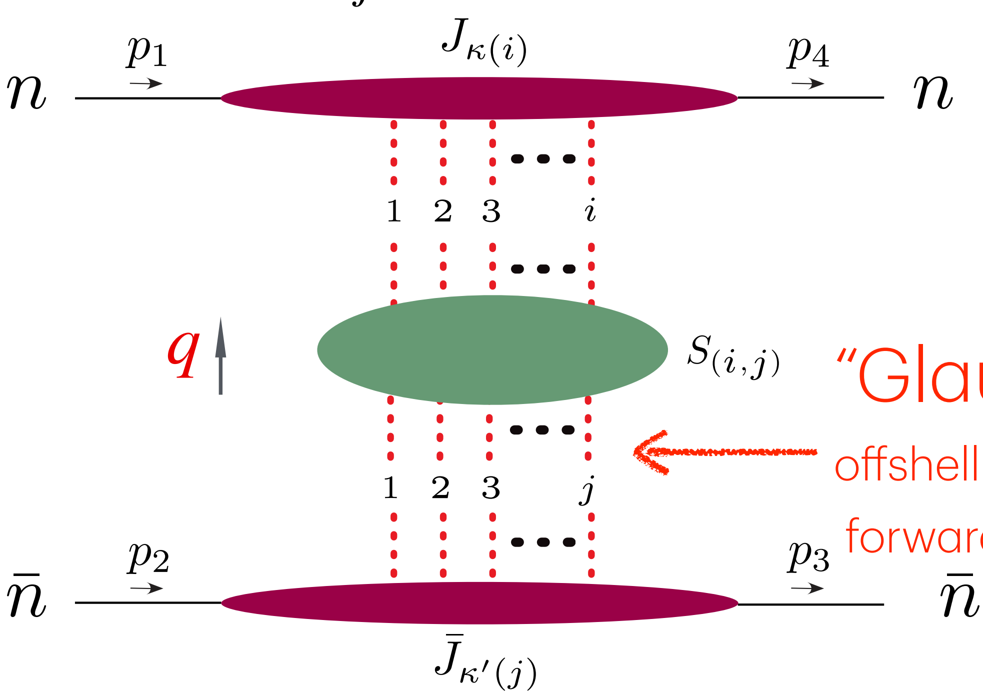
June 12, 2024

- Large $\log \frac{s}{-t}$ appears in the (QCD) amplitudes in the Regge limit $s \gg -t$
- Factorization of the Regge amplitudes from EFT (SCET) $\beta = a'_1 \dots a'_j$: color indices

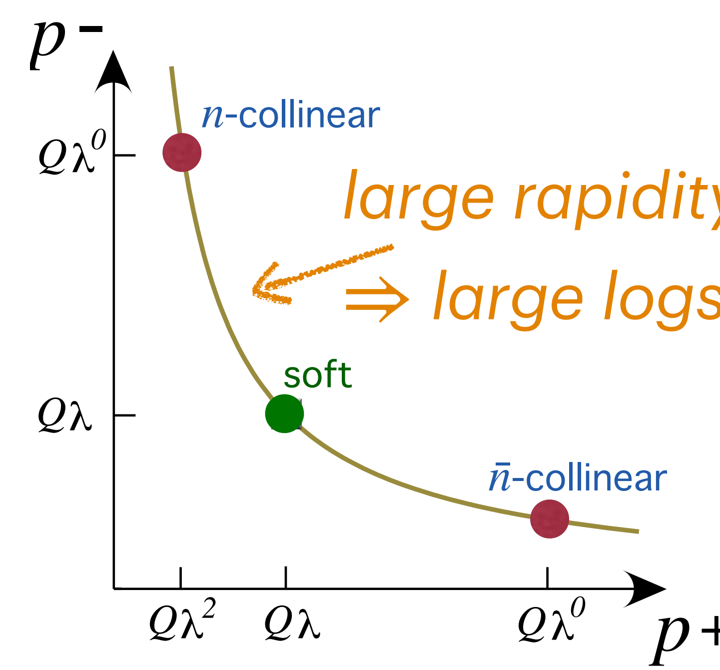
$$-i\mathcal{M} = \mathbf{J}_\kappa \cdot \mathbf{S} \cdot \bar{\mathbf{J}}_{\kappa'} = \sum_{ij} J^{\alpha}_{\kappa(i)}\big(\{\ell_{i\perp}\}\big) \otimes_\perp S^{\alpha\beta}_{(i,j)}\big(\{\ell_{i\perp}\};\{\ell'_{j\perp}\}\big) \otimes_\perp \bar{J}^{\beta}_{\kappa'(j)}\big(\{\ell'_{j\perp}\}\big)$$

$\kappa' = q, \bar{q}, g, gg, \dots$

$= \sum_{ij}$



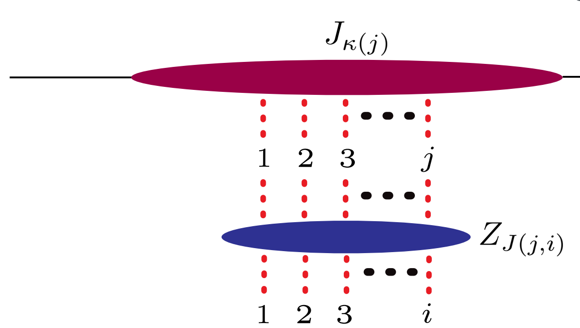
"Glauber gluons"
offshell mode mediates
forward scattering



large rapidity gaps
⇒ large logs

- Rapidity RG evolution
  - Use counterterms absorb rapidity divergences (RG consistency ⇒ $\mathbf{Z_S} = \mathbf{Z_J}^{-1}$)

$$\mathbf{J}_\kappa(\eta) = \mathbf{J}_\kappa(\nu) \cdot \mathbf{Z_J}(\eta,\nu)\,, \qquad \mathbf{S}(\eta) = \mathbf{Z_S}(\eta,\nu) \cdot \mathbf{S}(\nu) \cdot \mathbf{Z_S}(\eta,\nu)$$



rap. regulator

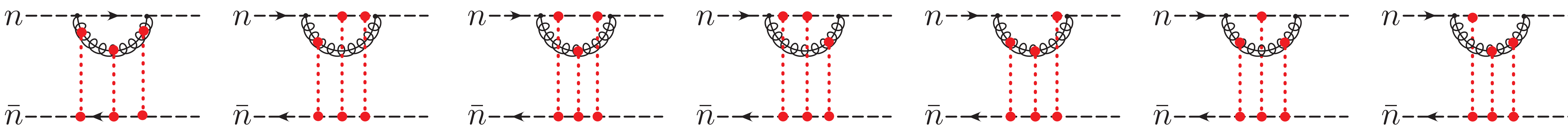
rap. ren. scale

  - Define anomalous dimension as $\mathbf{\Gamma} \equiv -(\nu\partial_\nu \mathbf{Z_J}) \cdot \mathbf{Z_J}^{-1} = \gamma_{(i,j)}$
  - Iteration of $\gamma_{(i,j)}$'s resums $\log \frac{s}{-t}$

Results highlight: $\gamma_{(2,3)}$ & $\gamma_{(3,3)}$ (relevant for amplitudes at NNLL)

Extract the rapidity divergence in the collinear loop

$$J^{[1]}_{(3)} \otimes_3 S^{[0]}_{(3,3)} \otimes_3 \bar{J}^{[0]}_{(3)} =$$

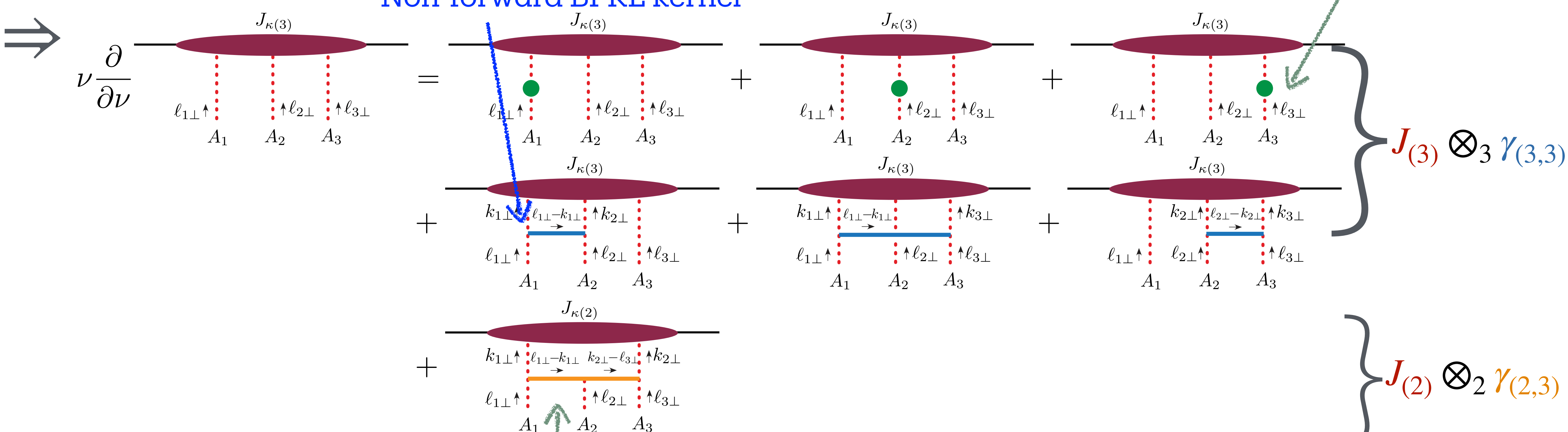


$$K_{\mathrm{NF}}(\ell_\perp, k_\perp) = \alpha_s N_c \left(-\frac{\vec{q}_\perp^{\,2}}{\vec{k}_\perp^2 (\vec{q}_\perp - \vec{k}_\perp)^2} + \frac{\vec{\ell}_\perp^{\,2}}{\vec{k}_\perp^2 (\vec{\ell}_\perp - \vec{k}_\perp)^2} + \frac{(\vec{q}_\perp - \vec{\ell}_\perp)^2}{(\vec{q}_\perp - \vec{k}_\perp)^2 (\vec{k}_\perp - \vec{\ell}_\perp)^2} \right)$$

Non-forward BFKL kernel

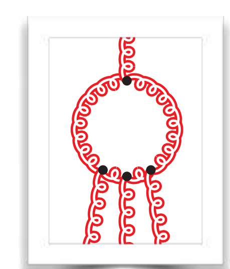
$$\omega_G(\ell_\perp) = -\alpha_s N_c \int \frac{d^{2-2\epsilon} k_\perp \, \vec{\ell}_\perp^{\,2}}{\vec{k}_\perp^2 (\vec{\ell}_\perp - \vec{k}_\perp)^2}$$

Regge trajectory



$$K_{\mathrm{TC}}(k_{i\perp}; \ell_{j\perp}) = \frac{(\ell_{1\perp} + \ell_{2\perp})^2}{k_{1\perp}^2 (\ell_{3\perp} - k_{2\perp})^2} + \frac{(\ell_{2\perp} + \ell_{3\perp})^2}{k_{2\perp}^2 (\ell_{1\perp} - k_{1\perp})^2} - \frac{\ell_{2\perp}^2}{(\ell_{1\perp} - k_{1\perp})^2 (\ell_{3\perp} - k_{2\perp})^2} - \frac{q_\perp^2}{k_{1\perp}^2 k_{2\perp}^2}$$

(The same convolution kernel appears in $1 \to 3$ transition in the Wilson line approach in e.g. [Falcioni et al, 2111.10664])

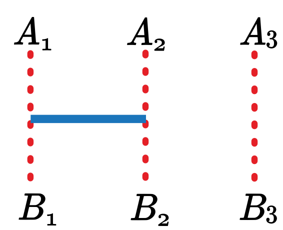


Results highlight: color evolution for $10 \oplus \overline{10}$ at NNLL

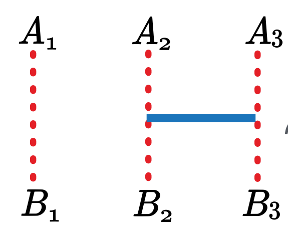
- Color channel $10 \oplus \overline{10}$ first appear in 3 Glauber exchange $\Rightarrow$ only need $\gamma_{(3,3)}$
- $8 \otimes 8 \otimes 8$ contains 4 copies of $10 \oplus \overline{10}$ (for N_c=3)

$\Rightarrow$ $\gamma_{(3,3)}$ *includes transition matrices within the 4d color space of* $10 \oplus \overline{10}$

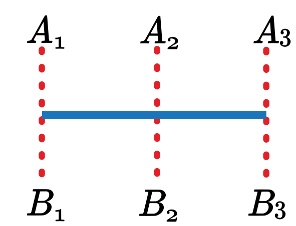
- Utilizing the orthogonal 6 gluon basis in [Sjodahl, Thorén 1507.03814], we compute



$\sim M_3^{10\oplus\overline{10}}$



$\sim M_1^{10\oplus\overline{10}}$



$\sim M_2^{10\oplus\overline{10}}$

- e.g. for $N_c = 3$, $M_1^{10\oplus\overline{10}} = \begin{bmatrix} 0 & -\frac{3\sqrt{2}}{2} & 0 & -3 \\ -\frac{3\sqrt{2}}{2} & 0 & -\sqrt{3} & 0 \\ 0 & -\sqrt{3} & \frac{5\sqrt{5}}{2} & \frac{\sqrt{3}}{2} \\ -3 & 0 & \frac{\sqrt{3}}{2} & \frac{3\sqrt{5}}{2} \end{bmatrix}$
- $N_c \to \infty$, $M_i^{10\oplus\overline{10}}$'s are orthogonal. Color space reduces to 3d, projected by $M_i^{10\oplus\overline{10}}$. We have a triple pole solution

$$\mathcal{M}^{10\oplus\overline{10}} \sim \int_\perp \left(\frac{s}{-t}\right)^{\omega_G(q_\perp-\ell_{1\perp})+\omega_G(\ell_{1\perp})} + \left(\frac{s}{-t}\right)^{\omega_G(q_\perp-\ell_{2\perp})+\omega_G(\ell_{2\perp})} + \left(\frac{s}{-t}\right)^{\omega_G(q_\perp-\ell_{3\perp})+\omega_G(\ell_{3\perp})}$$

Integrated negative geometries in ABJM

Martín Lagares

National University of La Plata and
Max Planck Institute for Physics

based on 2303.02996 and 2402.17432 with J.M. Henn and S.Q. Zhang

Why integrated negative geometries?

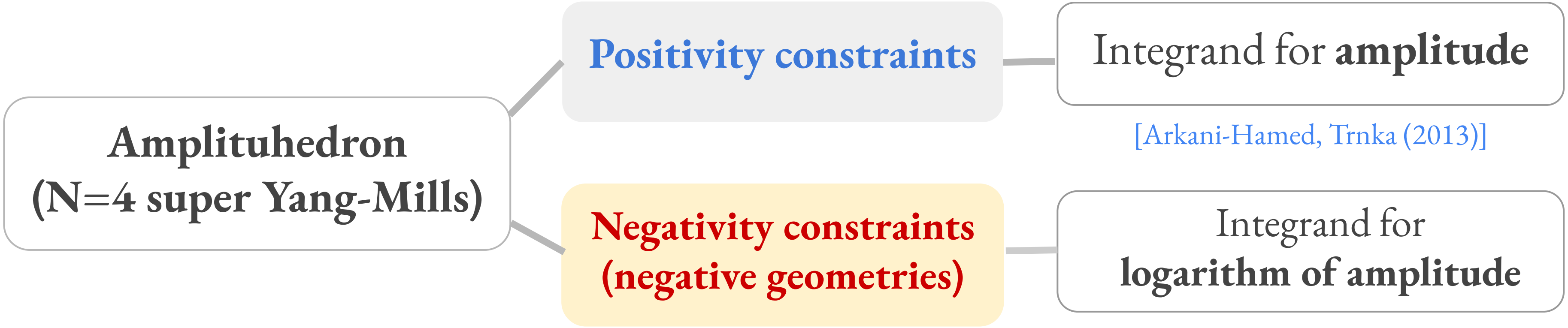
Amplituhedron (N=4 super Yang-Mills)

Positivity constraints

Integrand for **amplitude**

[Arkani-Hamed, Trnka (2013)]

Why integrated negative geometries?



Why integrated negative geometries?

Amplituhedron (N=4 super Yang-Mills)

Positivity constraints — Integrand for **amplitude**

[Arkani-Hamed, Trnka (2013)]

Negativity constraints (negative geometries) — Integrand for **logarithm of amplitude**

Performing **all but one** of the **loop integrations** — **IR finite!**

[Arkani-Hamed, Henn, Trnka (2021)]

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Performing **all but one** of the **loop integrations** — **IR finite!**

[Arkani-Hamed, Henn, Trnka (2021)]

Why ABJM?

Generalization to other theories

Fewer number of diagrams at each loop order

[He et al (2022, 2023)]

What results have we obtained?

Integrated results up to three loops

What results have we obtained?

Direct integration

Differential equations

Integrated results up to three loops

What results have we obtained?

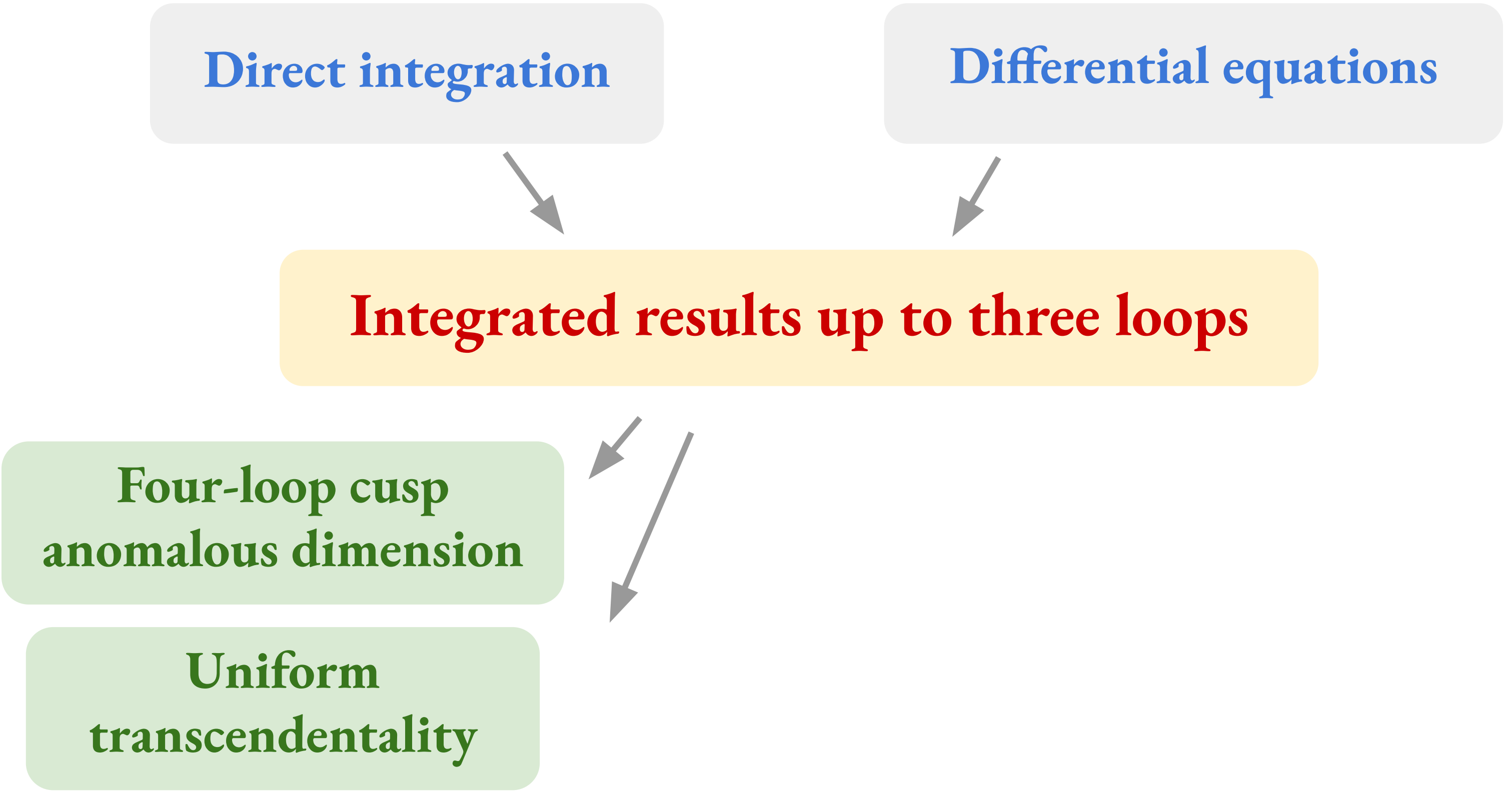
Direct integration

Differential equations

Integrated results up to three loops

Four-loop cusp anomalous dimension

What results have we obtained?



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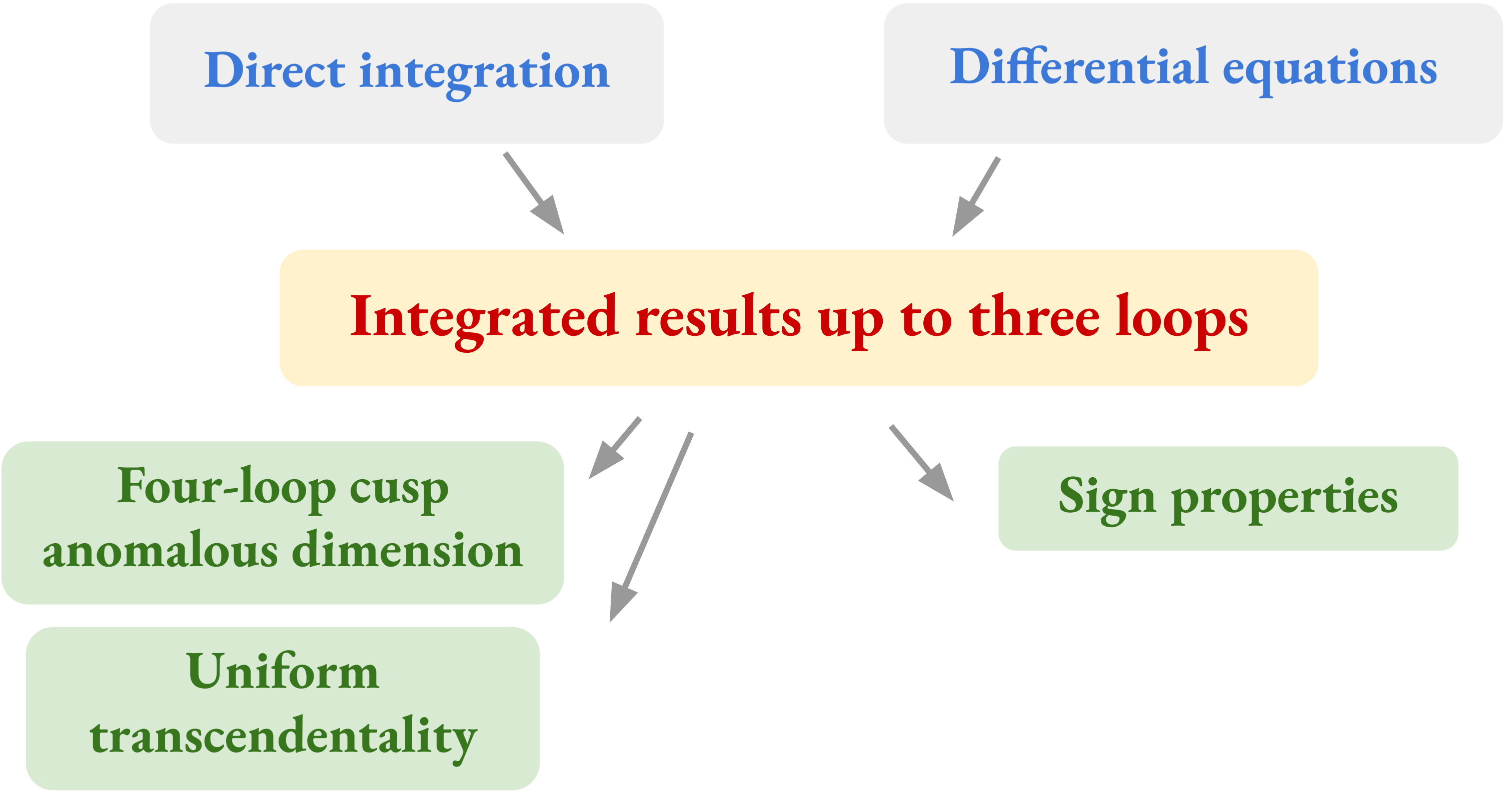
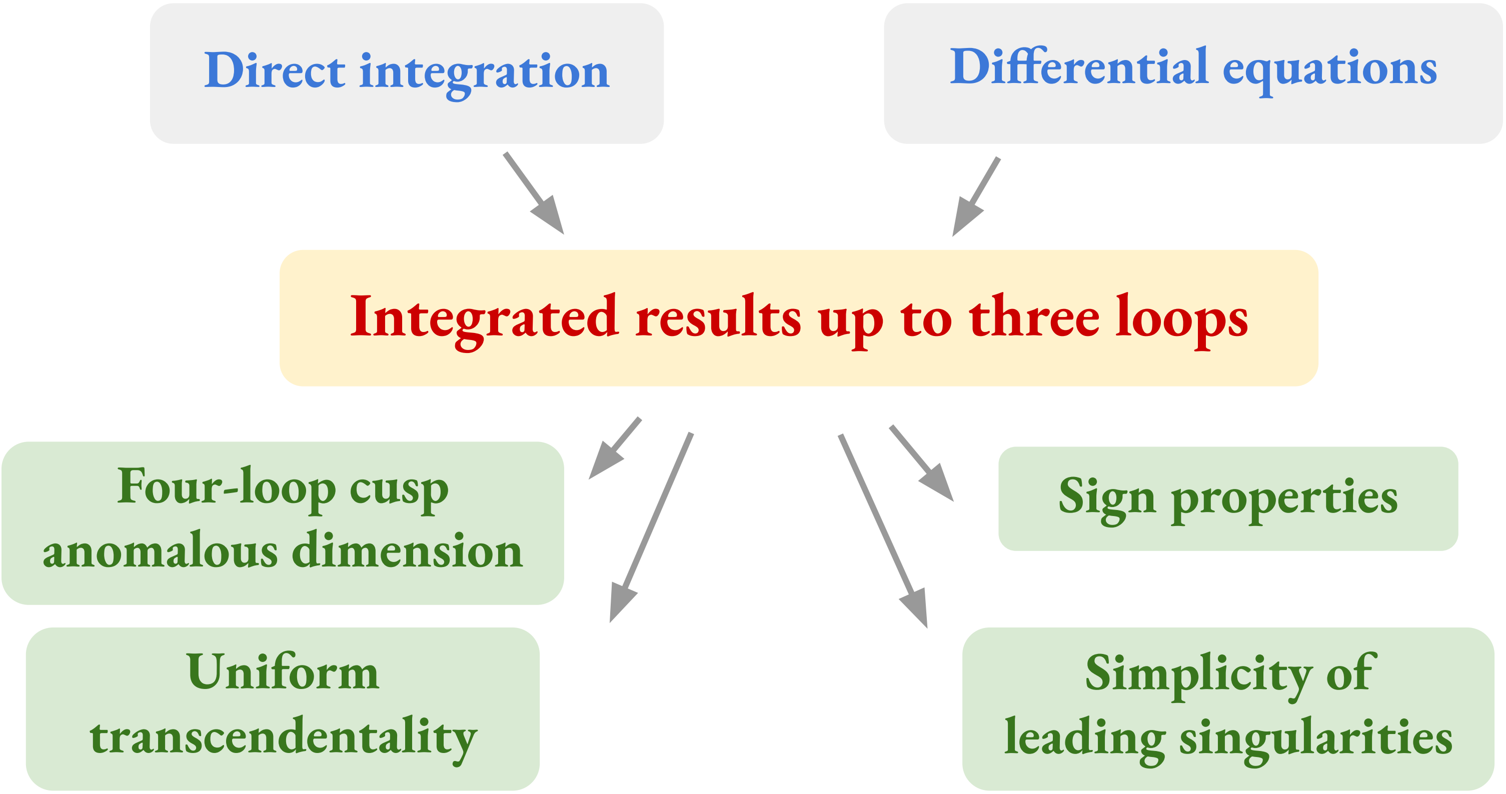


What results have we obtained?



Thank you!

Coon unitarity via partial waves or: how I learned to stop worrying and love the harmonic numbers

Based on 2401.13031

Bo Wang（王波）

Zhejiang Institute of Modern Physics, Zhejiang University

Amplitudes 2024 Conference Gong Show, IAS

June 12, 2024

Veneziano VS Coon

Veneziano amplitude [Veneziano '68]

- Crossing symmetric
- Polynomial residues
- Tame high-energy behaviour
- Linear Regge trajectories

Coon amplitude [Coon '69; Baker, Coon '70]

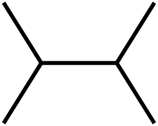
- Crossing symmetric
- Polynomial residues
- Tame high-energy behaviour
- Logarithmic Regge trajectories

Partial-wave analysis

From the well-known un-subtracted dispersion relation we have

$$\mathcal{A}(s,t) = \sum_{N=0}^{\infty} \frac{\text{residues}}{s - m_N^2} + \mathcal{A}_\infty(t).$$

Tame high-energy behaviour sets $\mathcal{A}_\infty(t) = 0$, and this is known as dual resonance. At each resonance, only a finite number of spins is exchanged



$$\sim \ \text{residues} = \sum_{\ell}^{n_0} f_{n,\ell}^2 \, C_\ell^{\frac{d-3}{2}} \left(1 + \frac{2t}{s_n - 4m^2}\right)$$

Harmonic Numbers As a Basis

Final result

$$f_{N,\ell}^2 = q^N \sum_{n,k=0}^{N} \underbrace{\binom{n}{\ell} \frac{\sqrt{\pi}}{\mathcal{K}(\ell,\alpha)} \frac{(-1)^\ell (\alpha)_{\frac{1}{2}+n}}{(\ell+2\alpha)_{1+n}}}_{\mathcal{T}_{n,\ell}^{-1}:\ \text{Gegenbauer polynomials}} \underbrace{\binom{k}{n}(-m^2)^{k-n}}_{\text{external mass}} \underbrace{\left(-s_N+4m^2\right)^n}_{\text{scattering angle}} \underbrace{Z_k^q(N)}_{\text{harmonic number}} . \quad (1)$$

The summation over n can be performed analytically but we don't present it here.

Manifest Positivity of Super String

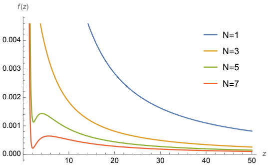
Consider the limit $q \to 1$ and $m^2 = 0$, the q-deformed harmonic numbers reduce to ordinary harmonic numbers and satisfy

$$\frac{1}{1-z}\frac{(-1)^n}{n!}\log^n(1-z) = \sum_{N=0}^{\infty} Z_n^1(N) z^N \ . \tag{2}$$

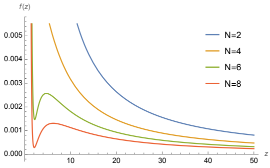
This allow us to express the partial-wave coefficients of super string

$$\begin{aligned} f^2_{N,\ell} = & \frac{1}{2\pi i}\oint \mathrm{d}z \frac{2(-1)^\ell}{z^{N+1}(1-z)}\frac{\ell+\alpha}{N+1}\Gamma(2\alpha) \\ & {}_2\tilde{F}_2\left(1, \alpha+\frac{1}{2}; 1-\ell, \ell+2\alpha+1; (N+1)\log(1-z)\right) \ . \end{aligned} \tag{3}$$

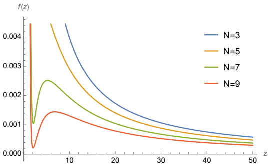
Manifest Positivity of Super String



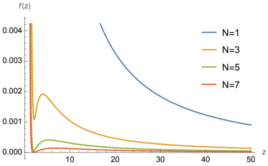
(a) $d = 4,\ \ell = 0$



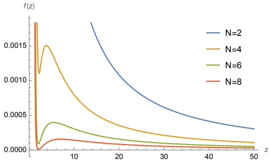
(b) $d = 4,\ \ell = 1$



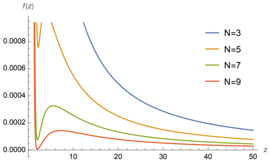
(c) $d = 4,\ \ell = 2$



(d) $d = 6,\ \ell = 0$



(e) $d = 6,\ \ell = 1$



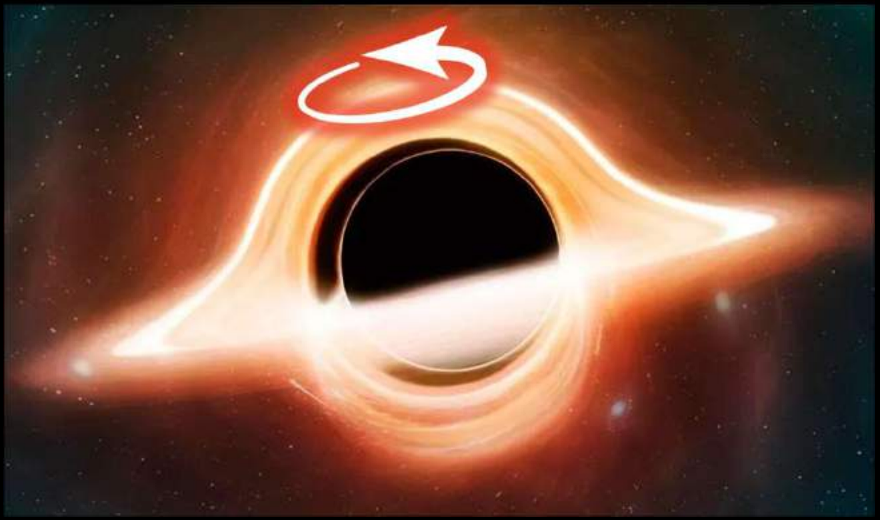
(f) $d = 6,\ \ell = 2$

Figure: More low-lying data upon $d \leq 6$ and $\ell \leq 2$ [BW '24].

Thanks

Complexified Equivalence Principle

$\Rightarrow$



through

Massive Twistors

Asymptotic Spinspacetime

Joon-Hwi Kim[1] [2309.11886]

[1] *Department of Physics, California Institute of Technology, Pasadena, CA 91125, U.S.A.*

(Dated: October 6, 2023)

See also: **JHK**, J.-W. Kim, S. Lee [2102.07063], **JHK**, S. Lee [2301.06203], **JHK**, J.-W. Kim, S. Lee [2405.17056], **JHK** [2405.09518]

Massive

The S-Matrix in Twistor Space

N. Arkani-Hamed[a], F. Cachazo[b], C. Cheung[a,c] and J. Kaplan[a,c]

[a] *School of Natural Sciences, Institute for Advanced Study, Princeton, NJ 08540, USA*
[b] *Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2J W29, CA*
[c] *Jefferson Laboratory of Physics, Harvard University, Cambridge, MA 02138, USA*

Spinor-helicity variables | ... and their conjugates

$$\lambda_\alpha{}^{I=0,1} \xrightarrow{\text{Half-Fourier}} \bar{\mu}_{I=0,1}{}^{\alpha}$$

Arkani-Hamed, Huang, Huang [1709.04891];
JHK, J.-W. Kim, S. Lee [2102.07063];
Penrose (1974);
Perjés (1975);

$$\mu^{\dot{\alpha}I} = z^{\dot{\alpha}\alpha}\lambda_\alpha{}^{I}$$

Massive **Incidence Relation**

$$z^\mu = x^\mu + i a^\mu$$

spin/mass

"Spinspacetime"

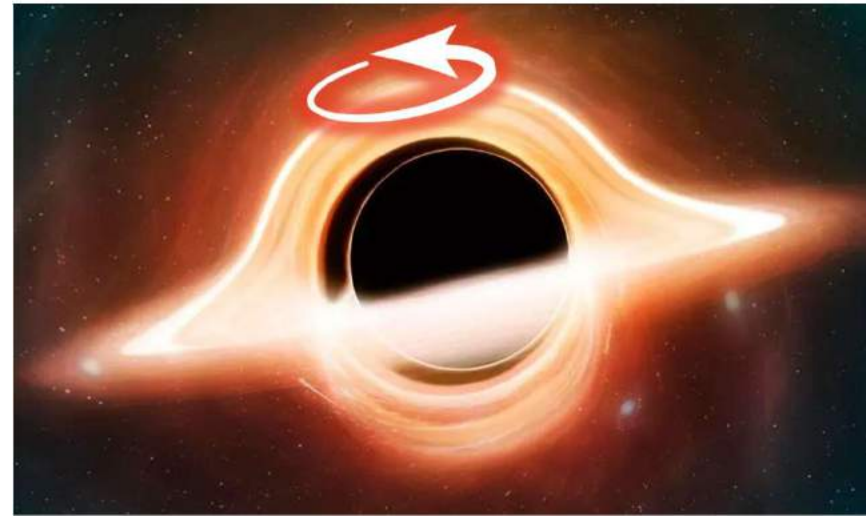
Newman (1974); **JHK** [2309.11886];
Born, Infeld (1935); Casalbuoni (1976);

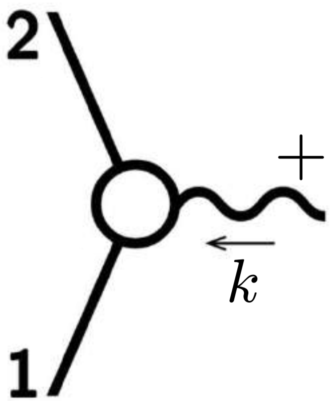
$$L^{\mu\nu} = (x \wedge p)^{\mu\nu} \quad S^{\mu\nu} = *(a \wedge p)^{\mu\nu} \quad J^{+\mu\nu} = ((x + ia) \wedge p)^{\mu\nu} \quad \rightsquigarrow \quad \mu^{\dot{\alpha}I}\bar{\lambda}_{I\dot{\beta}}$$

Complexified Equivalence Principle

(Minimal On-Shell Kinematics)

Half-Fourier $\Longrightarrow$



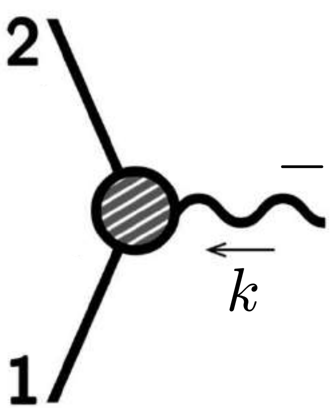


$\delta(\lambda_1 - \lambda_2)$

$\delta(\bar{\lambda}_1 - \lambda_1^{-1} k - \bar{\lambda}_2)$

$$\Longrightarrow \quad \mathcal{M}_{\rm BH}^{+} = e^{ik_3\cdot(x+ia)}$$

Very easy!!!
No spin coherent state smearing etc.



$\delta(\bar{\lambda}_1 - \bar{\lambda}_2)$

$\delta(\lambda_1 - k\bar{\lambda}_2^{-1} - \lambda_2)$

$$\Longrightarrow \quad \mathcal{M}_{\rm BH}^{-} = e^{ik_3\cdot(x-ia)}$$

Newman-Janis Shift = Black Holes !!

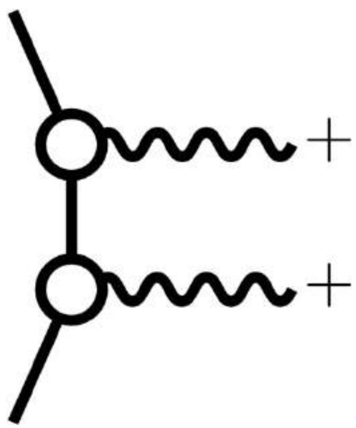
Guevara, Ochirov, Vines [1812.06895];
Arkani-Hamed, Huang, O'Connell [1906.10100]

Cool physics: Spacetime and spin unified — "spinspacetime"

Formalism: Massive twistor

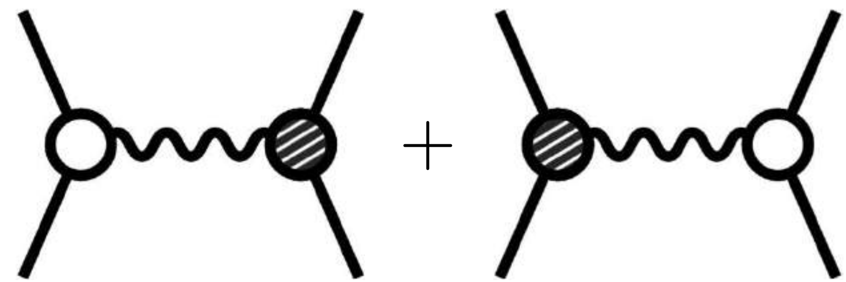
Output: Black holes obey minimal on-shell kinematics

Higher Multiplicity?



JHK [2309.11886]; JHK, S. Lee [2301.06203];
Aoude, Haddad, Helset [2001.09164];
Johansson, Ochirov [1906.12292];

Applications to PM potential



$$V^{1\mathrm{PM}} = \frac{e^{+h\varphi}}{\left| \vec{z}_1 - \vec{\bar{z}}_2 \right|} + \frac{e^{-h\varphi}}{\left| \vec{\bar{z}}_1 - \vec{z}_2 \right|}$$

JHK, J.-W. Kim, S. Lee [2405.17056]; JHK [2309.11886];

Stay Tuned for Future Works!

"Curved Spinspacetime" (**JHK** [2407.*****])

"Spinning BHs and the Dirac String" (**JHK** [2407.*****])

A Two-Loop Conundrum

Two Loops When One Loop Vanishes

Anthony Morales (SLAC/Stanford)

L. Dixon, AM, arXiv:2406.XXXXX

Amplitudes 2024 Gong Show
12 June 2024

A One-Loop Introduction

Self-dual Yang-Mills on twistor space is anomalous, with the anomaly being proportional to $\text{tr}_{\text{adj}}(X^4)$. This can be remedied by including fermions in a representation R such that [arXiv:2302.00770]

$$\text{tr}_{\text{adj}}(X^4) = \text{tr}_R(X^4).$$

For the gauge group $SU(N)$, a non-trivial choice is

$$R = 8\,\square \oplus 8\,\overline{\square} \oplus \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array} \oplus \overline{\begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}}$$

This forces the all-plus 1-loop amplitude to vanish, which in turn relates different permutations of the partial amplitudes

One-loop relation from anomaly cancellation (L. Dixon, AM, arXiv:2406.XXXXX)

$$8A^{[1]}(1,\ldots,n) = \sum_{k=1}^{n} \sum_{\sigma \in \alpha_k \sqcup\!\sqcup \beta_k} A_n^{[1]}(1,\sigma)$$

where $\alpha_k = (2,\ldots,k)$ and $\beta_k = (k+1,\ldots,n)$. These are the same relations conjectured to hold by Bjerrum-Bohr et al. [arXiv:1103.6190].

A Nice Two-Loop Result

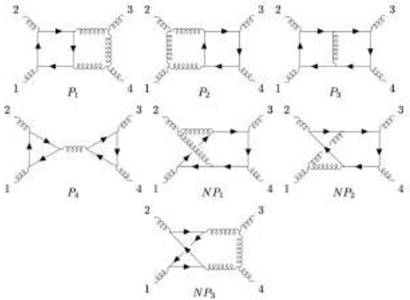
- Q: 1-loop vanishing $\implies$ 2-loop simplicity?
- A: K. Costello computed the n-point 2-loop amplitude in this theory, using OPEs of a chiral algebra, and found it to be both **finite** *and* **rational** [arXiv:2302.00770].

4-pt chiral algebra result

$$\mathcal{A}_{4,\text{sdYM}}^{\text{2-loop,all-}+} = ig^6 \frac{1}{(4\pi)^4} \frac{[12][34]}{\langle 12\rangle\langle 34\rangle} \left[\left(12N - 4\frac{s^2+4st+t^2}{st} - 24N^{-1} \right) \text{tr}(1234) \right.$$
$$\left. + (24+24N^{-1})\text{tr}(12)\text{tr}(34) + \text{cycles}(234) \right].$$

We set out to **check this result**.

A Two-Loop MISMATCH in Dim Reg



We used previous QCD results for the primitive amplitudes [arXiv:0001001,0201161,0202271] and dressed them with color factors for the antisym rep computed in a trace basis. The two methods disagree

Mismatch

$$\mathcal{A}^{\text{adj,all-+}}_{4,G,\epsilon} + \mathcal{A}^{R,\text{all-+}}_{4,F,\epsilon} \neq \mathcal{A}^{\text{2-loop,all-+}}_{4,\text{sdYM}}.$$

In fact, the left-hand side is *neither* finite nor purely rational. However, $\mathcal{A}^{\text{2-loop,all-+}}_{4,\text{sdYM}}$ is not necessarily incorrect.

- The 1-loop amplitude is actually $\mathcal{O}(\epsilon)$!!!
- So Catani's formula [arXiv:9802439] predicts non-vanishing IR divergent terms $\frac{1}{\epsilon^2} \times \mathcal{O}(\epsilon) = \mathcal{O}(1/\epsilon)$
- $\mathcal{A}^{\text{2-loop,all-+}}_{4,\text{sdYM}}$ actually gives the finite remainder to $\mathcal{A}^{\text{adj,all-+}}_{4,G,\epsilon} + \mathcal{A}^{R,\text{all-+}}_{4,F,\epsilon}$.

Conjecture

$$\left[\mathcal{A}^{\text{adj,all-+}}_{n,G,\epsilon} + \mathcal{A}^{R,\text{all-+}}_{n,F,\epsilon}\right]_{\text{finite}} = \mathcal{A}^{\text{2-loop,all-+}}_{n,\text{sdYM}} \quad \text{for } n \geq 4$$

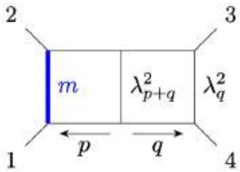
A MASSIVE Resolution

- Chiral algebra method is purely 4D $\Longrightarrow$ maybe mass regulator resolves disagreement
- Problem with dim-reg: ϵ-suppressed terms compete with $1/\epsilon^2$ IR poles.
- Doesn't happen with mass regulator: mass-suppressed terms vanish more quickly relative to the log-enhanced ones.

Example

The mass-regulated planar double-box integral is

$$\mathcal{I}^{P}_{4,m}[\lambda^2_{p+q}\lambda^2_q](s,t) = \mathcal{I}^{\text{1-loop}}_{3,m}[1](s)\ \mathcal{I}^{\text{1-loop}}_{4}[\lambda^4_p](s,t)$$
$$\xrightarrow{m,\epsilon\to 0} \frac{s^{-1}}{6(4\pi)^4}\left[-\frac{1}{2}\ln^2\left(\frac{m^2}{-s}\right) - 2\zeta_2\right]$$



When the divergent integrals are mass regulated, the mass dependence drops out of the full amplitude, and we get agreement with Costello's result:

Mass regulator gives agreement

$$\mathcal{A}^{\text{adj,all-+}}_{4,G,m} + \mathcal{A}^{R,\text{all-+}}_{4,F,m} = \mathcal{A}^{\text{2-loop,all-+}}_{4,\text{sdYM}}.$$

Fin

Please look out for our paper to appear. Thanks for listening!

Classical bound observables from amplitudes

Riccardo Gonzo

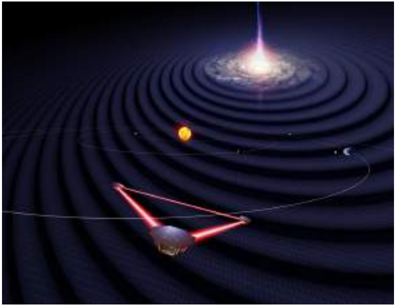
THE UNIVERSITY of EDINBURGH

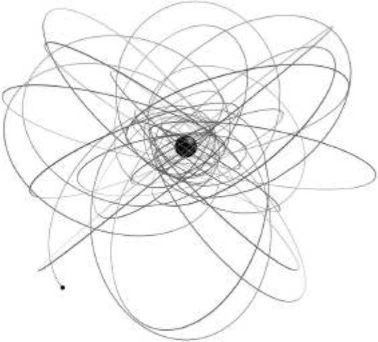
Gong show, Amplitudes 2024

Princeton, 12 June 2024

Motivation and introduction

- Analytic waveform templates are going to be necessary for extreme mass ratio inspirals, which are going to be detected by the LISA mission



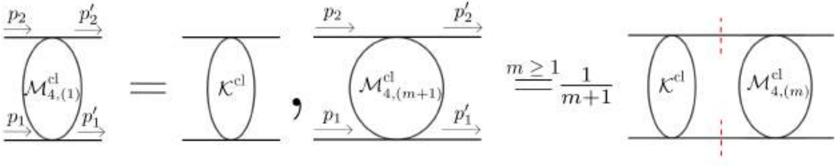


As theoretical physicists, we need to work hard to be ready for 2035!

- Idea: use particle field theory tools ($\rightarrow$ scattering amplitudes)

The classical Bethe-Salpeter equation: conservative

- We derive the classical Bethe-Salpeter equation describing binary gravitational systems with a two-body irreducible kernel $\widetilde{\mathcal{K}}_{\text{cl}}$: [Adamo, RG]



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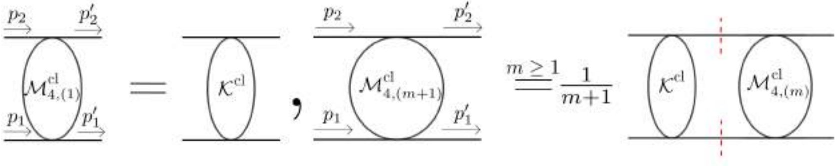

- We can solve the recursion in impact parameter space ($\sim$ partial wave basis)

$$\mathrm{i}\mathcal{M}_4^{\mathrm{cl}}(q_\perp) = \frac{4\sqrt{(p_A\cdot p_B)^2 - m_A^2 m_B^2}}{\hbar^2}\int \mathrm{d}^2 b\, \mathrm{e}^{-\mathrm{i}\bar{q}_\perp\cdot b}\left(\mathrm{e}^{\widetilde{\mathcal{K}}_{\mathrm{cl}}(b)} - 1\right).$$

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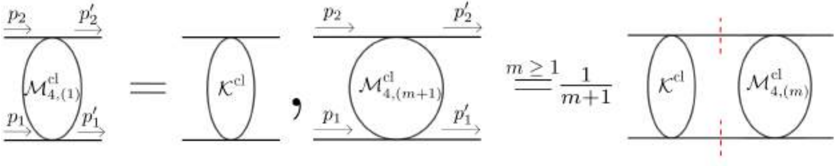

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- The conservative kernel is essentially the scattering radial action

$$\widetilde{\mathcal{K}}_{\text{cl}}^{>}(\{s_i^\mu, v_i^\mu\}; b(L)) = \underbrace{\frac{i}{\hbar} \oint_{\mathcal{C}^>} dr \, p_r(r, \{s_i^\mu, v_i^\mu\}, L)}_{I_r^>} + L\pi \, ,$$

[Bern et al.; Damgaard,Plante,Vanhove; Kol,O'Connell,Telem; Adamo,RG]

Impulse, spin kick and frequencies from the classical kernel

- We propose that Classical scattering observables can be extracted by recursively applying Dirac brackets to the radial action, [RG,Shi]

$$\Delta\lambda^\mu = \sum_{j=1} \frac{1}{j!} \underbrace{\{I_r^>, \{I_r^>, \ldots, \{I_r^>}_{j \text{ times}}, \lambda^\mu\} \ldots\}\}, \qquad \lambda^\mu \in \{v_1^\mu, v_2^\mu, s_1^\mu, s_2^\mu\}.$$

Complete proof for spinless particles in Kerr, checked beyond the probe limit! Another recent work independently confirm our proposal [Kim,Kim,Lee]

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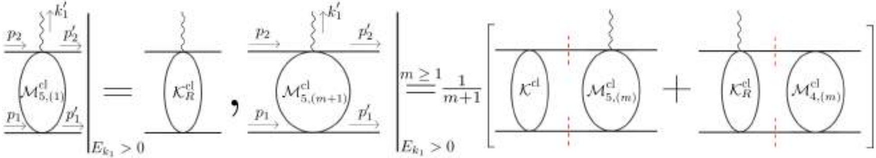
- With action-angle variables, we computed bound frequencies

Type of observable	Position space	Spin space
Scattering	Δv_1^μ $(\Delta\phi,\ \Delta\theta)$	Δs_1^μ
Bound	$K^{\phi r}$, $K^{\theta r}$	$K^{\phi_S r}$

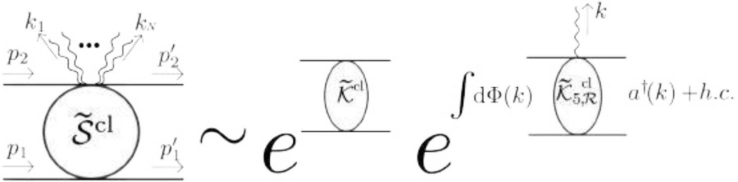
New scatter-to-bound map at 0SF order! [RG,Shi; Kälin,Porto]

The classical Bethe-Salpeter equation: radiation

- Extension of the Bethe-Salpeter with radiation [Adamo, RG, Ilderton]



which gives the exponential form of the S-matrix



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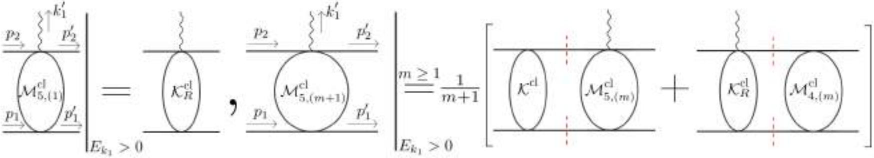

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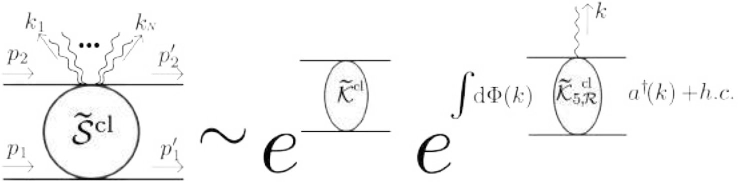

- All scattering and bound observables for the two-body problem can derived from a gauge-invariant representation with 2MPI kernels $\widetilde{\mathcal{K}}^{\text{cl}}$ and $\widetilde{\mathcal{K}}^{\text{cl}}_{5,\mathcal{R}}$!

Summary table of the scatter-to-bound dictionary

- For aligned-spin binaries where the motion remains on the equatorial plane we find a conjectural dictionary [Kälin,Porto;Saketh,Vines,Steinhoff, Buonanno;Cho,Kälin,Porto;Adamo,RG; Heissenberg;Adamo,RG,Ilderton]

Bound observable	Scattering observable ($p_\infty^2 = -\tilde{p}_\infty^2 = \frac{E^2-(m_1+m_2)^2}{2m_1 m_2}$),
$\Delta\Phi(\tilde{p}_\infty, L, a, c_X)$	$\chi(-i\tilde{p}_\infty, L, a, c_X) + \chi(+i\tilde{p}_\infty, L, a, c_X)$
$\Delta E^{<}_{\text{rad}}(\tilde{p}_\infty, L, a, c_X)$	$\Delta E^{>}_{\text{rad}}(-i\tilde{p}_\infty, L, a, c_X) + \Delta E^{>}_{\text{rad}}(+i\tilde{p}_\infty, L, a, c_X)$
$\Delta J^{<}_{\text{rad}}(\tilde{p}_\infty, L, a, c_X)$	$\Delta J^{>}_{\text{rad}}(-i\tilde{p}_\infty, L, a, c_X) + \Delta J^{>}_{\text{rad}}(+i\tilde{p}_\infty, L, a, c_X)$

which is valid at least up to 3PM (G_N^3) order for the scattering angle $\Delta\chi$/periastron advance $\Delta\Phi$ and for the fluxes $\Delta E_{\text{rad}}, \Delta J_{\text{rad}}$.

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- New waveform map (up to 1PN and tree-level)[Adamo,RG,Ilderton]

$$h^{<\text{dyn}}(u; \tilde{p}_\infty, L, a, c_X) = h^{>\text{dyn}}(u; +i\tilde{p}_\infty, L, a, c_X)$$

  in agreement with the prescription for the orbital elements [Damour,Deruelle]

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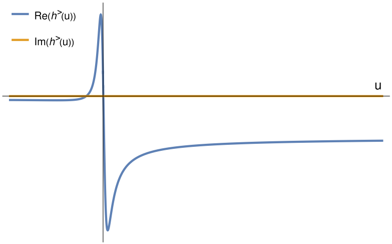
  in agreement with the prescription for the orbital elements [Damour,Deruelle]

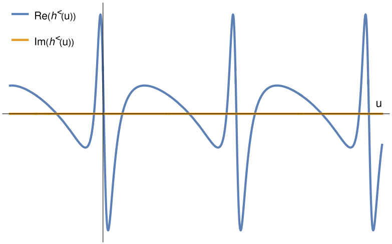
- Need to study tail effects appearing at higher orders! [Cho,Kälin,Porto]

From scattering to bound waveforms via resummation

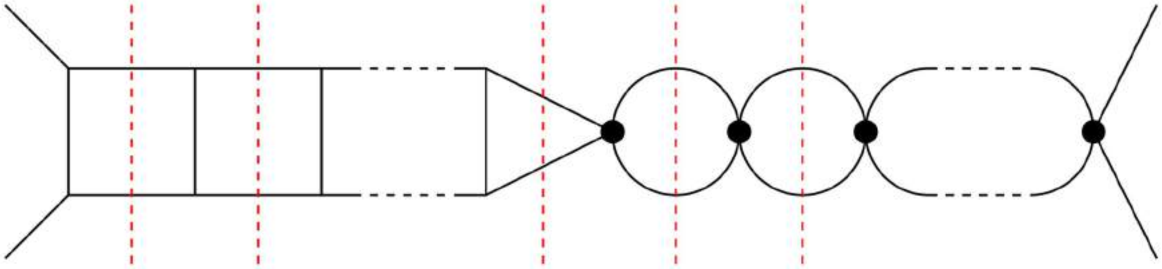
- The analytic continuation of the waveform computed for eccentric orbits requires a resummation in the eccentricity to recover the bound waveform periodicity in the time u [Adamo,RG,Ilderton]

$$n^{>} t = e_t^{>} \sinh(\mathrm{v}) - \mathrm{v} + \mathcal{O}(1/c) \,, \qquad n^{<} t = \mathrm{u} - e_t^{<} \sin(\mathrm{u}) + \mathcal{O}(1/c) \,.$$





Non-analytic terms of string amplitudes from partial waves



Hynek Paul

KU LEUVEN

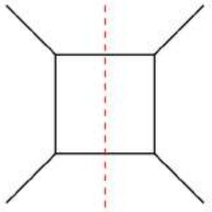
Based on WIP [2406.xxxxx]
w/ Yu-tin Huang & Michele Santagata

Amplitudes Conference
12/06/2024

Recall from unitarity cuts: the leading log(-s) part of the L-loop amplitude is obtained by gluing trees

Recall from unitarity cuts: the leading log(-s) part of the L-loop amplitude is obtained by gluing trees

at 1-loop:



$$\Rightarrow \quad \text{Disc}_s[\mathcal{A}^{(1)}(s,\cos\theta)] = \int d\Omega_2\, \mathcal{A}^{(0)}(p_1,p_2,\ell_1,\ell_2)\mathcal{A}^{(0)}(-\ell_1,-\ell_2,p_3,p_4)$$

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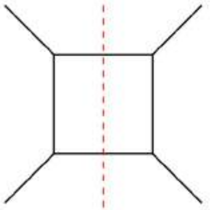

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$\rightarrow$ **let's use the good old partial-wave decomposition:**

$$\mathcal{A}(s,\cos\theta) = \sum_{\ell \text{ even}} a_\ell(s) P_\ell^{\frac{d-3}{2}}(\cos\theta)$$

scattering angle

partial-wave coefficients

Gegenbauer polynomials

Recall from unitarity cuts: the leading log(-s) part of the L-loop amplitude is obtained by gluing trees

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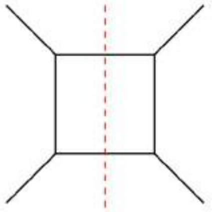

$$\Rightarrow \quad \mathrm{Disc}_s[\mathcal{A}^{(1)}(s,\cos\theta)] = \int d\Omega_2\, \mathcal{A}^{(0)}(p_1,p_2,\ell_1,\ell_2)\mathcal{A}^{(0)}(-\ell_1,-\ell_2,p_3,p_4)$$

→ **let's use the good old partial-wave decomposition:** $\mathcal{A}(s,\cos\theta) = \sum_{\ell \text{ even}} a_\ell(s) P_\ell^{\frac{d-3}{2}}(\cos\theta)$

scattering angle

partial-wave coefficients

Gegenbauer polynomials

→ **key property: partial-waves trivialise the 2-particle phase space integration** $d\Omega_2$**!**

Recall from unitarity cuts: the leading log(-s) part of the L-loop amplitude is obtained by gluing trees

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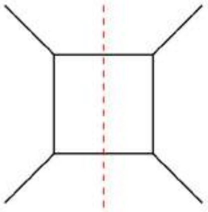

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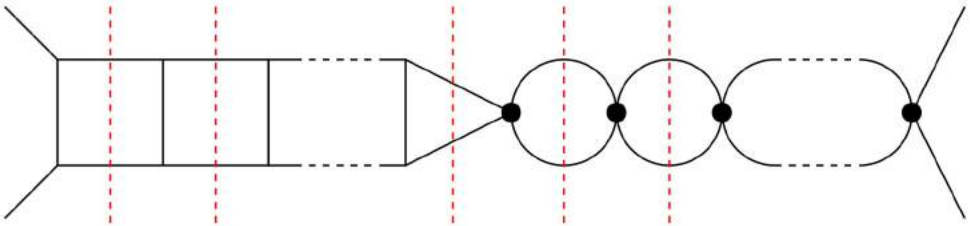
Generalisation to L-loops:

$$\text{Disc}_s^L[\mathcal{A}^{(L)}(s,\cos\theta)] \quad =$$



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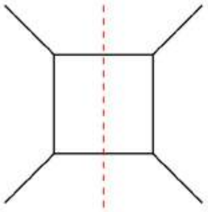

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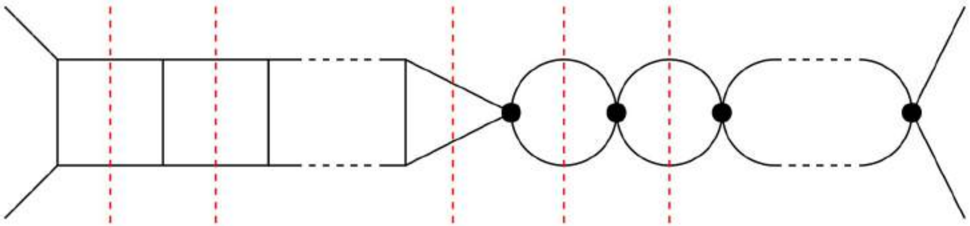

→ iterated s-channel discontinuity simply computed by: $a_\ell^{\mathrm{Disc},L} = (a_\ell^{(0)})^{L+1}$

Application to type IIB four-graviton amplitude

Application to type IIB four-graviton amplitude

genus 0:

$$\mathcal{A}^{(0)} = \frac{1}{stu}\frac{\Gamma(1-s)\Gamma(1-t)\Gamma(1-u)}{\Gamma(1+s)\Gamma(1+t)\Gamma(1+u)}$$

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extract tree-level
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[Green,Russo,Vanhove'08],[Edison,Guillen,Johansson,Schlotterer,Teng'21],[Eberhardt,Mizera'22]

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⋮

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[Green,Russo,Vanhove'08],[Edison,Guillen,Johansson,Schlotterer,Teng'21],[Eberhardt,Mizera'22]

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→ new predictions for leading logarithmic terms in string amplitudes!

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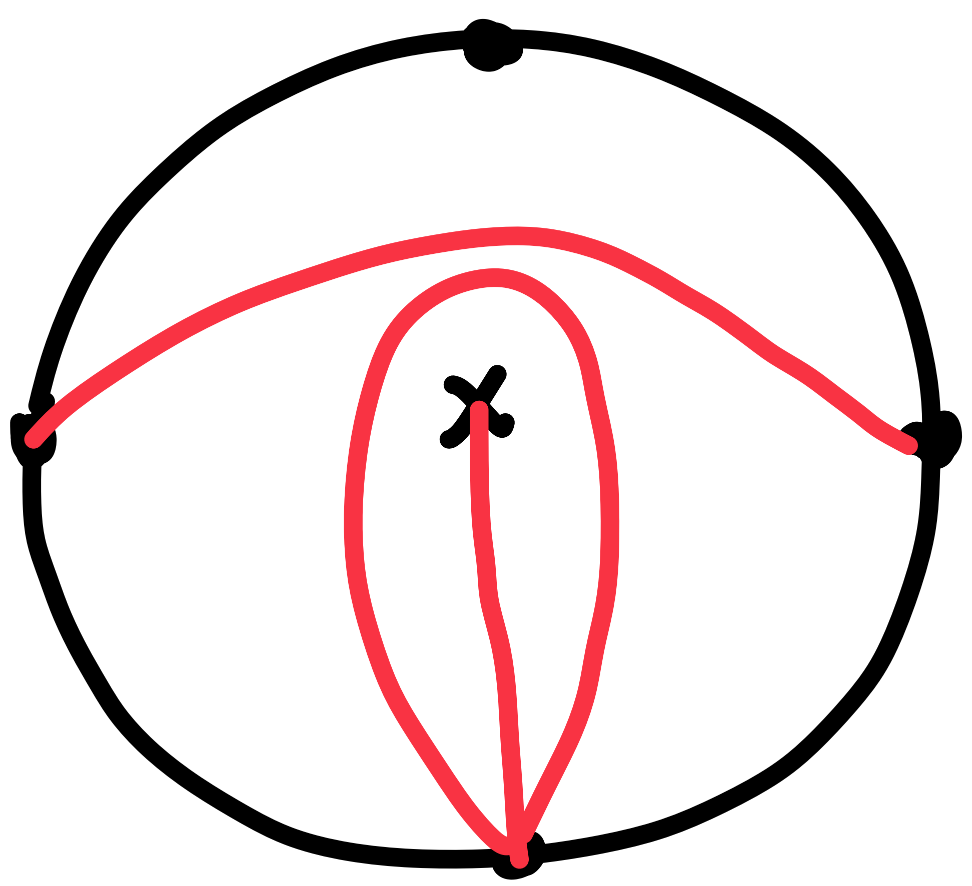
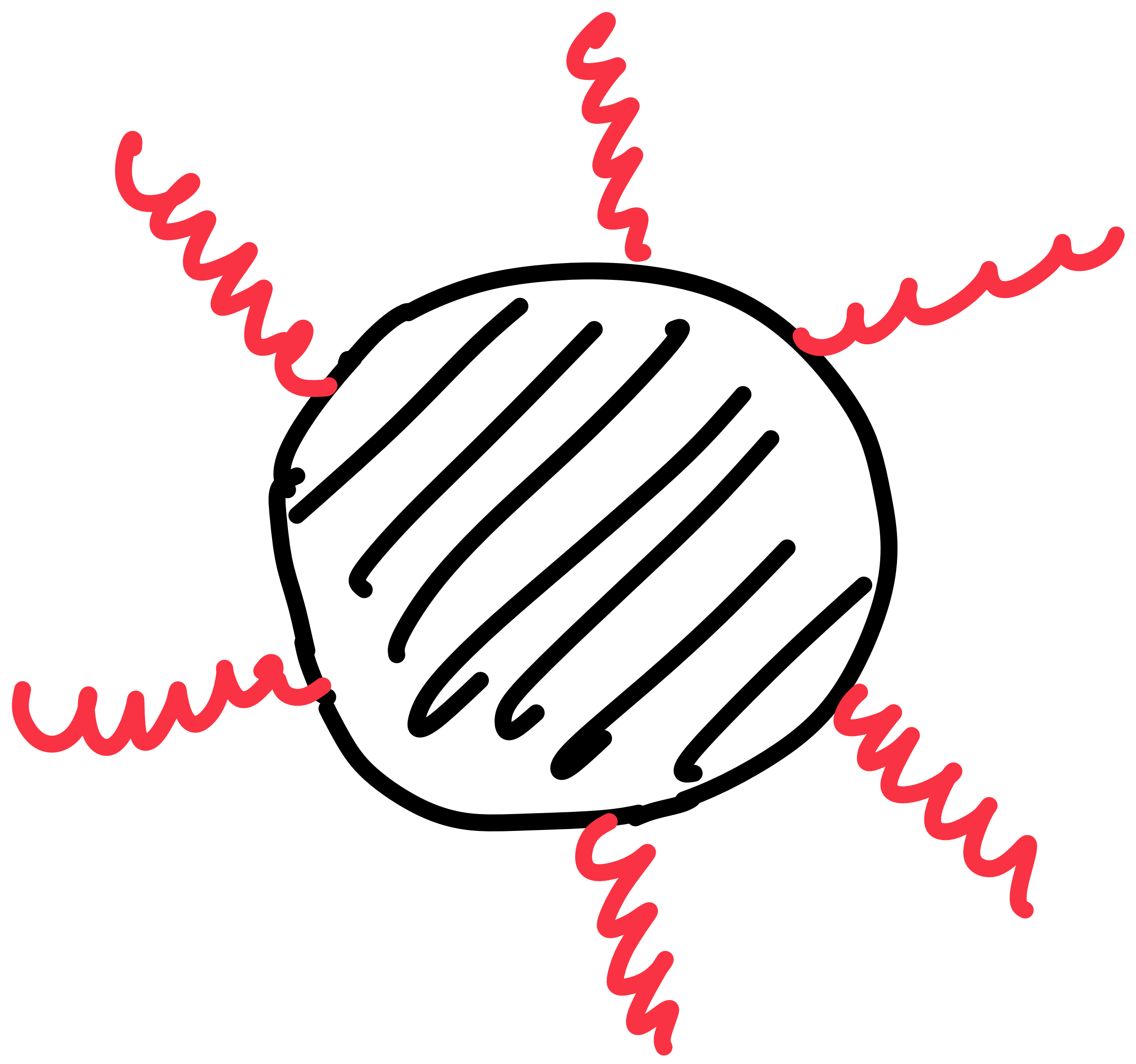
→ new predictions for leading logarithmic terms in string amplitudes!

To be matched by future worldsheet calculations by (aspiring) string-amplitudeologists...

— Novel Structures — in the Soft Gluon — Limit —





Jeffrey Backus

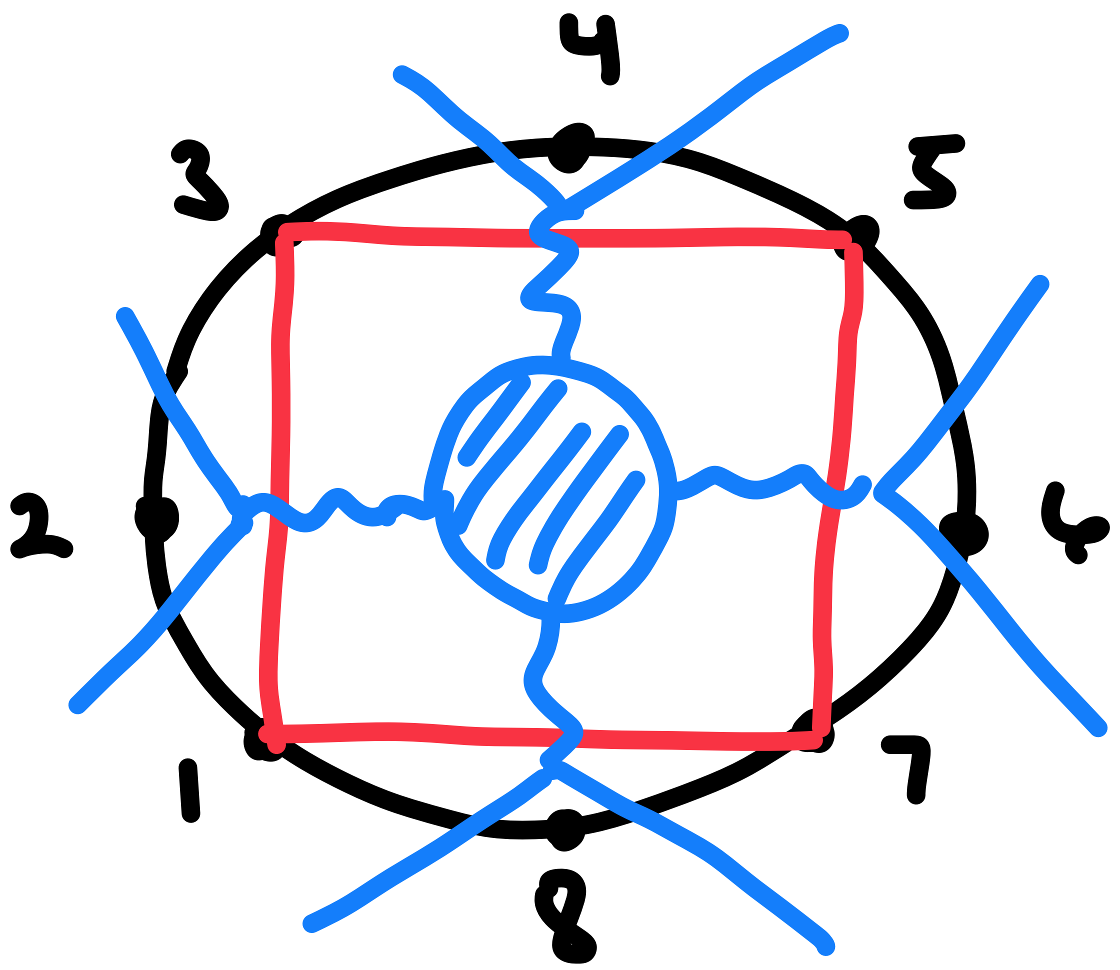
Amplitudes 2024 Gong Show

work-in-progress w/
N. Arkani-Hamed
C. Figueiredo

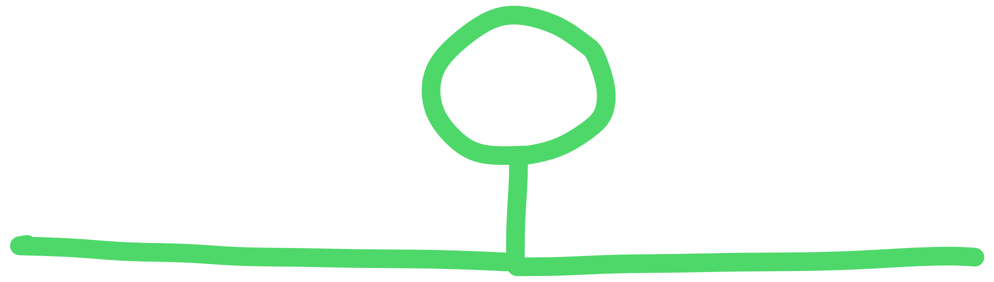
Yang-Mills from tr ϕ^3

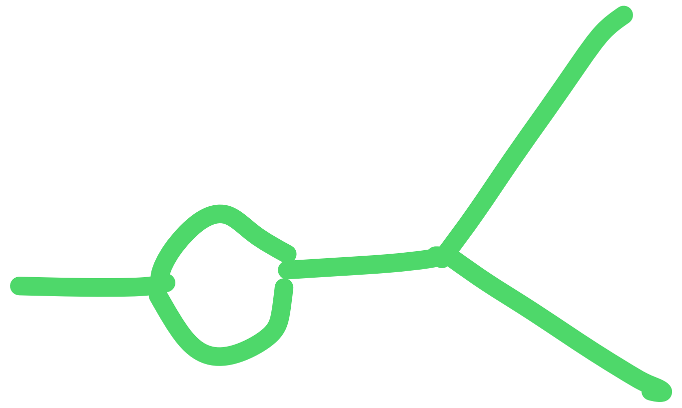
N. Arkani-Hamed, Q. Cao, J. Dong, C. Figueiredo, S. He 2401.00041

* $\mathcal{M}_n^{YM}[X_{i,j}] = \text{Res}\, \mathcal{M}_{2n}^{\text{tr}\phi^3}[X_{i,j}]$



* At one-loop and higher, includes

→ Tadpoles 

→ External Bubbles 

* Defines loop integrand notion of gauge invariance!

Take a Gluon Soft —

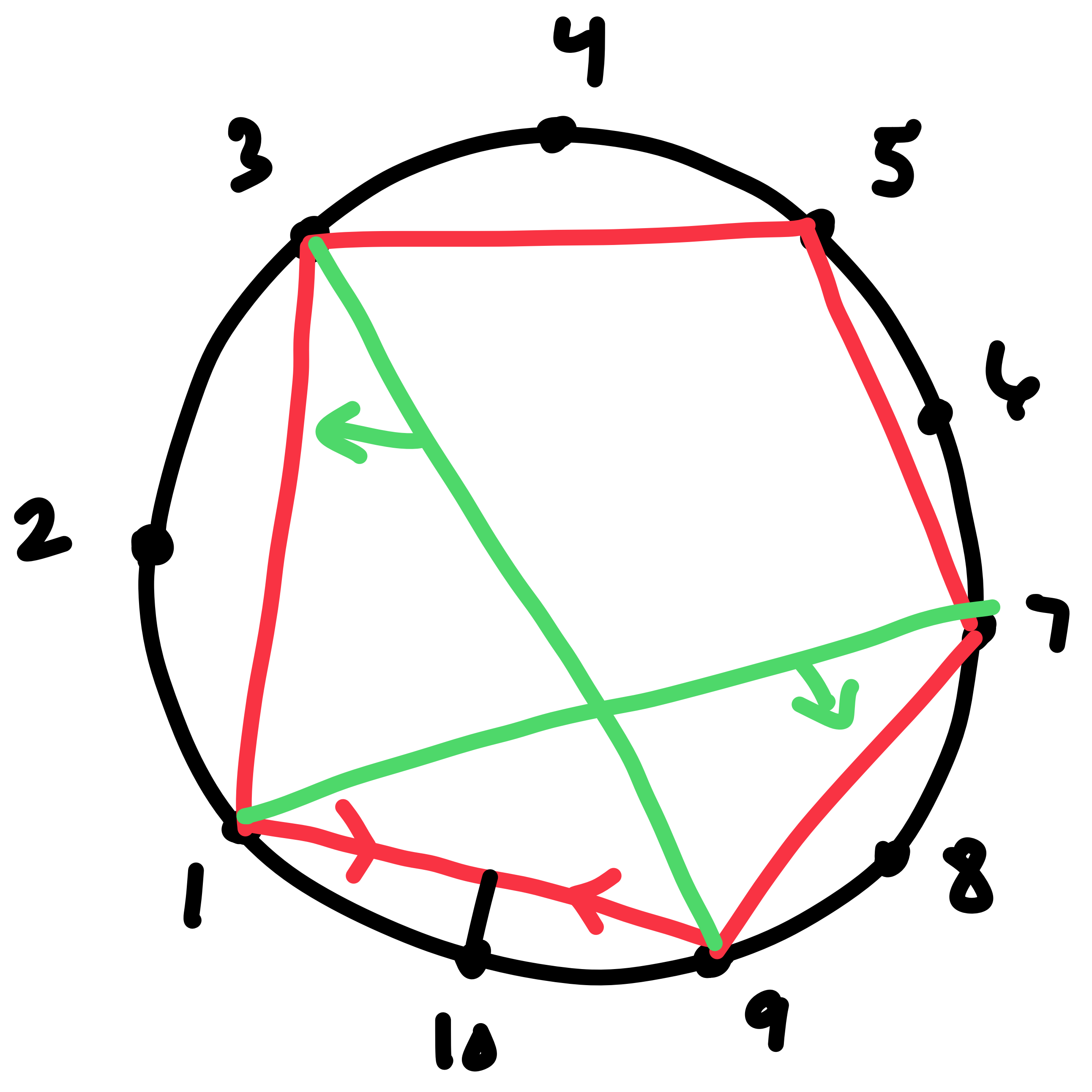
S. Weinberg, PR 140, B1049 (1964)

F. Cachazo, A. Strominger 1404.4091

* $A_n^{YM} = \sum_{i=-1}^{\infty} S_i \, A_{n-1}^{YM}$

* Known: S_{-1}, S_0

* Using the surface, we
  - → upgraded leading and sub-leading theorems to loop-integrand level.
  - → found structure in S_i for all $i>0$ and at loop-integrand level: sum rules and numerators of two poles exactly known.



Take $\gamma_5 \to 0$, "pinch" 1 and 9 together.

Thanks!

Giant Correlators at Quantum Level

Work with Yunfeng Jiang and Yang Zhang

arxiv: 2311.16791

Presented by Yu Wu
University of Science and Technology of China

Determinant Operator

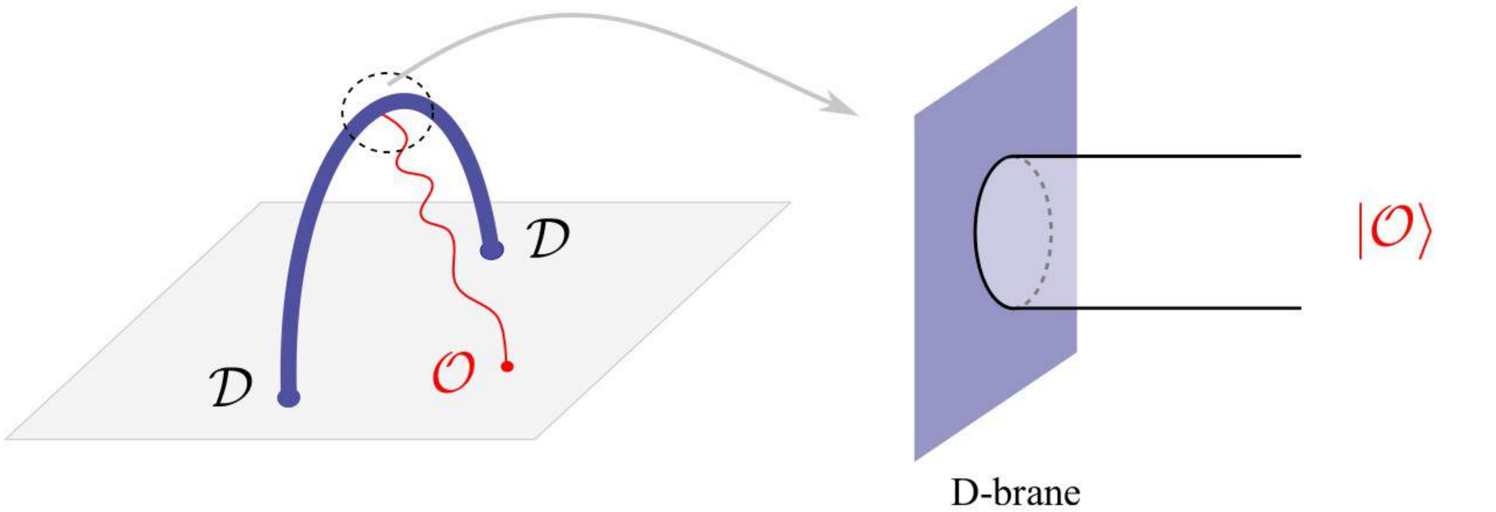
$$\langle \mathcal{D}(x_1)\mathcal{D}(x_2)\mathcal{O}_2(x_3)\mathcal{O}_2(x_4)\rangle$$

$$\mathcal{D}_i(x_i) = \det(Y_i^I \Phi_I)(x_i)$$

- Determinant-like operators known as giant gravitons
- Dual to D-branes in the bulk

Motivation

$$\langle \mathcal{D}_1 \mathcal{D}_2 \mathcal{O}_3 \rangle = \text{spacetime dependence} \times d_{\mathcal{O}}$$



Y. Jiang, S. Komatsu, E. Vescovi JHEP 07 (2020) 07, 037

- Structure constant as $\langle B|\psi\rangle$
- Known as exact g-function, can be computed by TBA
- Checked up to two-loop, higher loop checks needed

General Ansatz

$$\langle \mathcal{D}(x_1)\mathcal{D}(x_2)\mathcal{O}_2(x_3)\mathcal{O}_2(x_4)\rangle$$

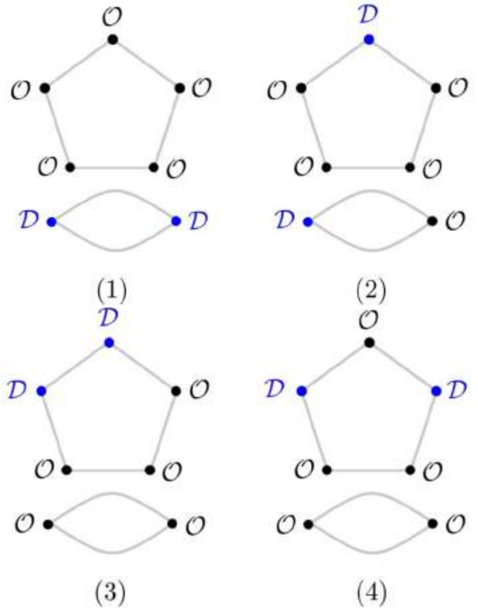
↓ Lagrangian Insertion

$$G^{(\ell)}_{\{2,2\}} = \int \mathrm{d}^4x_5 \ldots \mathrm{d}^4x_{4+\ell} \langle \mathcal{D}(x_1) \ldots \mathcal{O}_2(x_4)\mathcal{L}(x_5) \ldots \mathcal{L}(x_{4+\ell})\rangle_0$$

↓ Symmetry Constraint

$$\mathcal{G}^{(\ell)}_{\{2,2\}} = R_{1234}(d_{12})^{N-2} \frac{P^{(\ell)}}{\prod_{\substack{1\le p\le 4\\ 5\le q\le 4+\ell}} x_{pq}^2 \prod_{5\le p<q\le 4+\ell} x_{pq}^2}$$

↓ Planarity



$$P_1^{(3)}(x_i) = (x_{12}^4)(x_{34}^2x_{45}^2x_{56}^2x_{67}^2x_{73}^2) + S_2 \times S_5 \text{ permutations}\,,$$

$$P_2^{(3)}(x_i) = (x_{13}^4)(x_{24}^2x_{45}^2x_{56}^2x_{67}^2x_{72}^2) + S_2 \times S_5 \text{ permutations}$$

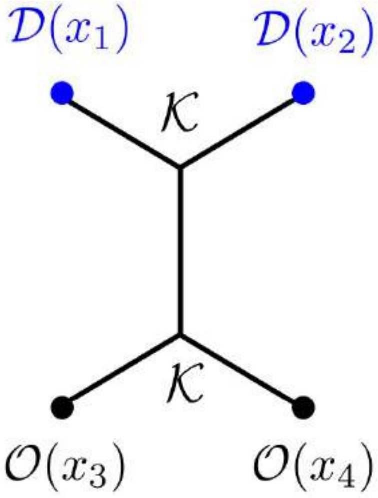
$$P_3^{(3)}(x_i) = (x_{67}^4)(x_{12}^2x_{23}^2x_{34}^2x_{45}^2x_{51}^2) + S_2 \times S_5 \text{ permutations}\,,$$

$$P_4^{(3)}(x_i) = (x_{67}^4)(x_{13}^2x_{32}^2x_{24}^2x_{45}^2x_{51}^2) + S_2 \times S_5 \text{ permutations}\,.$$

OPE Limit

- s-channel

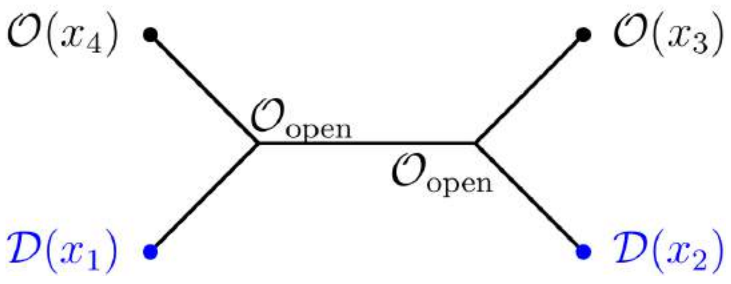
$x_1 \to x_2$ and $x_3 \to x_4$



- Totally 6 equations for 4 variables

- t-channel

$x_1 \to x_4$ and $x_2 \to x_3$



$$\mathcal{O}_{\rm open} \sim \epsilon^{j_1 \dots j_{N-2} a_1 a_2}_{i_1 \dots i_{N-2} b_1 b_2} (\mathcal{Z}_1)^{i_1}_{j_1} \cdots (\mathcal{Z}_1)^{i_{N-2}}_{j_{N-2}} (\Phi_I)^{a_1}_{b_1} (\Phi_I)^{a_2}_{b_2}$$

Result:

$$\frac{G^{(3)}_{\{2,2\}}}{\tilde{R}_{1234}(d_{12})^{N-2}} = 4\left[gh(1,3;2,4) + gh(1,4;2,3) - gh(1,2;3,4)\right] + 12\left[L(1,3;2,4) + L(1,4;2,3)\right] + 8E(1,2;3,4) + 2\left(1 - \frac{u}{v}\right) H(1,3;2,4) + 2(1-u)H(1,4;2,3)$$

OPE Coefficients at Three-loop

spin-S	$\mathsf{d}_S \mathsf{c}_{22S}\vert_{O(g^6)}$
2	$-768 + 112\zeta_3 - 160\zeta_5$
4	$-\frac{442765625}{3500658} + \frac{386}{27}\zeta_3 - \frac{400}{21}\zeta_5$
6	$-\frac{1183056555847}{88944075000} + \frac{48286}{37125}\zeta_3 - \frac{56}{33}\zeta_5$
8	$-\frac{1270649655622342732745039}{1075922954067591630000000} + \frac{1039202363}{9932422500}\zeta_3 - \frac{6088}{45045}\zeta_5$
10	$-\frac{7465848687069712820911408164847}{77747563297936585275804036000000} + \frac{8295615163}{1049947353000}\zeta_3 - \frac{2684}{264537}\zeta_5$

Surprisingly,

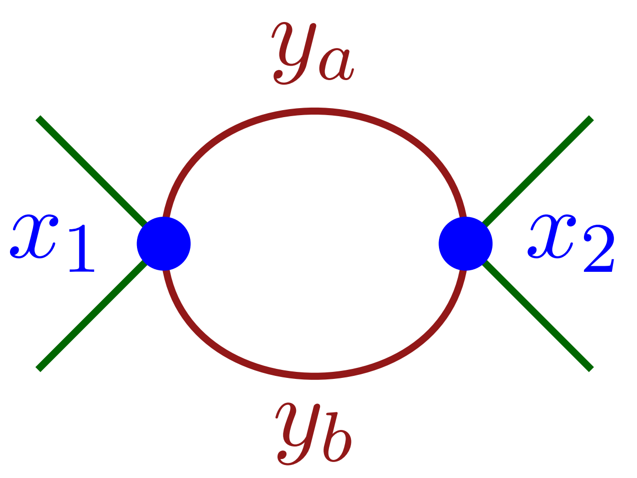
$$\mathsf{d}_S \mathsf{c}_{22S} = \mathsf{c}_{44S}^2$$

- Connection between the worldsheet g-function approach and the hexagon form factor approach

Loop Integrals in Expanding Universes

Giacomo Brunello, PhD student [in collaboration with: P. Benincasa, M. K . Mandal, P. Mastrolia, F. Vazão]

- **Cosmological Wavefunctions coefficients** as **Twisted Period Integrals:** [Arkani-Hamed, Benincasa] [Arkani-Hamed, Baumann, Hillman, Joyce, Lee, Pimentel] [Pokraka,De] [Benincasa, Vazão]



$$\Leftrightarrow \quad J = \int d\mathbf{x}\ \mathbf{x}^{\alpha} \int_\Gamma \mathscr{B}^\gamma(\mathbf{y},\mathbf{P}) \frac{d\mathbf{y}}{D_1 \cdots D_n} \qquad \text{where} \qquad \mathscr{B}\Big|_{\partial\Gamma} = 0$$

Twist
Degree 4 polynomial

Differential form φ

- Vector space?
- Geometrical structure?
- Space of functions?

- **Linear relations to decompose into a basis of Master Integrals:**

$$J = \sum_{i=1}^{\nu} c_i\, I_i \quad \Leftrightarrow \quad \langle \varphi | \Gamma] = \sum_{i=1}^{\nu} c_i\, \langle e_i | \Gamma]$$

- Algebraic Geometry methods for IBP identities

$$\sum_{a=1}^{n_y} \left(\partial_{y_a} \mathscr{B}\right) n_a = h_0 \mathscr{B}$$

- Sygyzy equations [Gluza, Kajda, Kosower] [Larsen, Zhang]
- Groebner bases
- FiniteFlow [Peraro]

- Intersection Theory [Mastrolia, Mizera] [G.B., Chestnov, Crisanti, Frellesvig, Mandal, Mastrolia] [Pokraka,De]

$$c_i \sim \langle \varphi | e_i \rangle$$

- Decomposition in terms of Intersection numbers [Mastrolia, Mizera]
- Relative cohomology [Caron-Huot, Pokraka]
- Polynomial division [Fontana, Peraro]

- **Canonical form of DEs satisfied by the MIs:** [Henn]

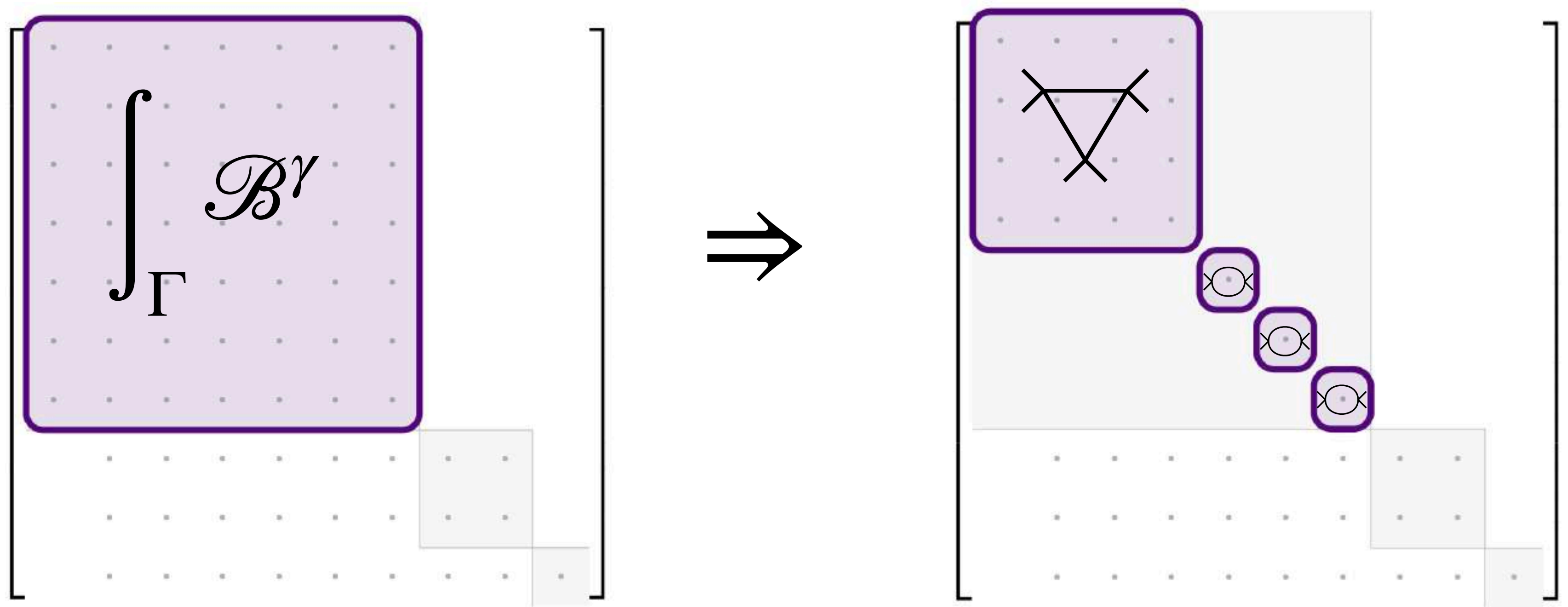
$$d\ \mathbf{I} = \epsilon\ d\hat{A}\ \mathbf{I} \qquad d = 3 + 2\epsilon$$

- **Solution in terms of Iterated integrals**

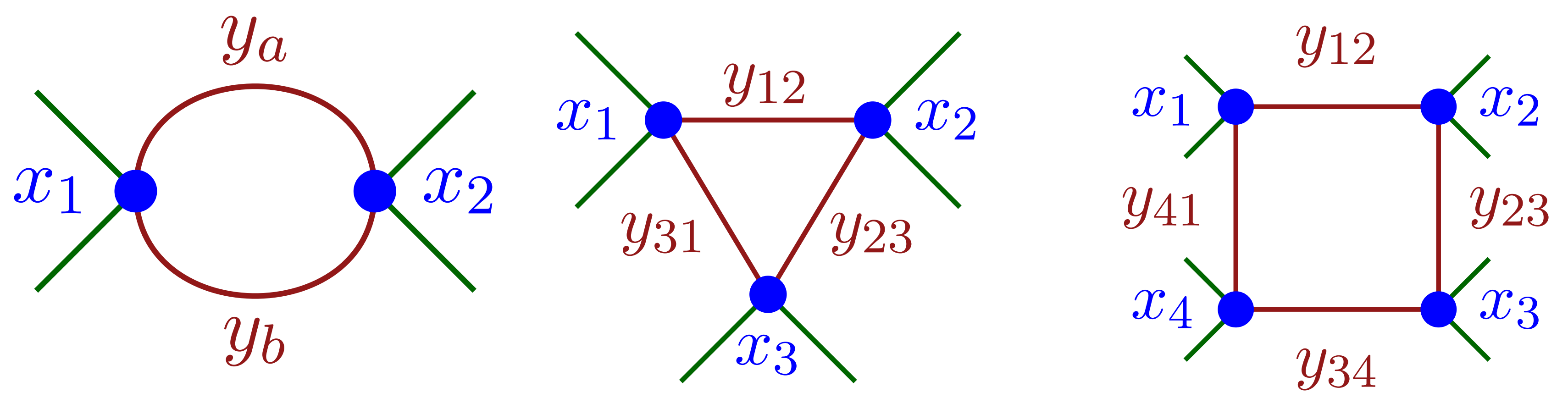
$$I = \mathbb{P}exp\left(\epsilon \int_\gamma d\hat{A}\right) \mathbf{I}\Big|_{\partial\gamma} \qquad d\hat{A} = \sum_i \mathbb{M}_i\ d\log(w_i)$$

- **BCs fixed via:**
  - Flat spacetime limit
  - Regularity
  - Method of regions

- **Symmetries of integrals appearing at the level of DEs:**



- **Which are the properties of 1-loop Cosmological Integrals?**



Thanks!

BROWN PHYSICS

INSTITUTE FOR ADVANCED STUDY

On Unitarity of Bespoke Amplitudes

He-Chen Weng, Brown University

Joint work with Rishabh Bhardwaj, Marcus Spradlin, and Anastasia Volovich

Amplitudes 2024 Gong Show

Construction of bespoke amplitude

- The bespoke amplitude [Cheung, Remmen '23] is a deformation of the Veneziano amplitude.

$$A^a_{\text{bespoke}} = \sum_{\alpha,\beta} A^a_V(\nu_\alpha(s), \nu_\beta(t))$$

- The deformation is generated by the roots of the spectral function

$$f(\mu, \nu) = P(\nu) - \mu Q(\nu) = 0$$

- The spectrum of the model is customizable $s_n = \frac{P(n)}{Q(n)}$
- The amplitude exhibits dual resonance

Partial wave unitarity at large n limit

- Unitarity implies that the residues of the amplitude must be non-negatively expanded on Gegenbauer polynomials (Legendre Polynomial for D=4).

- Utilizing the techniques developed in [Arkani-Hamed, Eberhardt, Huang, Mizera '22] [Bhardwaj, De, Spradlin, Volovich '22], we derived the asymptotic form for the partial wave coefficients in the large n limit.

Conclusion

- For the asymptotically non-linear spectrum, the bespoke amplitude is non-unitary.
- For the linear spectrum (linear shift of string spectrum), the mass gap must be non-positive.
- For the post-linear Regge spectrum, we can rule out region in the parameter space: $\kappa_1 > 3/2 \cup \delta + \kappa_1 > 0$.

$$s_n \sim (n+\delta) + \kappa_1 + \frac{\kappa_2}{n+\delta} + O\left(\frac{1}{n^2}\right)$$

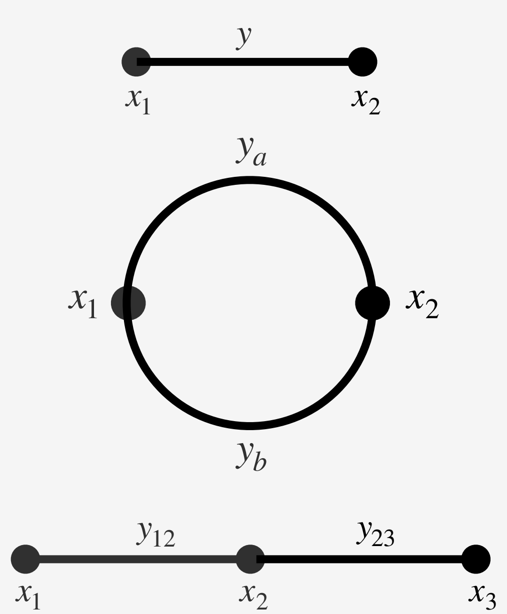
Divergent Structure of Cosmological Integrals

Francisco Vazão

Funded by the European Union
European Research Council

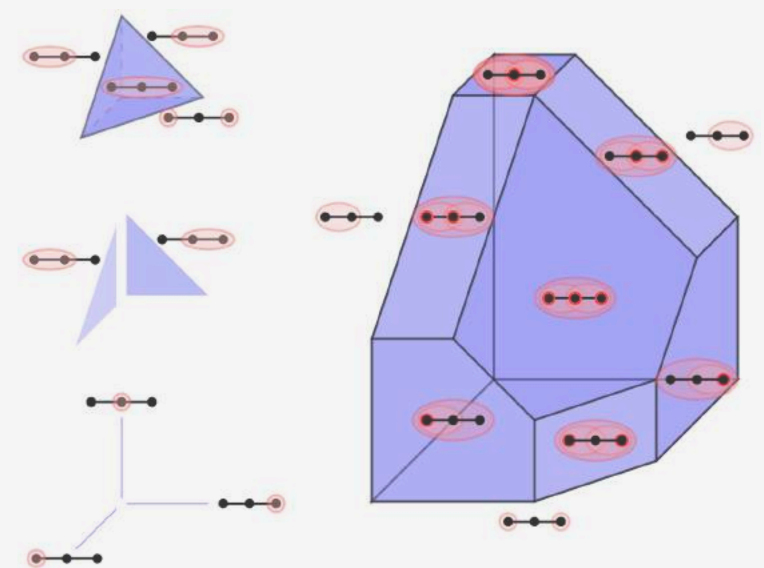
universe+

$$\mathcal{I}_{\mathcal{G}}[\alpha,\, \beta;\, \mathcal{X}] = \prod_{s\in\mathcal{V}}\left[\int_0^{+\infty}\prod_{s\in\mathcal{V}} dx_s\, x_s^{\alpha_s-1}\right]\int_\Gamma \prod_{e\in\mathcal{E}^{(L)}} dy_e\, y_e^{\beta_e-1}\, \mu_d(y_e;\mathcal{X})\, \frac{\mathfrak{n}_\delta(z,\, \mathcal{X})}{\prod_{\mathfrak{g}\subseteq\mathcal{G}}[q_{\mathfrak{g}}(z,\, \mathcal{X})]^{\tau_{\mathfrak{g}}}}$$

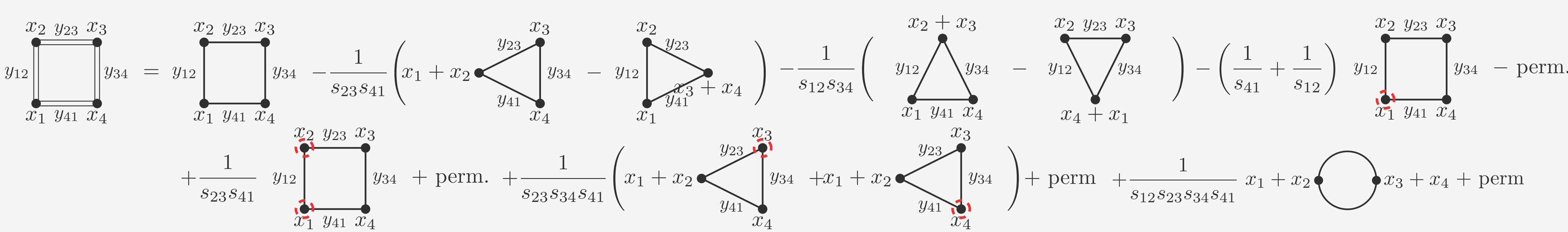


Examples of "Feynman Graphs"

Graph structure encodes the Newton Polytope



Newton Polytope: *Nestohedra*



Infrared-safe Computables

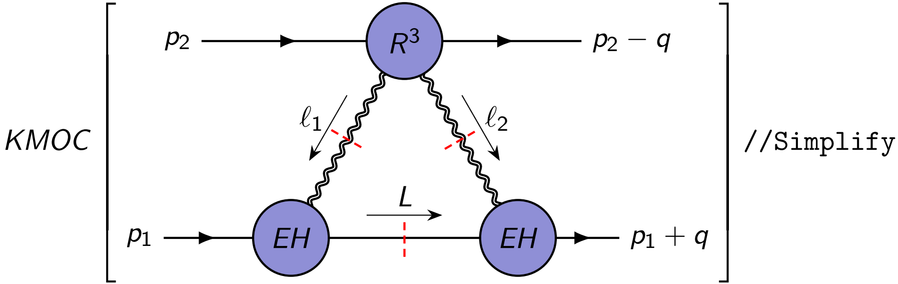
Spinning binary dynamics in cubic EFTs of gravity

Based on 2405.13826 with Andreas Brandhuber, Graham R. Brown, Paolo Pichini and Gabriele Travaglini

First relevant higher-derivative corrections to GR:

$$I_1 = R^{\alpha\beta}{}_{\mu\nu} R^{\mu\nu}{}_{\rho\sigma} R^{\rho\sigma}{}_{\alpha\beta}\ , \qquad G_3 = I_1 - 2R^{\mu\nu\alpha}{}_{\beta} R^{\beta\gamma}{}_{\nu\sigma} R^{\sigma}{}_{\mu\gamma\alpha}\ ,$$

(+ parity-odd versions)



Obtained the **impulse** Δp_1^μ and **spin-kick** Δa_1^μ.
Coming soon: the tree-level **waveform** from the 5-point amplitude.

An Area Law for Entanglement Entropy in Particle Scattering

Based on 2405.08056 with Ian Low

Zhewei Yin

Northwestern University and Argonne National Laboratory

June 12, 2024

$$\mathcal{E}_2 \doteq 2 I_0 \sigma_{\text{el}}$$

- $\mathcal{E}_2$ is the **linear entropy** for the **entanglement** between 2 particles in the final state of $2 \to 2$ scattering
- The result is of the leading order in the **plane wave limit**, i.e. for initial state **wave packets** approaching momentum eigenstates
- σ_{el} is the **total elastic cross section**; **non-perturbative** in coupling strength for any theory with at least 2 degrees of freedom
- I_0 is theory independent; size given by the inverse of the transverse **area** for the initial wave packets in the position space
- Can be generalized to the nth Tsallis and Rényi entropies by replacing the factor of 2 with $n/(n-1)$

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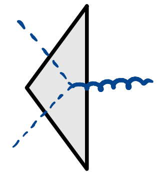
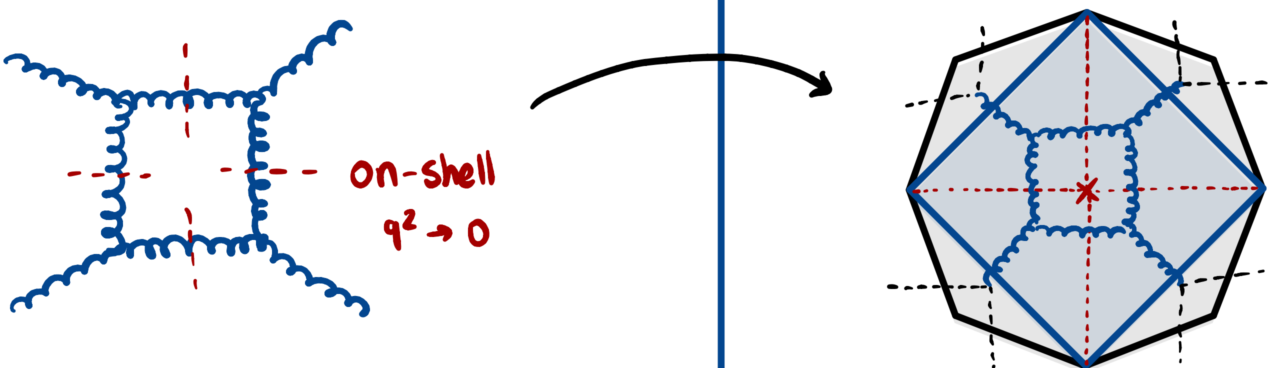
YM Leading Singularities from a Scaffolding Perspective



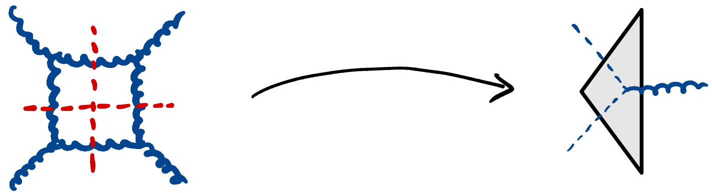
1 gluon = 2 collinear Scalars
(arXiv: 2401.00041)



has $\varepsilon . p$; $p . p$; $\varepsilon . \varepsilon$

Only has $X_{ij} = (p_i + \ldots + p_{j-1})^2$

YM Leading Singularities from a Scaffolding Perspective



1 gluon = 2 collinear Scalars
(arXiv: 2401.00041)

Why Scaffold?

* $\{\varepsilon.p, \; p.p, \; \varepsilon.\varepsilon\}$ are all $X_{i,j}$ [Variables of a Scalar Problem]

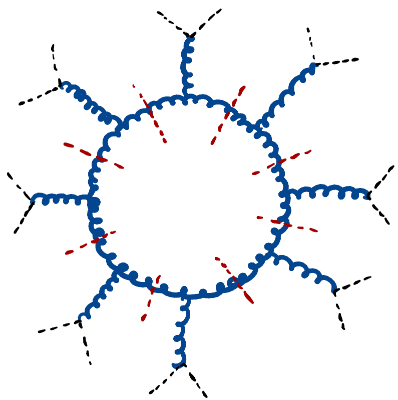
* D dimensional

* Gluing 3pt amplitudes only needs $\{+, \cdot, \partial_x\}$ INSTEAD OF $\eta_{\mu\nu}, \varepsilon_\nu, \ldots$

COMPLICATED LORENTZ STRUCTURES

YM Leading Singularities from a Scaffolding Perspective

Hint of magic ?



~ 165 000 terms !

BUT

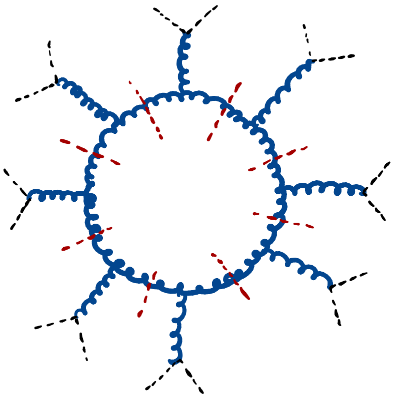
$$LS_{oct}[X_{ij} = X \, ; \, Y_i = Y] = \underbrace{(2-D)\,Y^8 - X^8 + 8X^7Y - 20X^6Y^2 + 16X^5Y^3 - 2X^4Y^4}$$

NOT random → has a recursive structure

→ Lucas Polynomials

YM Leading Singularities from a Scaffolding Perspective

Hint of magic ?



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→ Lucas Polynomials

Thank You !

Funded by the European Union | erc European Research Council | universe+

Sérgio Carrôlo | Amplitudes 2024

MAX-PLANCK-INSTITUT FÜR PHYSIK

Efficient sampling of large Feynman graphs in ϕ^4 theory

Paul-Hermann Balduf, U of Waterloo Math & Perimeter Institute

Amplitudes 2024, Princeton

based on JHEP 11 (2023) 160
and arXiv 2403.16217 (with Kimia Shaban)

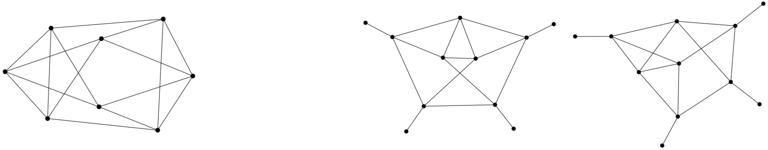
More Slides, references, dataset etc. available from paulbalduf.com/research

Background

- Massless scalar ϕ^4-theory if 4 dimensions

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{\lambda}{4!}\left(\phi^2\right)^2.$$

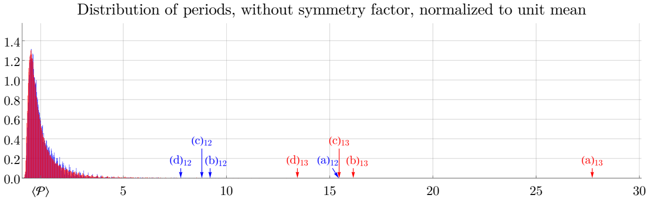
- We want to compute the beta function of this theory $\Rightarrow$ need vertex-type graphs (arising from vacuum graphs upon removing one vertex).



- We restrict ourselves to subdivergence-free (=primitive) graphs ("**Periods**") [Broadhurst and Kreimer 1995; Schnetz 2010; many others]

Distribution of Feynman integrals

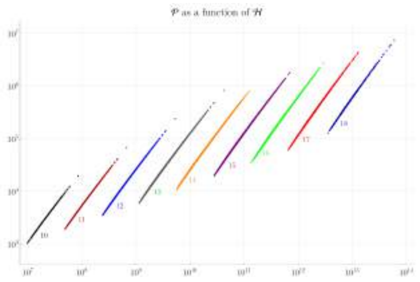
- Compute periods numerically [Borinsky 2023; Borinsky, Munch, and Tellander 2023], exploit various symmetries [Schnetz 2010; Panzer 2022; Hu et al. 2022].
- 2 Problems:
  - **Standard deviation of distribution is large**, similar to mean ⇒ uniform sample has large statistical uncertainty [B. 2023]. E.g. for 3 significant digits ($\Delta_{\text{samp}} \lesssim 0.1\%$) we need sample size $n \approx 10^6$.

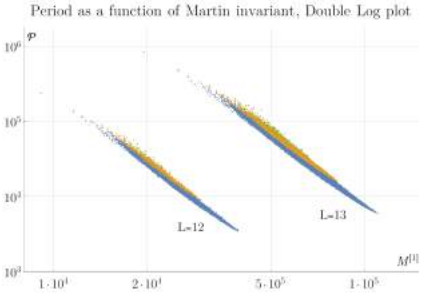


  - **Number of graphs grows factorially**, 750k graphs at 13 loops, 950M at 16 loops [Cvitanović, Lautrup, and Pearson 1978; Borinsky 2017] ⇒ impossible to compute all of them, need random sample, *Monte Carlo* algorithm.
- Solution: Importance sampling of graphs.

Approximating the Feynman integral

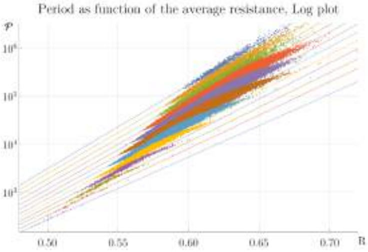
- Need quantity that is strongly correlated with Feynman integral, and fast to compute.
- Examined ≈ 150 different graph-theoretical quantities empirically, for a data set of ≈ $1.5M$ periods with $L \leq 18$ loops [B. 2023, available from my website].
- Strongly correlated: *Hepp bound* $\mathcal{H}(G)$ [Hepp 1966; Panzer 2022] (tropicalization of period integral upon sector decomposition), *Martin invariant* $M^{[k]}$ [Panzer and Yeats 2023] (derivative of $O(N)$ symmetry factor at $N = -2$).

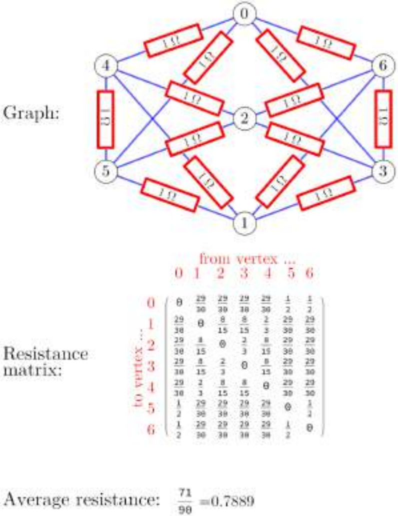




Average resistance (Kirchhoff index)

- Assign unit electrical resistance to every edge. Resistance r_{v_i,v_j} between vertices v_i and v_j. *Kirchhoff index* = average resistance between pairs of vertices.
- Extremely fast to compute due to matrix linear algebra operations ($\sim 100\mu$s per graph), predicts period to $\approx 5\%$ accuracy.

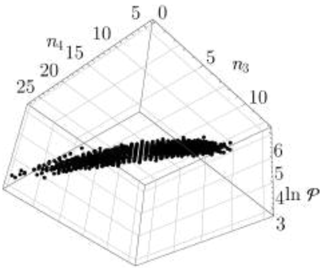


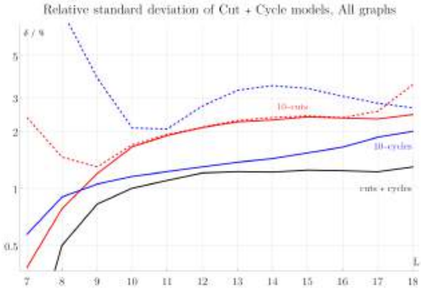


to \ from	0	1	2	3	4	5	6
0	0	$\frac{29}{30}$	$\frac{29}{30}$	$\frac{29}{30}$	$\frac{29}{30}$	$\frac{1}{2}$	$\frac{1}{2}$
1	$\frac{29}{30}$	0	$\frac{8}{15}$	$\frac{8}{15}$	$\frac{2}{3}$	$\frac{29}{30}$	$\frac{29}{30}$
2	$\frac{29}{30}$	$\frac{8}{15}$	0	$\frac{2}{3}$	$\frac{8}{15}$	$\frac{29}{30}$	$\frac{29}{30}$
3	$\frac{29}{30}$	$\frac{8}{15}$	$\frac{2}{3}$	0	$\frac{8}{15}$	$\frac{29}{30}$	$\frac{29}{30}$
4	$\frac{29}{30}$	$\frac{2}{3}$	$\frac{8}{15}$	$\frac{8}{15}$	0	$\frac{29}{30}$	$\frac{29}{30}$
5	$\frac{1}{2}$	$\frac{29}{30}$	$\frac{29}{30}$	$\frac{29}{30}$	$\frac{29}{30}$	0	$\frac{1}{2}$
6	$\frac{1}{2}$	$\frac{29}{30}$	$\frac{29}{30}$	$\frac{29}{30}$	$\frac{29}{30}$	$\frac{1}{2}$	0

Cut & Cycle counts

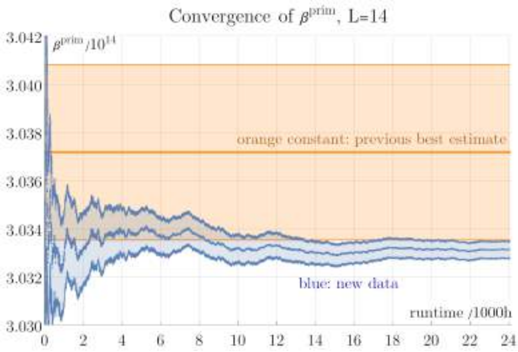
- Count number n_j of cycles of length $j \in \mathbb{N}$ (this is not the loop number).
- Count number c_j of vertex-induced j-edge cuts.
- Period is (almost) multi-linear function of n_j and $\ln(c_j)$, include $j \leq 10$.
- Average error $< 1.5\%$, takes $\sim 20\,\text{ms}$ per graph.





Example results: Primitive beta function for $L = 14$

- Reached 4 significant digits (120ppm standard deviation) after 24k CPU core h ($<$ 2 weeks walltime).



- Previous work with uniform random sampling took 400k CPU core h for 1063ppm.
- $\Rightarrow$ **Weighted sampling is $\approx 1000\times$ faster than uniform random sampling**, or reaches $\approx 35\times$ the accuracy at the same runtime.

Conclusion: Weighted Monte-Carlo sampling works!

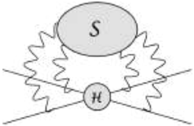
- Subdivergence-free Feynman integrals are correlated to various properties of the graph. Can approximate their value to $\sim 1\%$ in less than 1 second.
- Using these approximations in weighted sampling reduces the time for numerically computing the primitive beta function by a factor ~ 1000.
- Dataset of 2 million Feynman integrals available from paulbalduf.com/research.

Thank you!

Factorization and Wilson lines

Starting point: Factorization theorem (SCET, ...)

$$\mathcal{A}(1,2,3,4) = \mathcal{H}(1,2,3,4) \otimes \mathcal{S}(1,2,3,4)$$



LP in soft scale λ of $\mathcal{S}$ is generated by Wilson line operators $W(i) = \mathcal{P}\exp\left\{i\int_{\gamma_i} dx^\mu A_\mu^a \mathfrak{t}_a\right\}$
N^{n}LP of $\mathcal{S}$ is generated by Generalized Wilson line operators

$$\widetilde{W}(i) = \mathcal{P}\exp\left\{i\int_{\gamma_i} dx^\mu A_\mu^a \mathfrak{t}_a + \textbf{subleading}\right\}$$

Exponential can be derived from quantized particles

Applications for gravity

Recipe: Take worldline particle in fixed representation of Lorentz group and couple with curved background spacetime

Nice: $\widetilde{W}$ **automatically** organizes into powers of the soft scale

$$h_{\mu\nu}^{\text{soft}} = \int_{\text{soft}} \frac{d^D k}{(2\pi)^D} \tilde{h}_{\mu\nu}(k) e^{-ikx} \quad \text{where} \quad k \sim \lambda$$

Compare to $k = \hbar \bar{k}$ for classical limit $\rightarrow$ due to softness of $\widetilde{W}$ we automatically cover almost only potential graviton modes (**methods of regions**)

One can define $\tilde{W}^{\text{class.}} \subseteq \widetilde{W}$, such that (**automatically exponentiates iteration terms**)

$$\mathcal{A}^{\text{class.}} = \int \mathcal{D}[h_{\mu\nu}] \prod_{i=1}^{4} \tilde{W}^{\text{class.}}(i)$$

Domenico Bonocore, Anna Kulesza, and Johannes Pirsch. "Classical and quantum gravitational scattering with Generalized Wilson Lines." In: *JHEP* 03 (2022), p. 147. DOI: 10.1007/JHEP03(2022)147. arXiv: 2112.02009 [hep-th] and upcoming work

University of Southampton

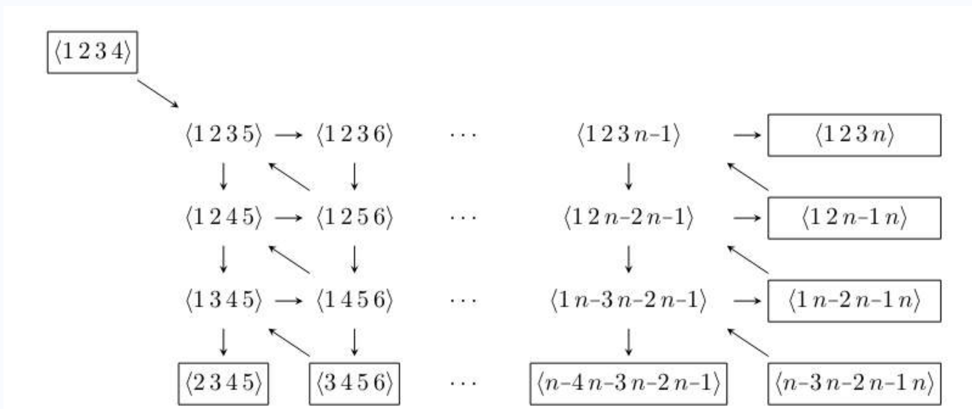
Cluster Algebras Beyond Dual Conformal Symmetry

Rowan Wright

Supervised by James Drummond and Ömer Gürdoğan

Grassmannian Cluster Algebras

- **Grassmannian cluster algebras** provide the symbol alphabet of amplitudes in **planar N=4 super Yang-Mills.**



- **Cluster adjacency** constrains consecutive discontinuities**.**
- Kinematics in terms of **Plücker coordinates** -- a consequence of **dual conformal symmetry**.

$$\langle ijkl\rangle \equiv \langle Z_i Z_j Z_k Z_l\rangle$$

Beyond the Dual Conformal Case

- **Plücker coordinates** no longer sufficient to describe the kinematics for **non-dual conformal observables**.

$$\langle ij \rangle \equiv \lambda_i^{\alpha} \lambda_{j,\alpha} \qquad [ij] \equiv \tilde{\lambda}_{i,\dot{\alpha}} \tilde{\lambda}_j^{\dot{\alpha}}$$

- By introducing the **infinity bi-twistor**, can still use four-brackets to parametrize the kinematics.

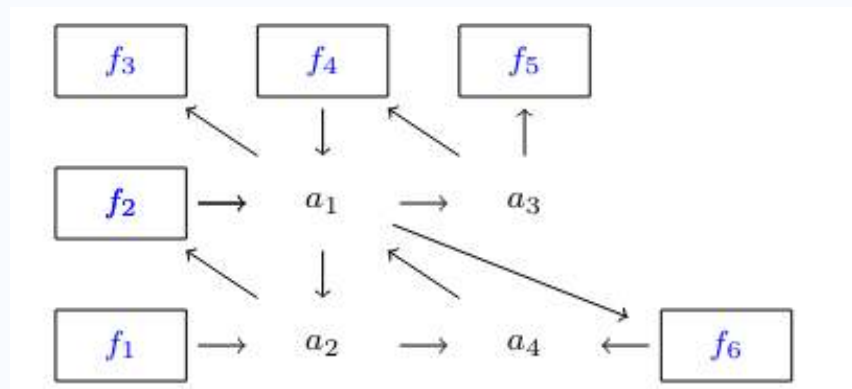
- e.g. for **five-point massless scattering**,

$$\langle ij67 \rangle \equiv \langle ij \rangle$$

Partial Flag Varieties

- For five-point massless scattering, need a cluster algebra where the polynomials in Plückers have 6 and 7 together, or not at all – **partial flag variety,** e.g. F(2,4,5).

$$\{a_1,\ldots,a_4\} = \{p_{14}p_{1235} - p_{15}p_{1234}, p_{1345}, p_{1235}, p_{1245}\},$$
$$\{f_1,\ldots,f_6\} = \{p_{2345}, p_{34}p_{1235} - p_{35}p_{1234}, p_{45}, p_{15}, p_{1234}, p_{12}\}.$$



- Can be embedded in Gr(4,7).

Application

- **Symbol alphabet** - **25 out of 26 letters** of the **planar two-loop pentagon alphabet** **are recovered** once completing F(2,4,5)'s A-coordinates under permutations on particle labels
- **Adjacencies** – F(2,4,5) predicts certain symbol letters cannot appear adjacently. **Obeyed by all known planar two-loop five-point data.**
- **Triples rule** – F(2,4,5) **predicts what sits between the symbol letters forbidden to appear adjacently**, as for dual conformal case!

Ongoing work

- **Bootstrap calculations** – can these observations be used to bootstrap three-loop five-point observables, e.g. pentagonal Wilson loop with Lagrangian operator insertion at **three loops**?
- Does a similar story hold for other observables e.g. **form factors with a massive operator**, or **six-point massless scattering**? Some evidence in favour!
- What is the precise role played by **permutations**?

Thank you for listening!

Loops of loops Expansion in the Amplituhedron

Amplitudes 2024, Institute for Advanced Study

Taro V. Brown

Center for Quantum Mathematics and Physics, UC Davis

Based on [2312.1773] with U. Oktem, S. Parajape, J. Trnka

June 12, 2024

Motivation

- Goal is to calculate the n-point L-loop amplitude

- Approach a simpler problem:

1) Restrict to planar $\mathcal{N}=4$ sYM at 4-points

2) Consider the logarithm $\ln M = \int \widetilde{\Omega}_L$

3) At L-loop $\ln M = \frac{\gamma_{cusp}}{\epsilon^2} + \mathcal{O}\left(\frac{1}{\epsilon^2}\right)$

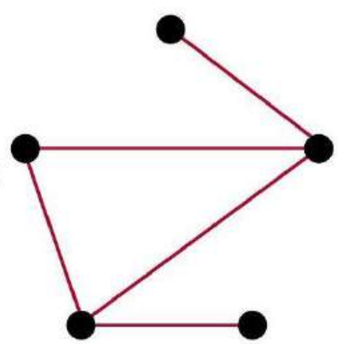
4) Each integration comes with $1/\epsilon^2$

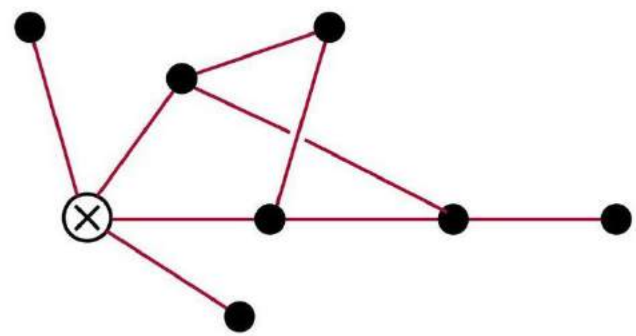
5) Define IR finite function $\mathcal{F}_L(z) = \int \widetilde{\Omega}_L$

Amplituhedron

- F can be calculated by doing an expansion in *negative* geometries, and leaving one marked point (loop unintegrated)

$$\widetilde{\Omega}_L = \sum_G (-1)^{E(G)}$$





Punchline

- Result is at all-loop order but is an expansion in ”loops” (cycles) in negative geometries.

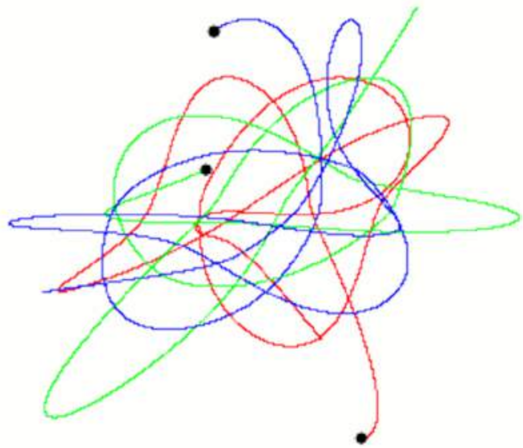
- Tree results, [Arkani-Hamed, Henn, Trnka]

	g^2	g^4	g^6	g^8	g^{10}	g^{12}	g^{14}	$\cdots$
$r_{\text{cusp/tree}} =$	1	1	0.92	0.83	0.74	0.63	0.53	$\cdots$

- Using cuts we add 1-cycle results

Thank you for your attention!

UCLA Mani L. Bhaumik Institute for Theoretical Physics



THE HIERARCHICAL 3-BODY PROBLEM AT 2PM

Anna Wolz

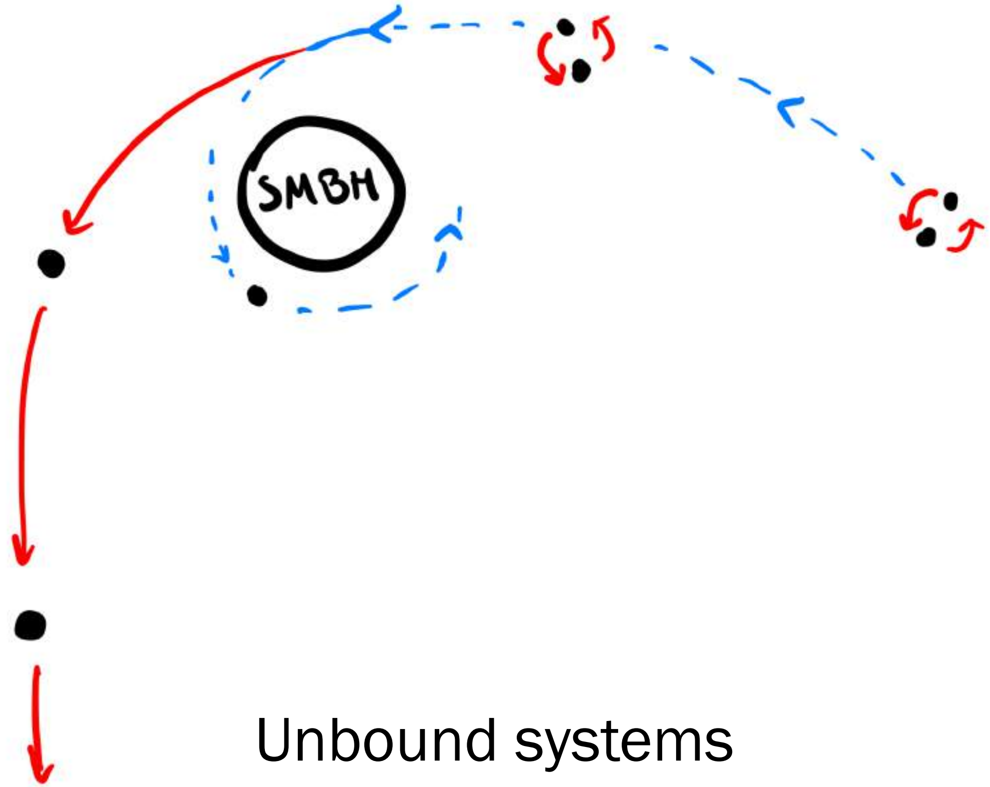
Amplitudes 2024

Based on:
24xx.xxxxx with Mikhail Solon
2208.02281 Callum Jones and Mikhail Solon

Goal

Conservative potential for hierarchical triples $V^{(3)}(\{\boldsymbol{p}, \boldsymbol{r}\})$ at 2PM

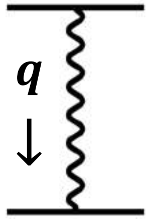
Bound systems



Unbound systems

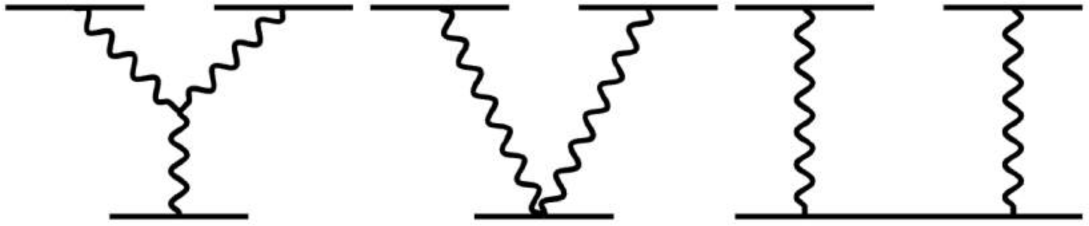
Method

- Connection between 2-2 scattering amplitudes and binary dynamics



$+\cdots$ — Classical limit + EFT / Fourier transform $\boldsymbol{q} \to \boldsymbol{r}$ → $V^{(2)} = \frac{Gm_1m_2}{r} + \cdots$

- Same for 3-body dynamics (but more complicated)



Classical limit + EFT / Fourier transform: hierarchical limit using method of regions → $V^{(3)}$

Amplitudes ⇒ 3-body potential in hierarchical limit at 2PM

The KLT kernel in twistor space

Sonja Klisch

School of Mathematics, University of Edinburgh

Amplitudes 2024 Gong Show, 12/06/2024

Based on a new paper with T. Adamo

The KLT kernel in twistor space

Out now!

- We find the double copy structure between the RSVW formula and the Cachazo-Skinner formula for N^{d-1}MHV scattering

$$\mathcal{A}^{\mathrm{YM}}_{n,d}[\sigma] = \int \mathrm{d}\mu_d \, |\tilde{\mathbf{g}}|^4 \, \mathrm{PT}[\sigma] \prod_i a_i^{\pm}(Z)$$

$$\mathcal{M}^{\mathrm{GR}}_{n,d} = \int \mathrm{d}\mu_d \, |\tilde{\mathbf{h}}|^8 \, \det{}'(\mathbb{H}) \det{}'(\mathbb{H}) \prod_i h_i^{\pm}(Z)$$

- This relates the integrands through a *helicity-graded* momentum kernel valued on maps $Z : \mathbb{PT} \to \mathbb{P}^1$

$$\det{}'(\mathbb{H}) \det{}'(\mathbb{H}) = \sum_{\substack{\tilde{\sigma},\tilde{\rho}\in S_d \\ \sigma,\rho\in S_{n-d-2}}} \mathrm{PT}[\tilde{\sigma}\,\sigma] \underbrace{S_{n,d}[\tilde{\sigma}\,\sigma|\tilde{\rho}^T\rho]}_{\substack{\text{kernel in} \\ \text{twistor space}}} \mathrm{PT}[\tilde{\rho}^T\rho]$$

The KLT kernel in twistor space

Out now!

$$\det{}'(\mathbb{H})\,\det{}'(\mathbb{H}) = \sum_{\substack{\tilde{\sigma},\tilde{\rho}\in S_d \\ \sigma,\rho\in S_{n-d-2}}} \mathrm{PT}[\tilde{\sigma}\,\sigma]\; \underbrace{S_{n,d}[\tilde{\sigma}\,\sigma|\tilde{\rho}^T\rho]}_{\text{kernel in twistor space}}\; \mathrm{PT}[\tilde{\rho}^T\rho]$$

- We prove that the inverse of the kernel is a twistor space *integrand* for bi-adjoint scalar (BAS) theory. This yields a new representation of the BAS amplitude

$$m_n[\tilde{\sigma}\,\sigma|\tilde{\rho}^T\rho] = \int \mathrm{d}\mu_d\, S^{-1}_{n,d}[\tilde{\sigma}\,\sigma|\tilde{\rho}^T\rho]\,\prod_i \phi_i(Z)$$

The KLT kernel in twistor space

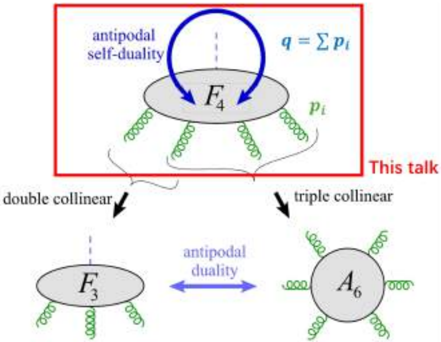
Out now!

Ask me about:

- How can BAS care about degree and chirality?
- Implications for double copy on backgrounds: AdS and self-dual radiative backgrounds?
- Connection to the field theory KLT and CHY double copy?

Two-Loop Four-Gluon Form Factor: antipodal duality and function level information

- Symbols encode iterated log-differentials: $F^x d \ln x \to F^x \otimes x$
- Antipodal self-duality: $a_1 \otimes \ldots \otimes a_m \to (-1)^m g(a_m) \otimes \ldots \otimes g(a_1)$
- F_4 is now bootstrapped to function level (*Dixon, SX, 2406.nnnnn*)



F_4 depends on 8 dimensionless ratios

$$u_i = \frac{s_{i,i+1}}{q^2}$$

$$v_i = \frac{s_{i,i+1,i+2}}{q^2}$$

5 of them are independent, e.g.

$$(u_1, u_2, u_3, v_1, v_2)$$

Figure: Antipodal self-duality of F_4. Relations to 3-gluon form factor and 6-gluon amplitude. F_3 and A_6 are dual to each other.[1, 2]

From symbol to function

- The 3-parameter "rational" kinematics, defined by $u_2 \to v_1 v_2, u_1 \to 0$, rationalizes all letters in the symbol alphabet, and allows a simple G-function representation.

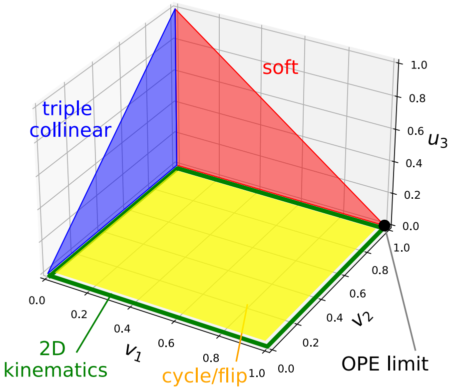


Figure: The rational kinematics interpolates between soft/collinear limits and fixes the function level information.

Numerics

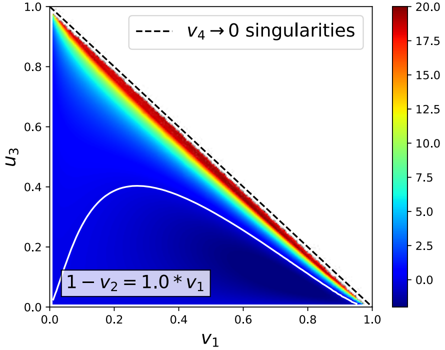


Figure: Numerical values of the remainder function on a slice of rational kinematics.

$u_i \otimes \zeta_3 \leftrightarrow \zeta_3 \otimes g(u_i)$ duality

- Antipodal duality is checked beyond the symbol for $u_i \otimes \zeta_3$ and $v_i \otimes \zeta_3$ terms. $(u_i = s_{i,i+1}/q^2, v_i = s_{i,i+1,i+2}/q^2)$

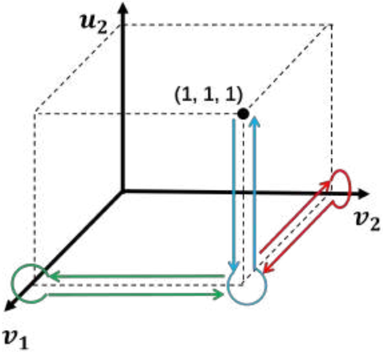


Figure: integrating along paths off the rational surface to obtain discontinuities.

References

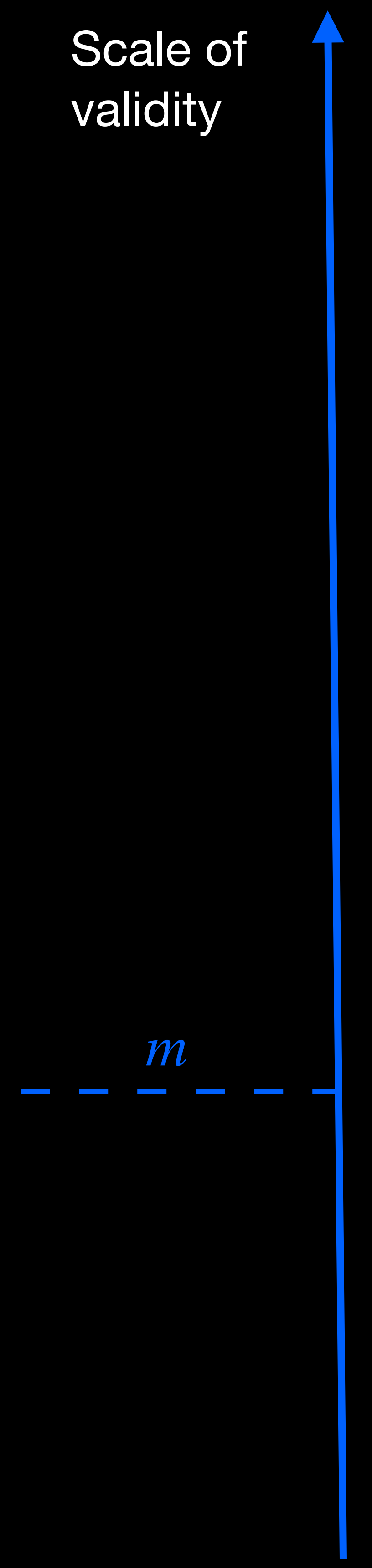
Lance J. Dixon, Ömer Gürdoğan, Andrew J. McLeod, and Matthias Wilhelm.
Folding Amplitudes into Form Factors: An Antipodal Duality.
Phys. Rev. Lett., 128(11):111602, 2022.

Lance J. Dixon, Ömer Gürdoğan, Yu-Ting Liu, Andrew J. McLeod, and Matthias Wilhelm.
Antipodal Self-Duality for a Four-Particle Form Factor.
Phys. Rev. Lett., 130(11):111601, 2023.

Integrating Out Heavy Multipoles in EM and GR

Edoardo Alviani, Adam Falkowski

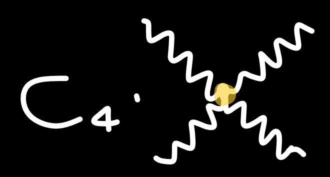
Integrating Out Heavy Multipoles in EM and GR



EFT

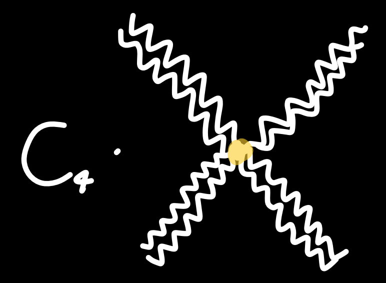
Euler Heisenberg

$$\mathscr{L} \supset \frac{C_1}{16} F_{\mu\nu}F^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} + \frac{C_2}{16} F_{\mu\nu}\tilde{F}^{\mu\nu}F_{\rho\sigma}\tilde{F}^{\rho\sigma}$$

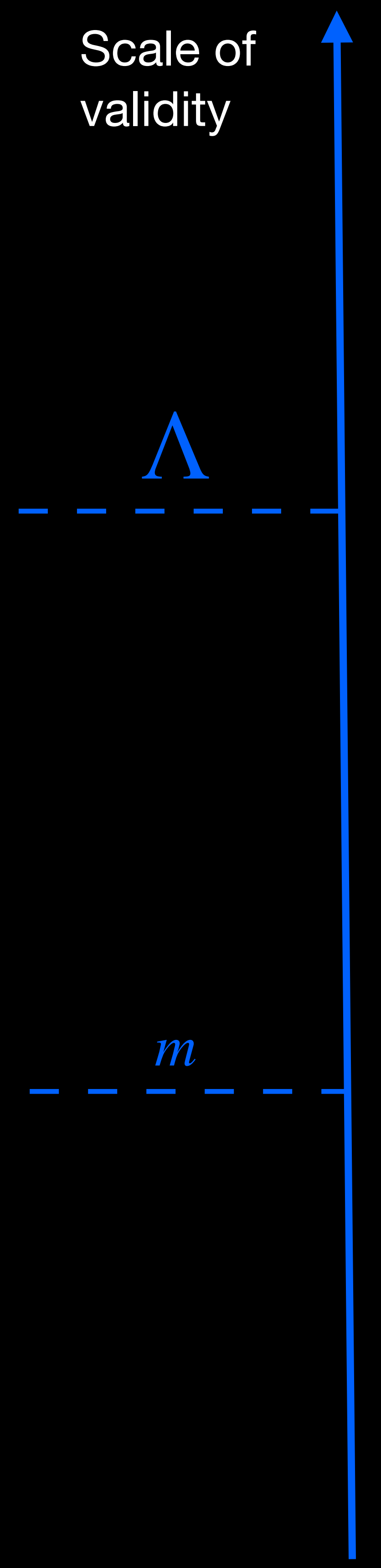


GREFT

$$\mathscr{L} \supset \frac{C_3}{6} C_{\mu\nu\alpha\beta}C_{\alpha\beta\rho\sigma}C_{\rho\sigma\mu\nu} + \frac{C_{4,1}}{8}\left(C_{\mu\nu\alpha\beta}C_{\mu\nu\alpha\beta}\right)^2 + \frac{C_{4,2}}{8}\left(C_{\mu\nu\alpha\beta}\tilde{C}_{\mu\nu\alpha\beta}\right)^2$$

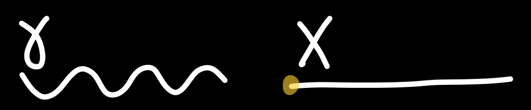


Integrating Out Heavy Multipoles in EM and GR

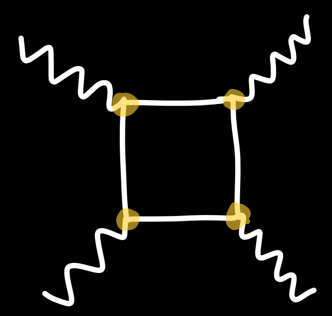


Partially UV complete theory

Photon + Multipole

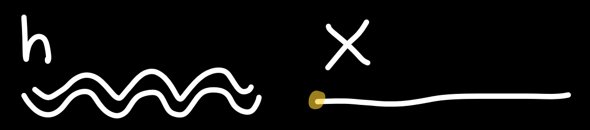


Mass m
Spin $S \leq 1$
Multipoles g_i

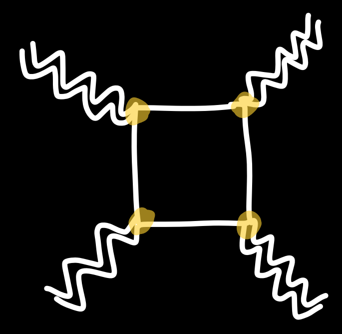


$m \to +\infty$

Graviton + Multipole



Mass m
Spin $S \leq 2$
Multipoles g_i



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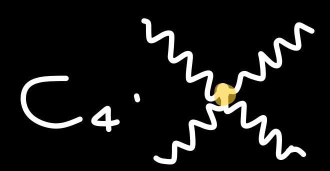

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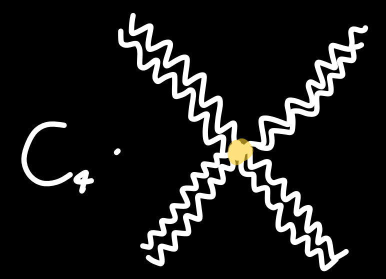


Edoardo Alviani

Integrating Out Heavy Multipoles in EM and GR

Scale of validity

Λ

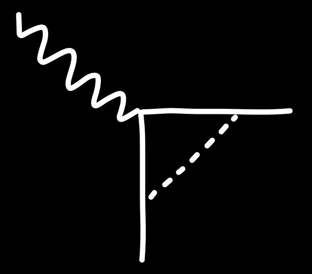
m

UV complete theory

Photon + Massive particles

γ ~~~ X —— m

Y - - - - - - - Λ

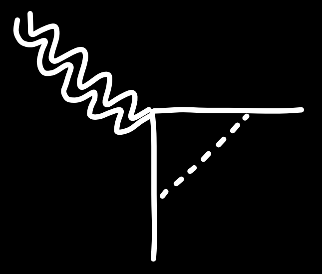


$\Lambda \to +\infty$

Graviton + Massive particles

h ≈≈≈ X —— m

Y - - - - - - - Λ



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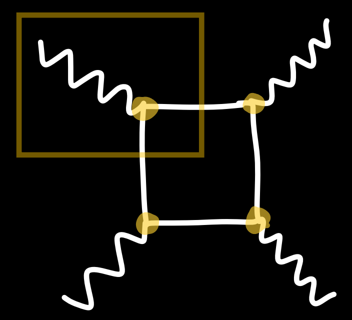


$m \to +\infty$

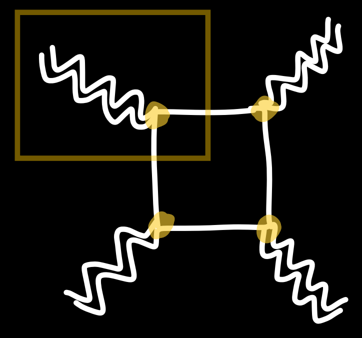
Graviton + Multipole

h ≈≈≈ X ——

Mass m

Spin $S \leq 2$

Multipoles g_i



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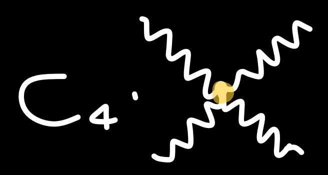

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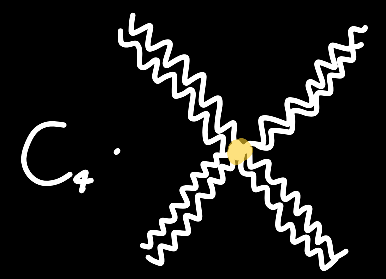

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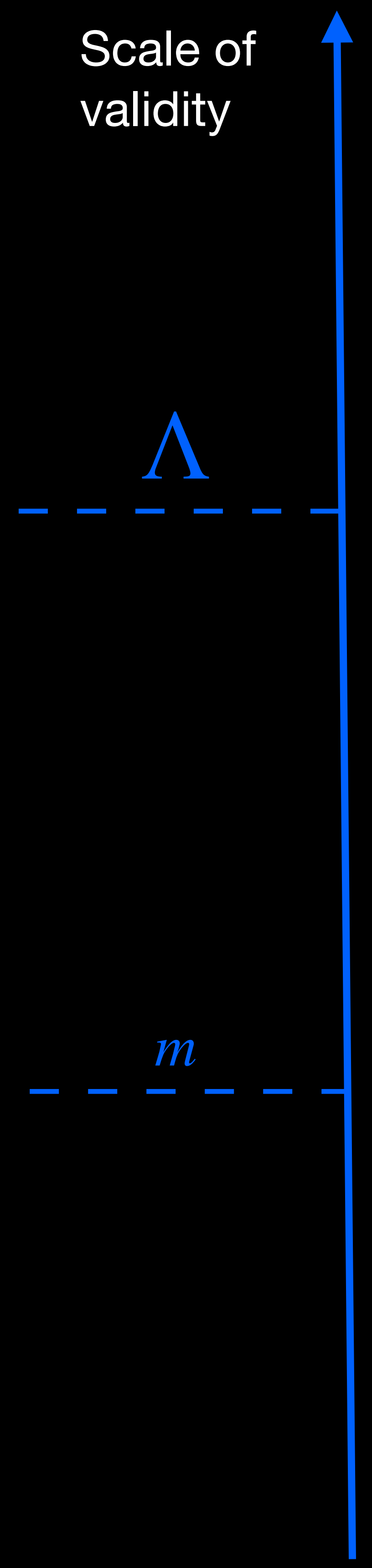

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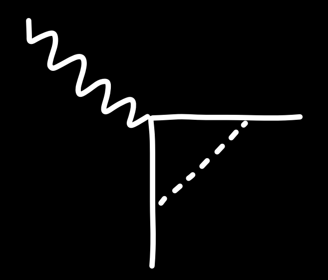

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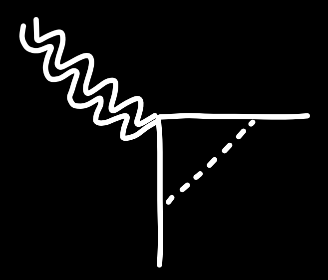

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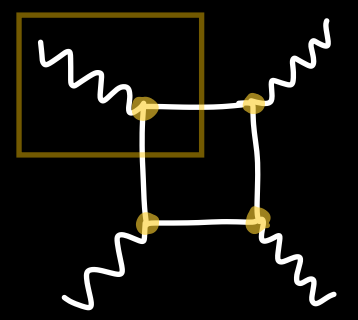

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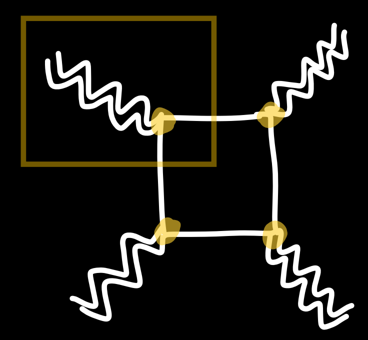

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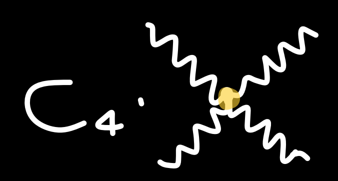

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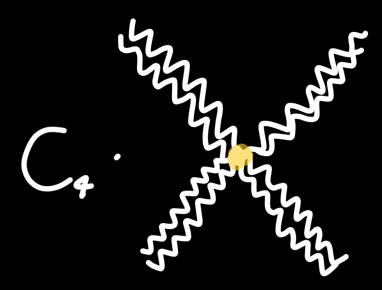
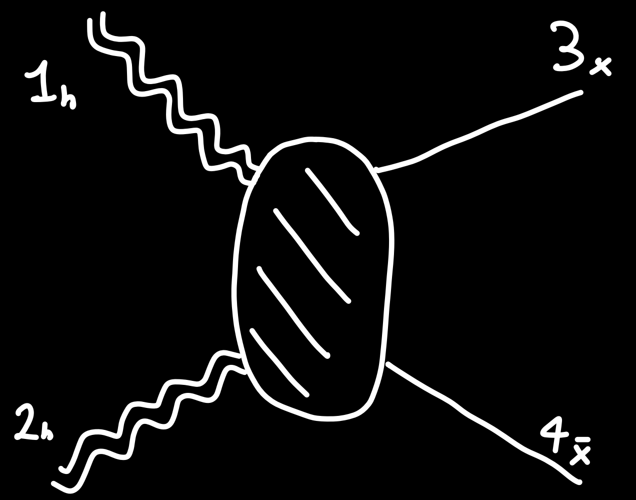
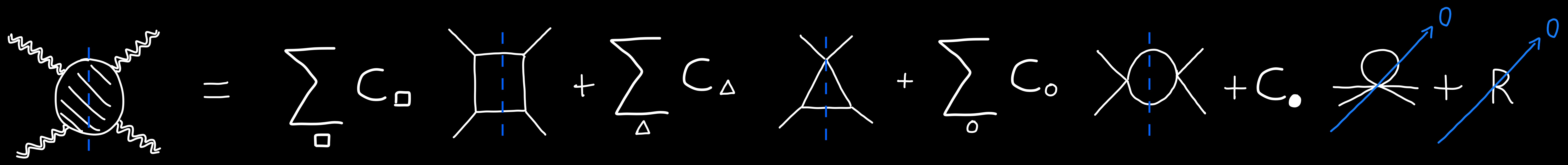




$$M\left(1_h^-, 2_h^+, 3_X, 4_{\bar{X}}\right) = \frac{\langle 1|p_3|2]^{4-2S}}{M_{Pl}^2 s(t-m^2)(u-m^2)}\left[\left(\langle 13\rangle[24] + \langle 14\rangle[23]\right)^{2S} + (-1)^{2S}\sum_{n=2}^{2S} \frac{g_n\left([12]\langle 13\rangle\langle 14\rangle\right)^n + \bar{g}_n\left(\langle 12\rangle[23][24]\right)^n}{m}\left(\langle 13\rangle[24] + \langle 14\rangle[23]\right)^{2S-n}\right]$$

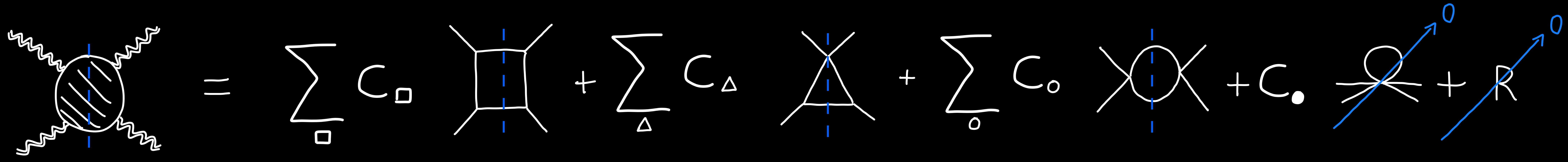
Edoardo Alviani

Matching is done through general unitarity



All C are found with up to two double cuts, by matching independent functional structures.

Matching is done through general unitarity



All C are found with up to two double cuts, by matching independent functional structures.

RESULTS

- Find Compton amplitudes for generic spins and non-minimal coupling.
- Obtain the Wilson coefficients corresponding to a new class of UV completions

$$C_{4,1}^{\gamma,S=1} = \frac{g_0^3(87g_0 + 440g_1 + 840g_2)}{16\pi^2 \cdot 30m^4} \qquad C_{4,1}^{h,S=1} = \frac{(12 + 245g_2)}{16\pi^2 \cdot 2100m^4}$$

- Find which class of multipoles is coherent with existing positivity constraints.

Corners and Islands in the S-matrix Bootstrap of the Open Superstring

Justin Berman

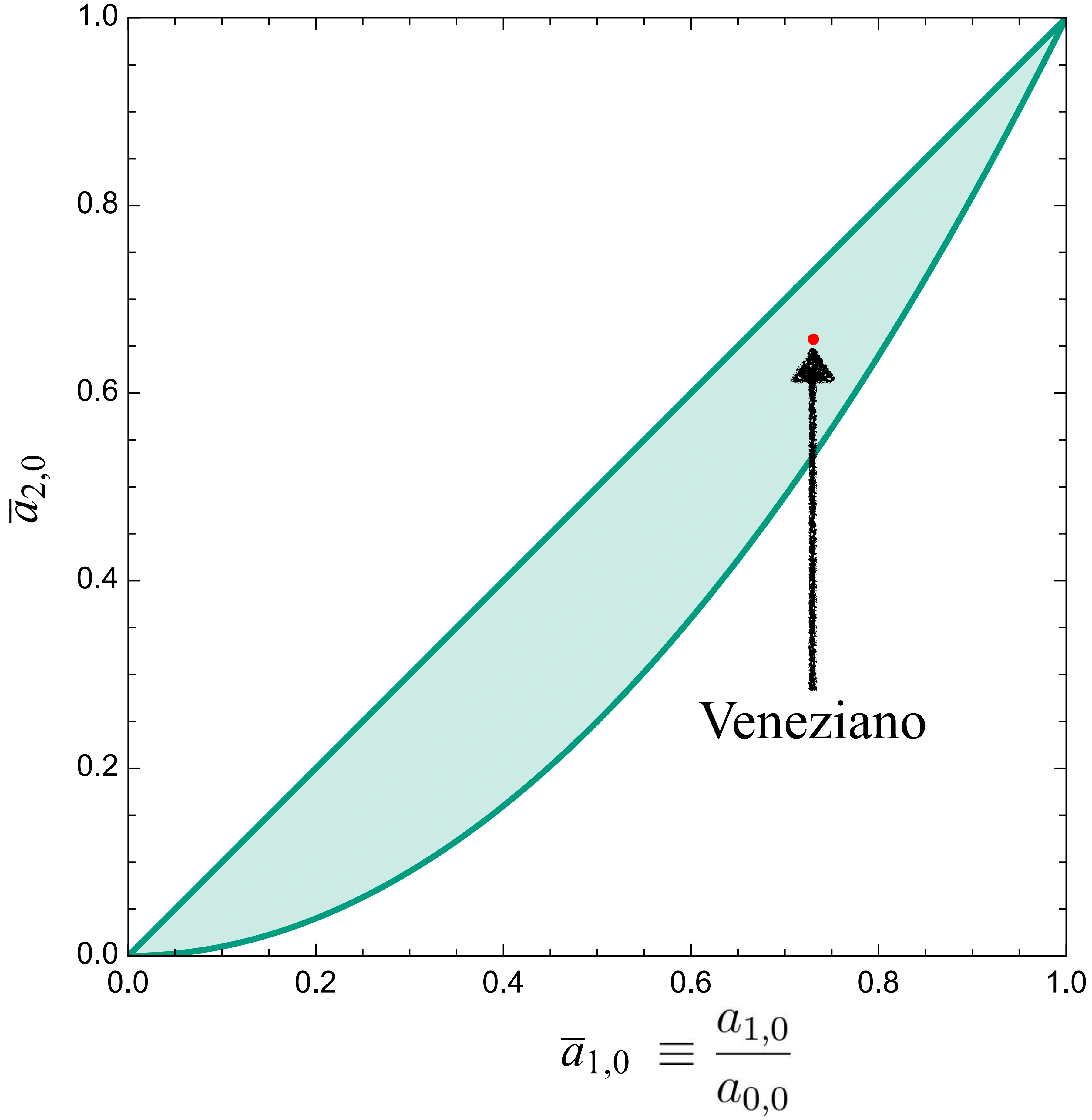
Based on arXiv: 2310.10729 w/ Henriette Elvang and Aidan Herderschee
and arXiv: 2406.03543 with Henriette Elvang

Bounding low energy Wilson Coefficients

$$A(s,u) = -\frac{s}{u} + s^2\Big[a_{0,0} + a_{1,0}(s+u) + a_{2,0}(s^2+u^2) + a_{2,1}su + \dots\Big]$$

<u>Universal Bounds</u>

1. Partial Wave Decomposition
2. Unitarity (Positivity)
3. Froissart Bound
4. Maximal Supersymmetry



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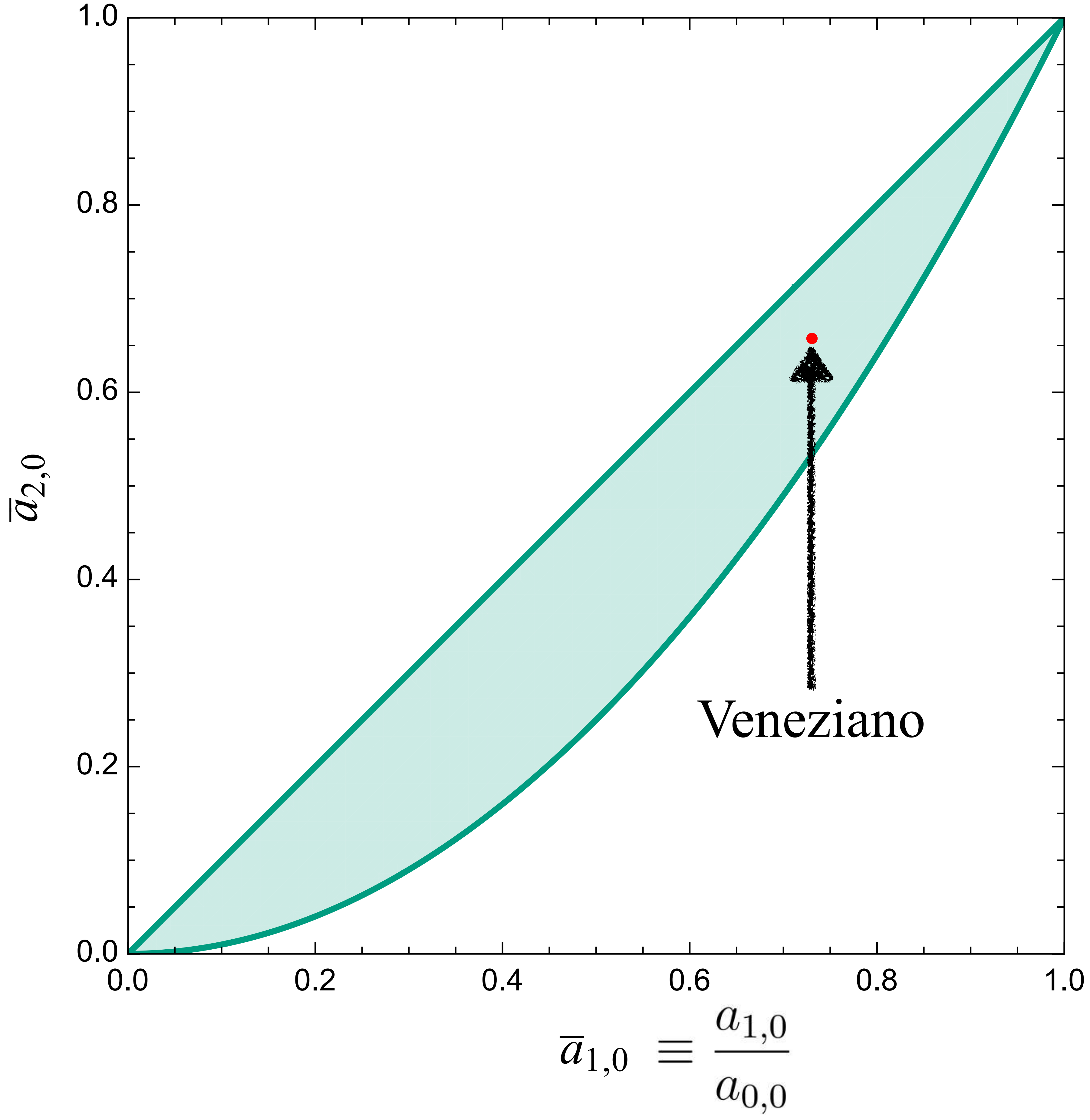
What more can we do?



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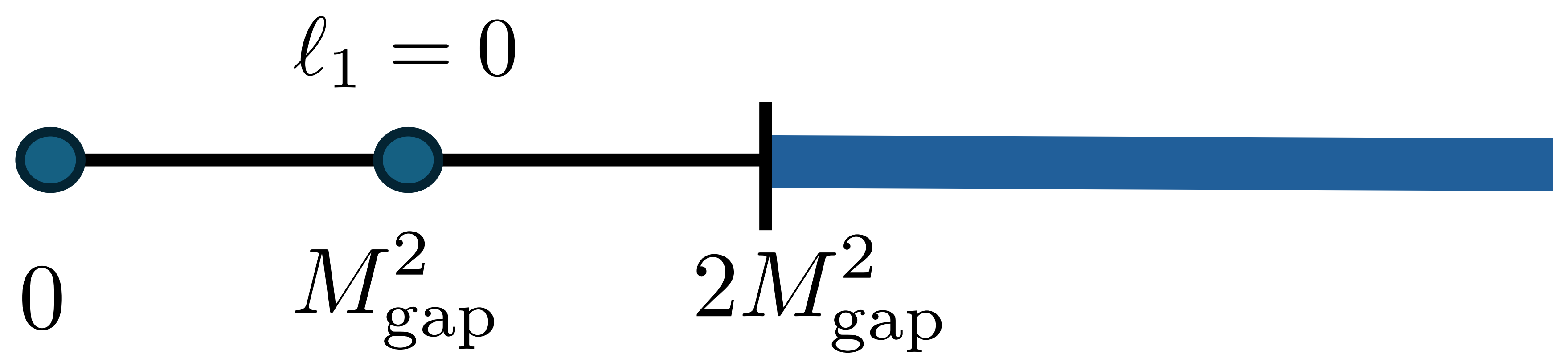
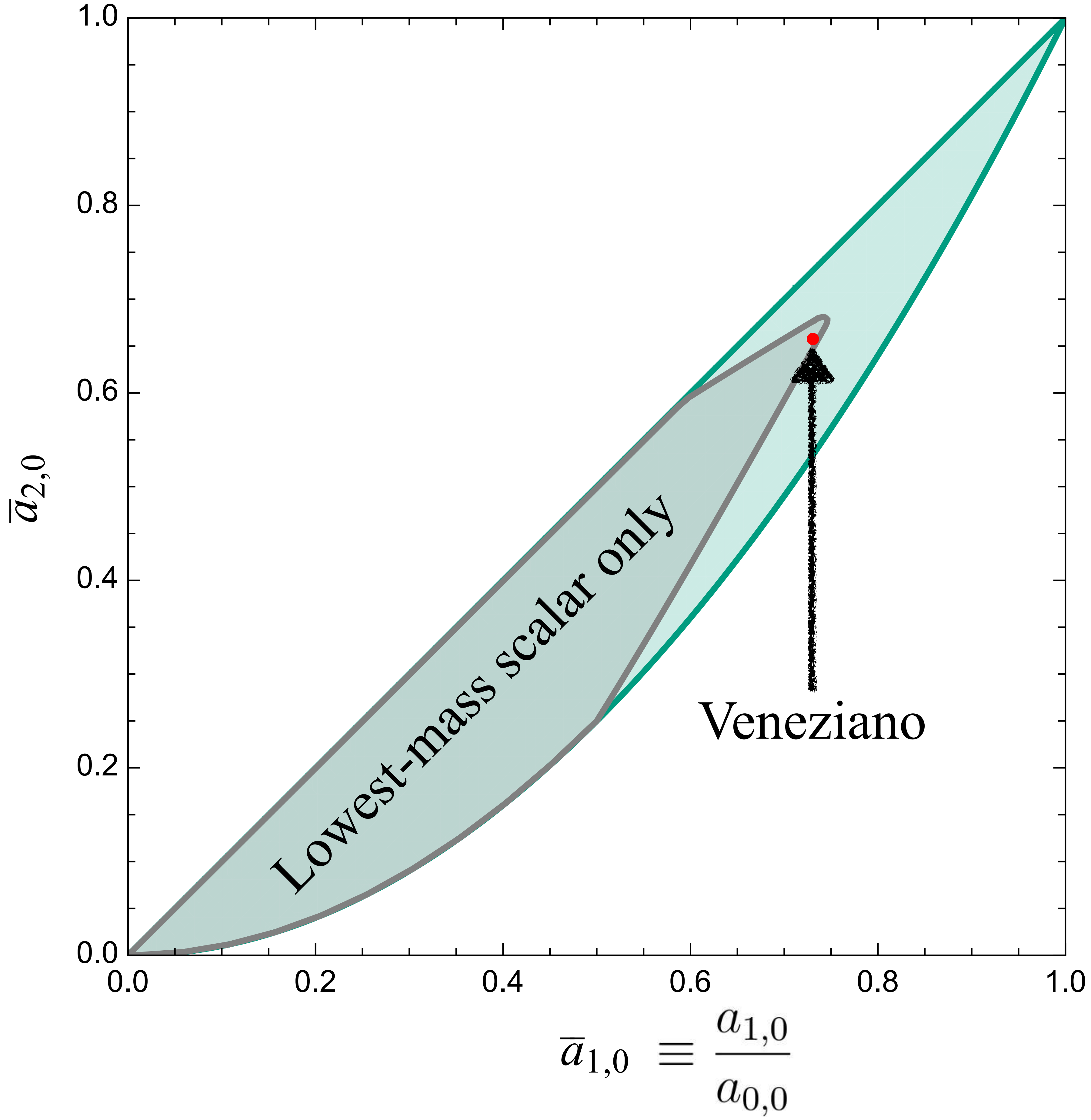
<u>"Experimental" Perspective: Low Mass Spectrum Input</u>





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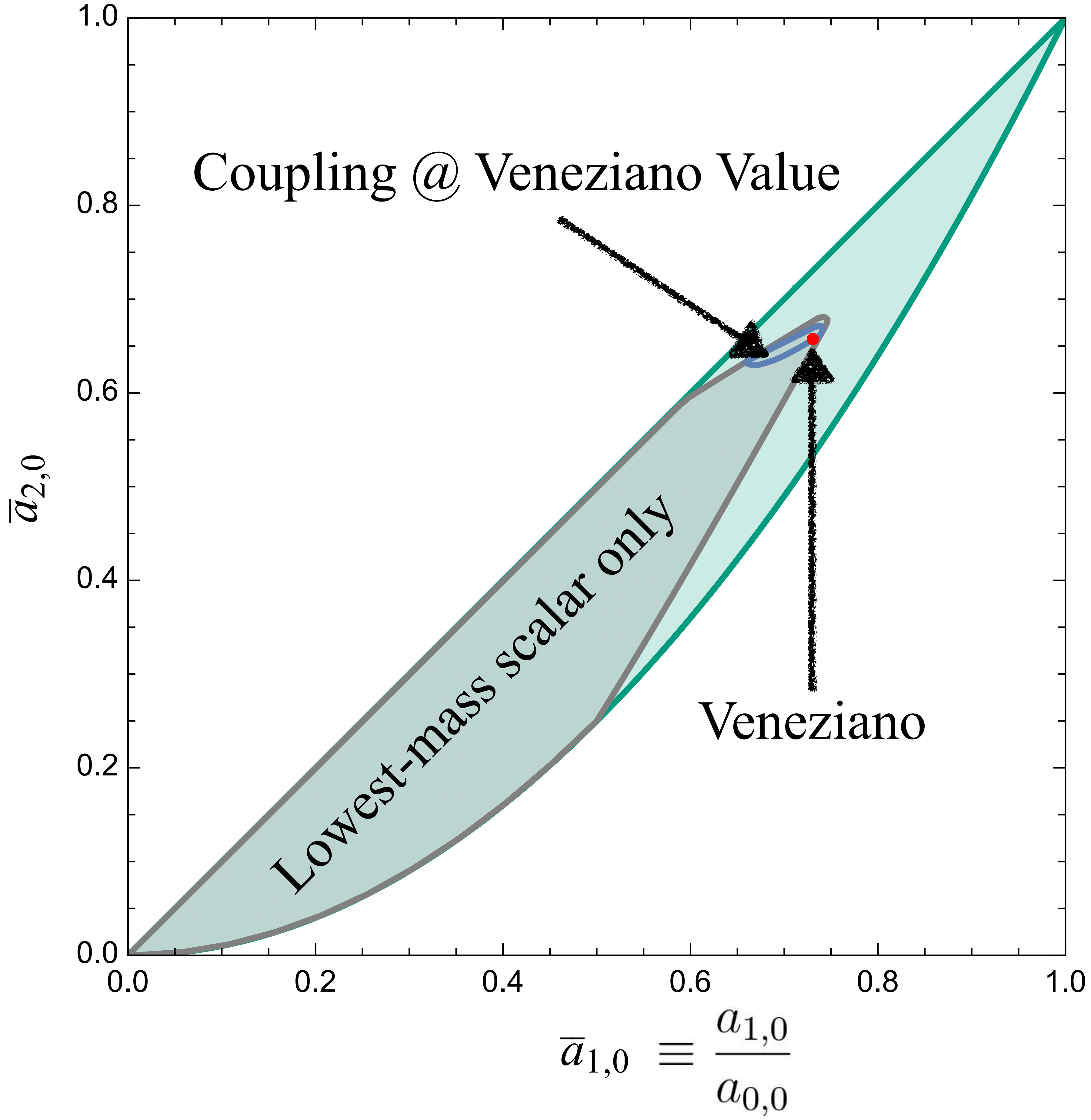

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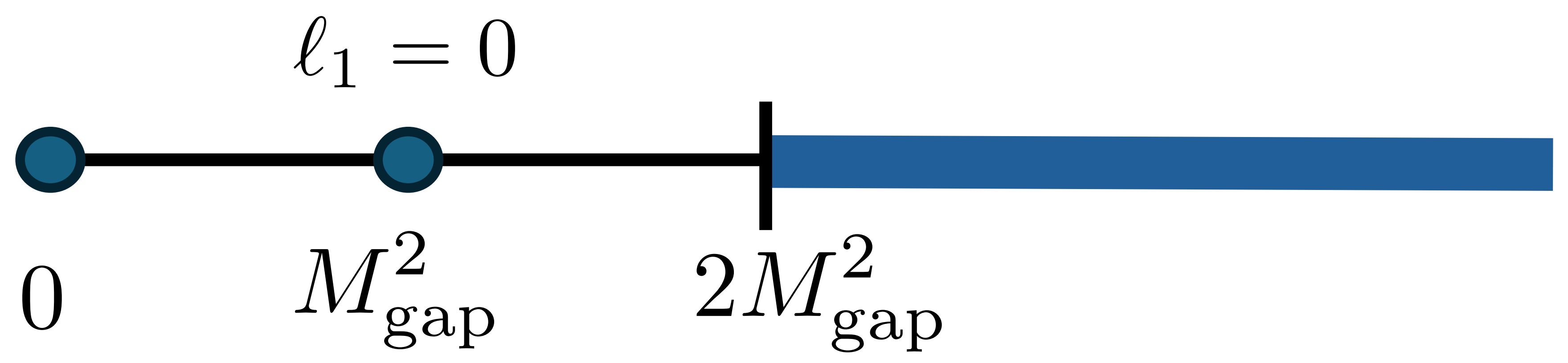
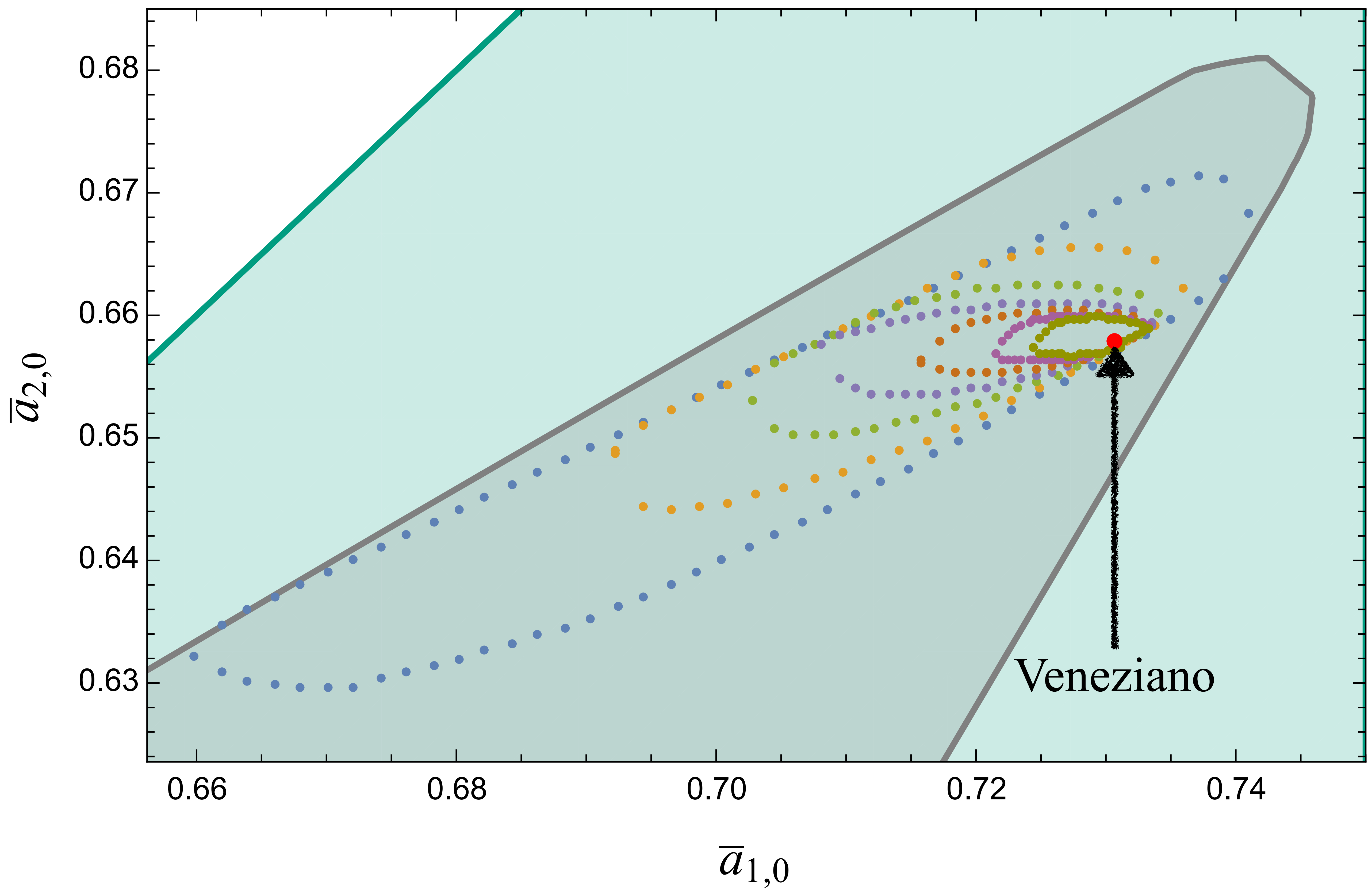
<u>“Experimental” Perspective: Low Mass Spectrum Input</u>



More SUSY Constraints → Smaller Islands



Conjecture: Particle theoretic input

- single state input at lowest massive level
- coupling and lowest Wilson coefficient match Veneziano

⬇

Veneziano the unique unitary, maximally SUSY UV completion!

Constructibility of AdS supergluon amplitudes

Qu Cao (曹趣), Song He (何颂), **<u>Yichao Tang</u>** (唐一朝)
Institute of Theoretical Physics, CAS, Beijing
See [2312.15484][2406.xxxxx] and poster

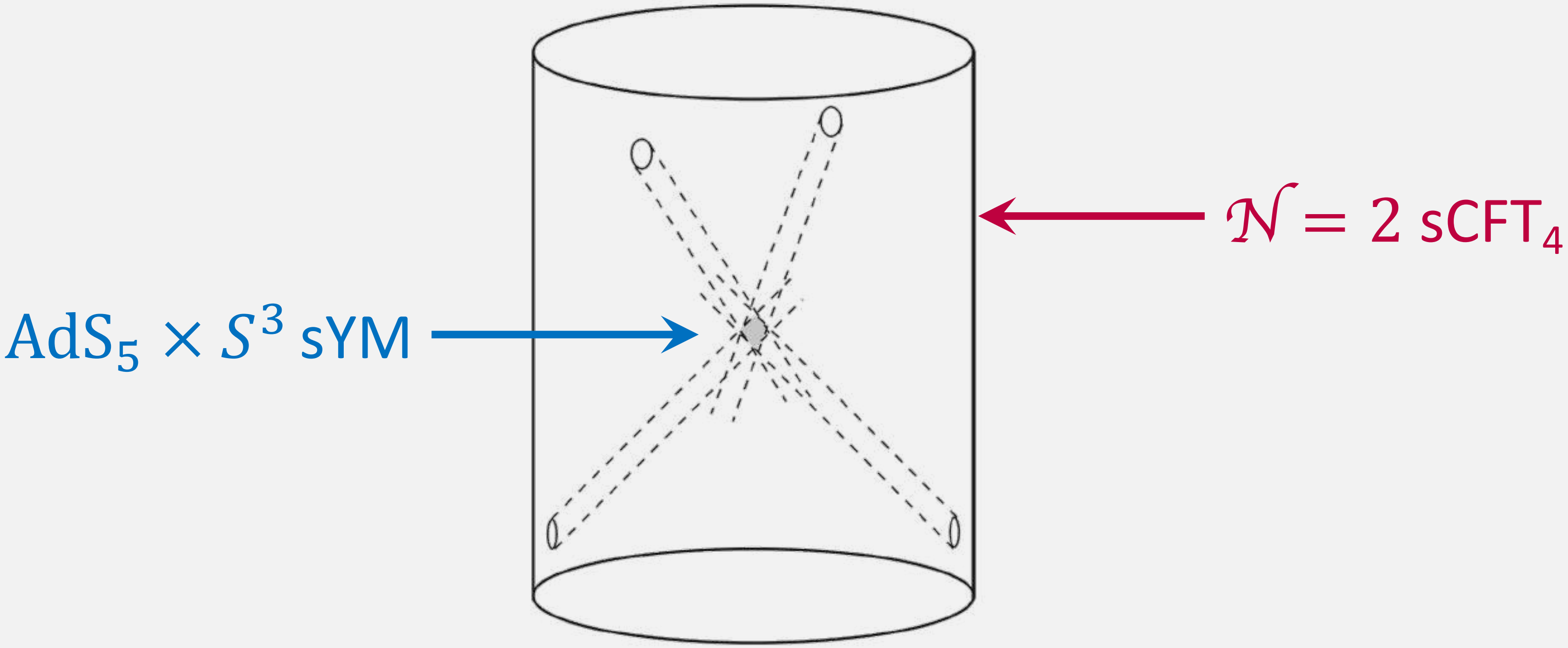
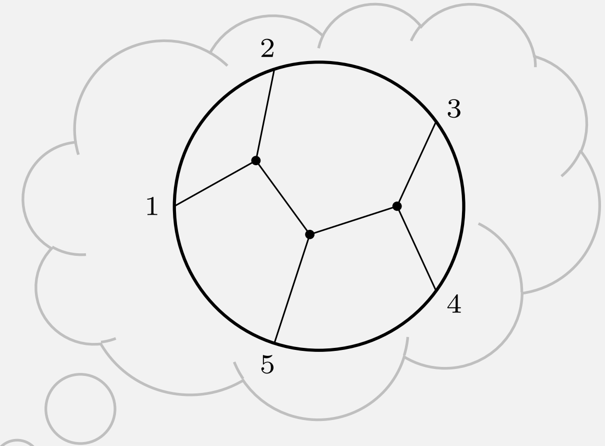
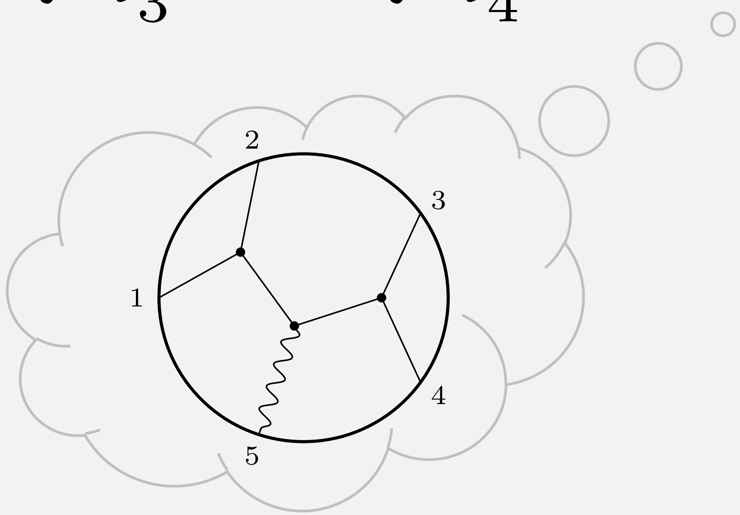


Figure from [0907.0151]

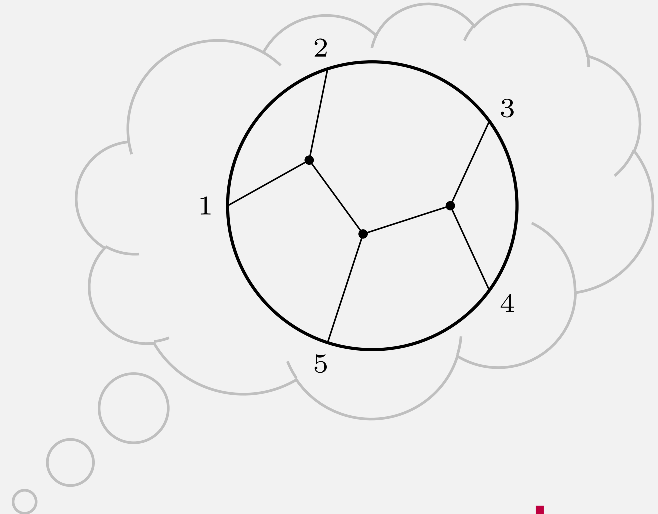
Before …



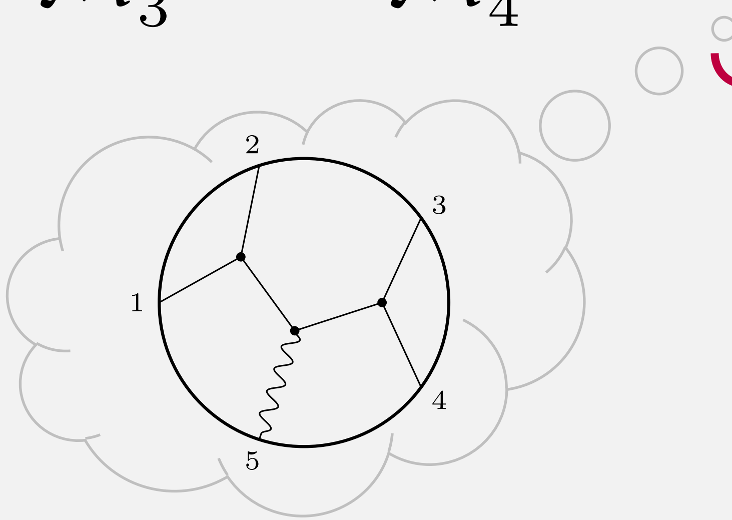
$$
\begin{array}{ccccccccc}
\mathcal{M}_3^{(s)} & \longrightarrow & \mathcal{M}_4^{(s)} & \longrightarrow & \mathcal{M}_5^{(s)} & \longrightarrow & \mathcal{M}_6^{(s)} & \longrightarrow & \cdots \\
 & & \uparrow & & \uparrow & & \uparrow & & \\
 & & \mathcal{M}_3^{(v)} & & \mathcal{M}_4^{(v)} & & \mathcal{M}_5^{(v)} & & \cdots
\end{array}
$$



Before ...

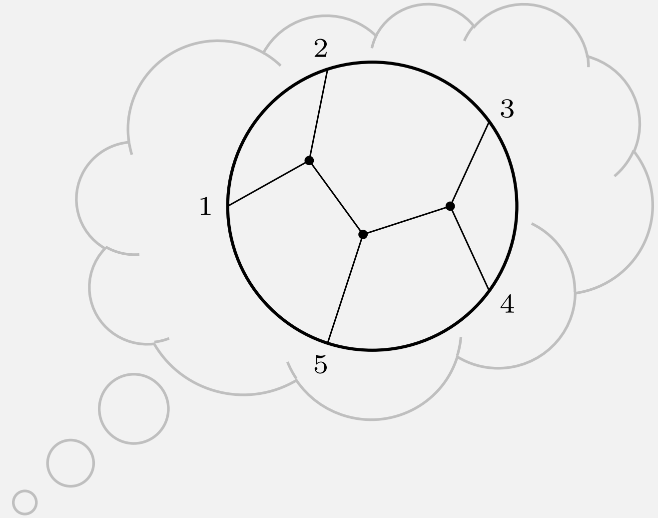


$$\begin{array}{ccccccccc}
\mathcal{M}_3^{(s)} & \longrightarrow & \mathcal{M}_4^{(s)} & \longrightarrow & \mathcal{M}_5^{(s)} & \longrightarrow & \mathcal{M}_6^{(s)} & \not\longrightarrow & \cdots \\
 & & \uparrow & & \uparrow & & \uparrow & & \\
 & & \mathcal{M}_3^{(v)} & & \mathcal{M}_4^{(v)} & & \mathcal{M}_5^{(v)} & & \cdots
\end{array}$$

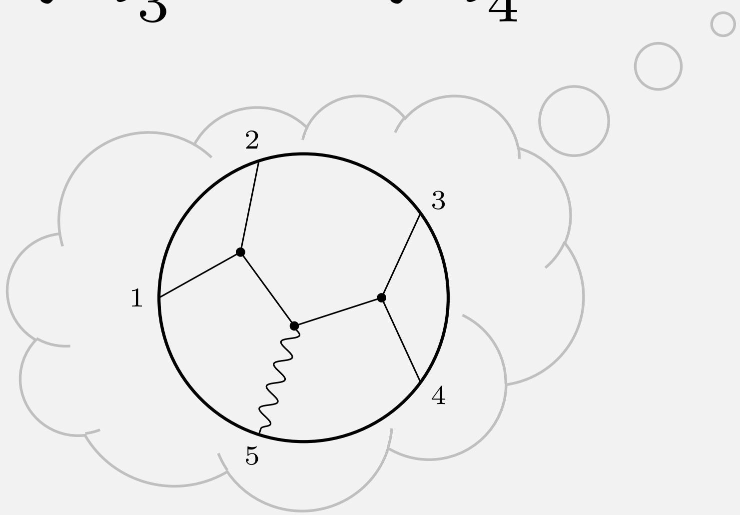


Unknown
Hard to compute

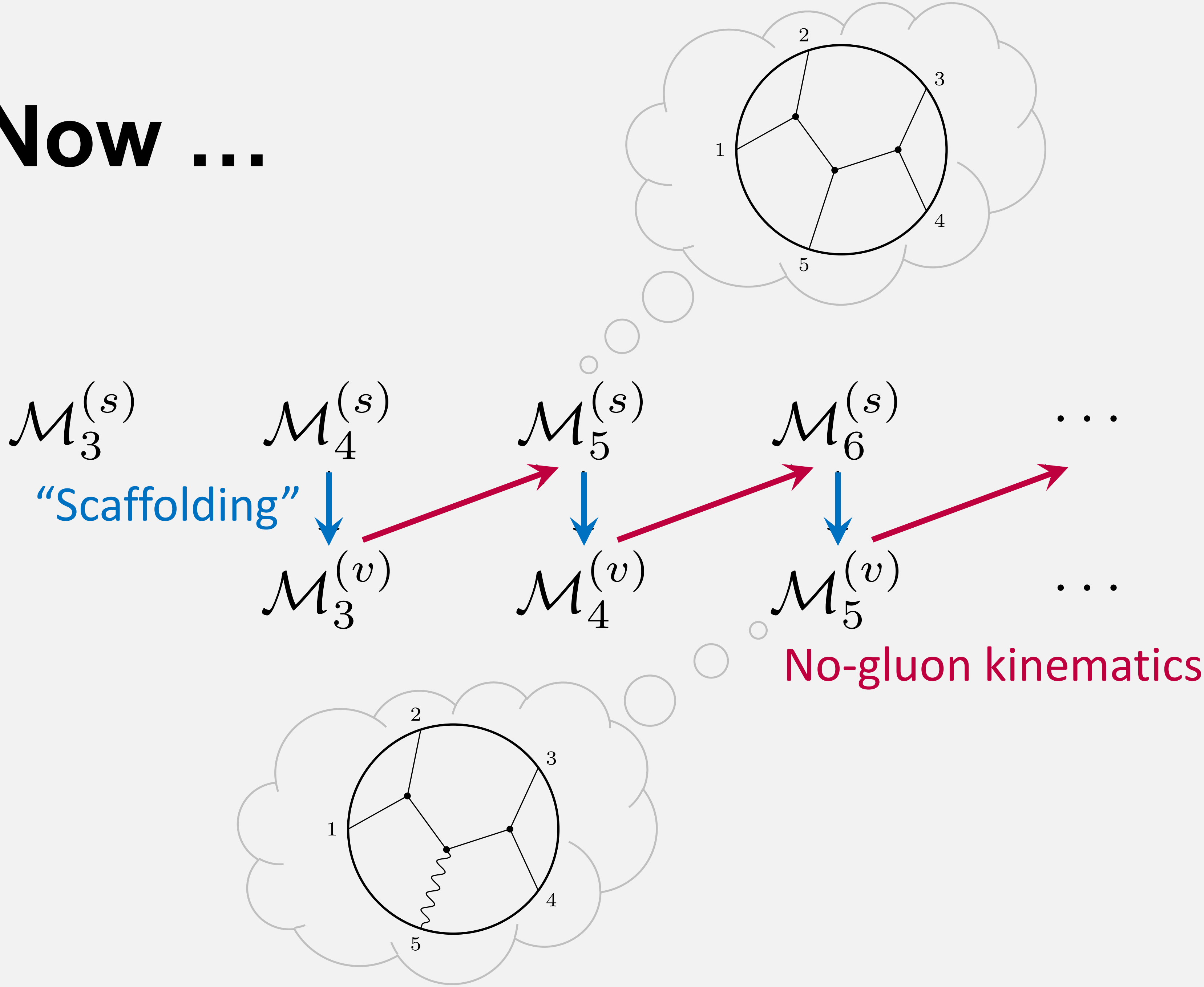
Now ...



$$\begin{array}{ccccccccc} \mathcal{M}_3^{(s)} & & \mathcal{M}_4^{(s)} & & \mathcal{M}_5^{(s)} & & \mathcal{M}_6^{(s)} & & \cdots \\ & & \downarrow & \nearrow & \downarrow & \nearrow & \downarrow & \nearrow & \\ & & \mathcal{M}_3^{(v)} & & \mathcal{M}_4^{(v)} & & \mathcal{M}_5^{(v)} & & \cdots \end{array}$$



Now ...



2
3
1
4
5
$\mathcal{M}_3^{(s)}$
$\mathcal{M}_4^{(s)}$
$\mathcal{M}_5^{(s)}$
$\mathcal{M}_6^{(s)}$
...
"Scaffolding"
$\mathcal{M}_3^{(v)}$
$\mathcal{M}_4^{(v)}$
$\mathcal{M}_5^{(v)}$
...
No-gluon kinematics
2
3
1
4
5

David Prinz, MPI f. Mathematics, Bonn (Germany):

Renormalization of Gauge Theories and Gravity

Renormalization Theory:

- Connes–Kreimer framework
- Hopf algebra for subdivergences
- Algebraic Birkhoff decomposition for renormalized Feynman rules

Quantum Gauge Theories:

- Slavnov–Taylor identities as obstructions for gauge anomalies
- Implemented via Hopf ideals and Feynman graph cohomology
- Relate to BV-BRST formalism w. PhD student *Jonah Epstein*

Feynman Graph Complex:

- Perturbative BRST cohomology
- Differential projects external edges onto longitudinal d.o.f.
- Encodes cancellation identities via homological algebra

Quantum General Relativity:

- "Generalized gauge theory" w.r.t. diffeomorphism group
- Non-renormalizable by power counting
- Construct higher-derivative counterterms via gravitational Slavnov–Taylor identities

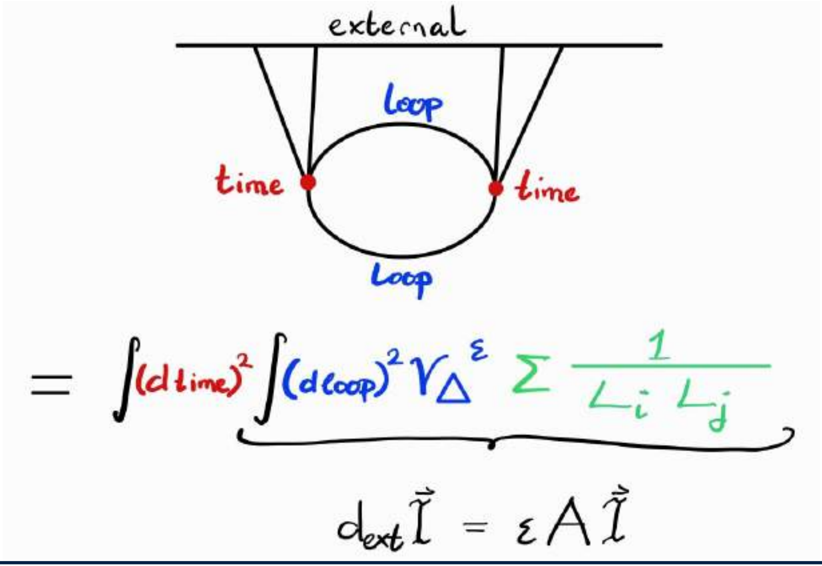
Loops in Cosmology

SCUOLA
NORMALE
SUPERIORE

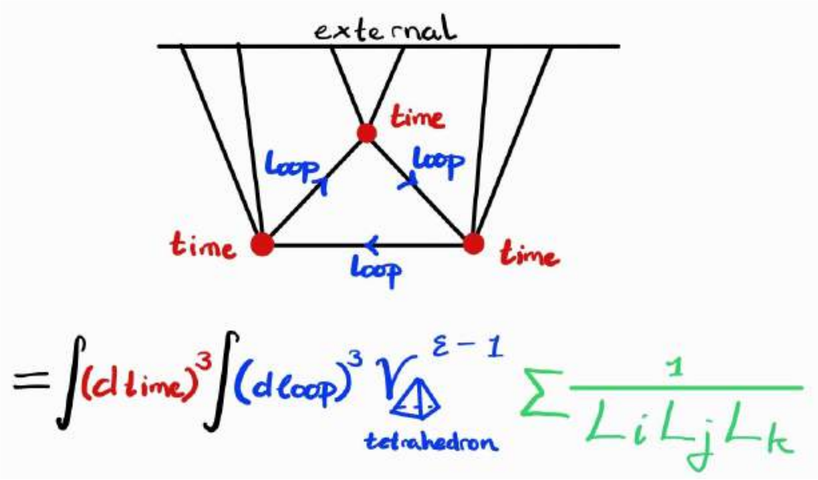
Tom Westerdijk

Advisor: Guilherme Pimentel

June 7, 2024


external
loop
time
time
Loop
$= \int (d\,\text{time})^2 \int (d\,\text{loop})^2 \, r_\Delta^{\varepsilon} \sum \frac{1}{L_i L_j}$
$d_{ext} \vec{I} = \varepsilon A \vec{I}$

Triangle


external
time
loop
time
loop
loop
time
= ∫(d time)^3 ∫(d loop)^3 V^{ε-1} Σ 1/(L_i L_j L_k)
tetrahedron

Higher-Point Loop Integrands and Ten-Dimensional Null Limits

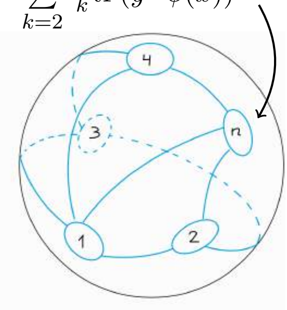
Till Bargheer[1], Albert Bekov[1], Carlos Bercini[1], Frank Coronado[2]

[1] Deutsches Elektronen-Synchrotron DESY, Notkestr. 85, 22607 Hamburg, Germany

[2] Institut für Theoretische Physik, ETH Zurich, CH-8093 Zürich, Switzerland

half-BPS operators

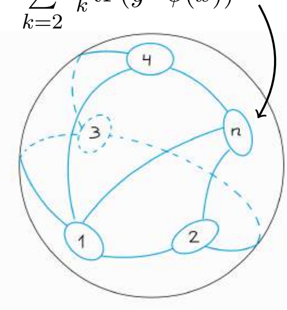
$$\sum_{k=2}^{\infty} \frac{1}{k} \mathrm{tr}\,(y \cdot \phi(x))^k$$



Generating function G_n

half-BPS operators

$$\sum_{k=2}^{\infty} \frac{1}{k} \mathrm{tr}\,(y \cdot \phi(x))^k$$



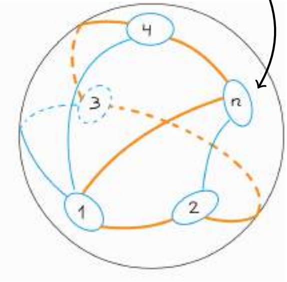
Generating function G_n

Loop-integrands of $G_{5,1}$, $G_{6,1}$ and $G_{5,2}$!

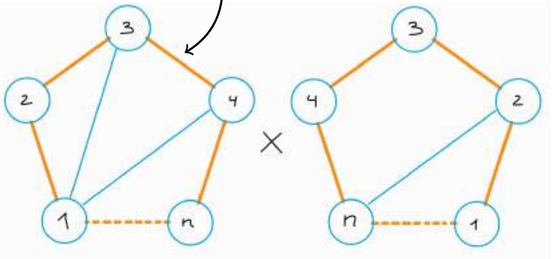
half-BPS operators

$$\sum_{k=2}^{\infty} \frac{1}{k} \mathrm{tr}\,(y \cdot \phi(x))^k$$

$$X_{i,i+1}^2 = x_{i,i+1}^2 + y_{i,i+1}^2 \to 0$$



10D null polygonal limit



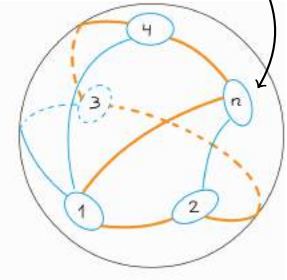
Generating function G_n

Polygons $(M_n)^2$

Loop-integrands of $G_{5,1}$, $G_{6,1}$ and $G_{5,2}$!

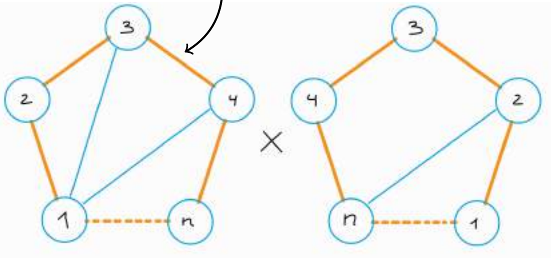
half-BPS operators

$$\sum_{k=2}^{\infty} \frac{1}{k} \mathrm{tr}\,(y \cdot \phi(x))^k$$



10D null polygonal limit $\longrightarrow$

$$X^2_{i,i+1} = x^2_{i,i+1} + y^2_{i,i+1} \to 0$$



Generating function G_n

Loop-integrands of $\boldsymbol{G_{5,1}}$**,** $\boldsymbol{G_{6,1}}$ **and** $\boldsymbol{G_{5,2}}$ **!**

Polygons $(M_n)^2$

Integrands of all $\boldsymbol{n}$**-point polygons up to 2 loops:** $\boldsymbol{M_{n,1}}$ **and** $\boldsymbol{M_{n,2}}$ **!**

Three-point energy correlator in N=4 and QCD

Xiaoyuan Zhang
Harvard University

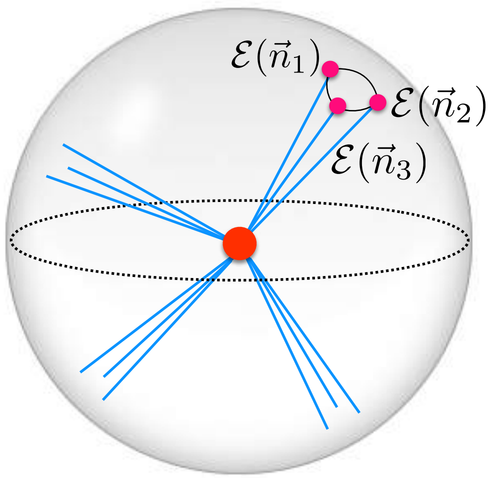
Based on:

- 2203.04349 [PhysRevLett.129.021602]: with Kai Yan [Shanghai Jiao Tong University]
- 2208.01051 [JHEP09(2022)006],
  2402.05174 [PhysRevD.XXX]: with Tong-Zhi Yang [Universität Zürich]

Energy correlators

- Energy correlator is defined as the Wightman correlation function of the energy flow operators

$$\mathcal{E}(\vec{n}) = \int_{-\infty}^{\infty} d\tau \lim_{r\to\infty} r^2 n^i T_{0i}(t=\tau+r, r\vec{n})$$



N-point energy correlator ⇒ (N+2)-point correlation function

[Hofman, Maldacena, 0803.1467]

$$\int [d\Omega_{\vec{n}_i} \delta(\vec{n}_i \cdot \vec{n}_{i+1} - \cos\chi_i)] \times \int d^4x e^{iqx} \langle \Omega | J^\mu(x) \mathcal{E}(\vec{n}_1) \cdots \mathcal{E}(\vec{n}_N) J_\mu(0) | \Omega \rangle$$

- Perturbative: defined as the energy-weighted differential cross section

2-point correlator: [Basham, Brown, Ellis, Love, 1978]

$$\mathrm{EEC}(\chi) = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta(\vec{n}_i \cdot \vec{n}_j - \cos\chi)$$

n-point correlator: [Chen, Luo, Moult, Yang, XYZ, Zhu, 1912.11050]

$$\frac{d\sigma}{dx_{12}\cdots dx_{(n-1)n}} = \sum_m \sum_{1\le i_1,\cdots i_n \le m} \int d\sigma_m \times \prod_{1\le k\le n} \frac{E_{i_k}}{Q} \prod_{1\le j<l\le n} \delta\left(x_{jl} - \frac{1-\cos\theta_{i_j i_l}}{2}\right)$$

3-point correlator in N=4 SYM

- Calculation setup at LO:

tree-level $1 \to 4$ squared form factor $\left|\langle p_1 p_2 p_3 p_4 | \mathrm{tr}[\phi^2] | \rangle\right|^2$

$$\frac{1}{\sigma_0}\frac{d^3\sigma}{dx_1 dx_2 dx_3} = \sum_{i,j,k}\int \mathrm{dPS}_4\, |\mathcal{M}_{\mathcal{N}=4}|^2 \frac{E_i E_j E_k}{Q^3}$$
$$\times\, \delta\left(x_3 - \frac{1-\cos\theta_{12}}{2}\right)\delta\left(x_2 - \frac{1-\cos\theta_{13}}{2}\right)\delta\left(x_1 - \frac{1-\cos\theta_{23}}{2}\right)$$

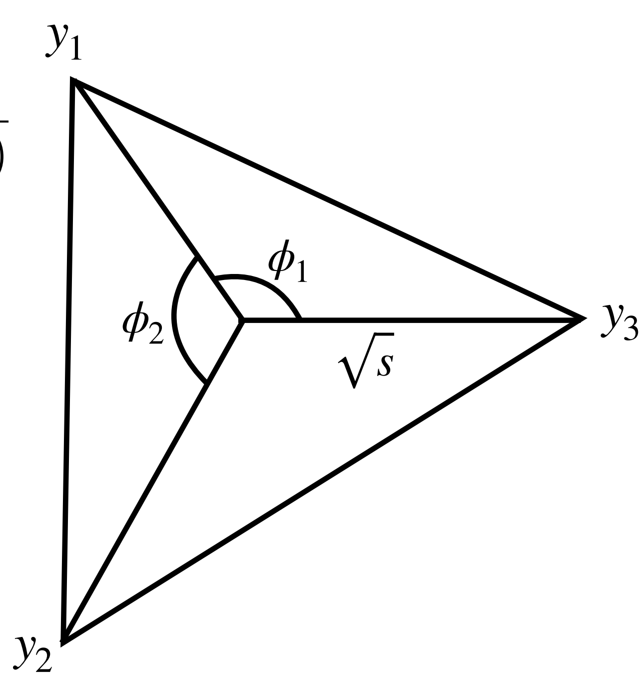
- Need to rationalize two square roots:

$$\sqrt{x_1^2+x_2^2+x_3^2-2x_1x_2-2x_1x_3-2x_2x_3} = \sqrt{\widetilde{\Delta}_4^{\mathrm{coll}}}\,,$$
$$\sqrt{x_1^2+x_2^2+x_3^2-2x_1x_2-2x_1x_3-2x_2x_3+4x_1x_2x_3} = \sqrt{\widetilde{\Delta}_4}$$

- The angles x_i are mapped onto the distances between three points on celestial sphere

$$x_1 = \frac{|y_2-y_3|^2}{(1+|y_2|^2)(1+|y_3|^2)},\; x_2 = \frac{|y_1-y_3|^2}{(1+|y_1|^2)(1+|y_3|^2)},\; x_3 = \frac{|y_1-y_2|^2}{(1+|y_1|^2)(1+|y_2|^2)}$$

$$y_1 = \sqrt{s}\,\underbrace{e^{i\phi_1}}_{\equiv \tau_1},\; y_2 = \sqrt{s}\,\underbrace{e^{i(\phi_1+\phi_2)}}_{\equiv \tau_1\tau_2},\; y_3 = \sqrt{s}$$



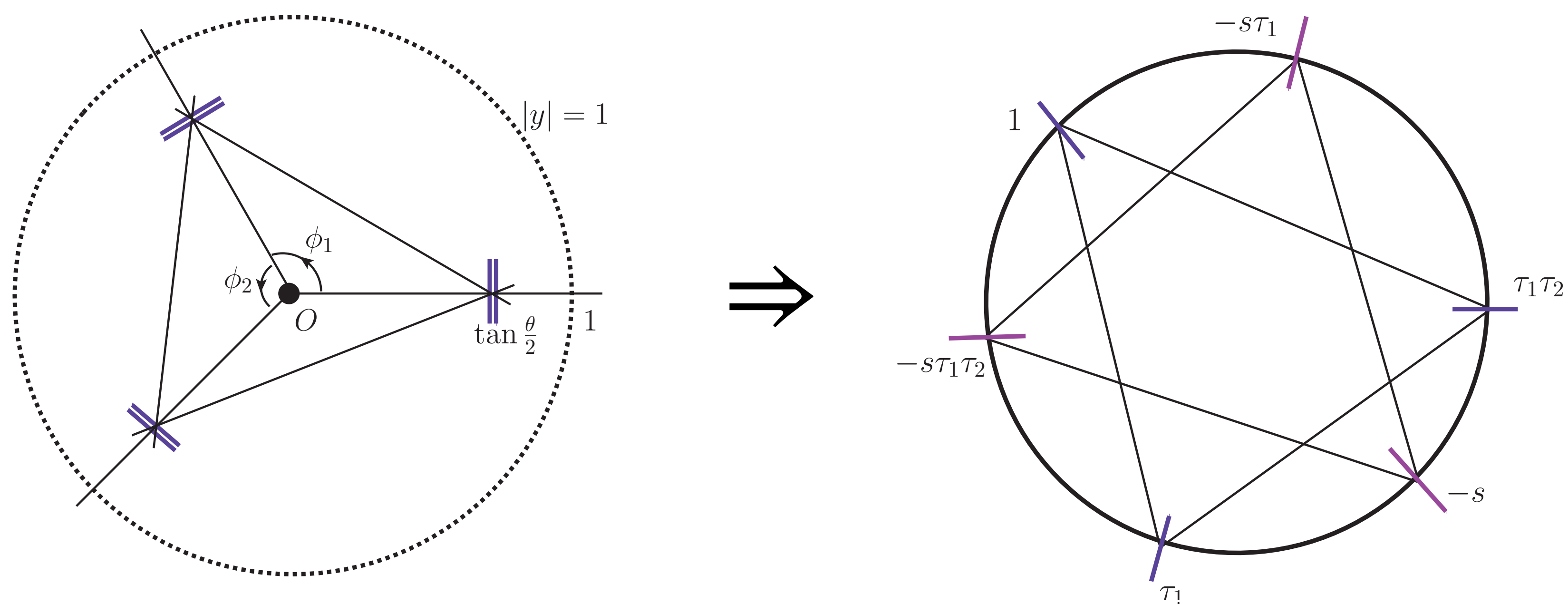
The $\{s, \tau_1, \tau_2\}$ variable allows direct integration and expressing the result in terms of generalized polylogarithms (GPLs)

Embedding

- The kinematic variable can be embedded into a hexagon on a unit circle. Defining

$$x_k = \frac{q^2(p_i \cdot p_j)}{2(q \cdot p_i)(q \cdot p_j)} = \frac{\langle p_i p_j \rangle \langle \xi_i \xi_j \rangle}{\langle p_i \xi_j \rangle \langle p_i \xi_j \rangle}, \quad |\xi_j\rangle \equiv q|j]$$

- We can embed: $|p_i\rangle \equiv |2i-1\rangle, |\xi_i\rangle \equiv |2i+2\rangle$



Symmetry: D_6 dihedral group $\qquad Z_i = \{1, -s\tau_1\tau_2, \tau_1, -s, \tau_1\tau_2, -s\tau_1\}, \ I = \infty$

- Cyclic permutation (σ) : $Z(a+2) = Z(a)$
- Parity (P) : $Z(a+3) = Z(a)$
- Flip (τ) : $Z(8-a) = Z(a)$

Symbol alphabet

- Original symbol alphabet (16 independent letters)

$$\{s-1,\ s,\ s+1,\ \tau_1-1,\ \tau_1,\ \tau_2-1,\ \tau_2,\ s+\tau_1,\ 1+s\tau_1,\ s+\tau_2,\ 1+s\tau_2,\ s+\tau_1\tau_2,\ 1+s\tau_1\tau_2,\ \tau_1\tau_2-1,\ \tau_1-\tau_2,\ \tau_1^2\tau_2-1,\ \tau_1\tau_2^2-1\}$$

- This can be written as a close set under D_6 using three conformal invariant ratios:

$$u_1 \equiv -\frac{\langle 51\rangle\langle 62\rangle\langle 43\rangle}{\langle 35\rangle\langle 16\rangle\langle 24\rangle} = -\frac{s+\tau_1}{1+s\tau_1},$$
$$u_2 \equiv -\frac{\langle 31\rangle\langle 5I\rangle}{\langle 15\rangle\langle I3\rangle} = \frac{\tau_1-1}{1-\tau_1\tau_2},$$
$$u_3 \equiv -\frac{\langle 13\rangle\langle 56\rangle}{\langle 35\rangle\langle 61\rangle} = \frac{(1-\tau_1)(s+\tau_2)}{(1-\tau_2)(1+s\tau_1)},$$

$$\{u_1\,,1+u_1\,,u_2\,,1+u_2\,,u_3\,,1+u_3\,,u_1+u_3\,,$$
$$1+u_1+u_3\,,u_2+u_3+u_2u_3\,,u_1+u_2+u_3+u_2u_3\,,$$
$$1+u_1+u_2+u_3+u_2u_3\,,1+u_1+u_2+2u_3+u_2u_3\,,$$
$$u_2+u_1u_2+u_2^2+u_3+2u_2u_3+u_2^2u_3\,,$$
$$1+u_2+u_1u_2+u_2^2+u_3+2u_2u_3+u_2^2u_3\,,$$
$$1+u_1+u_2+u_1u_2+u_2^2+u_3+2u_2u_3+u_2^2u_3\,,$$
$$1+u_1+u_2+u_1u_2+u_2^2+2u_3+2u_2u_3+u_2^2u_3\,\}$$

- Polylogarithm arguments: 15 conformal invariant ratios that cover the χ- coordinate in [Golden, Paulos, Spradlin, Volovich, 1401.6446]; additional variable $r_1 \equiv -\frac{\langle 14\rangle\langle 3I\rangle}{\langle 13\rangle\langle 4I\rangle}$ and the images under D_6
- Eventually, we find 3 weight-1 basis and 11 weight-2 basis

Function space

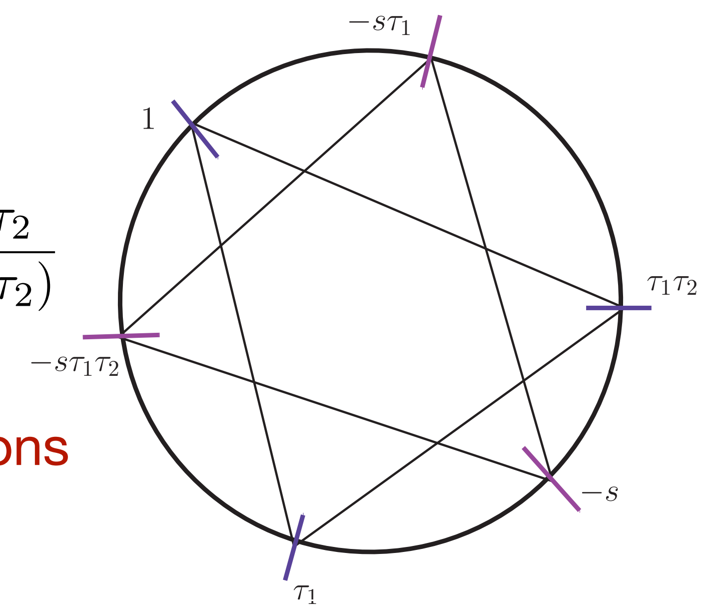
- For weight-2 functions, we need to introduce a few variables from the embedding

$$z_1 \equiv \frac{\langle 56\rangle\langle 13\rangle}{\langle 35\rangle\langle 16\rangle} = \frac{(\tau_1-1)(s+\tau_2)}{(1-\tau_2)(1+s\tau_1)}, \quad \bar{z}_1 \equiv P(z_1) = \frac{\langle 23\rangle\langle 46\rangle}{\langle 26\rangle\langle 34\rangle} = \frac{(1+s\tau_2)(1-\tau_1)}{(s+\tau_1)(1-\tau_2)}$$

$$w_1 \equiv \frac{\langle 16\rangle\langle 25\rangle}{\langle 56\rangle\langle 12\rangle} = -\frac{(1+s)(1+s\tau_1)\tau_2}{(s+\tau_2)(1+s\tau_1\tau_2)}, \quad \bar{w}_1 \equiv P(w_1) = \frac{\langle 34\rangle\langle 25\rangle}{\langle 23\rangle\langle 45\rangle} = -\frac{(1+s)(s+\tau_1)\tau_2}{(1+s\tau_2)(s+\tau_1\tau_2)}$$

$$v_1 \equiv \frac{\langle 26\rangle\langle 35\rangle}{\langle 56\rangle\langle 23\rangle} = -\frac{s(1-\tau_2)^2}{(s+\tau_2)(1+s\tau_2)}$$

$\{z_2, z_3, \cdots\}$ is obtained by cyclic permutations



$$\begin{aligned}
g_1 &= \mathrm{Li}_2(-v_2) \\
g_2 &= \mathrm{Li}_2(1+w_3) + \mathrm{Li}_2(1+\bar{w}_3) + 2\,\mathrm{Li}_2(-v_3) \\
&\quad - \mathrm{Li}_2(1+w_1) - \mathrm{Li}_2(1+\bar{w}_1) - 2\,\mathrm{Li}_2(-v_1) \\
g_3 &= \mathrm{Li}_2(-z_2) - \mathrm{Li}_2(-\bar{z}_2) + \frac{1}{2}\ln|z_2|^2 \ln\frac{1+z_2}{1+\bar{z}_2} \\
g_4 &= \mathrm{Li}_2(1+w_1) - \mathrm{Li}_2(1+\bar{w}_1) + \mathrm{Li}_2(1+w_2) \\
&\quad - \mathrm{Li}_2(1+\bar{w}_2) + \mathrm{Li}_2(1+w_3) - \mathrm{Li}_2(1+\bar{w}_3) \\
g_5 &= \pi^2 \\
g_6 &= \ln^2\frac{\bar{w}_1}{w_1} \\
g_7 &= \ln\frac{\bar{w}_1}{w_1}\,\ln|z_2|^2 \\
g_8 &= \ln(1+v_3)\ln|z_1|^2 - \ln(1+v_1)\ln|z_3|^2
\end{aligned}$$

Additional variable under D_6

$$r_a = \frac{\langle a\,a+3\rangle\langle a+2\,I\rangle}{\langle a\,a+2\rangle\langle I\,a+3\rangle}, \quad \bar{r}_a = \frac{\langle a+5\,a+2\rangle\langle a+3\,I\rangle}{\langle a+5\,a+3\rangle\langle I\,a+2\rangle}$$

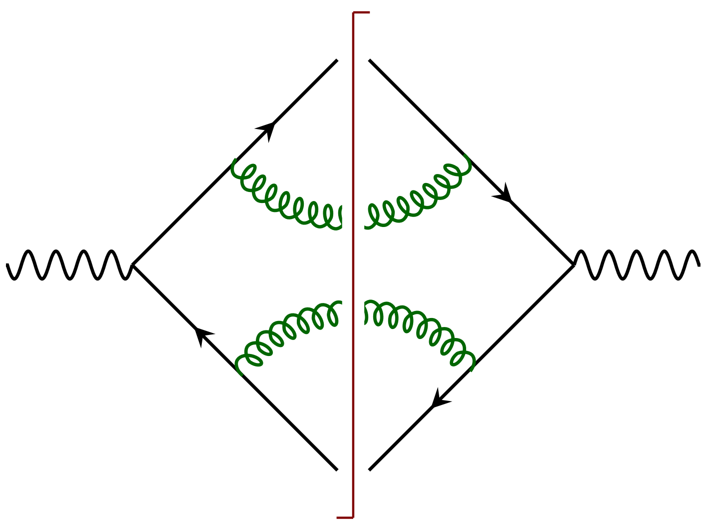
$$\begin{aligned}
g_{11} &= \sum_{i=1}^{6} \mathrm{Li}_2(-r_i) - \mathrm{Li}_2(-\bar{r}_i) + \frac{1}{2}\ln|r_i|^2 \ln\frac{1+r_i}{1+\bar{r}_i} \\
g_9 &= \frac{1}{2}\mathrm{Li}_2\left(1-\frac{\bar{w}_1}{w_1}\right) - \frac{1}{2}\mathrm{Li}_2\left(1-\frac{w_1}{\bar{w}_1}\right) \\
g_{10} &= \mathrm{Li}_2(1-|z_2|^2) + \frac{1}{2}\ln|z_2|^2\ln|1-z_2|^2
\end{aligned}$$

With rational coefficients in front of them

Identical structure in QCD

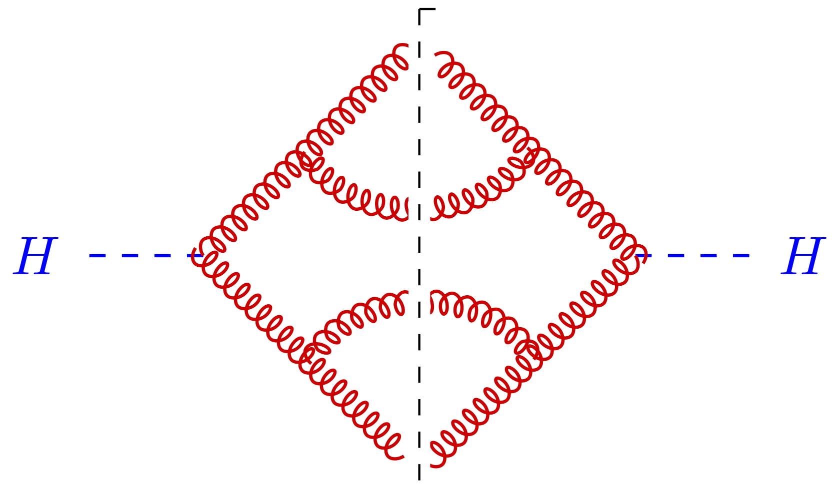
- For QCD, we calculate both e^+e^- annihilation and Higgs decays

$e^+e^- \to$ hadrons



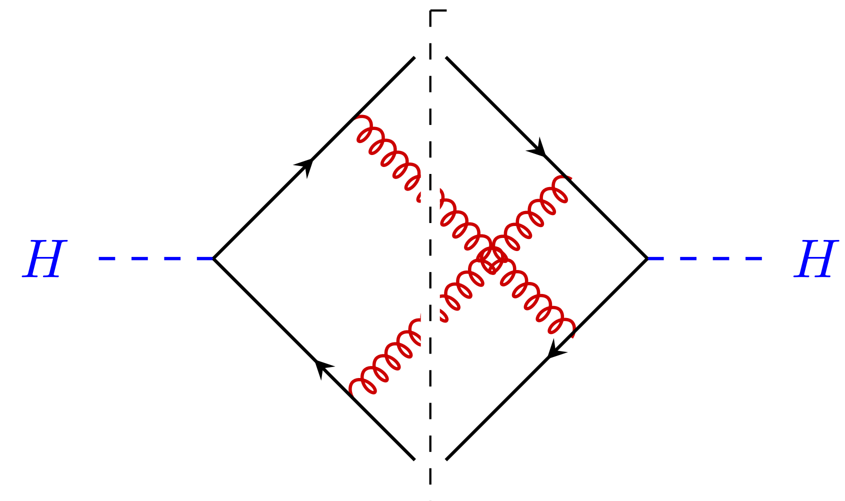
$\gamma^\star \to q\bar{q}q'\bar{q}',\ q\bar{q}q\bar{q},\ q\bar{q}gg$

$H \to gg + X$



$H \to gggg,\ q\bar{q}gg,\ q\bar{q}q'\bar{q}',\ q\bar{q}q\bar{q}$

$H \to b\bar{b} + X$



$H \to q\bar{q}gg,\ q\bar{q}q'\bar{q}',\ q\bar{q}q\bar{q}$

- Identical alphabet and function space as in N=4 SYM, but different rational coefficients
- Same observation for 2-point correlator at both LO and NLO
- From N=4 to QCD:

  energy correlator could be the observables that allow us to borrow the tools developed in N=4 and easily generalize them to QCD

Evaluation of two–loop six–point Feynman integrals

Yingxuan Xu, Humboldt University of Berlin

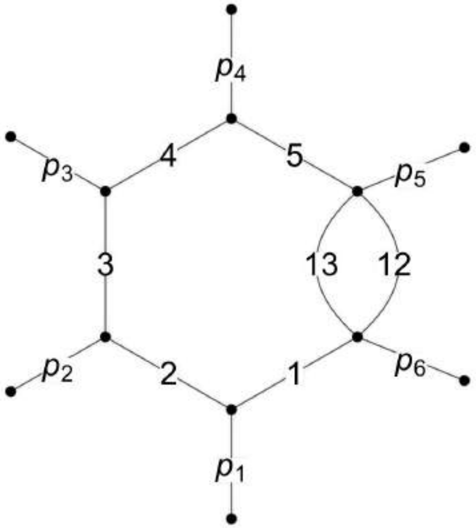
Based on:

ArXiv: 2403.19742, J. Henn, A. Matijašić, J. Miczajka, T. Peraro, Y. Xu, Y. Zhang

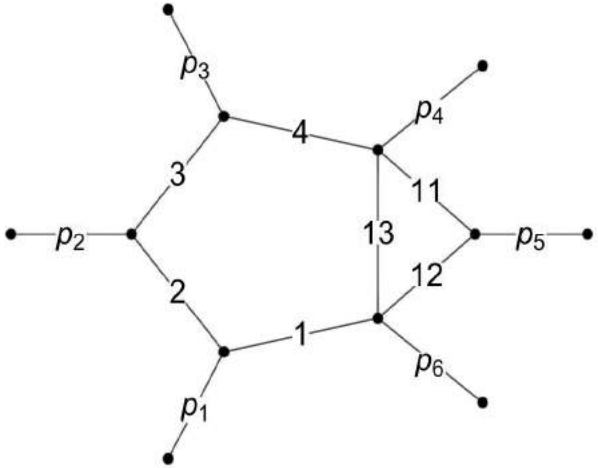
Motivation

- LHC data accumulation drives interest in multi-jet production, requiring more precise theoretical predictions ✓
- Two–loop five–point integrals play an important role in phenomenology calculation ✓
- Higher-point sYM amplitudes show similarities with lower-point QCD amplitudes ✓
- Knowing the full alphabet would open the door to potential bootstrap application ✓

2l6p UT basis

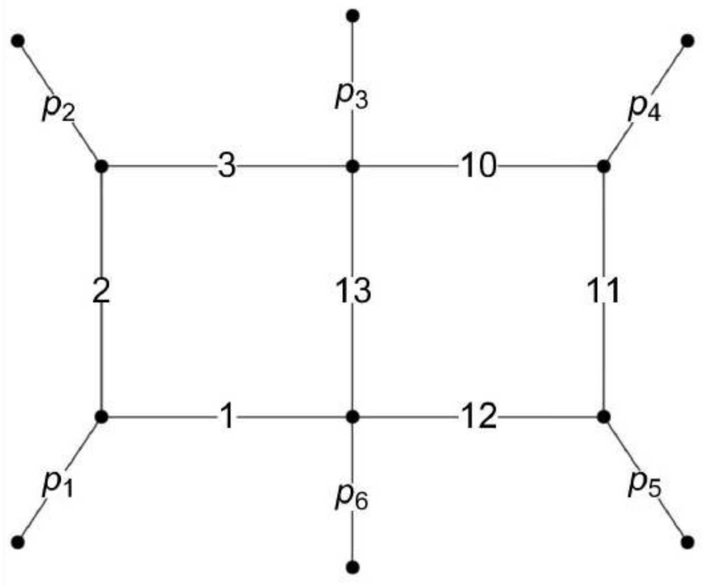


$$N_{\mathrm{hb}} = \frac{1}{32} \frac{G\begin{pmatrix} l_1 & p_1 & p_2 & p_3 & p_4 \\ l_1 & p_1 & p_2 & p_3 & p_4 \end{pmatrix} D_6}{\epsilon_{1234}}$$



$$N_{\mathrm{pt}} = \frac{1}{32} \frac{G\begin{pmatrix} l_1 & p_1 & p_2 & p_3 & p_5 \\ l_1 & p_1 & p_2 & p_3 & p_5 \end{pmatrix}}{\epsilon_{1235}}$$

2l6p UT basis



$$N_1 = -s_{12}s_{45}s_{156},$$

$$N_2 = -s_{12}s_{45}(l_1+p_5+p_6)^2,$$

$$N_3 = \frac{s_{45}}{\epsilon_{5126}} G\begin{pmatrix} l_1 & p_1 & p_2 & p_5+p_6 \\ p_1 & p_2 & p_5 & p_6 \end{pmatrix},$$

$$N_4 = \frac{s_{12}}{\epsilon_{1543}} G\begin{pmatrix} l_2-p_6 & p_5 & p_4 & p_1+p_6 \\ p_1 & p_5 & p_4 & p_3 \end{pmatrix},$$

$$N_5 = -\frac{1}{4}\frac{\epsilon_{1245}}{G(1,2,5,6)} G\begin{pmatrix} l_1 & p_1 & p_2 & p_5 & p_6 \\ l_2 & p_1 & p_2 & p_5 & p_6 \end{pmatrix},$$

$$N_6 = \frac{1}{8} G\begin{pmatrix} l_1 & p_1 & p_2 \\ l_2-p_6 & p_4 & p_5 \end{pmatrix} + \frac{D_2D_{11}(s_{123}+s_{126})}{8},$$

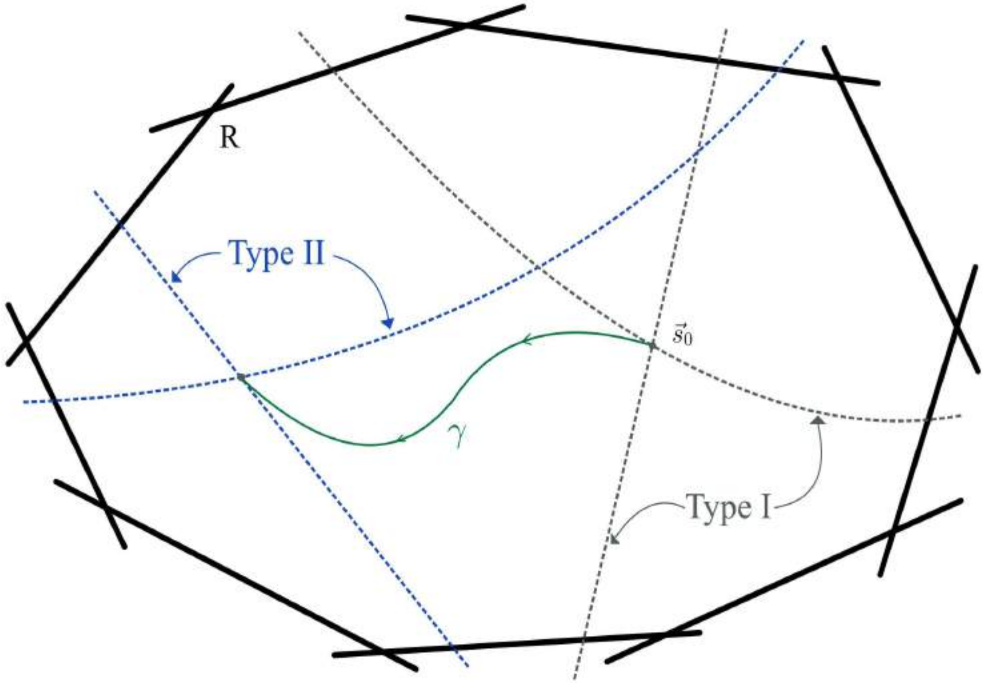
$$N_7 = -\frac{1}{2\epsilon}\frac{\Delta_6}{G(1,2,4,5)} G\begin{pmatrix} l_1 & p_1 & p_2 & p_4 & p_5 \\ l_2 & p_1 & p_2 & p_4 & p_5 \end{pmatrix}.$$

The UT basis are obtained via CDE w.r.t 8 momentum twistor variables.

Alphabets

	Double–box	Pentagon–triangle	Hexagon–bubble
Weight 1	$\alpha_1, \ldots, \alpha_9$	$\alpha_1, \ldots, \alpha_9$	$\alpha_1, \ldots, \alpha_9$
Weight 2	$\alpha_{16}, \ldots, \alpha_{27}$, $\alpha_{49}, \ldots, \alpha_{51}$, $\alpha_{115}, \ldots, \alpha_{118}$	$\alpha_{16}, \ldots, \alpha_{27}$, $\alpha_{49}, \ldots, \alpha_{51}$, $\alpha_{115}, \ldots, \alpha_{118}$	$\alpha_{16}, \ldots, \alpha_{27}$, $\alpha_{49}, \ldots, \alpha_{51}$, $\alpha_{115}, \ldots, \alpha_{118}$
Weight 3	$\alpha_{28}, \ldots, \alpha_{33}$, $\alpha_{46}, \ldots, \alpha_{48}$, $\alpha_{76}, \ldots, \alpha_{81}$, α_{94}, α_{95}, $\alpha_{119}, \ldots, \alpha_{124}$, $\alpha_{140}, \ldots, \alpha_{148}$, $\alpha_{152}, \ldots, \alpha_{154}$, $\alpha_{164}, \ldots \alpha_{169}$, $\alpha_{176}, \ldots, \alpha_{178}$, $\alpha_{209}, \ldots, \alpha_{212}$	$\alpha_{28}, \ldots, \alpha_{33}$ $\alpha_{46}, \ldots, \alpha_{48}$, α_{56}, α_{58}, $\alpha_{76}, \ldots, \alpha_{81}$, α_{94}, α_{95}, $\alpha_{119}, \ldots, \alpha_{124}$, $\alpha_{140}, \ldots, \alpha_{148}$, $\alpha_{152}, \ldots, \alpha_{154}$, $\alpha_{164}, \ldots, \alpha_{169}$, $\alpha_{176}, \ldots, \alpha_{178}$, $\alpha_{206}, \ldots, \alpha_{212}$	$\alpha_{28}, \ldots, \alpha_{33}$, $\alpha_{46}, \ldots, \alpha_{48}$, $\alpha_{76}, \ldots, \alpha_{81}$, α_{94}, α_{95}, $\alpha_{119}, \ldots, \alpha_{124}$, $\alpha_{140}, \ldots, \alpha_{157}$, $\alpha_{164}, \ldots, \alpha_{181}$, $\alpha_{209}, \ldots, \alpha_{214}$
Weight 4	α_{36}, α_{39}, α_{41}, α_{44}, $\alpha_{99}, \ldots, \alpha_{104}, \alpha_{114}$, α_{125}, α_{128}, α_{131}, α_{214}	α_{13}, α_{36}, α_{45}, α_{65}, α_{73}, α_{88}, $\alpha_{99}, \ldots, \alpha_{104}$, α_{114}, α_{155}, α_{160}, α_{188}	$\alpha_{99}, \ldots, \alpha_{104}$, α_{114}
Weight 5	α_{84}, α_{87}, α_{96} α_{134}, α_{137}, α_{184}, α_{187}, α_{217}, α_{220}	α_{105}	–
Total	101	95	100

Integration result



- Euclidean region: solid black lines; γ: integration path
- One-fold integration till weight-4

$$\frac{\mathbb{Q}-\text{Number}}{\epsilon^4}+\frac{\text{Log}}{\epsilon^3}+\frac{\text{Li}_2}{\epsilon^2}+\frac{\text{one}-\text{fold}}{\epsilon}+\text{one}-\text{fold integral}$$

THANK YOU

On the Calculation of The Three-Loop-Five-Point Rocket Feynman Integral

Yuanche Liu[c,d],Antonela Matijašić[a],Julian Miczajka[a],
Yingxuan Xu[b],Yongqun Xu[c,d] ,Yang Zhang[c,d]

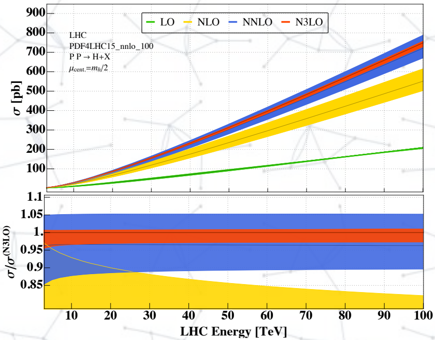
[a] Max-Planck-Institut für Physik, Werner-Heisenberg-Institut, München, Germany
[b] Humboldt–Universität zu Berlin, Institut für Physik, Berlin, Germany
[c] Interdisciplinary Center for Theoretical Study, University of Science and Technology of China, Hefei, China
[d] Peng Huanwu Center for Fundamental Theory, Hefei, China

Yongqun Xu
June 12 Amplitudes 2024

Why we do? Motivation & Background

Phenomenological Aspect

Toward the precision physics



■ The Path forward to N^3LO [Caola,Chen,Duhr,Liu, Mistlberger, Petriello,Vita,Weinzierl]

...

...

Computational Frontier

Extend the complexity border

Massless Feynman Integral in Dim-Reg

Loops / Legs	1	2	3	4
4	✓	✓	3l4p	4l4p
5	✓	2l5p	3l5p	
6	✓	2l6p		

State-of-the-Art Feynman Integrals:

- 2l5p [Chicherin,Gehrmann,Henn,Wasser,Zhang,Zoia]
- 2l6p [Henn,Matijašić,Miczajka,Peraro,Xu,Zhang]
- 3l4p [Henn,Mistlberger,Smirnov,Wasser]
- 4l4p [Dlapa,Henn,Yan]

see also:

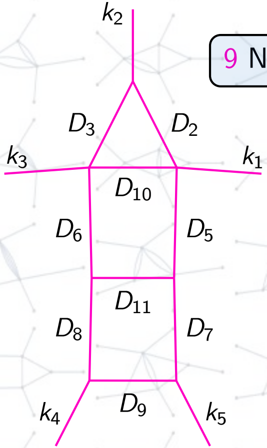
- 2l5p1m [Kardos,Papadopoulos,Smirnov,Syrrakos,Wever] [Abreu,Ita,Moriello,Page,Tschernow,Zeng]
- 3l4p1m [Henn,Lim,Bobadilla]

...

What we are calculating?

Massless Three-Loop-Five-Point

Rocket Feynman Integral in $d = 4 - 2\varepsilon$



9 Numerators + 9 Denominators = 18 Propagators

- Infeasible For FIRE6:

  Too many non-zero sectors!

- 50 cores $\times$ 5 days for a full run of KIRA
- 50 cores $\times$ 3 days for AMFlow

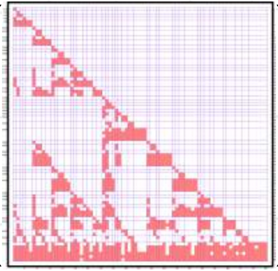
  to get 30 digits at one point till finite order

$$F(\alpha_1, \alpha_2 \cdots \alpha_{18}) = \mathrm{e}^{3\varepsilon\gamma_E} \int \frac{\mathrm{d}^d\ell_1}{\mathrm{i}\pi^{\frac{d}{2}}} \frac{\mathrm{d}^d\ell_2}{\mathrm{i}\pi^{\frac{d}{2}}} \frac{\mathrm{d}^d\ell_3}{\mathrm{i}\pi^{\frac{d}{2}}} \frac{1}{D_1^{\alpha_1} D_2^{\alpha_2} \cdots D_{18}^{\alpha_{18}}}$$

47 Distinctive Topologies. **80** Master Integrals with **6** in Top Sector.
5 Scales $\{s_{12}, s_{23}, s_{34}, s_{45}, s_{15}\}$, **1** $\sqrt{\text{Sqrt}}$, **31** Letters.

Completed UT basis $\Rightarrow$ Canonical Differential Equation:

$$\mathrm{d}\mathbf{J} = \varepsilon \underbrace{\left(\quad \right)}_{80\times 80 \ \textit{matrix}\ \mathrm{d}\tilde{A} = \sum_{i=1}^{31} a_i \mathrm{dlog} W_i} \mathbf{J}, \quad \mathbf{J} = \left(\quad \right)$$



Full Analytic Function Representation till Weight-**3**
Ongoing work for Higher Weight...

$$\frac{\mathbb{Q}\text{-Number}}{\varepsilon^6} + \frac{\text{Log}}{\varepsilon^5} + \frac{\text{Li}_2}{\varepsilon^4} + \frac{\text{Multiple-PolyLog}}{\varepsilon^3} + \frac{\text{Ongoing}}{\varepsilon^2} + \frac{\text{Ongoing}}{\varepsilon^1} + \text{Ongoing}$$

Thank You!

HOW CAN WE PREDICT ALGEBRAIC LETTERS OF SYMBOL ALPHABETS?

$$\mathbb{A} = \left\{ \mathbb{A}_{even}(\vec{v}), \sqrt{Q_i(\vec{v})}, W_{odd} \right\}?$$

[cf. talk by Xu]

$$W_{odd} = \frac{P - \sqrt{Q}}{P + \sqrt{Q}}$$

$$\left(P - \sqrt{Q}\right)\left(P + \sqrt{Q}\right) = c \prod_i W_i^{e_i}, \quad W_i \in \mathbb{A}_{even}$$

[Heller, von Manteuffel, Schabinger '19]

$$P(\vec{v})^2 = Q(\vec{v}) + c \prod_i W_i^{e_i}$$

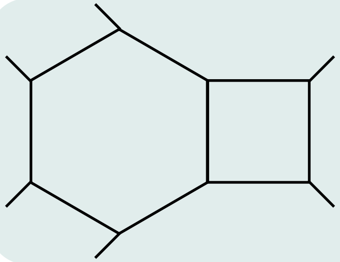
Study of singularities of Feynman integrals: **Landau analysis**

$$\mathbb{A}_{even}, Q_i \in \mathbb{Z}[v_1, \ldots, v_n]$$

[Bjorken; Landau; Nakanishi '59]

recent progress: [Fevola, Mizera, Telen '23,'24]
[Helmer, Papathanasiou, Tellander '24]
[He, Jiang, Liu, Yang '23; Jiang, Liu, Xu, Yang '24]

[cf. talk by Giroux]



116 even letters + 40 square roots

$\Rightarrow$ 121 odd letters

Analytic Continuation of Five-Point Two-Loop Master Integrals

based on work with Costas Papadopoulos,
in extension of [arXiv:2201.07509 [hep-ph]]

Nikos Dokmetzoglou

National Centre for Scientific Research "Demokritos"
Institute of Nuclear and Particle Physics

June 12, 2024

Amplitudes 2024 Gong Show

DEMOKRITOS
NATIONAL CENTRE FOR SCIENTIFIC RESEARCH

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Hellenic Foundation for Research & Innovation

Brief Review of the Simplified Differential Equations (SDE) Approach

In the SDE approach for Master Integrals (Papadopoulos 2014), the ordinary external momenta q_i are parametrized by introducing a dimensionless variable x, as follows

$$q_1 \to p_{123} - x\, p_{12},\;\; q_2 \to p_4,\;\; q_3 \to -p_{1234},\;\; q_4 \to x\, p_1$$

where the new momenta p_i, $i = 1 \ldots 5$, now satisfy $\sum_1^5 p_i = 0$, $p_i^2 = 0$, $i = 1 \ldots 5$, whereas $p_{i \ldots j} := p_i + \ldots + p_j$. The set of independent invariants is given by $\{S_{12}, S_{23}, S_{34}, S_{45}, S_{51}, x\}$, with $S_{ij} := (p_i + p_j)^2$.

Brief Review of the Simplified Differential Equations (SDE) Approach

For pure bases with rational alphabet letters or algebraic alphabet letters with rationalizable square roots, the differential equation for Master Integrals can be cast into its canonical form, easily solvable in terms of Goncharov Polylogarithms (GPLs):

$$\partial_x \mathbf{g} = \epsilon \left(\sum_{i=1}^{l_{max}} \frac{\mathbf{M}_i}{x - l_i} \right) \mathbf{g} \tag{1}$$

However, for some topologies there are alphabet letters with square roots that cannot be rationalized simultaneously, leading to the more general form:

$$\partial_x \mathbf{g} = \epsilon \left(\sum_{a=1}^{l_{max}} \frac{d\log L_a}{dx} \mathbf{M}_a \right) \mathbf{g} \tag{2}$$

5-Point 2-Loop 1-Mass Non-Planar Topologies

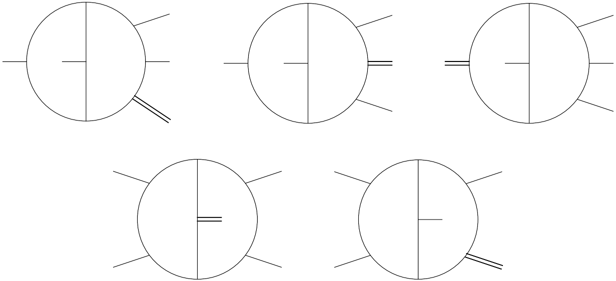

Figure: The five non-planar families with one external massive leg. The first row corresponds to the so-called hexabox topologies, whereas the diagrams of the second row are known as double-pentagons. We label them as follows: N_1 (top left), N_2 (top middle), N_3 (top right), N_4 (bottom left), N_5 (bottom right).

Hexa-Box Topologies: Semi-analytic above Weight 2

Above weight 2, we resort to a semi-numerical one-dimensional integral representation (Kardos, Papadopoulos, Smirnov, Syrrakos, Wever 2022). At weight 3,

$$\partial_x g_I^{(3)} = \sum_a (\partial_x \log L_a) \sum_J c^a_{IJ} g_J^{(2)} \tag{3}$$

$$g_I^{(3)} = g_{I,\mathcal{G}}^{(3)} + b_I^{(3)} + \int_0^{\bar{x}} \mathrm{d}x \left(\sum_a (\partial_x \log L_a) \sum_J c^a_{IJ} g_J^{(2)} - \sum_a \frac{l_a}{x} \sum_J c^a_{IJ} g_{J,0}^{(2)} \right) \tag{4}$$

with $b_I^{(3)}$ being the boundary terms at $\mathcal{O}(\epsilon^3)$ and $g_{I,\mathcal{G}}^{(3)} = \int_0^{\bar{x}} \mathrm{d}x \sum_a \frac{l_a}{x} \sum_J c^a_{IJ} g_{J,0}^{(2)}\Big|_{\mathcal{G}}$, with the subscript $\mathcal{G}$, indicating that the integral is represented in terms of GPLs.

Mathematica Implementation

$$\boxed{g_I^{(n+1)}} = g_{I,\mathcal{G}}^{(n+1)} + b_I^{(n+1)} + \int_0^{\bar{x}} \mathrm{d}x \left(\sum_a (\partial_x \log L_a) \sum_J c_{IJ}^a \, g_J^{(n)} - \sum_a \frac{l_a}{x} \sum_J c_{IJ}^a \, g_{J,0}^{(n)} \right)$$

PureBasisElementNumerical[*ElementIndex*, *TranscendentalWeight*, *PhaseSpacePoint***]** : numerical evaluation of pure basis elements at weight $n+1$ by integration of the analytic form of pure basis elements at weight n

Mathematica Implementation

$$g_I^{(n+1)} = g_{I,\mathcal{G}}^{(n+1)} + b_I^{(n+1)} + \int_0^{\bar{x}} \mathrm{d}x \left(\sum_a (\partial_x \log L_a) \sum_J c_{IJ}^a \boxed{g_J^{(n)}} - \sum_a \frac{l_a}{x} \sum_J c_{IJ}^a \boxed{g_{J,0}^{(n)}} \right)$$

PureBasisElementAnalytic[*ElementIndex*, *TranscendentalWeight*] : analytic form of pure basis elements, up to weight 2 for N_2 and N_3 hexa-box families

PureBasisElementAnalytic[*ElementIndex*, *TranscendentalWeight*, x $\to$ 0] : $x \to 0$ limit of analytic form of pure basis elements, e.g. $g_{J,0}^{(2)}$ are obtained by expanding $g_J^{(2)}$ around $x = 0$ and keeping terms up to order $\mathcal{O}(\log(x)^2)$

Mathematica Implementation

$$g_I^{(n+1)} = g_{I,\mathcal{G}}^{(n+1)} + \boxed{b_I^{(n+1)}} + \int_0^{\bar{x}} \mathrm{d}x \left(\sum_a \left(\partial_x \log L_a\right) \sum_J c_{IJ}^a \, g_J^{(n)} - \sum_a \frac{l_a}{x} \sum_J c_{IJ}^a \, g_{J,0}^{(n)} \right)$$

BoundaryTerm[*ElementIndex*, *TranscendentalWeight*] : analytic form of boundary terms at $\mathcal{O}(\epsilon^{(n+1)})$

Mathematica Implementation

$$g_I^{(n+1)} = g_{I,\mathcal{G}}^{(n+1)} + b_I^{(n+1)} + \int_0^{\bar{x}} \mathrm{d}x \left(\sum_a \boxed{(\partial_x \log L_a)} \sum_J c_{IJ}^a \, g_J^{(n)} - \sum_a \boxed{\frac{l_a}{x}} \sum_J c_{IJ}^a \, g_{J,0}^{(n)} \right)$$

DLogAlphabetLetter[*LetterIndex*] : derivatives of logs of alphabet letters with respect to parameter x
DLogAlphabetLetter[*LetterIndex*, x → 0] : $x \to 0$ limit of $\partial_x \log L_a$, obtained by expanding $\partial_x \log L_a$ around $x = 0$ and keeping terms up to order $\mathcal{O}(x^{-1})$
LogAlphabetLetter[*LetterIndex*] : logs of alphabet letters
LogAlphabetLetter[*LetterIndex*, x → 0] : $x \to 0$ limit of $\log L_a$, obtained by expanding $\log L_a$ around $x = 0$ and keeping terms up to order $\mathcal{O}(\log(x))$

Mathematica Implementation

$$g_I^{(n+1)} = g_{I,\mathcal{G}}^{(n+1)} + b_I^{(n+1)} + \int_0^{\bar{x}} \mathrm{d}x \left(\boxed{\sum_a \left(\partial_x \log L_a\right) \sum_J c_{IJ}^a \, g_J^{(n)}} - \boxed{\sum_a \frac{l_a}{x} \sum_J c_{IJ}^a \, g_{J,0}^{(n)}} \right)$$

DifferentialEquationRHS[*ElementIndex*, *TranscendentalWeight*]
DifferentialEquationRHS[*ElementIndex*, *TranscendentalWeight*, x → 0]

Mathematica Implementation

$$g_I^{(n+1)} = \boxed{g_{I,\mathcal{G}}^{(n+1)}} + b_I^{(n+1)} + \int_0^{\bar{x}} \mathrm{d}x \left(\sum_a \left(\partial_x \log L_a\right) \sum_J c_{IJ}^a \, g_J^{(n)} - \sum_a \frac{l_a}{x} \sum_J c_{IJ}^a \, g_{J,0}^{(n)} \right)$$

$$\boxed{g_{I,\mathcal{G}}^{(n+1)} = \int_0^{\bar{x}} \mathrm{d}x \sum_a \frac{l_a}{x} \sum_J c_{IJ}^a g_{J,0}^{(n)} \Big|_{\mathcal{G}}}$$

ClassicalToGoncharovPolyLogs[*expr*] and **GoncharovToClassicalPolyLogs[*expr*]** : converter between Goncharov Polylogarithms and Classical Logs and PolyLogs, making use of (Frellesvig et al. 2016)

Analytic integration performed using the function **GIntegrate** of the `PolyLogTools` package (Duhr, Dulat 2019).

Mathematica Implementation

Other auxiliary functions:

- **LettersToInvariants[*expr*]** and **InvariantsToLetters[*expr*]**
- **NumericalEvaluationMathematica[*expr*, *PhaseSpacePoint*]** and **NumericalEvaluationGiNaC[*expr*, *PhaseSpacePoint*]**
- **AssignImaginaryParts[*expr*, *PhaseSpacePoint*]** $\rightarrow$ Analytic Continuation

Need for Analytic Continuation

- Need to analytically continue the integrand of the numerical integration, i.e. the differential equation at weight n and its $x \to 0$ limit
- Types of singularities encountered:
  - $g_J^{(n\leq 2)}$ and $g_{J,0}^{(n\leq 2)}$: logarithmic and polylogarithmic branch points/cuts
  - $\partial_x \log L_a$: poles at points $x = \ell_i$
  - $\log L_a$ and LL_a (the $x \to 0$ limits of $\log L_a$) : logarithmic branch points/cuts
  - square-root branch points/cuts

Imaginary Parts for S_{ij} Invariants from $\mathcal{F}$ Symanzik Polynomial

Since the $\mathcal{F}$ Symanzik polynomial maintains the sign of the $i0$ prescription of Feynman propagators with all original invariants (s_{ij}), assuming $s_{ij}\,(p_{1s}) \to s_{ij}\,(p_{1s}) + i\,\eta$, we determine the corresponding infinitesimal imaginary parts for our invariants (S_{ij}) by recasting the $\mathcal{F}$ Symanzik polynomial for a given integral family in terms of our invariants, extending those by $S_{ij} \to S_{ij} + i\,\delta_{ij}\,\eta$, and imposing a positivity constraint on the coefficients of all the Feynman parameters.

Analytic Continuation of $\partial_x \log L_a$

As mentioned earlier, $\partial_x \log L_a$ have poles at points $x = \ell_i$. To control the numerical integration over the locations of these poles, we make this pole structure manifest

$$\frac{d \log L_a}{dx} \to \frac{f(x)}{\prod_i (x - \ell_i)} ,$$

where $f(x) \in \mathbb{R}$, except for factors of $\sqrt{\Delta} = i\sqrt{|\Delta|}$ for $\Delta < 0$, and we assign imaginary parts to all ℓ_i's using those we assigned to the S_{ij} invariants

$$\ell_i(S_{ij}) \to \ell_i(S_{ij} + i\,\delta_{ij}\,\eta) \equiv \ell_i + i\,\delta_i\,\eta \ .$$

Analytic Continuation of $g_J^{(n\leq 2)}$

Concerning $g_J^{(n\leq 2)}$, their logarithmic and polylogarithmic branch points/cuts appear through expressions of the form:

- $\log(x)$, $\log(\ell_a)$, $\log(1-\ell_a)$,
- $\mathcal{G}(\ell_a; x)$, $\mathcal{G}(0; x)$, $\mathcal{G}(\ell_a, \ell_b; x)$, $\mathcal{G}(0, \ell_a; x)$, $\mathcal{G}(0,0;x)$,

In some elements of the N_2 hexa-box family we also find:

- $\mathcal{G}\left(0,1;\tilde{\ell}_a(x)\right)$ and $\mathcal{G}\left(1;\tilde{\ell}_a(x)\right)$,

where $\tilde{\ell}_a(x)$ are **algebraic expressions of** x. To control the numerical integration over the locations of all branch points, we assign imaginary parts to all ℓ_i's similarly to before, and also to the $\tilde{\ell}_a(x)$'s, such that

$$\tilde{\ell}_a(\ell_i) \to \tilde{\ell}_a(\ell_i + i\,\delta_i\,\eta) \equiv \tilde{\ell}_a(\ell_i) + i\,\delta_a\,\eta\ .$$

Analytic Continuation of $\log L_a$

As for the logs of the alphabet letters, we isolate their logarithmic branch points in the following way:

$$\log L_a \to \log\left(L_a \frac{\prod_{i_D}(x-\ell_{i_D})}{\prod_{j_N}(x-\ell_{j_N})}\right) - \log\left(\prod_{i_D}(x-\ell_{i_D})\right) + \log\left(\prod_{j_N}(x-\ell_{j_N})\right)$$

where $\left(L_a \frac{\prod_{i_D}(x-\ell_{i_D})}{\prod_{j_N}(x-\ell_{j_N})}\right) \in \mathbb{R}$, except for factors of $\sqrt{\Delta} = i\sqrt{|\Delta|}$ for $\Delta < 0$, and has no zeroes or poles in $x \in (0, \bar{x})$, and then we assign imaginary parts in the ordinary way to all ℓ_i's.

Summary and Outlook

- Need for semi-analytic approach above transcendental weight 2 for some of the non-planar 5-point topologies $\rightarrow$ analytic continuation of the integrands
- Successful check for N_2 integrals family on a specific physical phase-space point, against the numerical results obtained from the literature (Abreu, Chicherin, Ita, Page, Sotnikov, Tschernow, Zoia 2024), up to weight 3
- Need to extend our Mathematica implementation to weight 4, as well as to the other non-planar topologies
- More work to be done on generalizing our analytic continuation methods

Thank you for your attention!

This research is supported by the Hellenic Foundation for Research and Innovation (H.F.R.I.), under the 2nd Call for H.F.R.I. Research Projects for the Support of Faculty Members and Researchers (Project Acronym: HOCTools-II, Project Number: 2674).

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Scalar radiation in scalar-tensor theories

Panagiotis Marinellis

In collaboration with Adam Falkowski **[24XX.XXXXX]**

IJCLab Irène Joliot-Curie Laboratoire de Physique des 2 Infinis

université PARIS-SACLAY

Image credit: Mark Garlick/Science Photo Library/Getty Images

Example of scalar-tensor theories: Scalar-Gauss Bonnet and Dynamical Chern Simons Gravity

$$S = S_{EH}[R] + S_{SGB,DCS}[\phi, g_{\mu\nu}] + S_m[\Psi_m, \mathscr{A}(\phi) g_{\mu\nu}] \quad ,$$

- $S_{EH} = \int d^4x \frac{M_{Pl}^2}{2}\sqrt{-g}R$

- $S_{SGB,DCS} = \int d^4x \sqrt{-g}\left[M_{Pl}\left(\frac{\alpha}{\Lambda^2}\phi\mathscr{G} + \frac{\tilde{\alpha}}{\Lambda^2}\phi R\tilde{R}\right) + \frac{1}{2}\left(\partial^\mu\phi\partial_\mu\phi\right)\right]$

Example of scalar-tensor theories: Scalar-Gauss Bonnet and Dynamical Chern Simons Gravity

$$S = S_{EH}[R] + S_{SGB,DCS}[\phi, g_{\mu\nu}] + S_m[\Psi_m, \mathscr{A}(\phi) g_{\mu\nu}] \quad ,$$

- $S_{EH} = \int d^4x \frac{M_{Pl}^2}{2}\sqrt{-g}R$

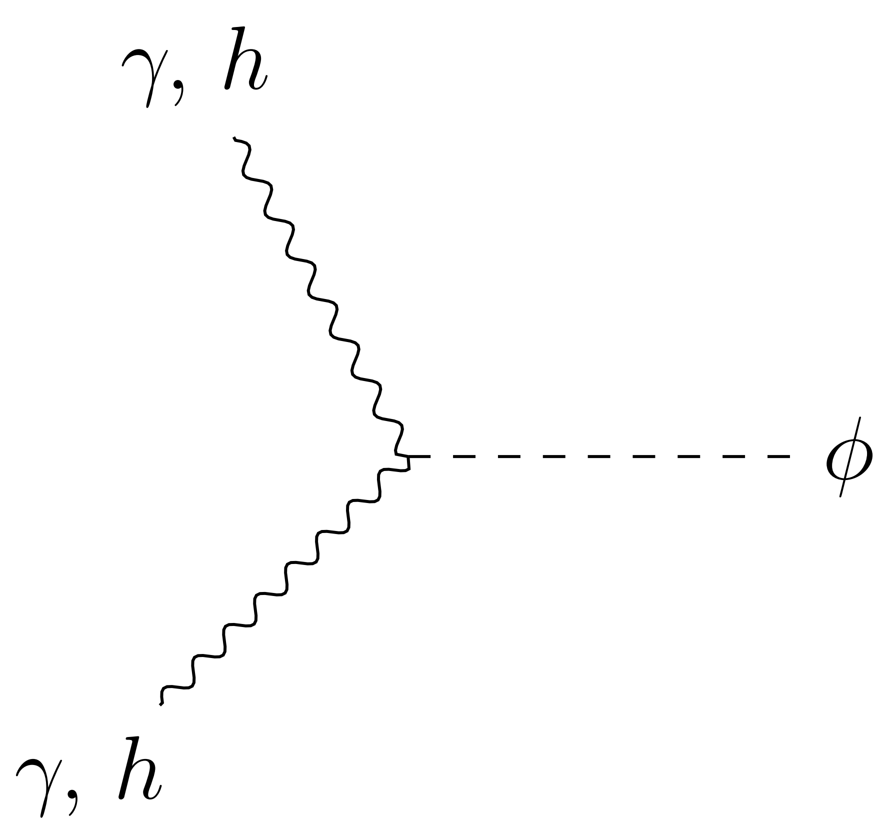
$R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2$

$R^{\mu\nu\rho\sigma}\tilde{R}_{\mu\nu\rho\sigma} \quad , \quad \tilde{R}^{\mu}{}_{\nu\rho\sigma} = \frac{1}{2}\varepsilon_{\rho\sigma}{}^{\alpha\beta}R^{\mu}{}_{\nu\alpha\beta}$

- $S_{SGB,DCS} = \int d^4x\sqrt{-g}\left[M_{Pl}\left(\frac{\alpha}{\Lambda^2}\phi\mathscr{G} + \frac{\tilde{\alpha}}{\Lambda^2}\phi R\tilde{R}\right) + \frac{1}{2}\left(\partial^\mu\phi\partial_\mu\phi\right)\right]$

Scalar radiation for spinning bodies

Building blocks:



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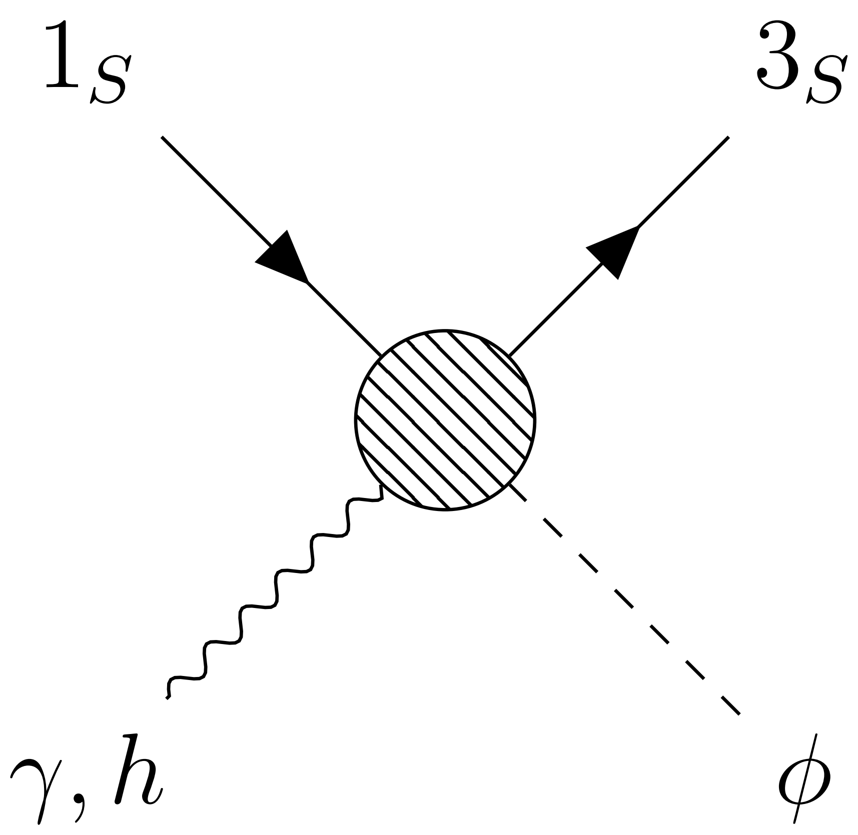
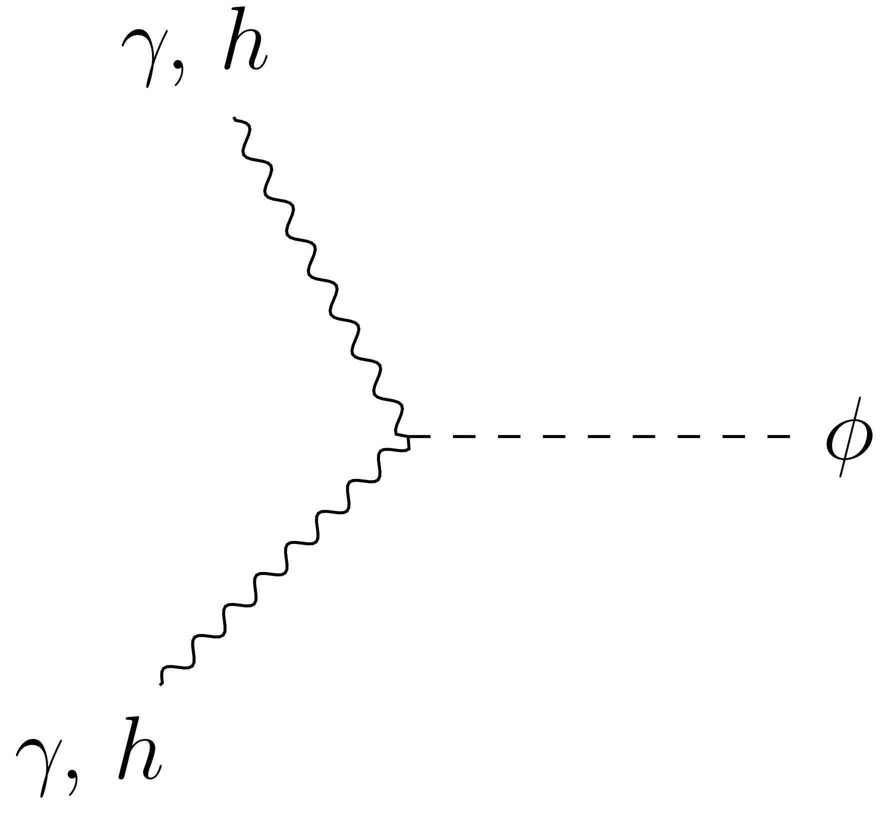


Includes:

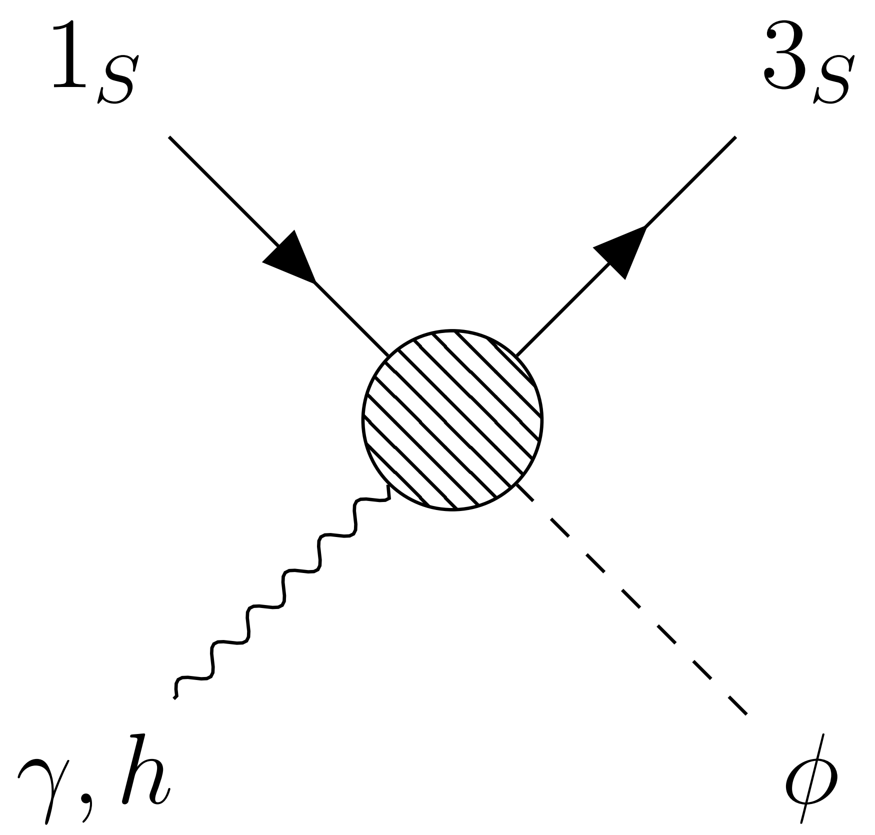
- **Graviton exchange**
- **Contact terms' contributions**

Scalar radiation for spinning bodies

Building blocks:



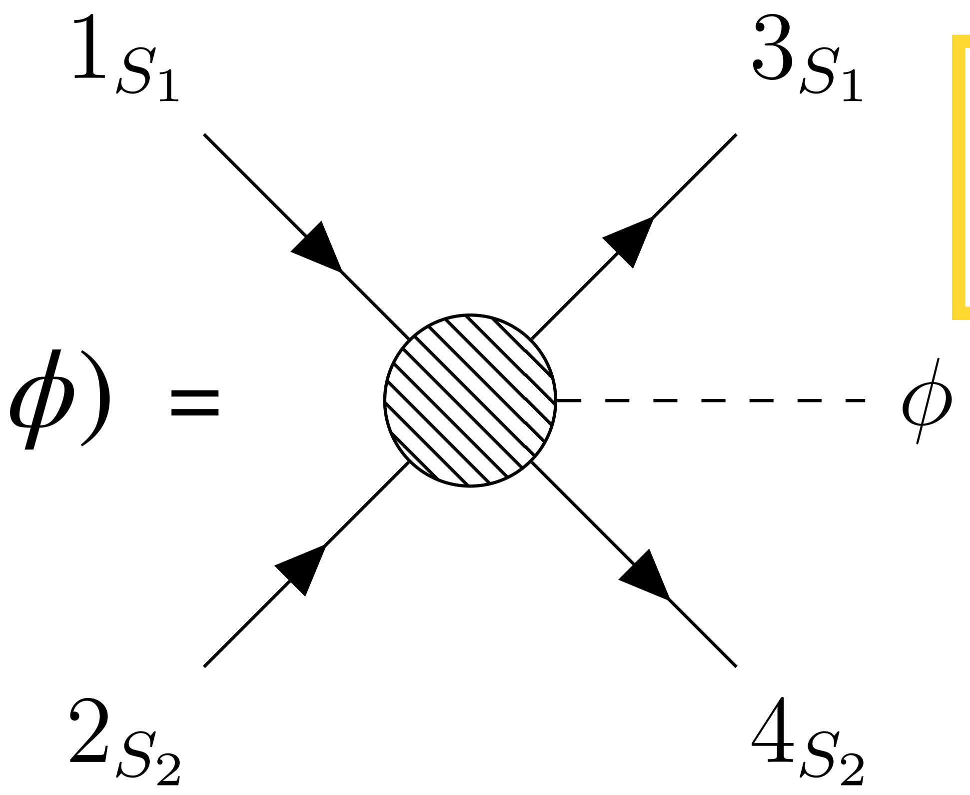
,



Includes:

- **Graviton exchange**
- **Contact terms' contributions**

$\mathscr{M}_5(1_{S_1},2_{S_2},3_{S_1},4_{S_2},\phi)$ =



KMOC formalism*, $\mathscr{M}_5 \to \mathscr{M}_{5,cl.}$

$$W^{\mathrm{LO}}(t,\hat{n}) = \frac{1}{4\pi}\int_0^\infty \hat{d}\omega e^{-i\omega t}\left\langle\!\left\langle \hbar^2 \prod_{i=1,2}\int \hat{d}^4\bar{w}_i\hat{\delta}(2p_i\cdot\bar{w}_i)e^{ib_i\cdot\bar{w}_i}\hat{\delta}^{(4)}(\bar{w}_1+\bar{w}_2-\bar{k}) \times \mathscr{M}^{(0)}_{5,cl.}(p_1+\hbar\bar{w}_1,p_2+\hbar\bar{w}_2\to p_1,p_2,\hbar\bar{k})|_{\bar{k}=\omega(1,\hat{n})}\right\rangle\!\right\rangle + h.c.$$

The 5-point Amplitude at any spin order!

*D. A. Kosower, B. Maybee, D. O'Connell, "Amplitudes, Observables, and Classical Scattering", JHEP 02 (2019) 137
A. Cristofoli, R. Gonzo, D. A. Kosower, D. O'Connell, "Waveforms from Amplitudes", Phys.Rev.D 106 (2022) 5, 0567007

Some results for Scalar Gauss Bonnet Gravity:

Thank you for your attention!

Gravitational Collapse in Large Dimensions

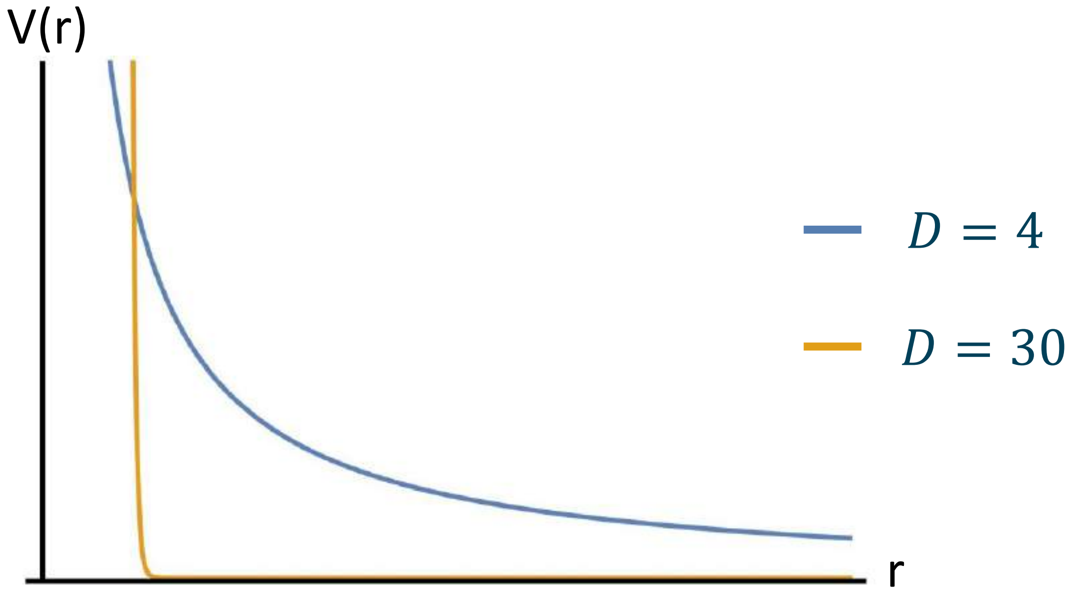
$$S = \frac{1}{2}\int d^D x (R - (\nabla\varphi)^2)$$

- $p > p_*$ Supercritical
- $p < p_*$ Subcritical
- $p = p_*$ Critical

- Universality, Enhanced Symmetry
  - CSS or DSS

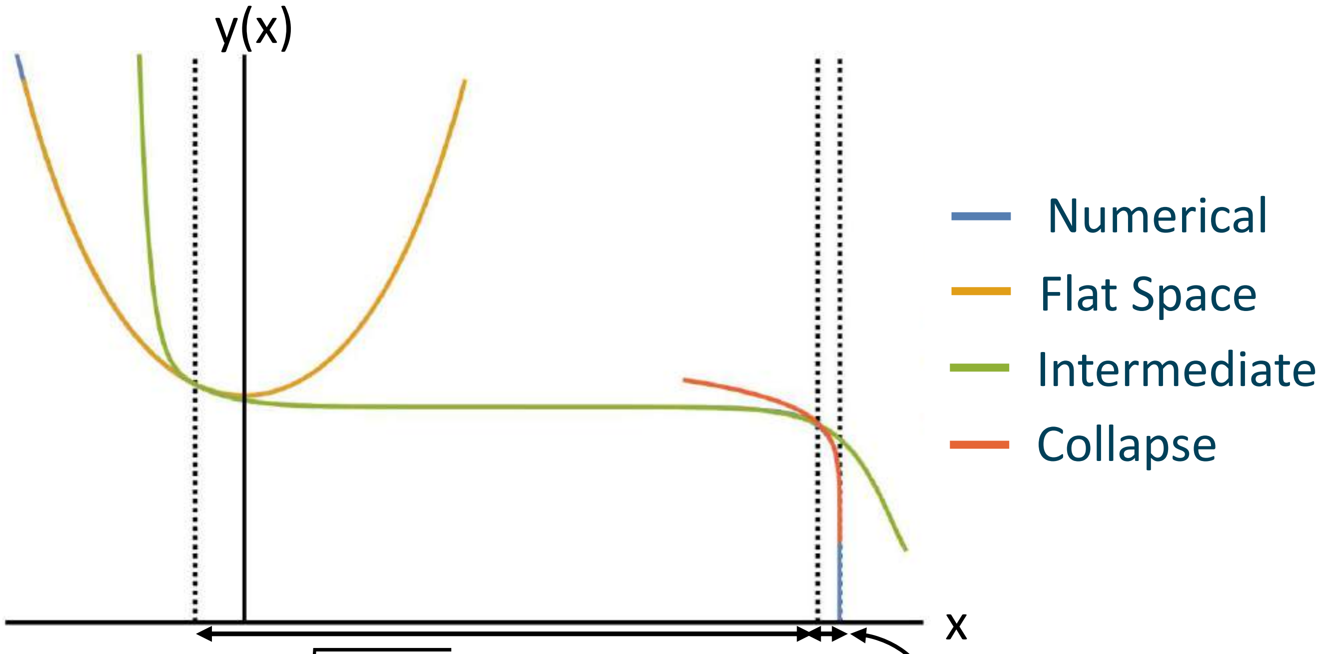
- Critical Exponent $M \sim (p - p_*)^\gamma$

Why Large Dimensions?



$$V(r) \propto \frac{1}{r^{D-3}}$$

E.g CSS Supercritical



$$\sim \sqrt{\frac{2}{D-2}\log\left(\frac{1}{p-p_*}\right)}$$

$$\sim \frac{2}{D-2}$$

Separation Into Regions

SCUOLA NORMALE SUPERIORE

Toward Chaos in String Scattering Amplitudes

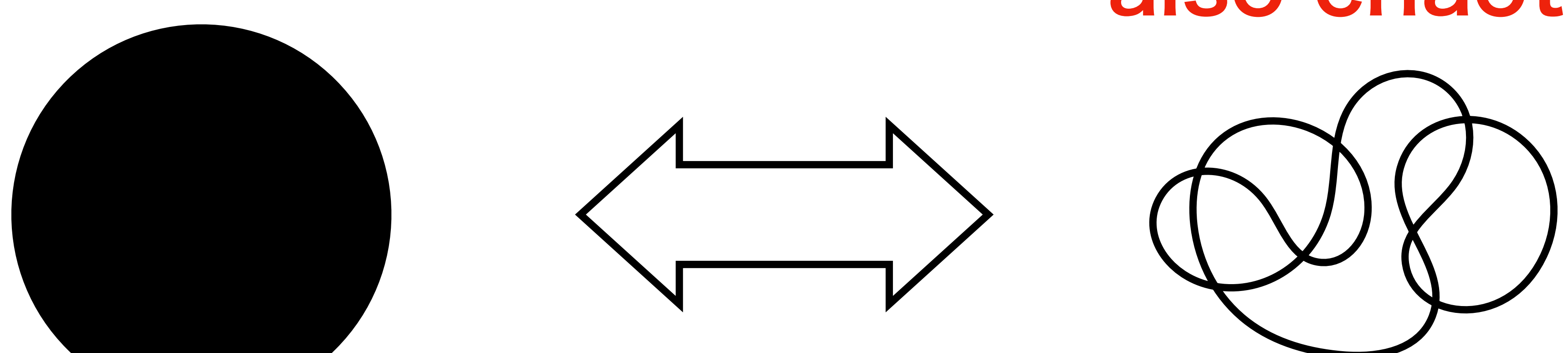
Takuya Yoda

Kyoto University, Japan

K. Hashimoto, Y. Matsuo, T. Yoda, Transient chaos analysis of string scattering, JHEP11(2022)147,
K. Hashimoto, Y. Matsuo, T. Yoda, String is a double slit, Prog. Theor. Exp. Phys. (2023) 043B04,
and some works in progress

Black hole is chaotic

Then, is string also chaotic?



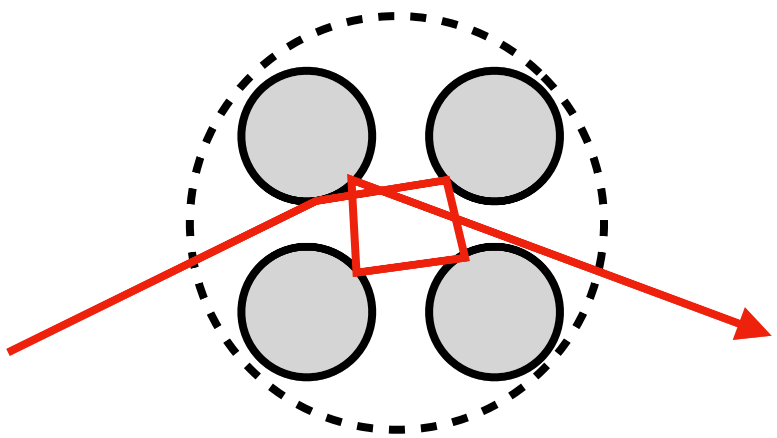
Black hole-string correspondence

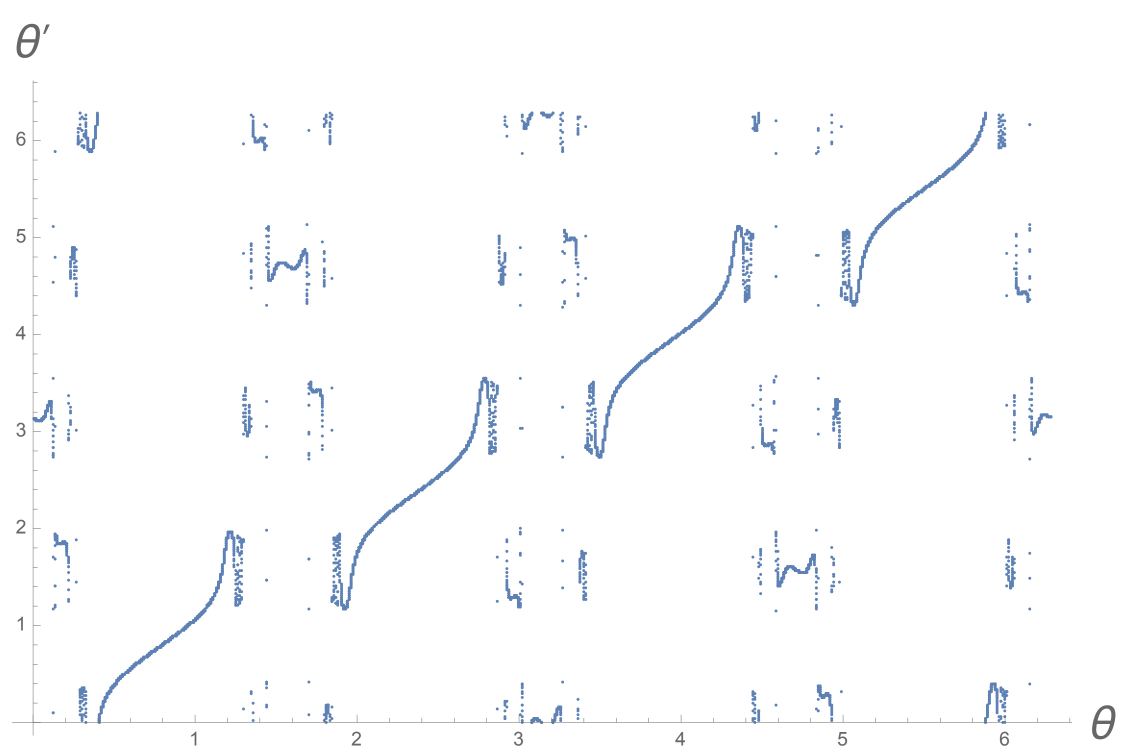
京都大学
KYOTO UNIVERSITY

Amplitudes 2024@IAS, 12 Jun. 2024

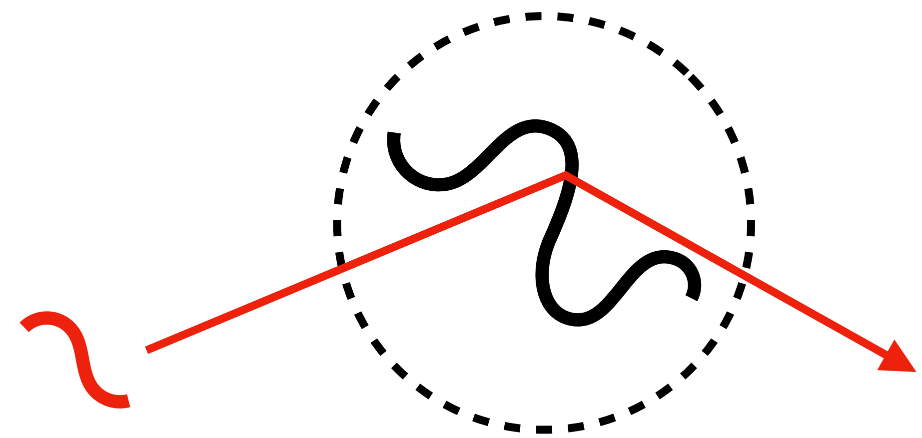
Comparison:

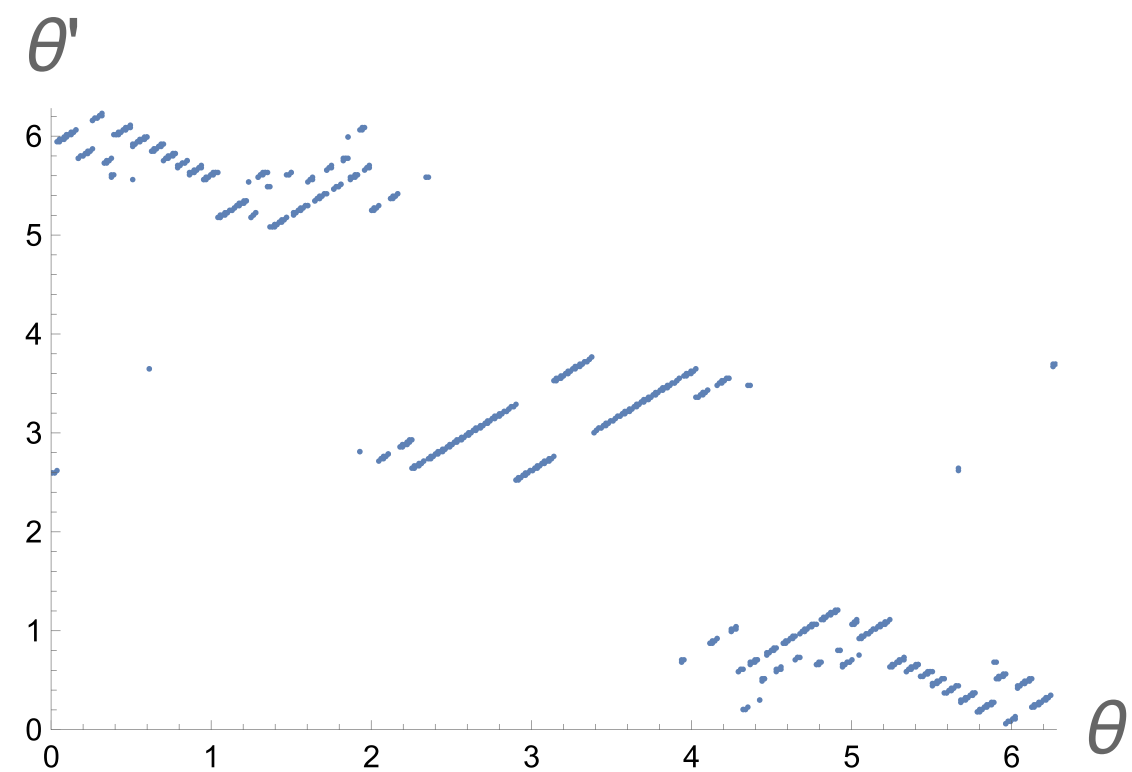
classical chaotic scattering





highly excited strings

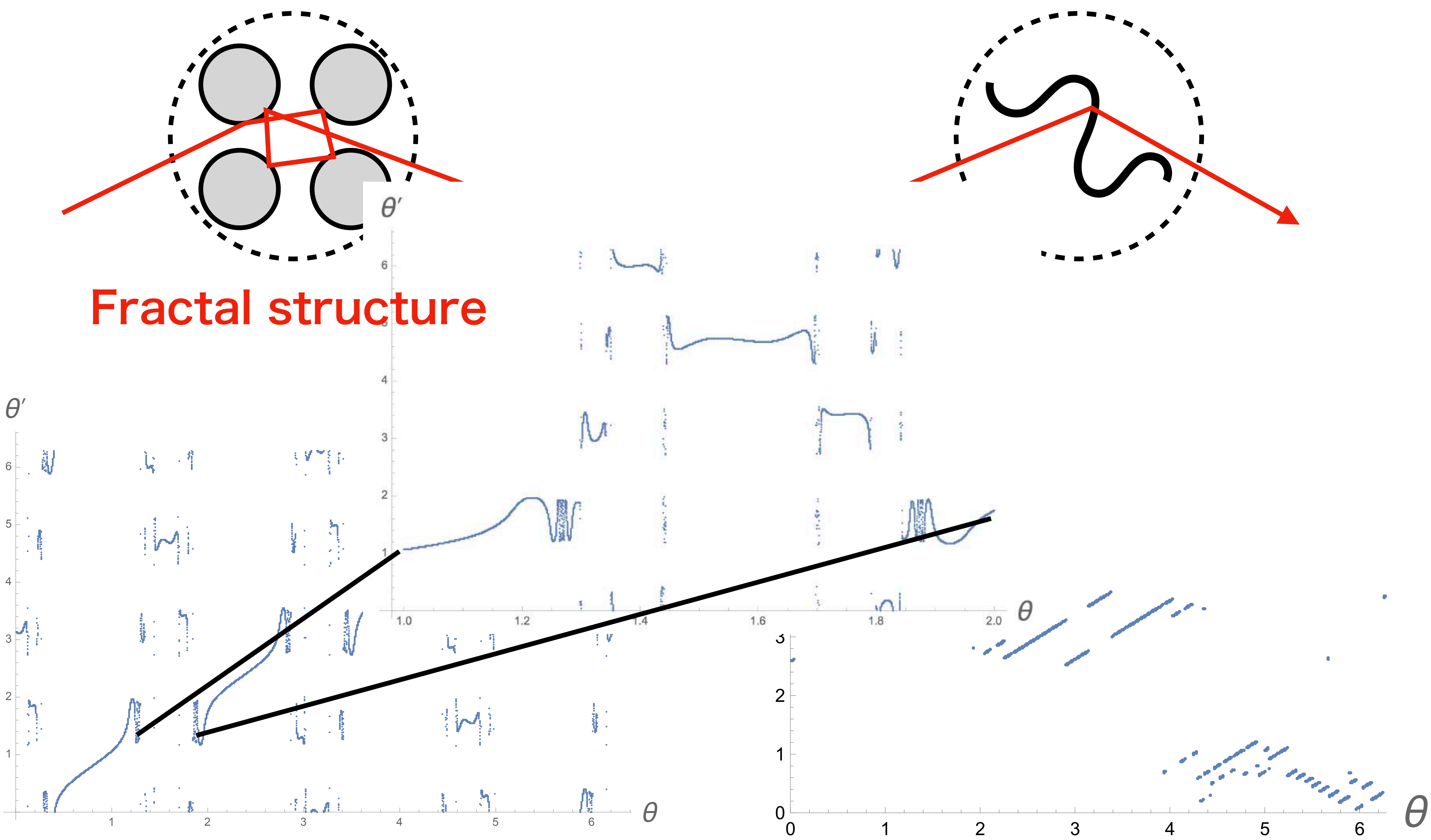




Comparison:

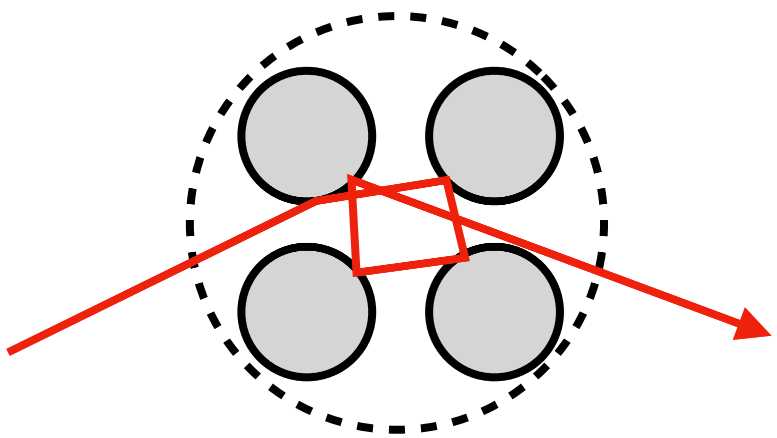
classical chaotic scattering

highly excited strings

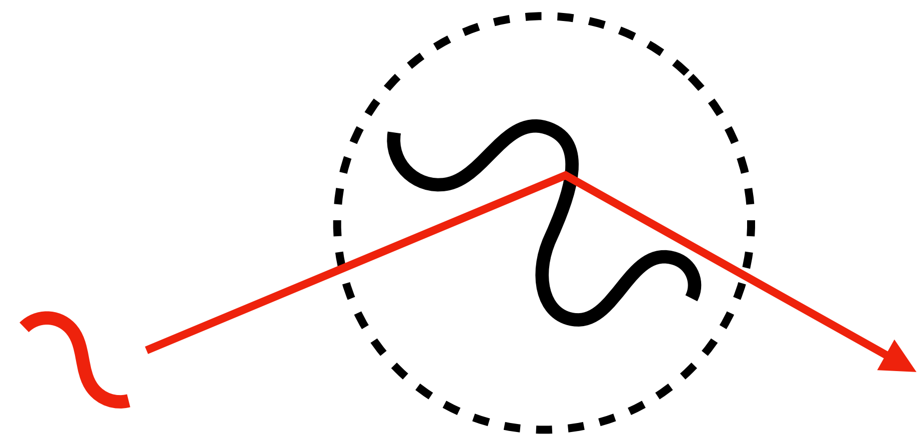


Comparison:

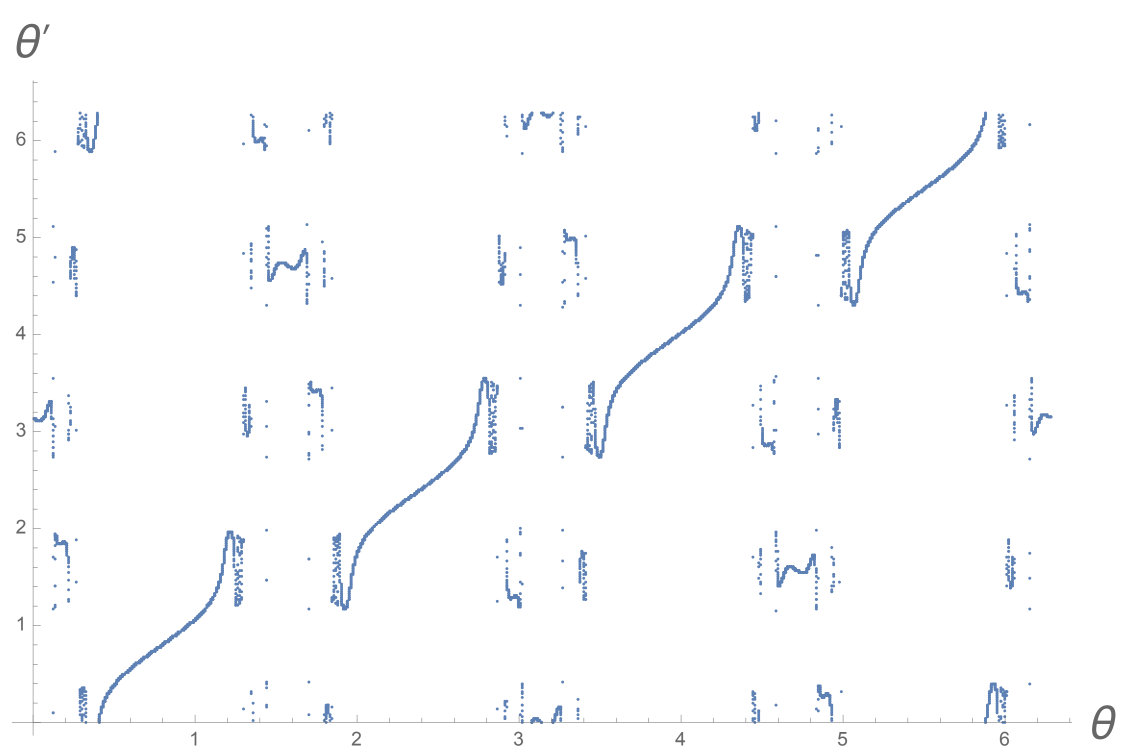
classical chaotic scattering

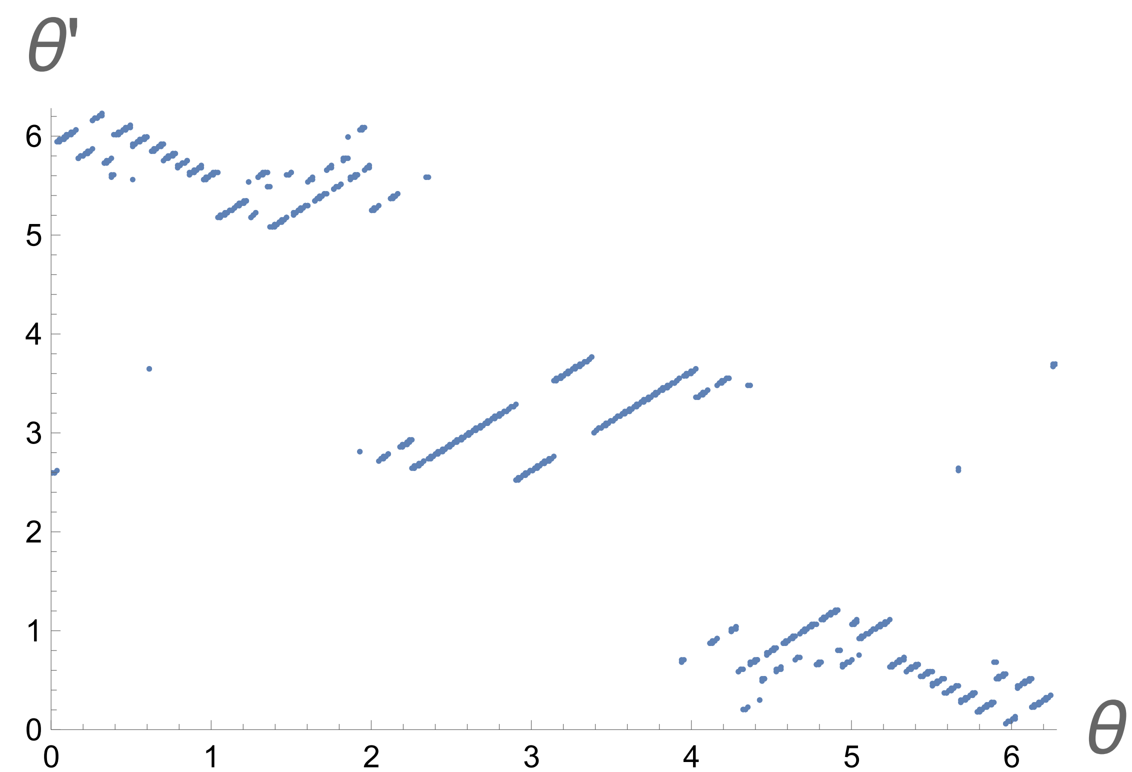


highly excited strings

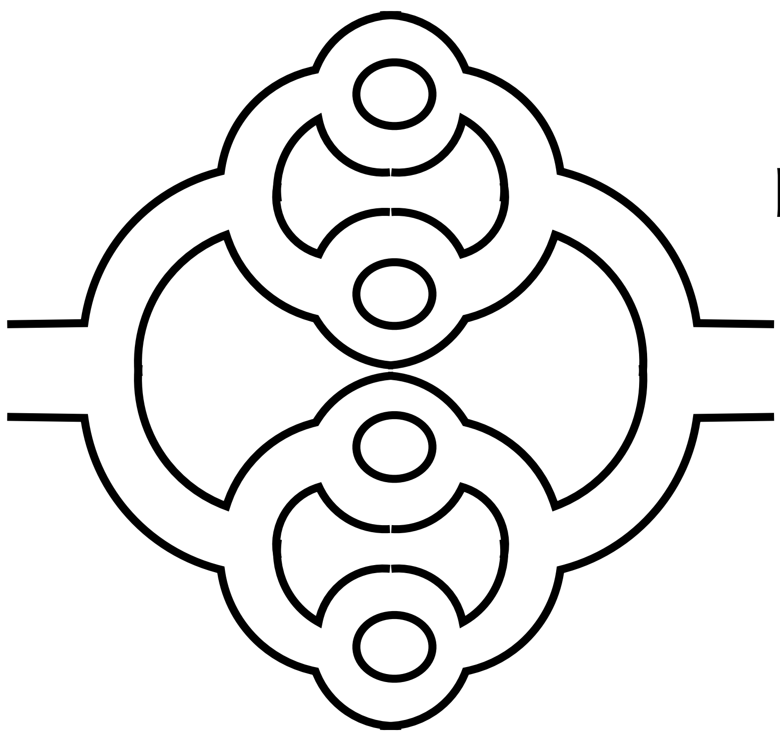


Not fractal at tree level



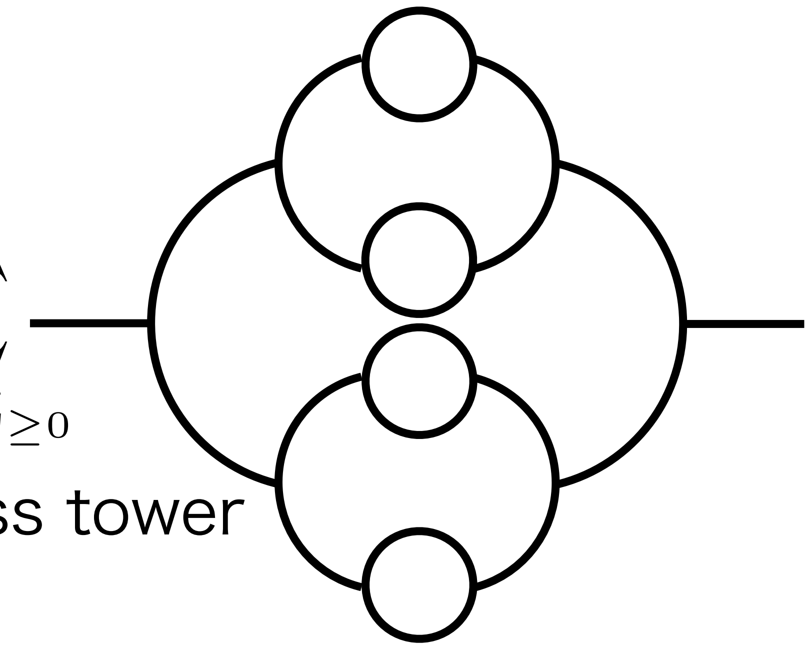


How about higher genus?

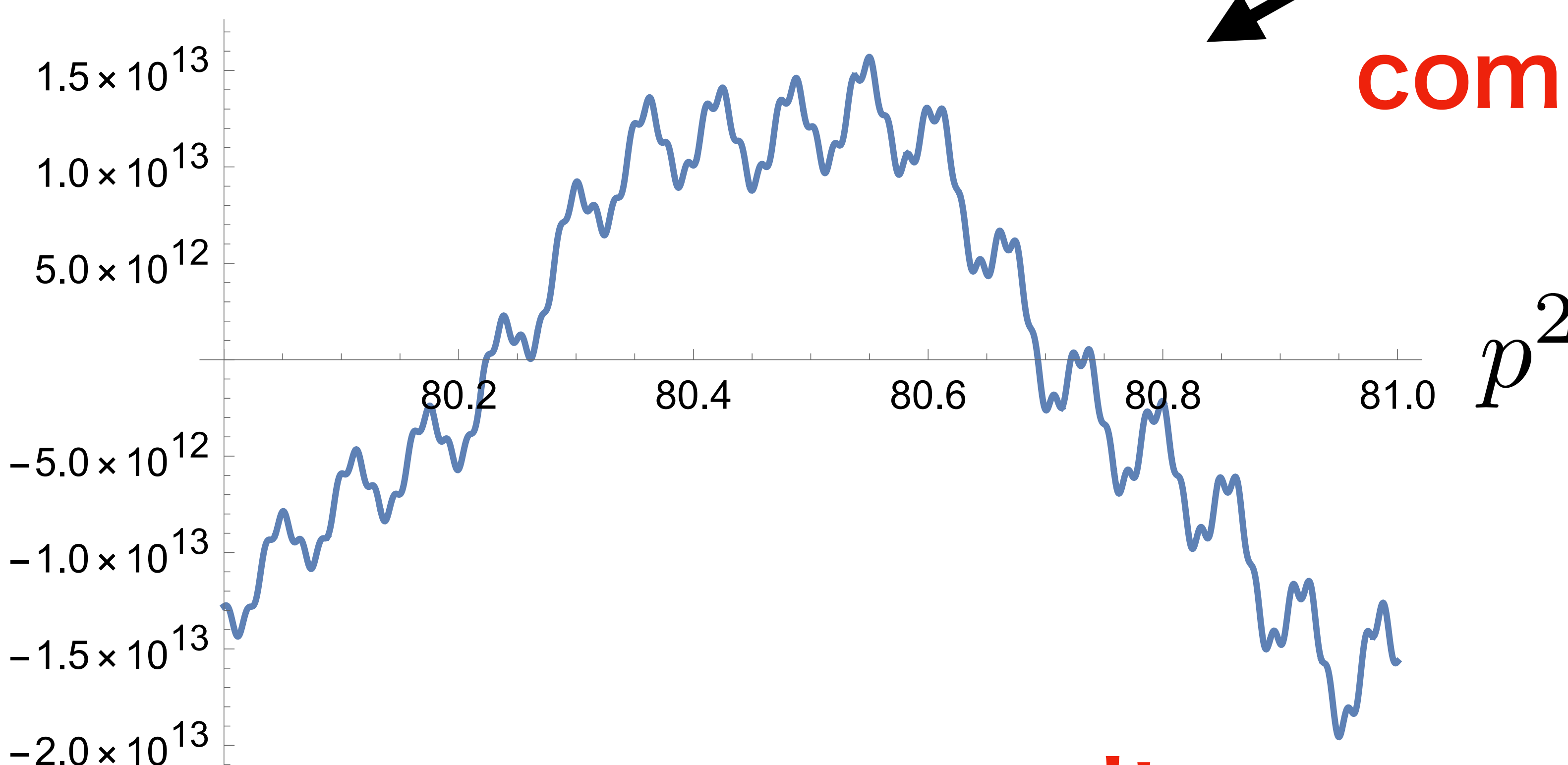


reduce by hand

$$\sum_{m_i^2 \in \mathbf{Z}_{\geq 0}}$$



stringy mass tower



complex saddle analysis

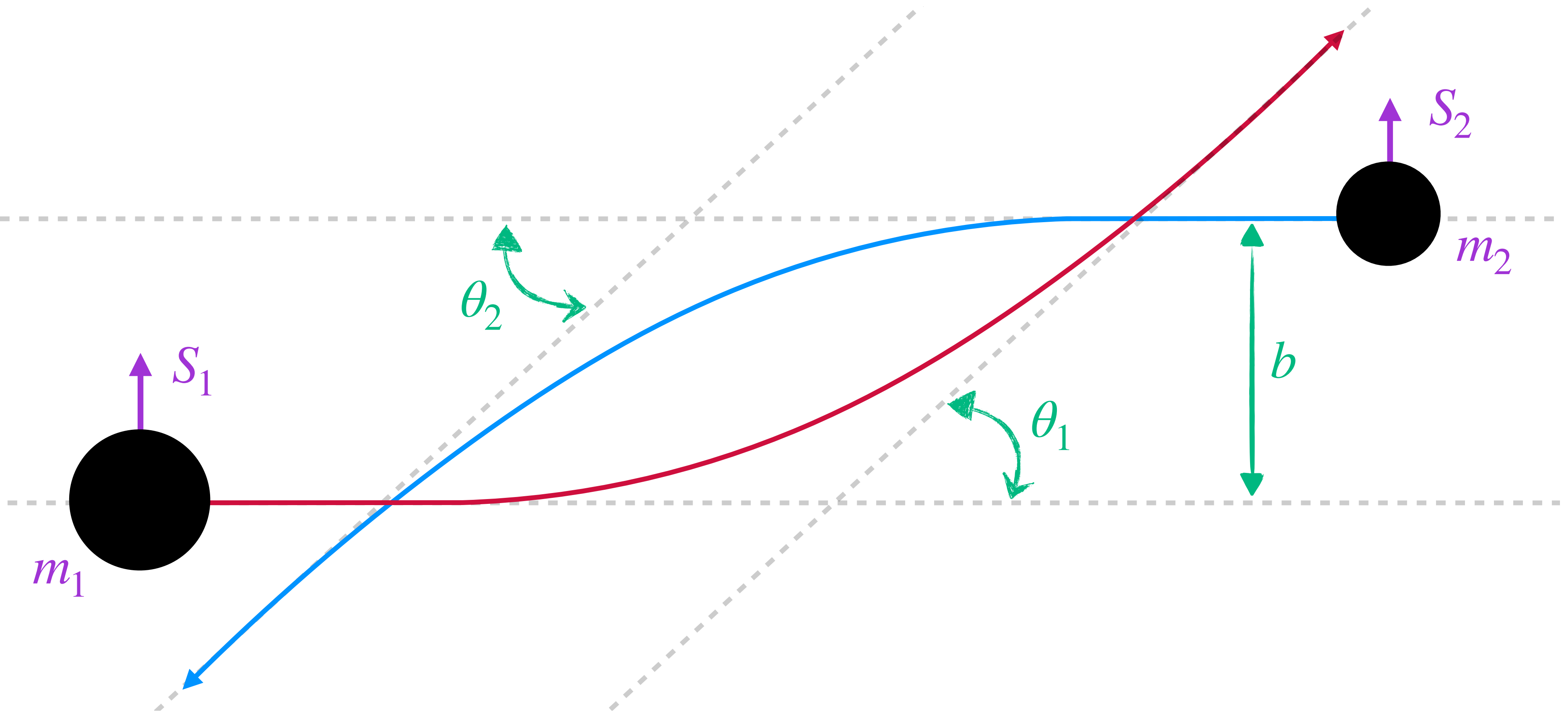
It approaches fractal function

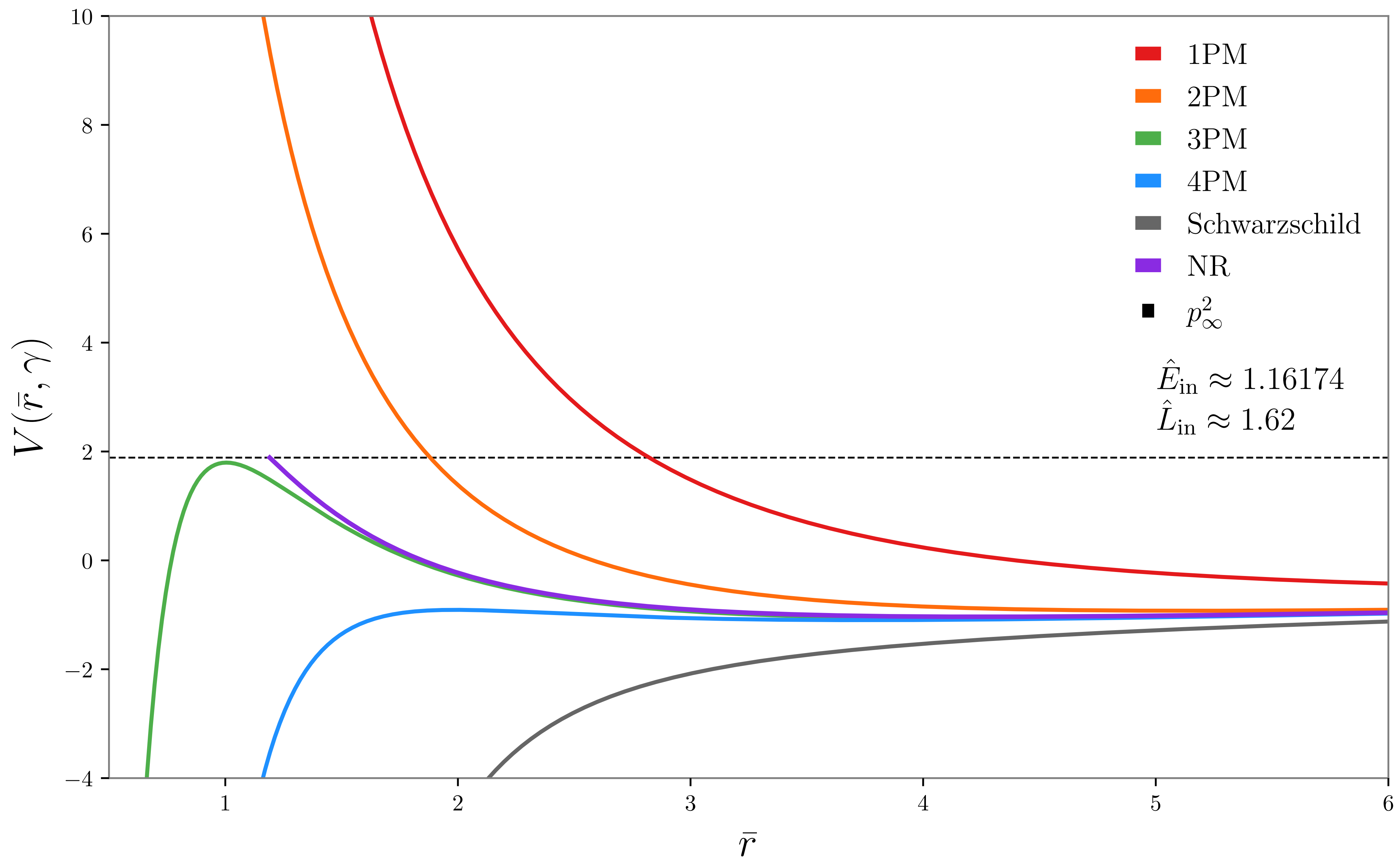
(preliminary result)

Scattering Black Holes: Post-Minkwoski Gravity & Numerical Relativity

Shaun Swain*, Geraint Pratten, Patricia Schmidt

$$\chi_{n\mathrm{PM}}(\gamma, j) = 2j \int_0^{\bar{u}_{\max}} \frac{d\bar{u}}{\sqrt{p_\infty^2 + w_{n\mathrm{PM}}(\bar{u},\gamma) - j^2\bar{u}^2}} - \pi \quad \longrightarrow \quad V_{n\mathrm{PM}}(\bar{u},\gamma,j) = j^2\bar{u}^2 - w_{n\mathrm{PM}}(\bar{u},\gamma)$$





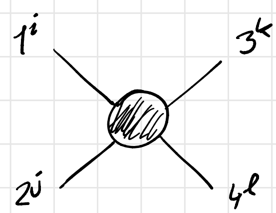
- Quantum scattering amplitudes → classical dynamical information
- NR provides non-perturbative information → validate post-Minkowski calculations
- Suite of *new* NR simulations at higher energies
- Best strategy for resummation of PM information into EOB framework?
- How many loops *required* for accuracy demands of GW data analysis?

THE ROYAL SOCIETY

UNIVERSITY OF BIRMINGHAM

Extremal HIGGS Couplings

[or a guide for bounding dim ≤6 low energy parameters]



presented by

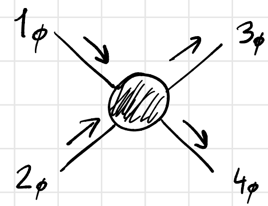
Mehmet Asim Gümüş (LAPTh Annecy)

joint with

Joan Elias Miró (ICTP)
Andrea Guerrieri (CERN)

Amplitudes 2024 @ IAS
Gong show 12.06.24

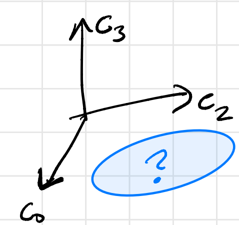
- Consider a real massive scalar and its $2 \to 2$ interacting scattering amplitude:



$$= T(s,t)$$

$s = (p_1 + p_2)^2$
$t = (p_1 - p_3)^2$

- Can $\{c_i\}$ be anything?
What are the allowed values?



- Low energy expansion

$$T(\bar{s},\bar{t}) \approx c_0 + c_2(\bar{s}^2 + \bar{t}^2 + \bar{u}^2) + c_3 \cdot \bar{s} \cdot \bar{t} \cdot \bar{u} + \cdots$$

$\bar{s} = (s - 4m^2/3)$, $\bar{t} = (t - 4m^2/3)$

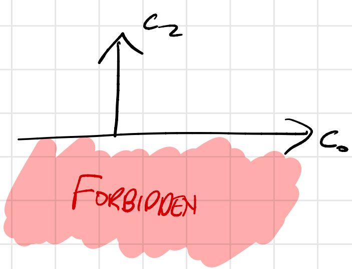
c_0, c_2, c_3: non-perturbatively defined

— No! c_2 is positive.

$$c_2 = \frac{1}{2\pi i}\oint \frac{d\bar{v}}{\bar{v}^3}\, T'(\bar{v},0)$$

$$= \frac{1}{\pi}\int_{8/3}^{\infty} \frac{d\bar{v}}{\bar{v}^3}\, \mathrm{Im}\, T'(\bar{v},0) \geq 0$$

$$\sum_{\ell=0}^{\infty} \underbrace{P_\ell\left(1+\frac{8/3}{v-4}\right)}_{\text{Legendre } \geq 0} \cdot \underbrace{\mathrm{Im}\, f_\ell(s)}_{\text{partial wave positivity}} \geq 0$$

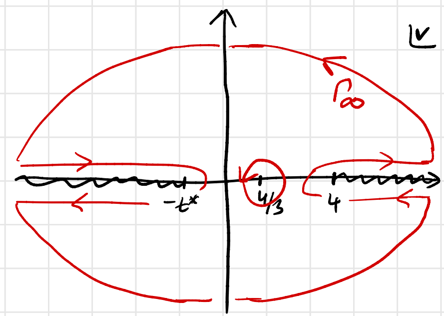


— What about c_0? It may not be positive...

$$c_0 = \frac{1}{2\pi i}\oint \frac{d\bar{v}}{\bar{v}^1}\, T'(\bar{v},0)$$

$$= c_\infty + \frac{1}{\pi}\int_{8/3}^{\infty} \frac{d\bar{v}}{\bar{v}^3}\, \mathrm{Im}\, T'(\bar{v},0)$$

↑ unknown constant from $|v| \to \infty$

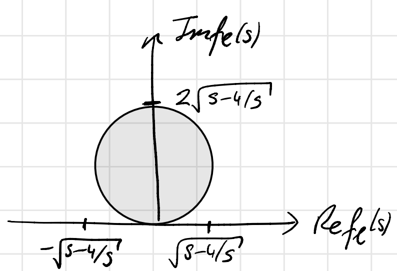


– The idea:

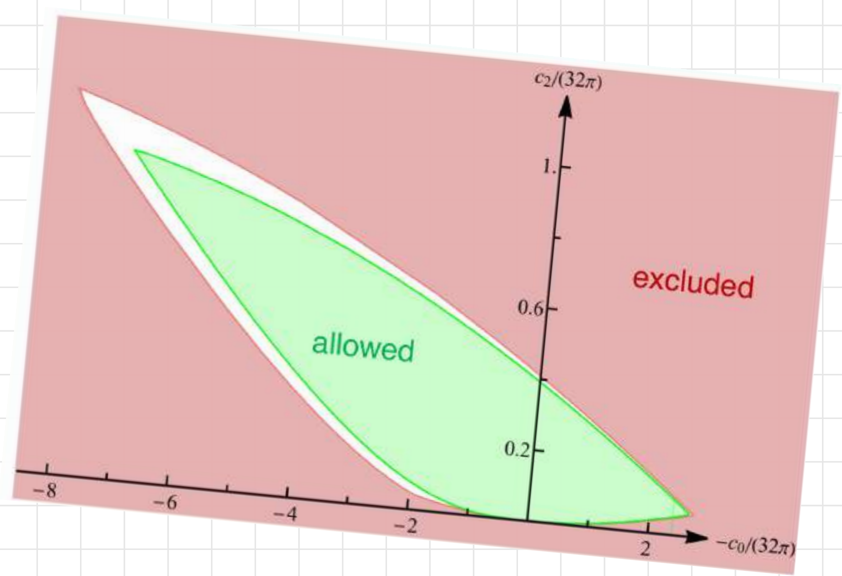
Use $\mathrm{Re}\, f_\ell(s)$ as the subtraction constant

\+ partial wave unitarity circle

$$\frac{c_0}{16\pi} = \underbrace{\mathrm{Re}\, f_0(s)}_{\text{bounded by unitarity circle!}} - \frac{1}{\pi}\, p.v. \int_{4m^2}^{\infty} dz\, \Sigma_{\ell=0}^{\infty}\, \mathrm{ker}_{0,\ell}(s,z)\, \underbrace{\mathrm{Im}\, f_\ell(z)}_{\text{bounded by unitarity circle!}}$$



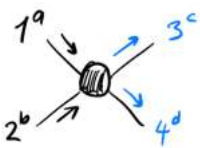
– The result:



– The same idea for Higgs scattering

① custodial symmetric limit

$SU(2)_L \times SU(2)_R \simeq O(4)$



$$\mathbf{M}_{ab}^{cd} = M(s|t,u)\,\delta_{ab}\delta^{cd} + M(t|u,s)\delta_a^c\delta_b^d + M(u|s,t)\delta_a^d\delta_b^c$$

$M(s|t,u)$ is symmetric only in $t \leftrightarrow u$.

$$\frac{M(s|t,u)}{(4\pi)^2} = c_\lambda + c_H\bar{s} + c_2(\bar{t}^2+\bar{u}^2) + c_2'\,\bar{s}^2 + O(\bar{s}^4,\bar{t}^4,\bar{u}^4)$$

② assume $g_{SM} \ll g_{BSM}$

a Lagrangian matching

$(\lambda, g_H) \rightarrow (c_\lambda, c_H)$

$$\mathcal{L} = \frac{1}{2}(\partial\vec{\phi})^2 - \frac{m^2}{2}\vec{\phi}^2 - \frac{\lambda}{8}\vec{\phi}^4 - \frac{g_H}{4}\vec{\phi}^2(\partial\vec{\phi})^2 + \cdots$$

$\partial_\mu(H^\dagger H)\partial^\mu(H^\dagger H) \equiv O_H$

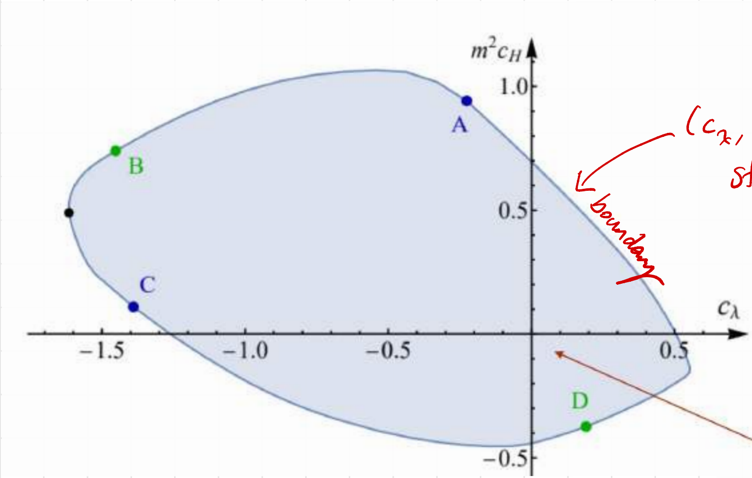
dim-6 kinetic correction term

③ Roy equations:

$$c_\lambda\, 2\pi = \text{Re}\, f_0^{(sym)}(s) - \frac{1}{\pi}\int_{4m^2}^{\infty} dz\, \Sigma_{\ell,rep}\, \text{K}_{1,\ell}^{(sym,rep)}(s,z)\,\text{Im}\, f_\ell^{(rep)}(z)$$

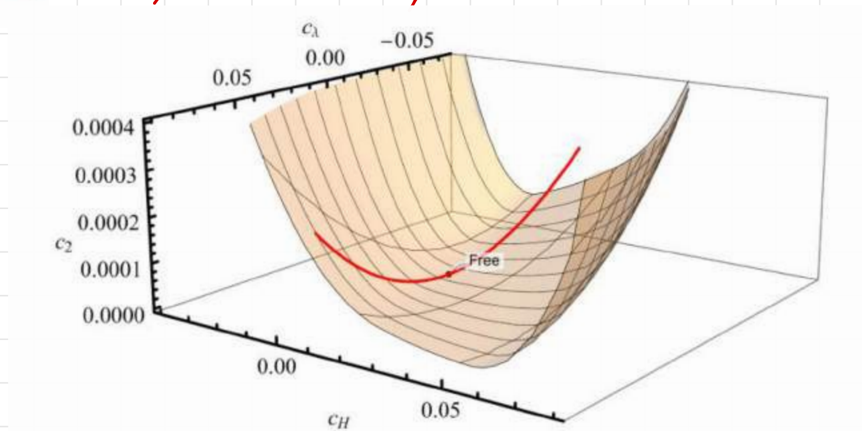
$$c_H\frac{\pi}{3}(s-4) = \text{Re}\, f_1^{(anti)}(s) - \frac{1}{\pi}\int_{4m^2}^{\infty} dz\, \Sigma_{\ell,rep}\, \text{L}_{1,\ell}^{(anti,rep)}(s,z)\,\text{Im}\, f_\ell^{(rep)}(z)$$

- The result for Higgs scattering

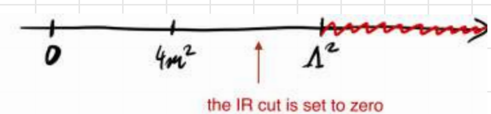


$(c_\lambda, c_H) \sim O(1) \cdot (m^2)^{\#}$ strongly coupled!

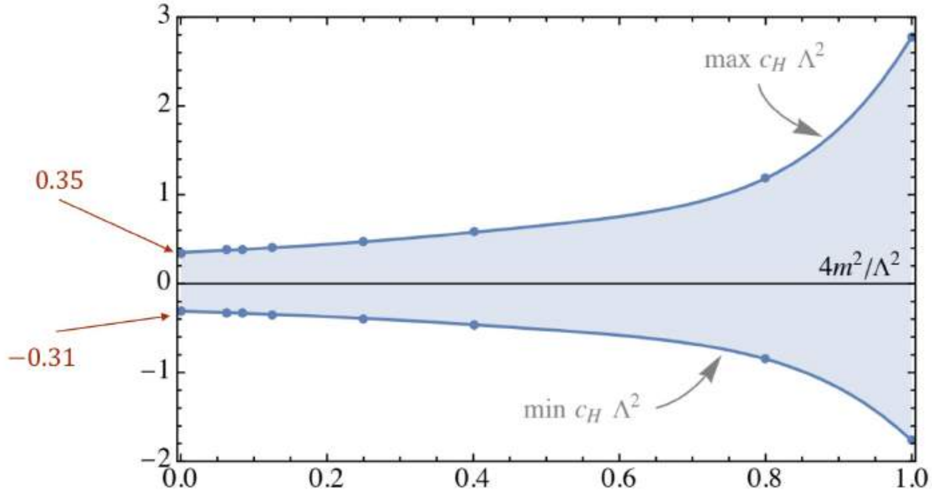
zooming in to find weakly coupled S-matrices



UV dominated EFT limit



– Dim. – 6 bounds on c_H



$$-0.31 \leq c_H \cdot \Lambda^2 \leq 0.35$$

Thanks
for
your Attention!

No On-shell Minimal Coupling in 5D

W. Wayne Zhao
Princeton
Amplitudes 2024

[cf. 2405.09533,2406.xxxx]

Lorentz Group: $\mathrm{Spin}(4,1) \sim \mathrm{USp}(2,2) \cong \mathrm{Sp}(4,\mathbb{C}) \cap \mathrm{U}(2,2)$

Massive Little Group: $\mathrm{Spin}(4) \cong \mathrm{SU}(2) \times \mathrm{SU}(2)$

Massless Little Group: $\mathrm{Spin}(3) \cong \mathrm{SU}(2)$

$$p_{AB} := p_\mu {\gamma^\mu}_{AB} \qquad\qquad p_{AB}\Omega^{AB} = 0$$

$$\text{(massive)} = \frac{1}{2}\left({\lambda_A}^a \lambda_{Ba} - \tilde{\lambda}_{A\dot{a}} {\tilde{\lambda}_B}{}^{\dot{a}}\right)$$

$$\text{(massless)} = \epsilon_{ab} {\lambda_A}^a {\lambda_B}^b$$

$s \gg 1$

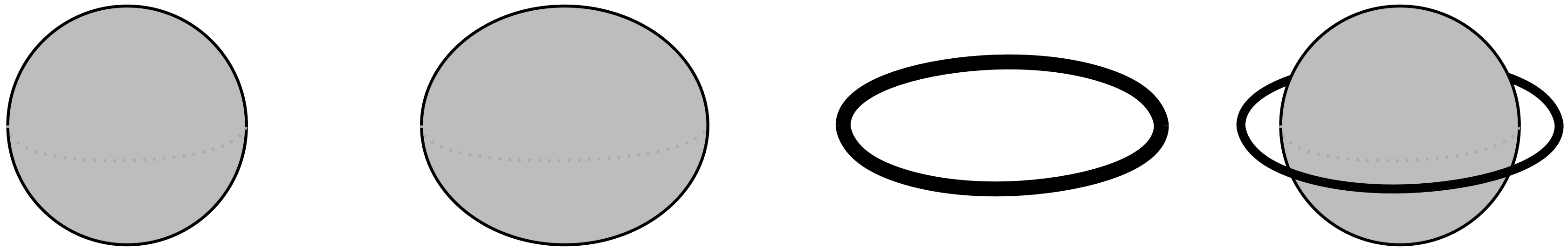
$$D = 4$$

Minimal coupling in 4D has an invariant on-shell meaning.

Kerr BH is minimally coupled to gravity in 4D

$$D = 5$$

No structure can survive the high energy limit that is minimally coupled

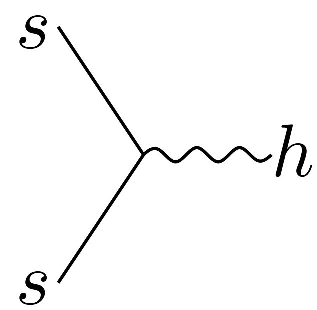


n-pt scattering:

$$\mathbf{\Lambda} = \left(\lambda^{(1)}{}_A{}^{a_1} \quad \cdots \lambda^{(n)}{}_A{}^{a_n} \right) \in \mathrm{Gr}(4, 2n)$$

$$\sum_i p_i = 0 \implies \mathbf{\Lambda} \text{ is Lagrangian}$$

$n = 3$: special 3-particle kinematics:



$$= \sum_{k=0}^{2\min(s,h)} E^{\frac{1}{2}-2s-h+k} \left(\hat{1}23 + 1\hat{2}3 + 12\hat{3}\right)^k (12)^{2s-k} 3^{2h-k}$$

No minimal coupling for $s > 2$ to gravity in general D.

$D = 4$ minimal structures are special $\overset{?}{\leftrightsquigarrow}$ no hair theorem
* 4D structures like $\mathcal{M}(+1/2, -1/2, 0) = \frac{[13]}{[23]}$ $\nexists$ in $D > 4$.

KK compactification of nonminimal 5D Myers-Perry is minimal in 4D?

Fourier Calculus from Intersection Theory

[Brunello, Crisanti, Giroux, Smith, Mastrolia, 2311.14432]

1. What is intersection theory?

[Matsumoto, 1998]

A mathematical framework that allows us to build inner products between twisted period integrals:

$$I_{\alpha_1\cdots\alpha_m} \sim \int u\,\varphi_{\alpha_1\cdots\alpha_m} \qquad \varphi_{\alpha_1\cdots\alpha_m} = \frac{dz_1\wedge\cdots\wedge dz_m}{z_1^{\alpha_1}\cdots z_m^{\alpha_m}} \qquad \langle\varphi_L|\varphi_R\rangle_u = \sum_{p\in\mathcal{P}} \mathrm{Res}_{z=p}(\psi\varphi_R)$$

2. How is it useful for Fourier integrals?

[Baikov, 1996]

Through a parametric representation, Fourier integrals can be written as twisted period integrals. Allows us to decompose to MIs using this inner product:

$$\tilde{f}(\{x_i\}) = \int f(\{q_i\})\prod_{j=1}^{L} \mathrm{e}^{iq_j\cdot x_j}\,\text{đ}^{\mathrm{D}}q_j \longrightarrow \int f(\mathbf{z})B(\mathbf{z})^{\gamma}e^{ig(\mathbf{z})}\mathrm{d}^n\mathbf{z}$$

$$\tilde{f} = \sum_{i=1}^{\nu} c_i J_i \longrightarrow c_i = \sum_{j=1}^{\nu}\left\langle \tilde{f}\middle| J_j\right\rangle (\mathbf{C}^{-1})_{ji}$$

3. Some Applications

[Balitsky, Chirilli, 2008] [Herderschee et al, 2023]

This allows us to use intersection theory to compute master integrals and differential equations, we applied this to three families of Fourier integrals arising in various corners of particle physics.

$$I_n = \int_{\mathcal{M}} d^dq \frac{e^{iq\cdot x}}{(q^2+m^2-i\varepsilon)^n}$$

$$\mathcal{I}_\alpha = \int_{\mathcal{M}} d^dq \frac{\delta(u_1\cdot q)\delta(u_2\cdot(q-k))e^{-iq\cdot b}}{[q^2-i\varepsilon]^\alpha}$$

$$G^{ij} = \int d^dq_1 d^dq_2 \frac{N_I^{ij}(q_1,q_2)e^{i(q_1\cdot x_1+q_2\cdot x_2)}}{(q_1+q_2)^2(q_1^2\tau+q_2^2)}$$

$$\mathcal{I}_\alpha = \int_{\mathcal{M}} d^dq \frac{\delta(u_1\cdot q)\delta(u_2\cdot(q-k))e^{-iq\cdot b}}{[q^2-i\varepsilon]^\alpha} \longrightarrow \mathcal{I}_\alpha = \int_{\mathcal{M}} \frac{\mathrm{d}^4\mathbf{z}}{z_3^\alpha}u(\mathbf{z})\delta(z_1)\delta(z_2)$$

Allows us to build DEQ systems and solve them!

$$\partial_s\boldsymbol{\mathcal{K}} = \boldsymbol{\Omega}_s\cdot\boldsymbol{\mathcal{K}}\,, \qquad \boldsymbol{\Omega}_s = \begin{pmatrix} 0 & \frac{-1}{y^2-1} \\ -\frac{1}{4s} & \frac{d-6}{2s}\end{pmatrix}$$

$$\partial_y\boldsymbol{\mathcal{K}} = \boldsymbol{\Omega}_y\cdot\boldsymbol{\mathcal{K}}\,, \qquad \boldsymbol{\Omega}_y = \begin{pmatrix} \frac{-y}{y^2-1} & \frac{2sy}{(y^2-1)^2} \\ \frac{y}{2(y^2-1)} & \frac{(5-d)y}{y^2-1}\end{pmatrix}$$

$$\longrightarrow$$

$$\mathcal{I}_1 = \frac{(b^2/w_2^2)^{\frac{4-d}{4}}}{2\pi\,(y^2-1)^{\frac{d-2}{4}}}K_{\frac{4-d}{2}}\left(\frac{\sqrt{b^2}w_2}{\sqrt{y^2-1}}\right)$$

$$\mathcal{I}_2 = \frac{(b^2/w_2^2)^{\frac{6-d}{4}}}{4\pi\,(y^2-1)^{\frac{d-4}{4}}}K_{\frac{6-d}{2}}\left(\frac{\sqrt{b^2}w_2}{\sqrt{y^2-1}}\right)$$