SCATTERING AMPLITUDES (AND FEYNMAN INTEGRALS) FOR COLLIDER PHYSICS

Amplitudes 2024

IAS (Princeton, NJ, USA) – June 10th 2024

Lorenzo Tancredi – Technical University Munich

Technische Universität München

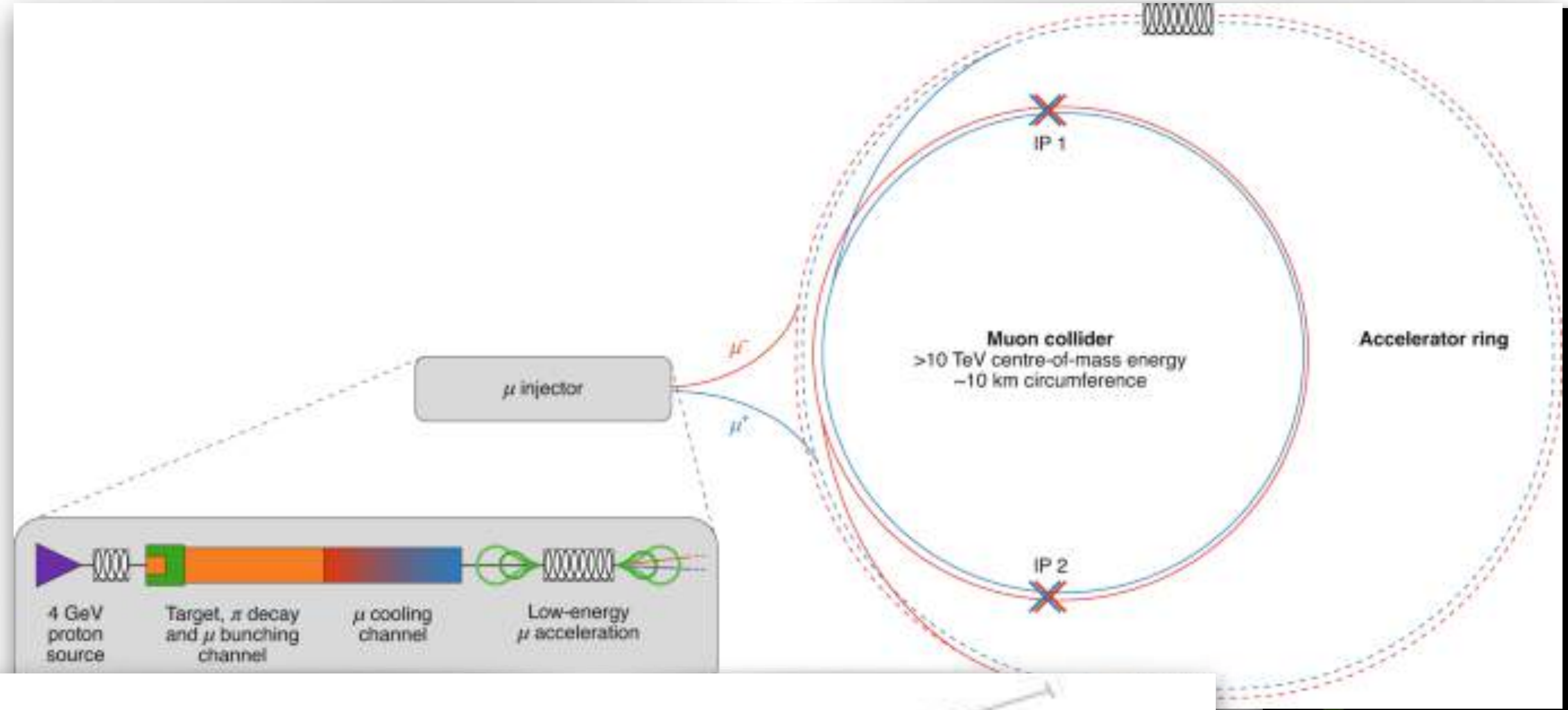
DISCLAIMER:

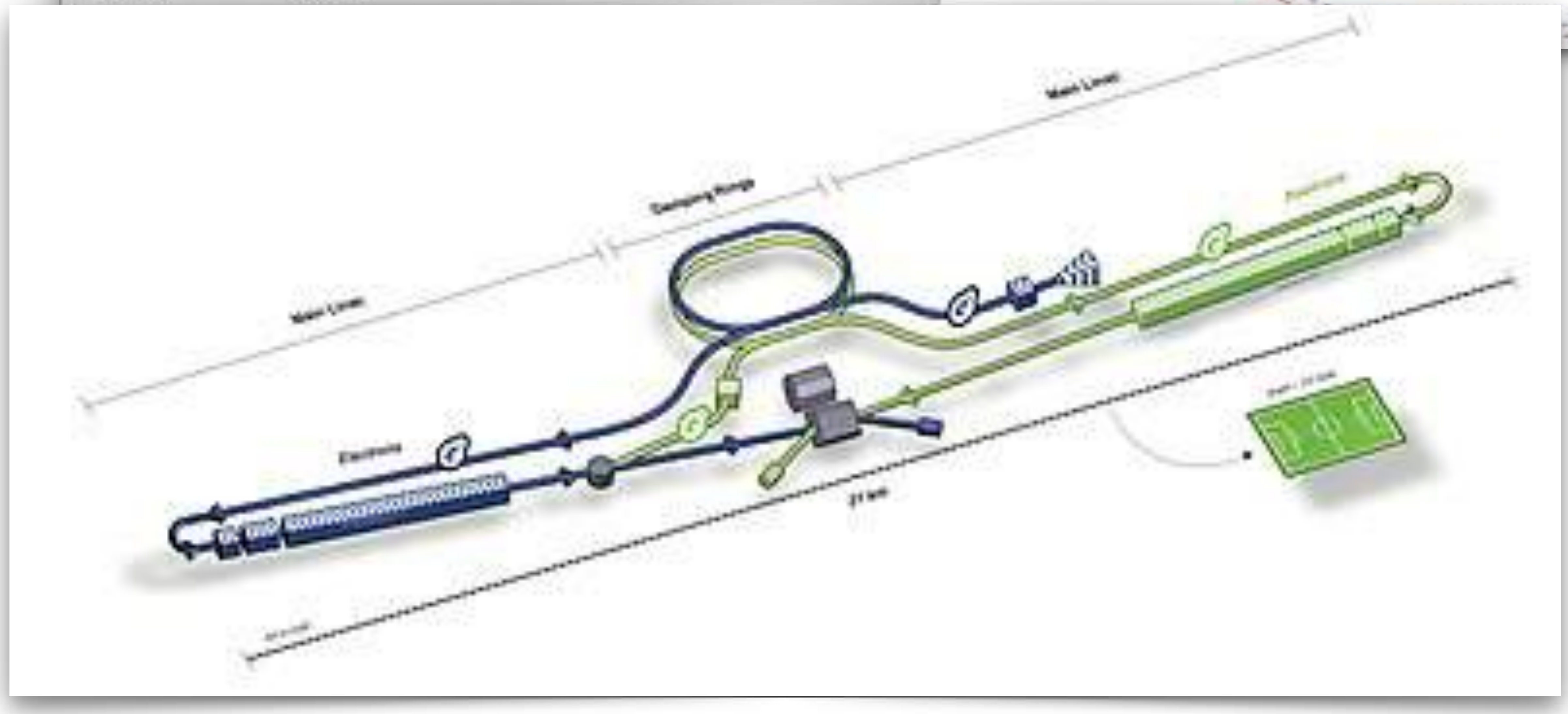
THIS IS NOT A REVIEW OF RESULTS !!

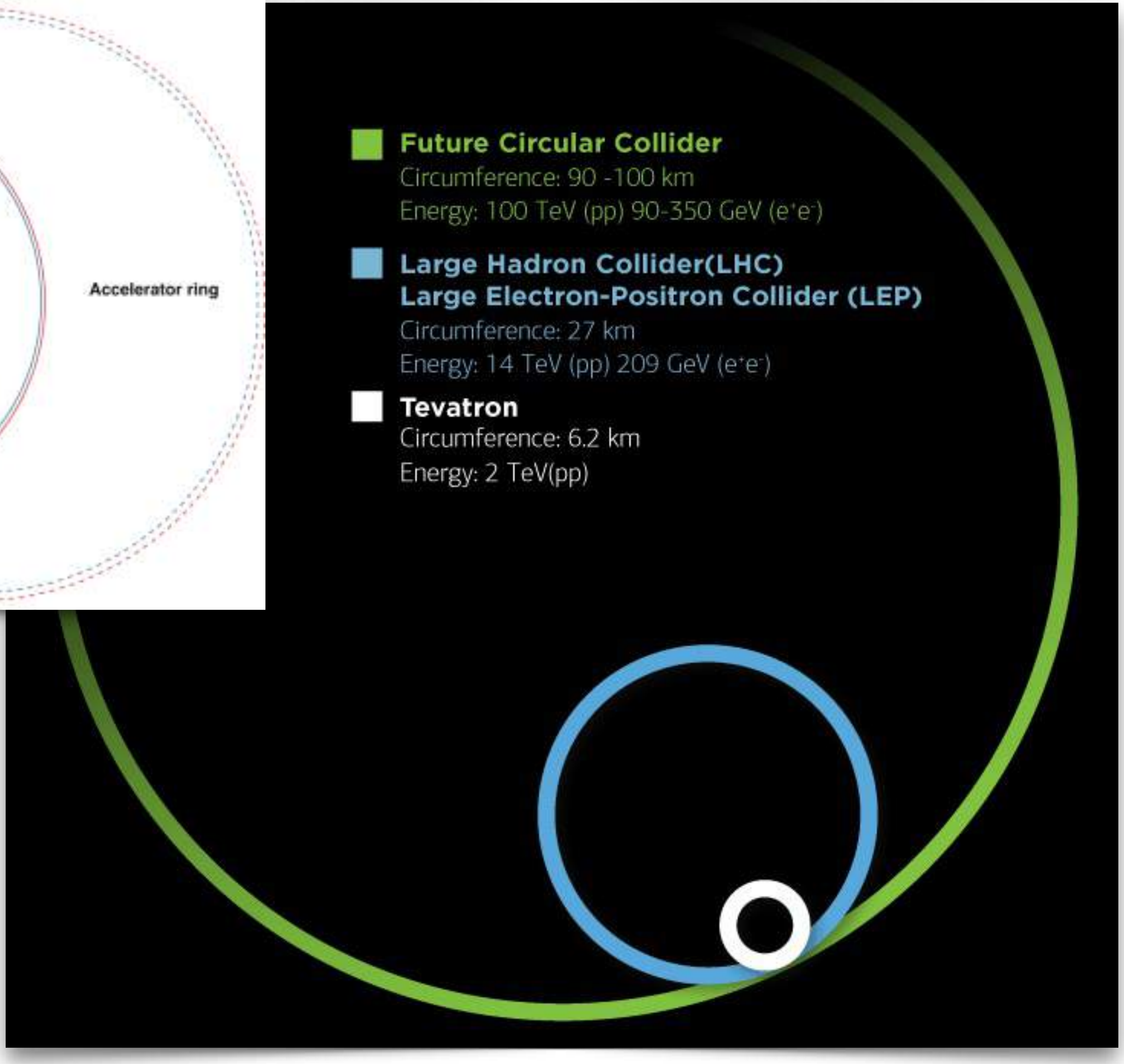
MORE OF A PERSONAL PERSPECTIVE ON SOME INTERESTING ISSUES THAT WE ARE DEALING WITH TODAY

IT WILL BE, BY CONSTRUCTION, BIASSED AND INCOMPLETE

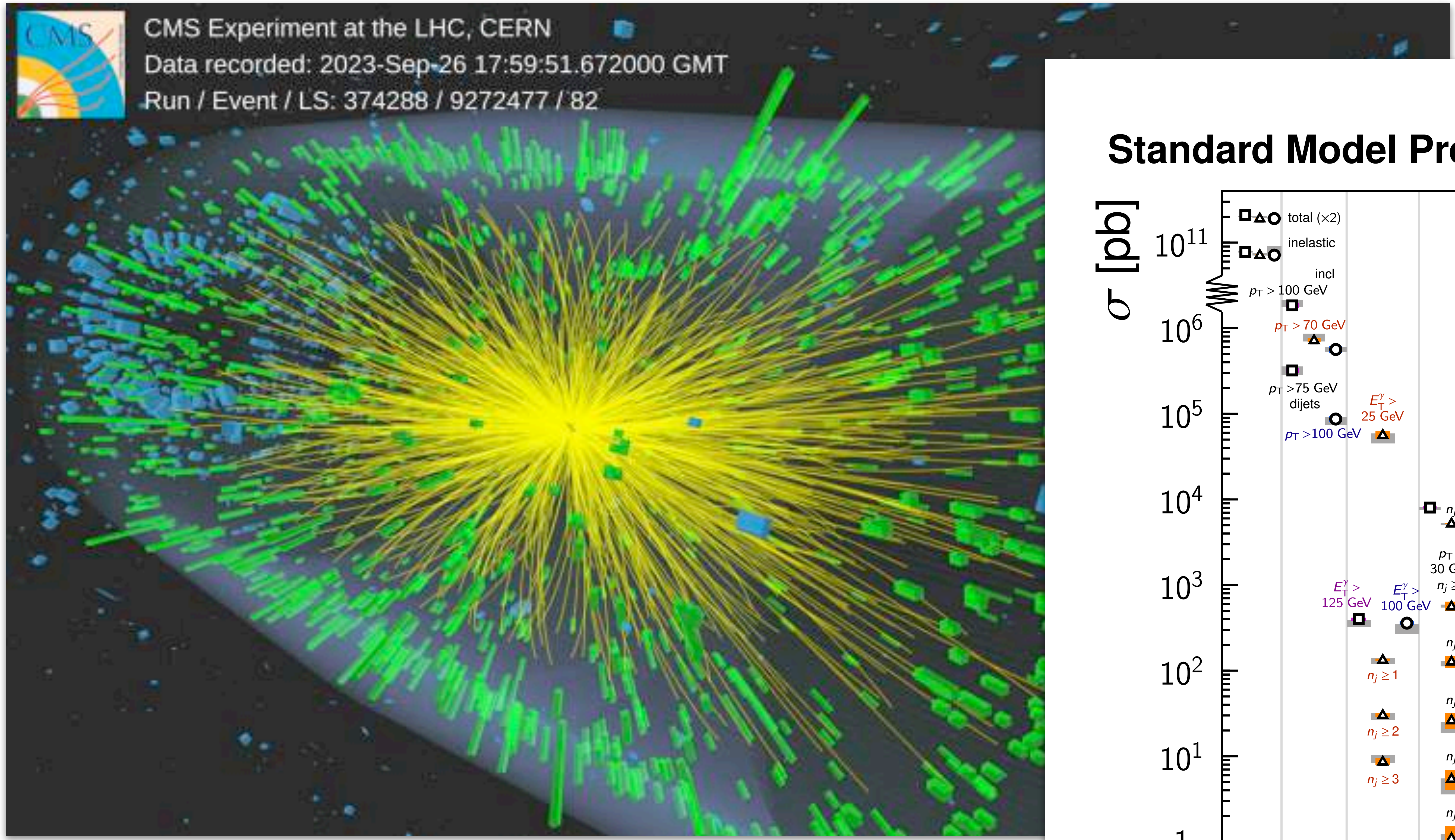
WHY (STILL) COLLIDERS? THE LHC (AND BEYOND)…

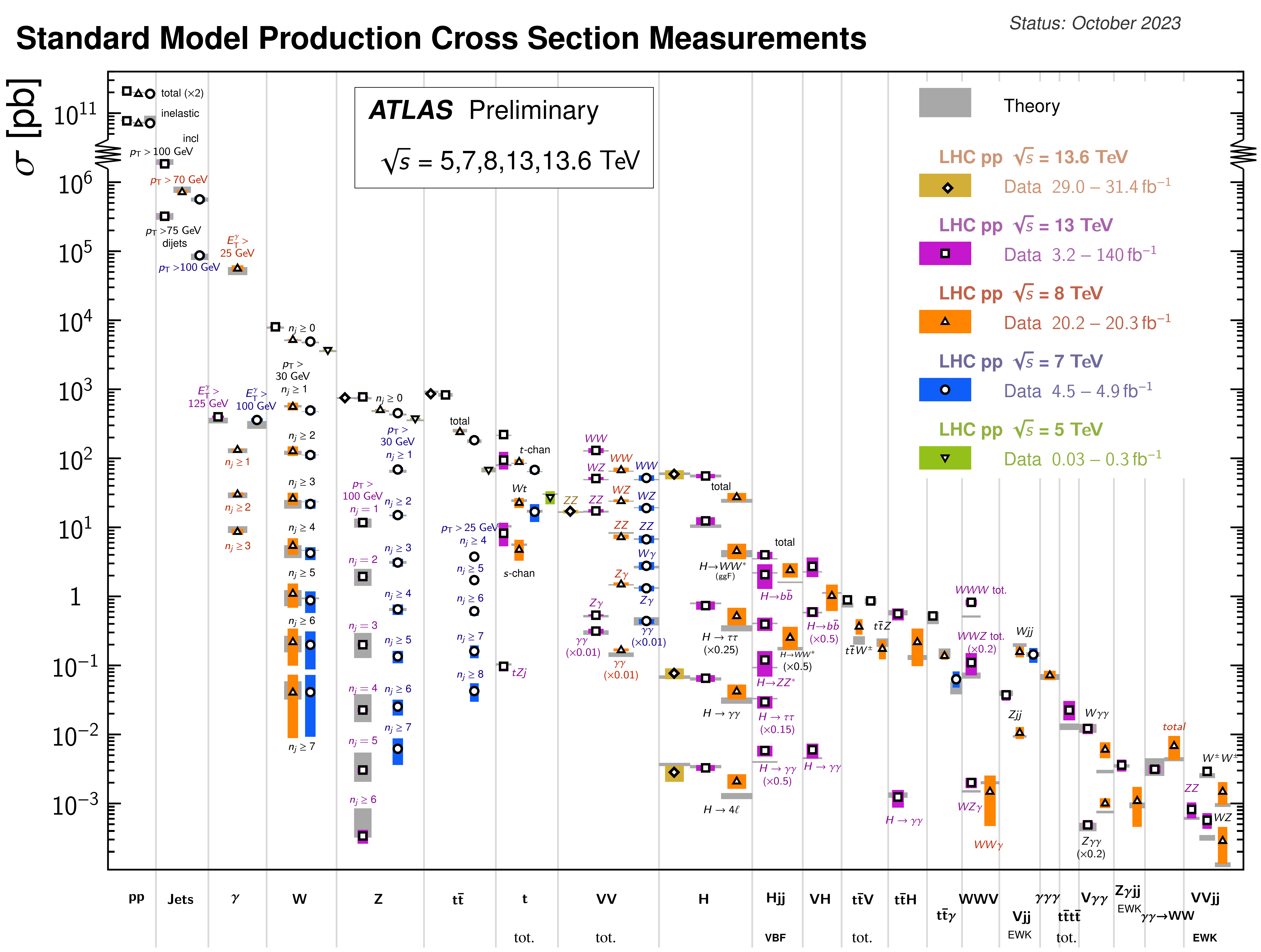






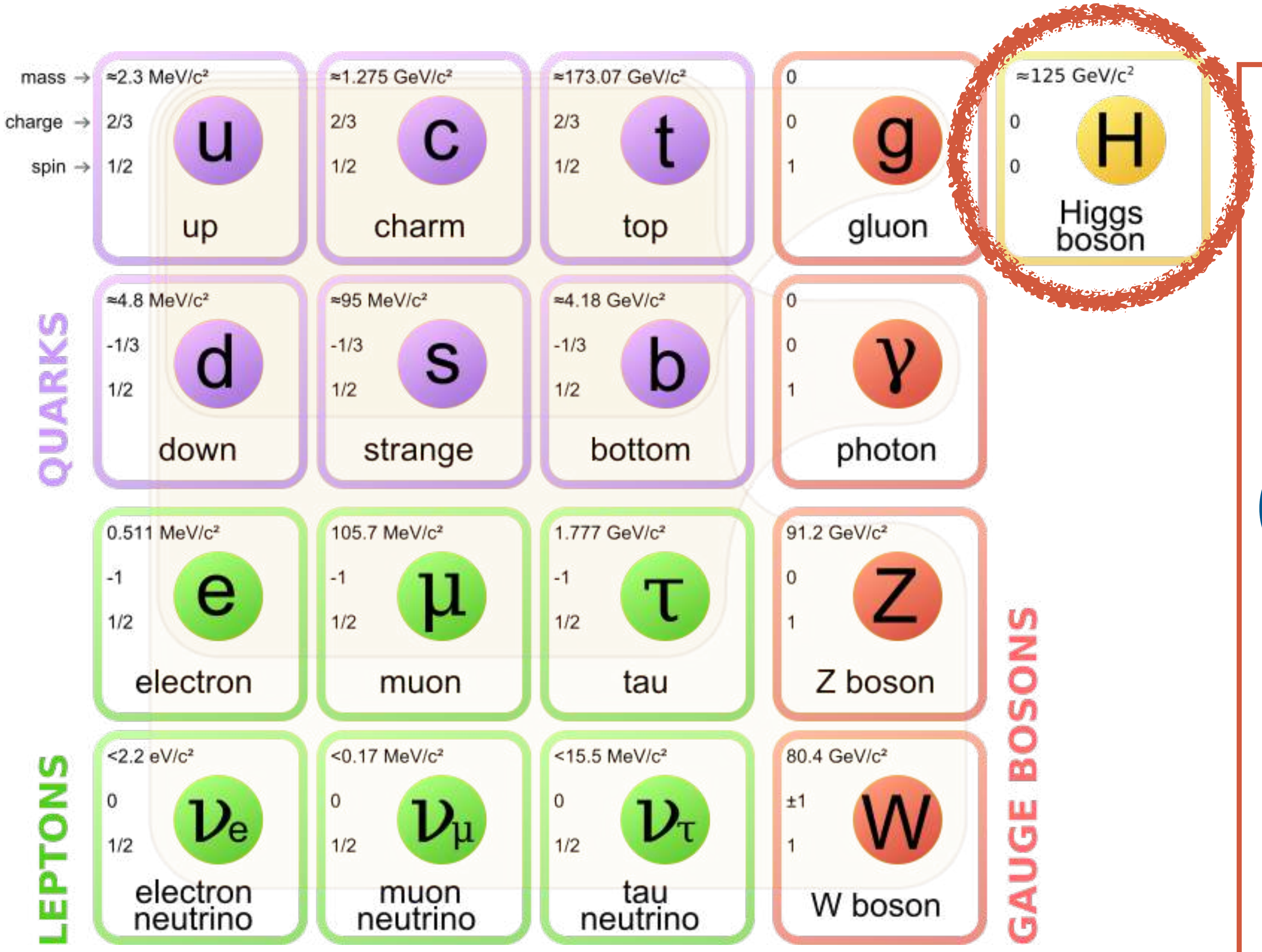
THE LHC HAS BECOME A PRECISION MACHINE

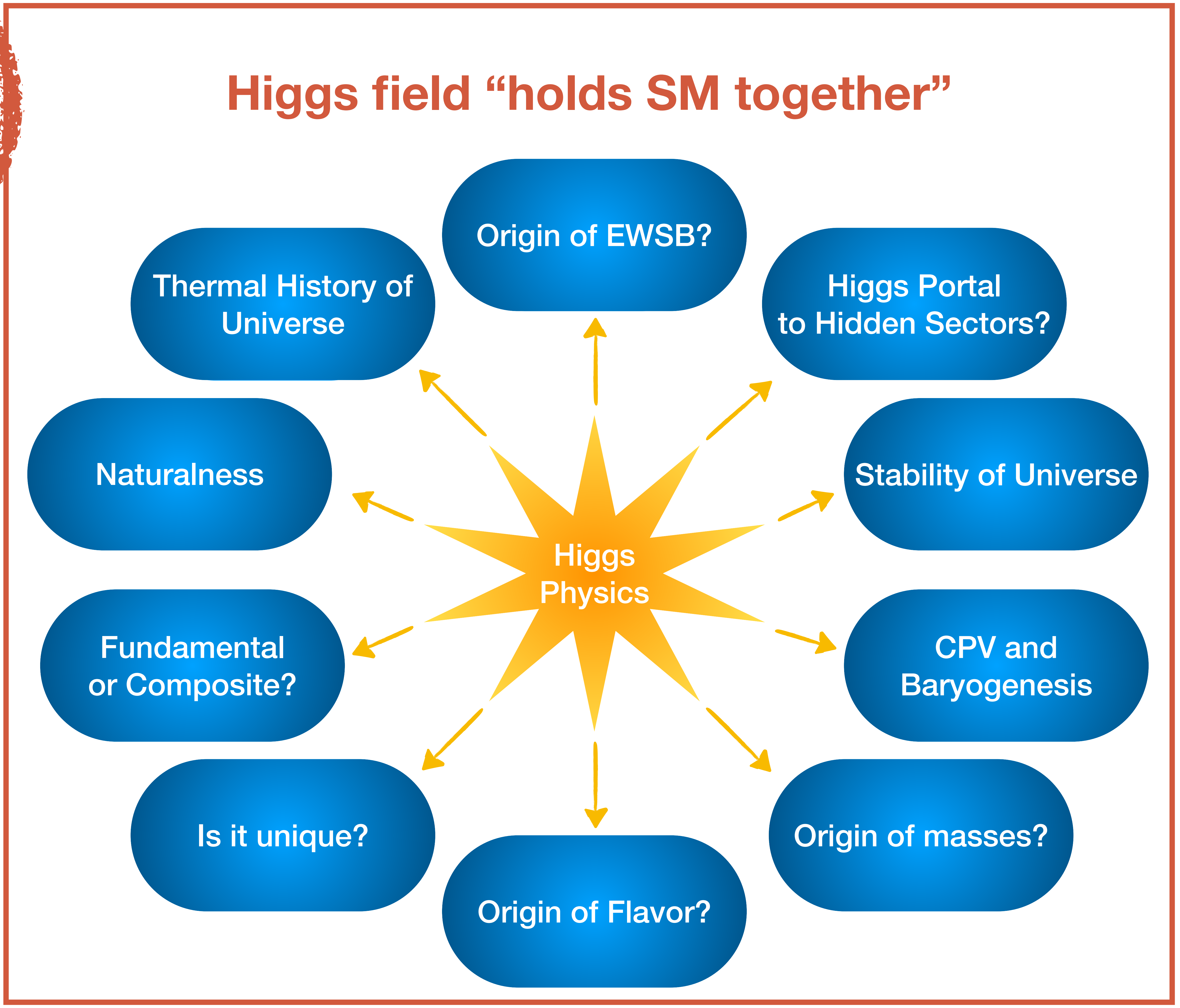




After its discovery in 2012, a lot (but not only) revolving around Higgs boson's properties

THE HIGGS BOSON: THE LAST MISSING PIECE

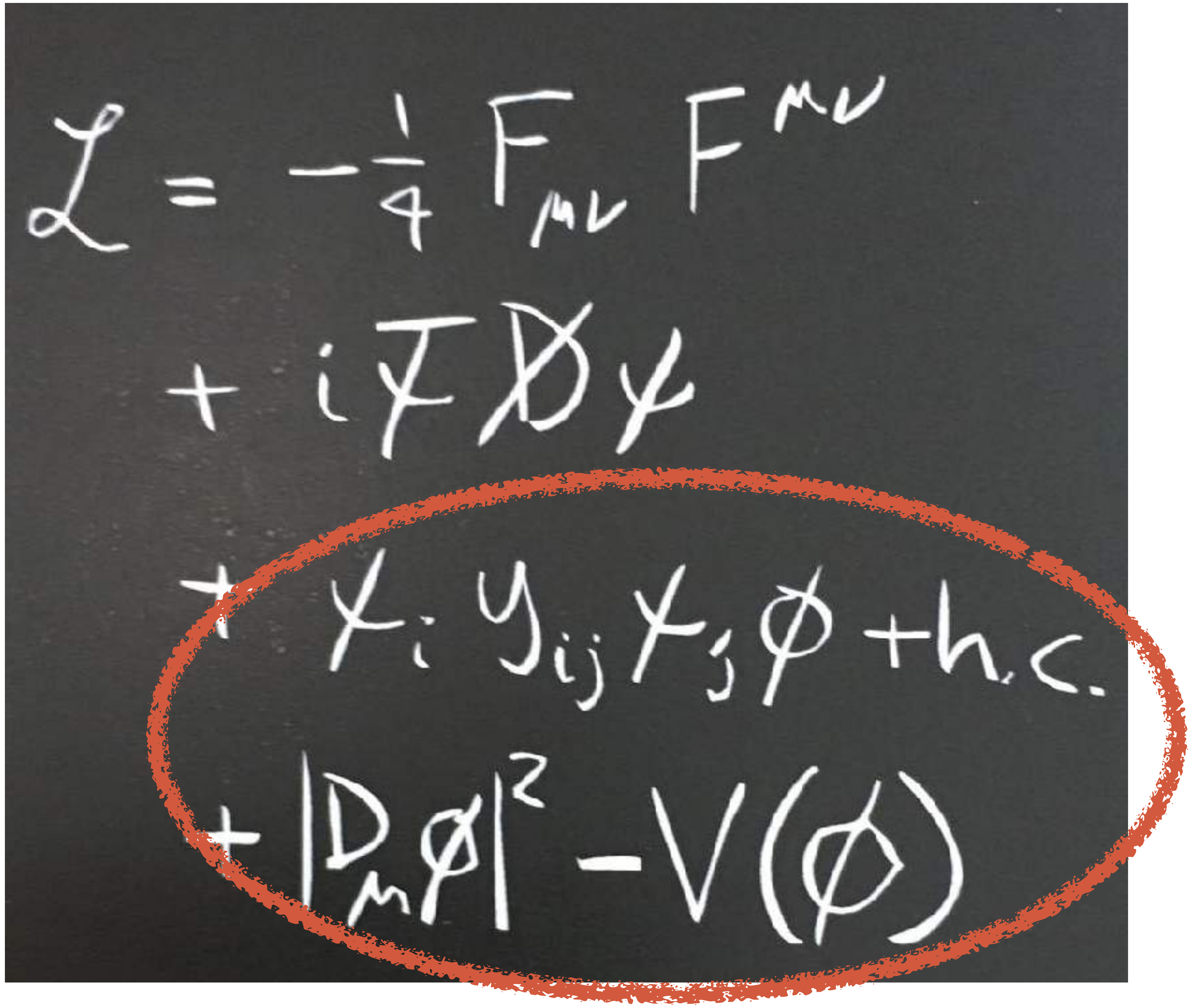




[Snowmass 2022 arXiv:2209.0751]

HIGGS INTERACTIONS AT THE LHC

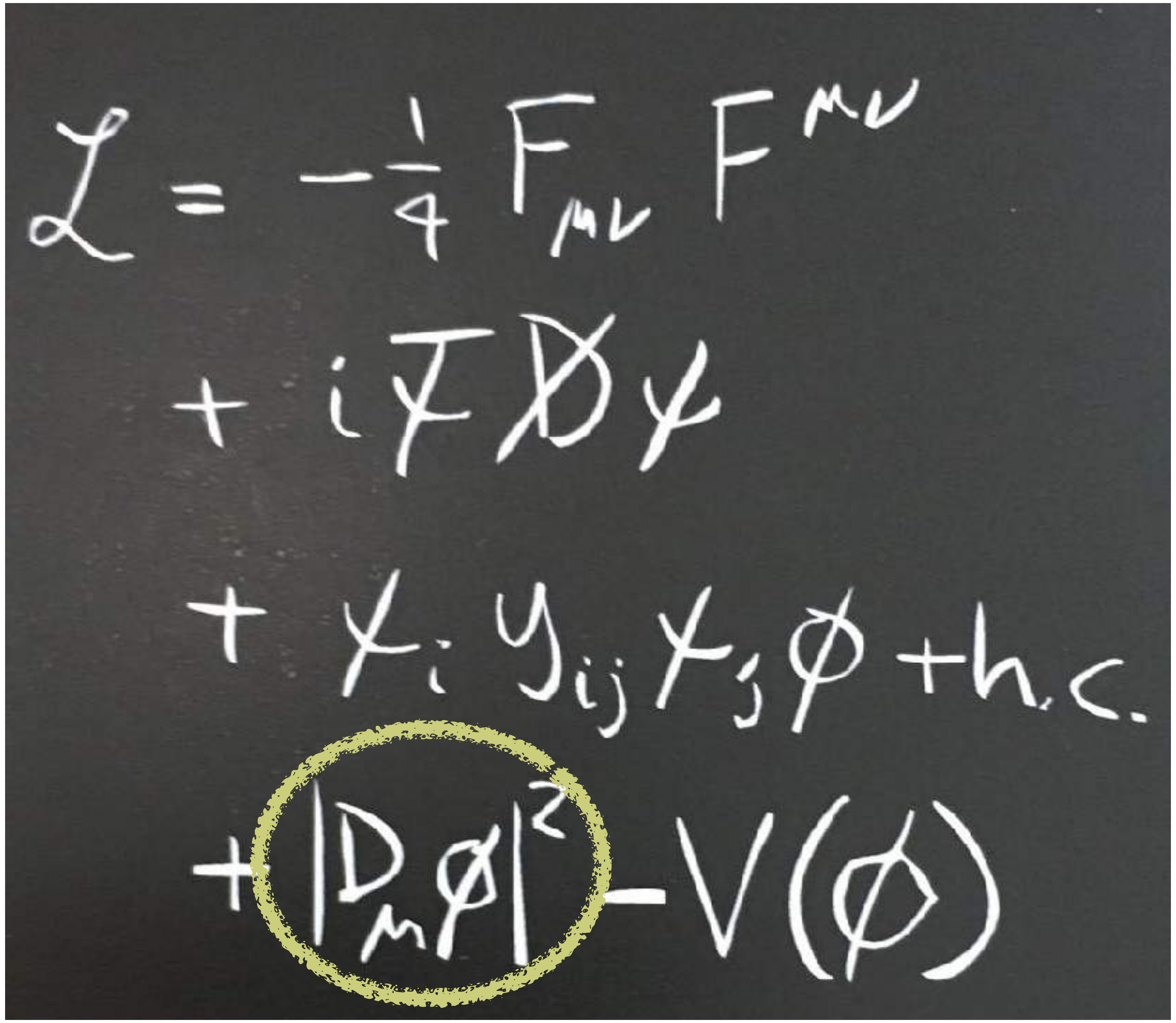
Hints to answer these questions hidden in the **details of Higgs interactions to SM particles**

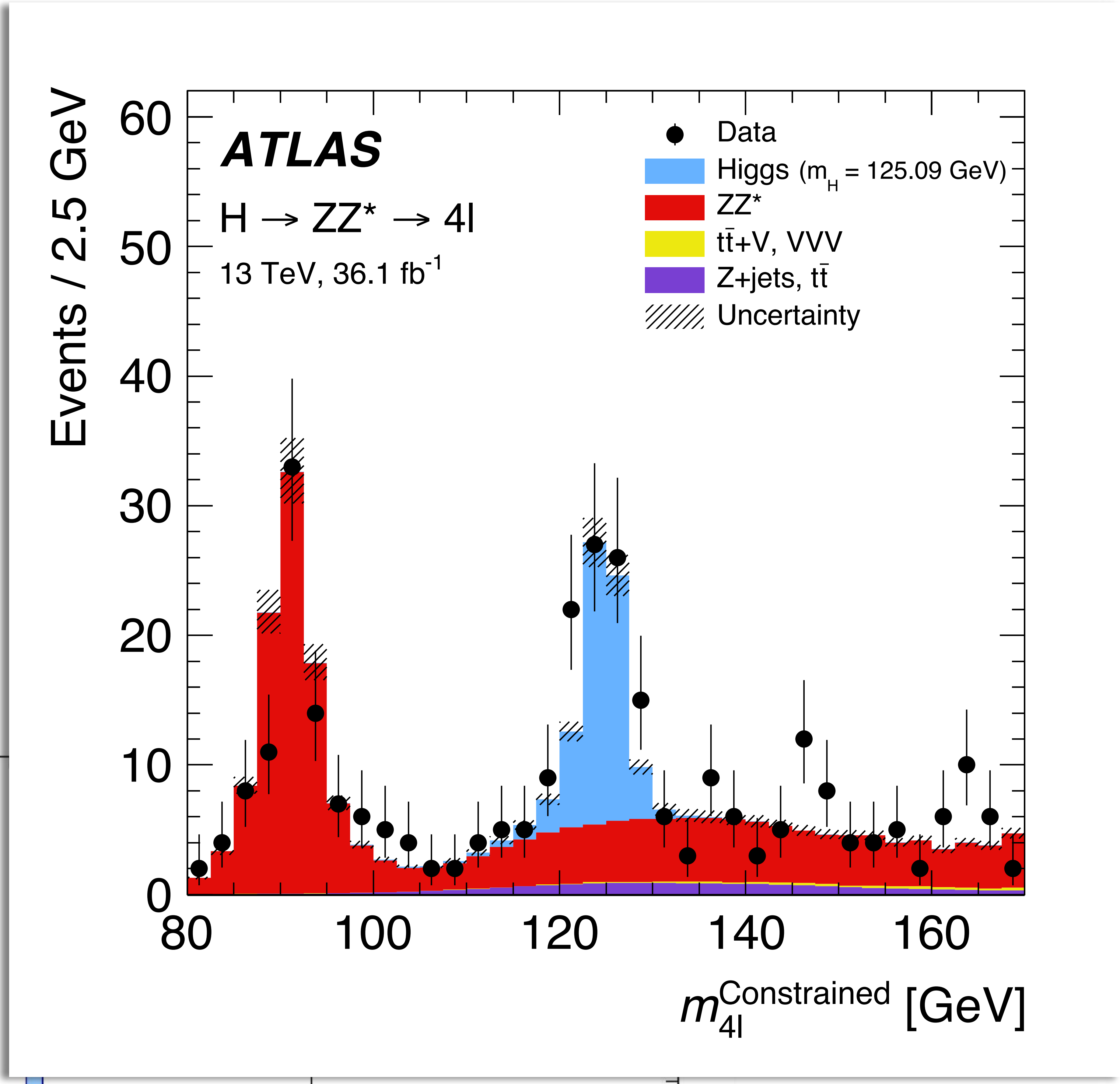


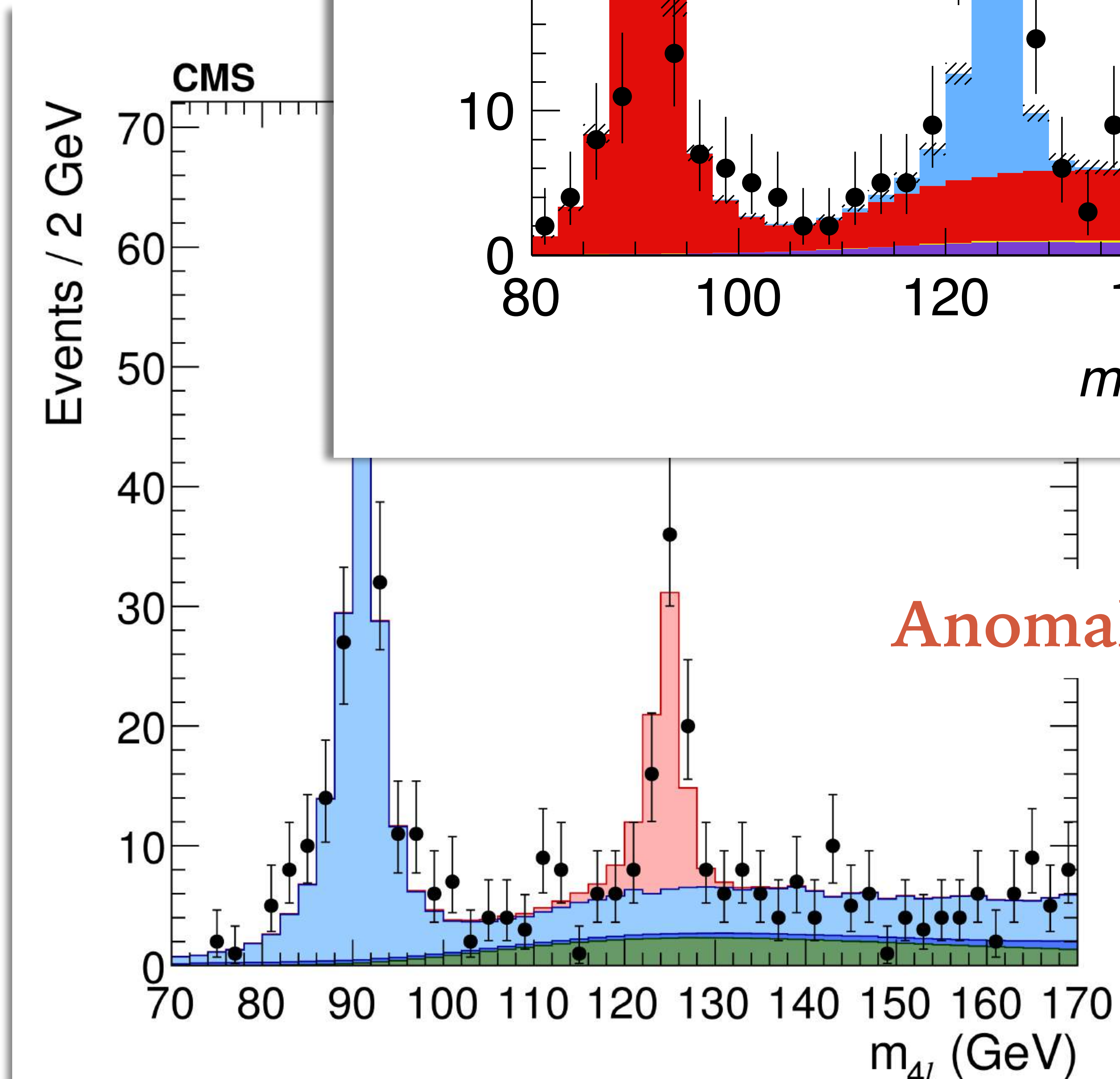
LHC has opened a window for us to peak at Higgs' interactions for the first time !

HIGGS INTERACTIONS THE GAUGE SECTOR

Higgs discovery through its **couplings to gauge sector**



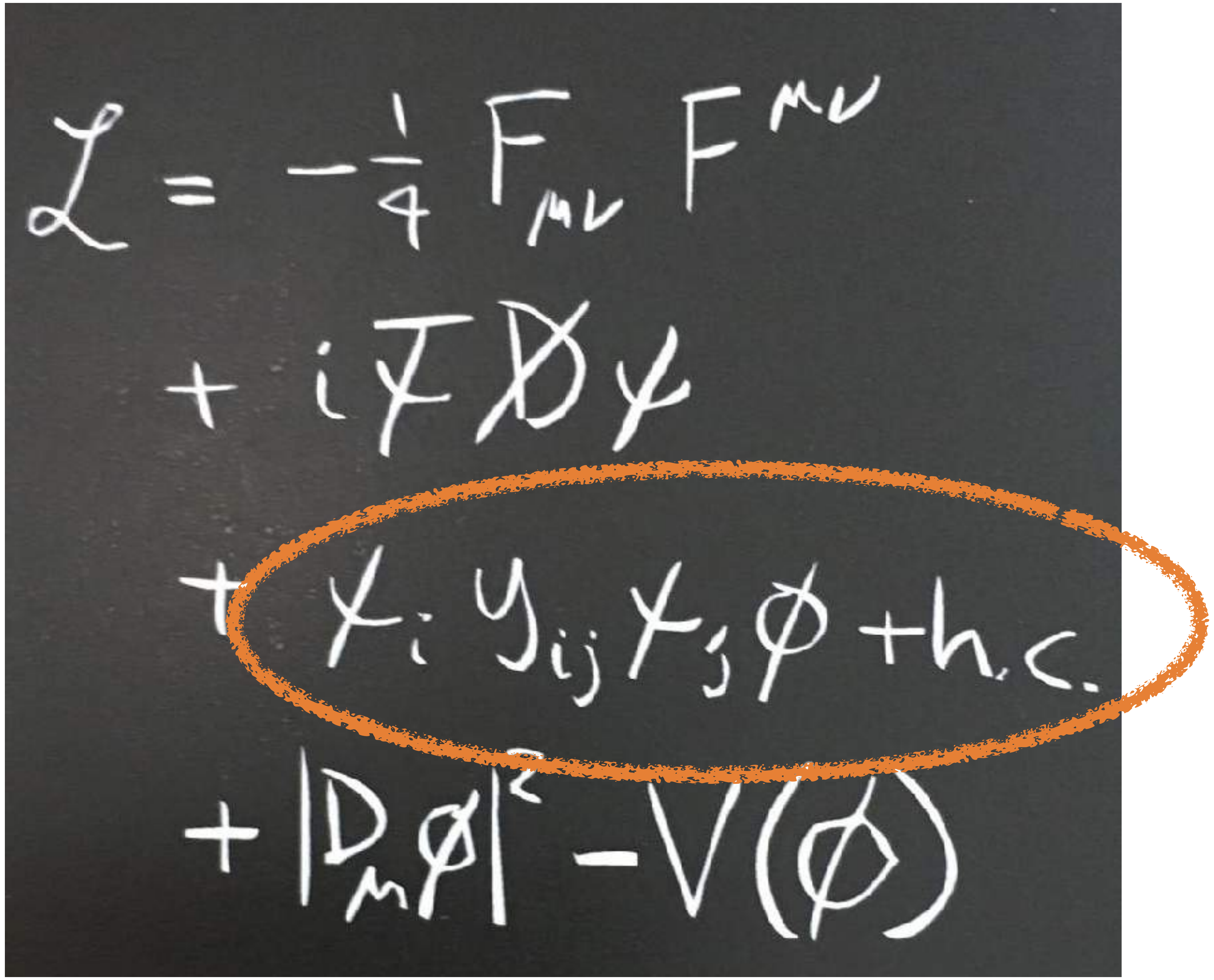


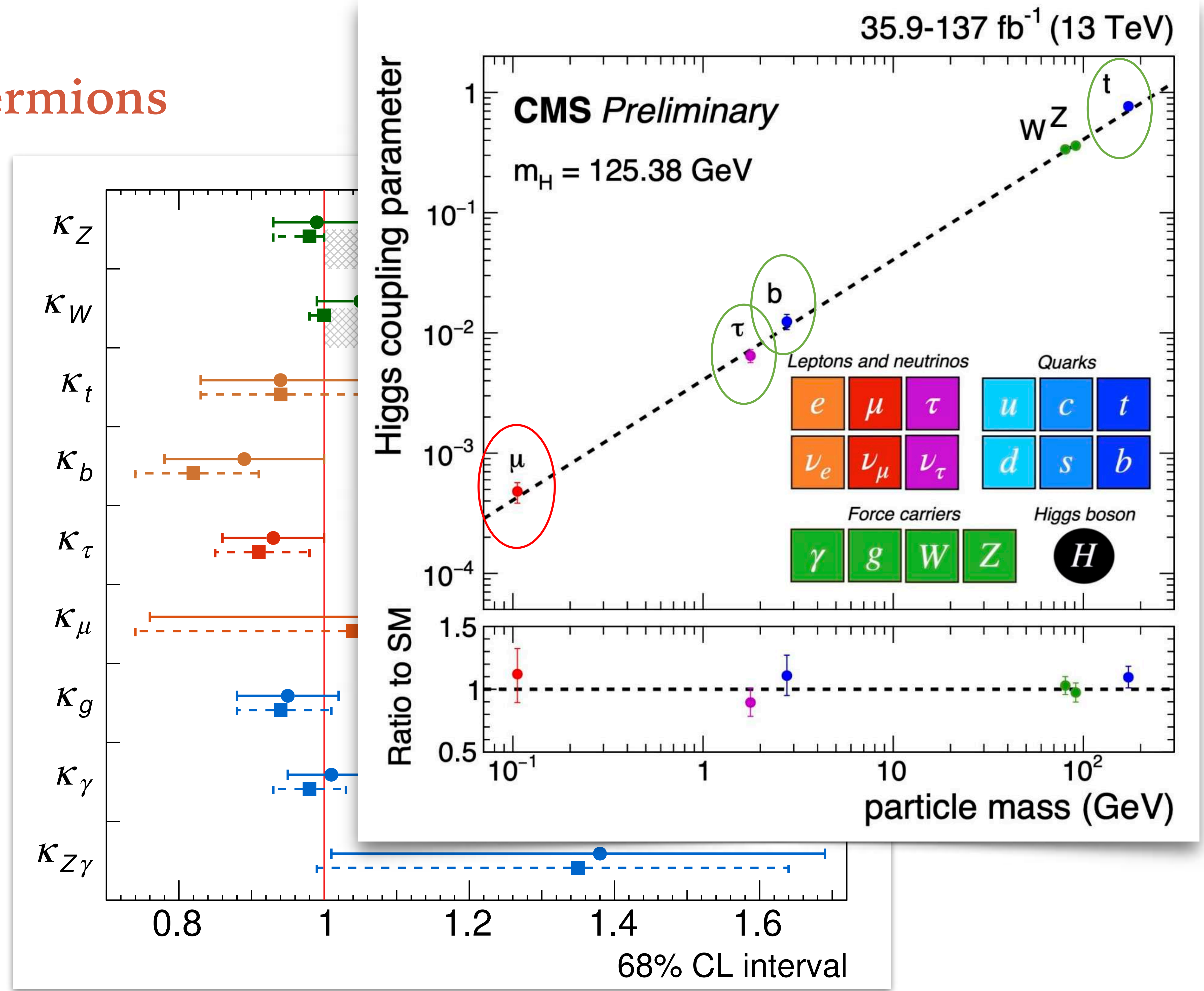


Anomalous couplings?

HIGGS INTERACTIONS THE YUKAWA SECTOR

Run 2 direct observation of H coupling to **third family fermions**





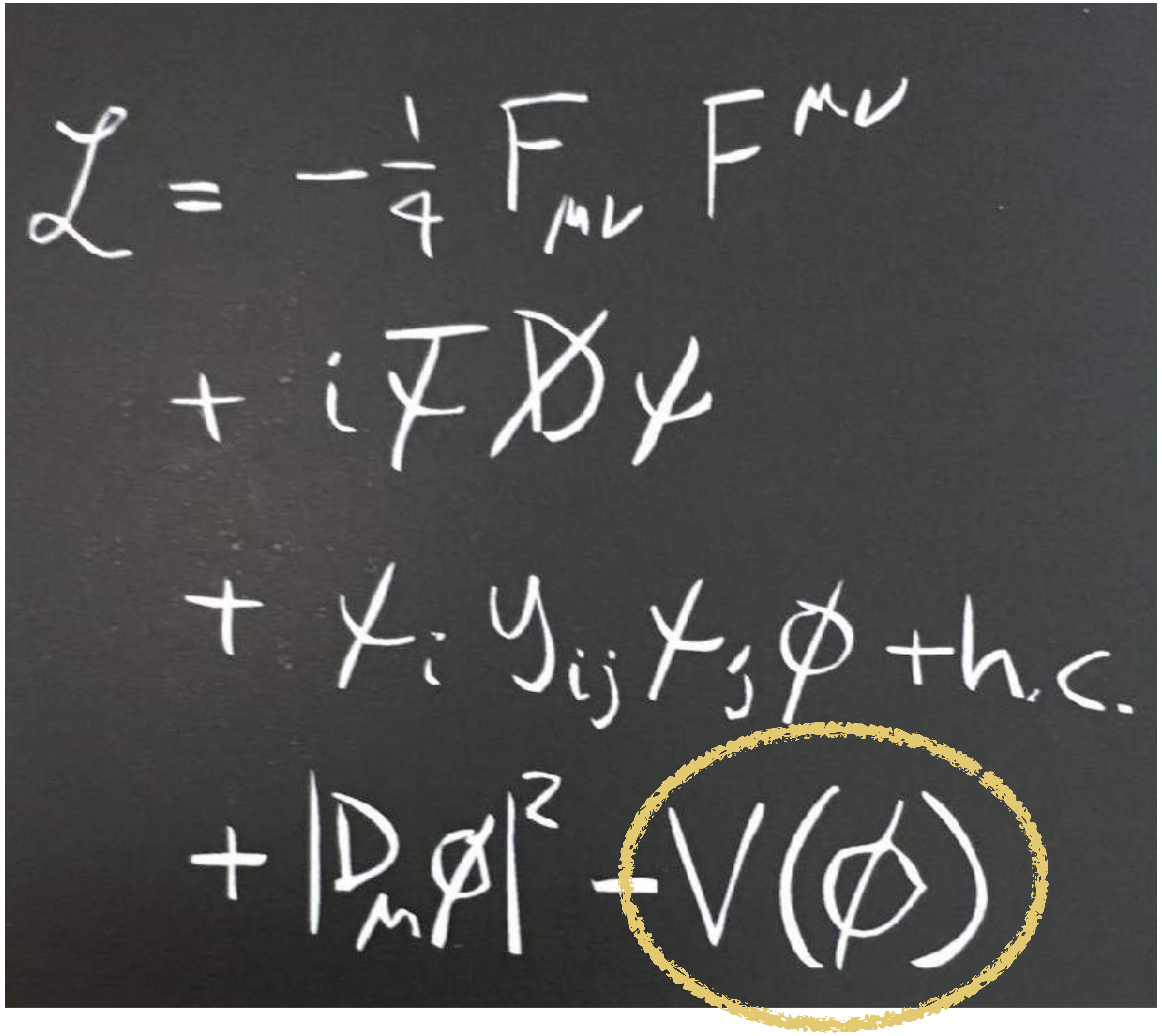
Run 3 and **HL potential:**

1. **Precision** measurements for **third family**

2. Discovery couplings to **second family** (μ & c)

HIGGS SELF INTERACTIONS THE MOST MYSTERIOUS?

HL-LHC first to see the triple-H coupling



We have seen the Higgs but

$$V(\phi) = -\mu^2\phi^2 + \frac{\lambda}{4!}\phi^4$$

is a “toy model”!

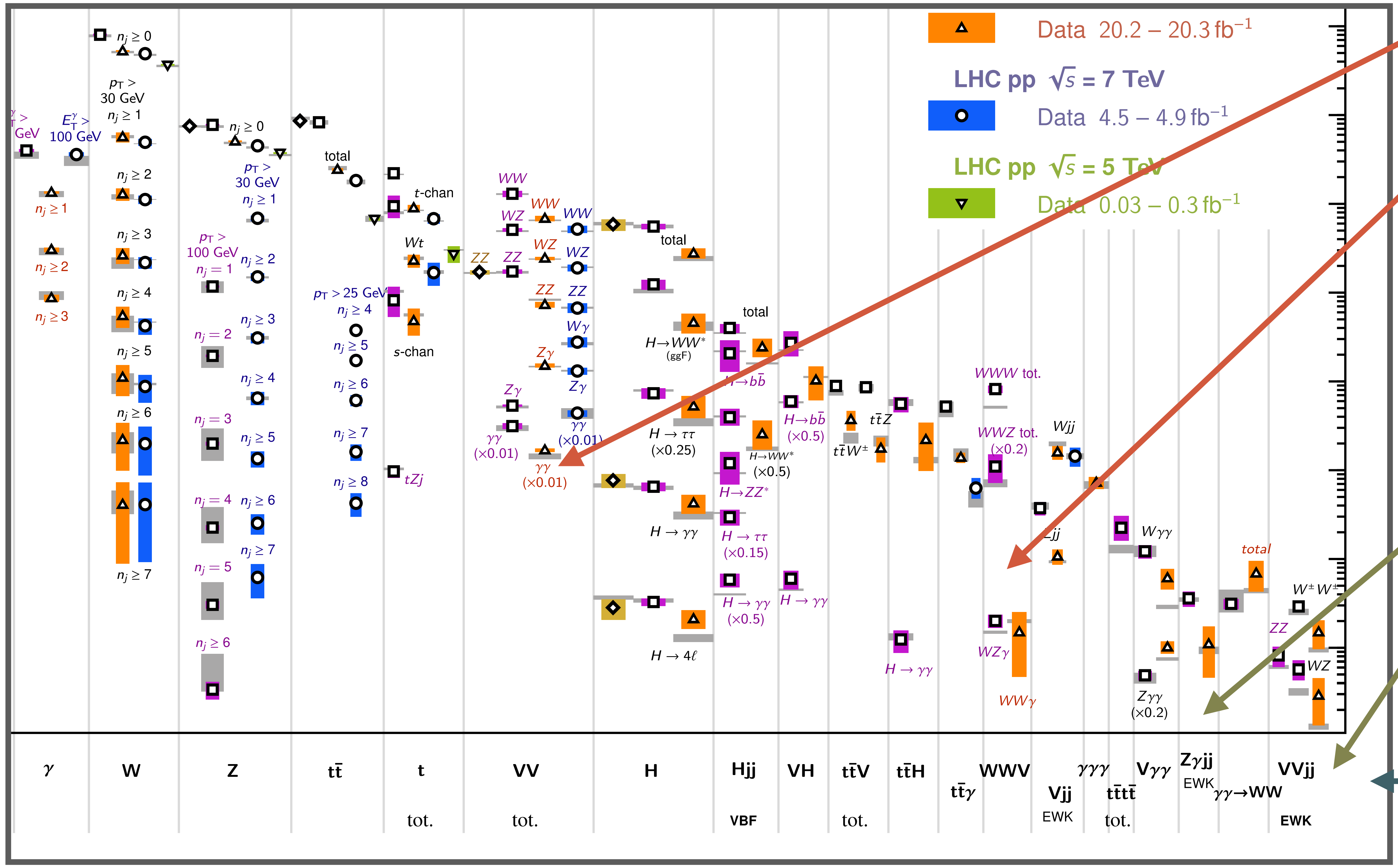
1. more minima?
2. more Higgses?
3. microscopic model of SSB?
4. …

Higgs self coupling extremely difficult to measure.

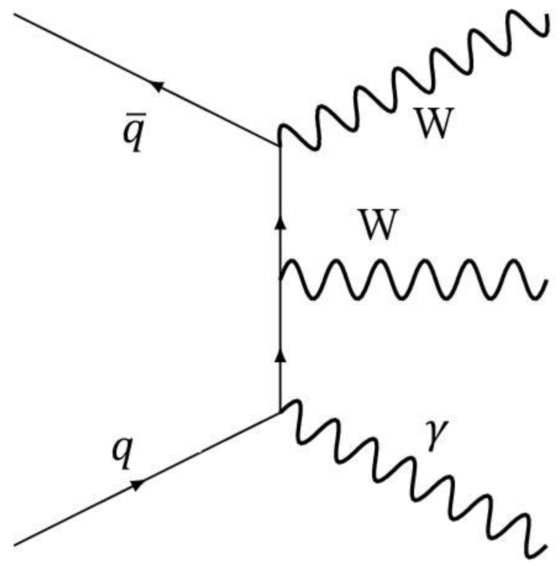
With 2018 estimates 4σ ATLAS+CMS

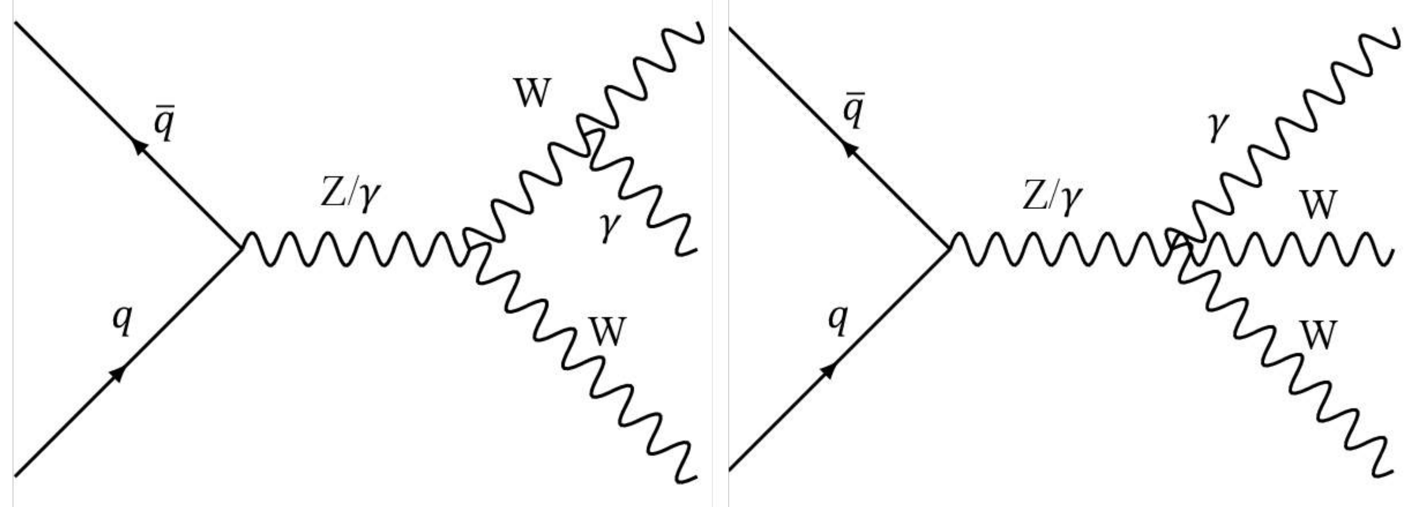
PROBING THE GAUGE SECTOR

Multiboson final states as probe of electroweak sector of SM

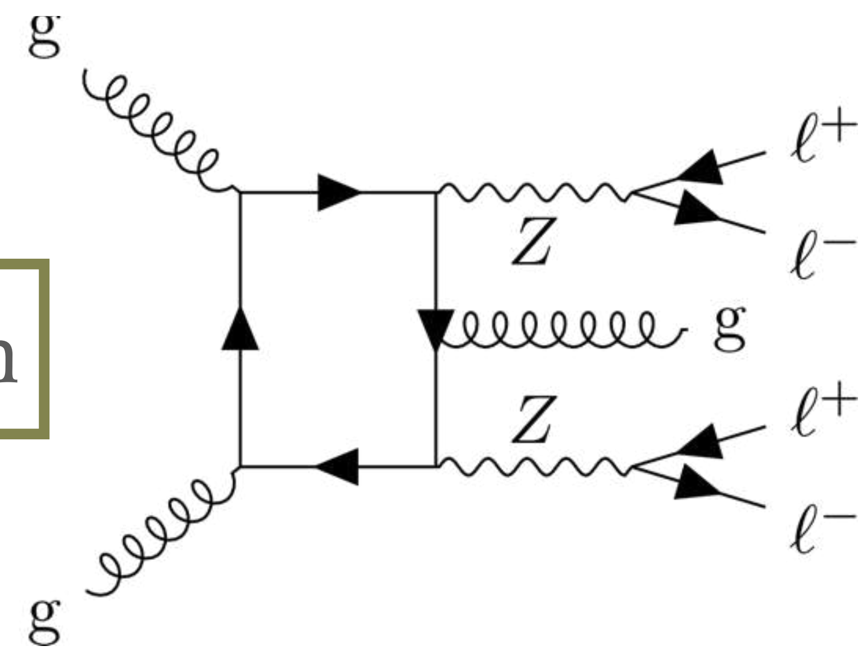


VV & *VVV* production

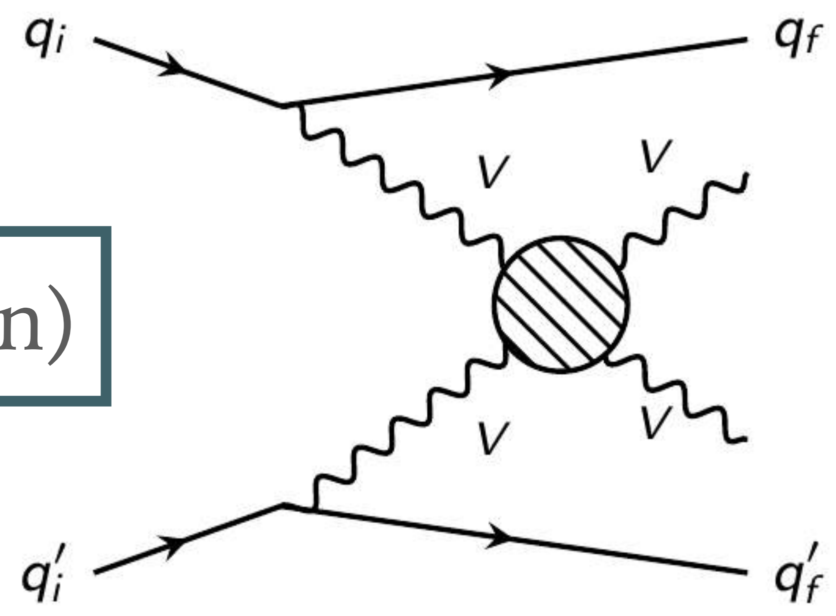




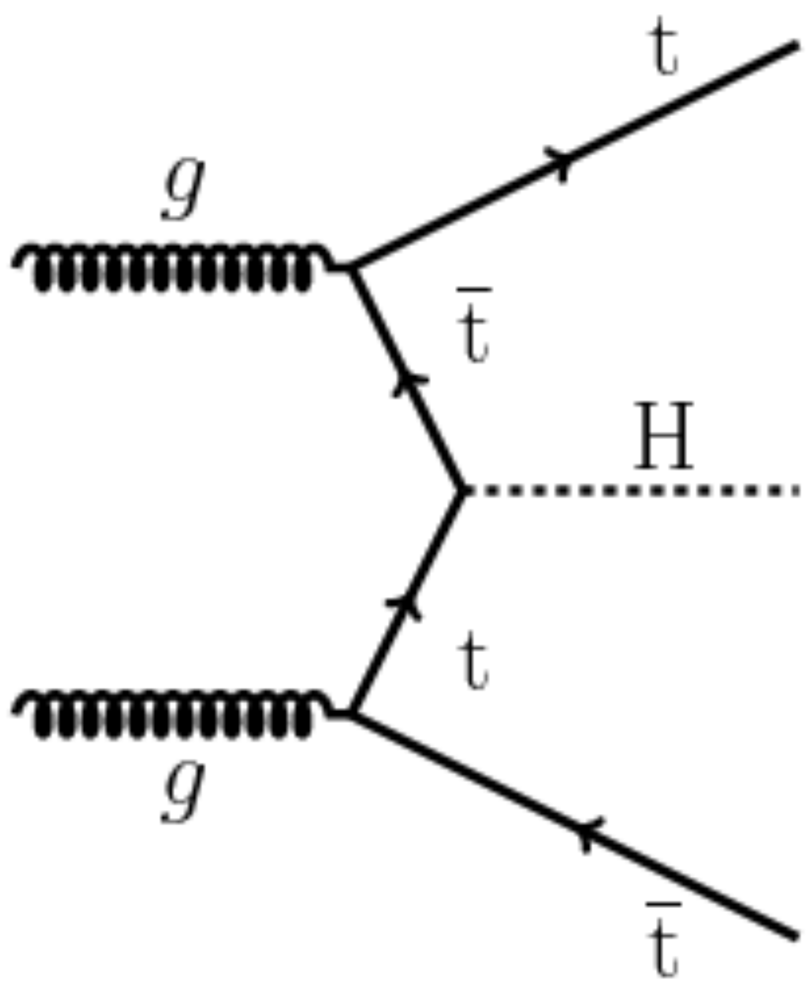
V & *VV* + jets production



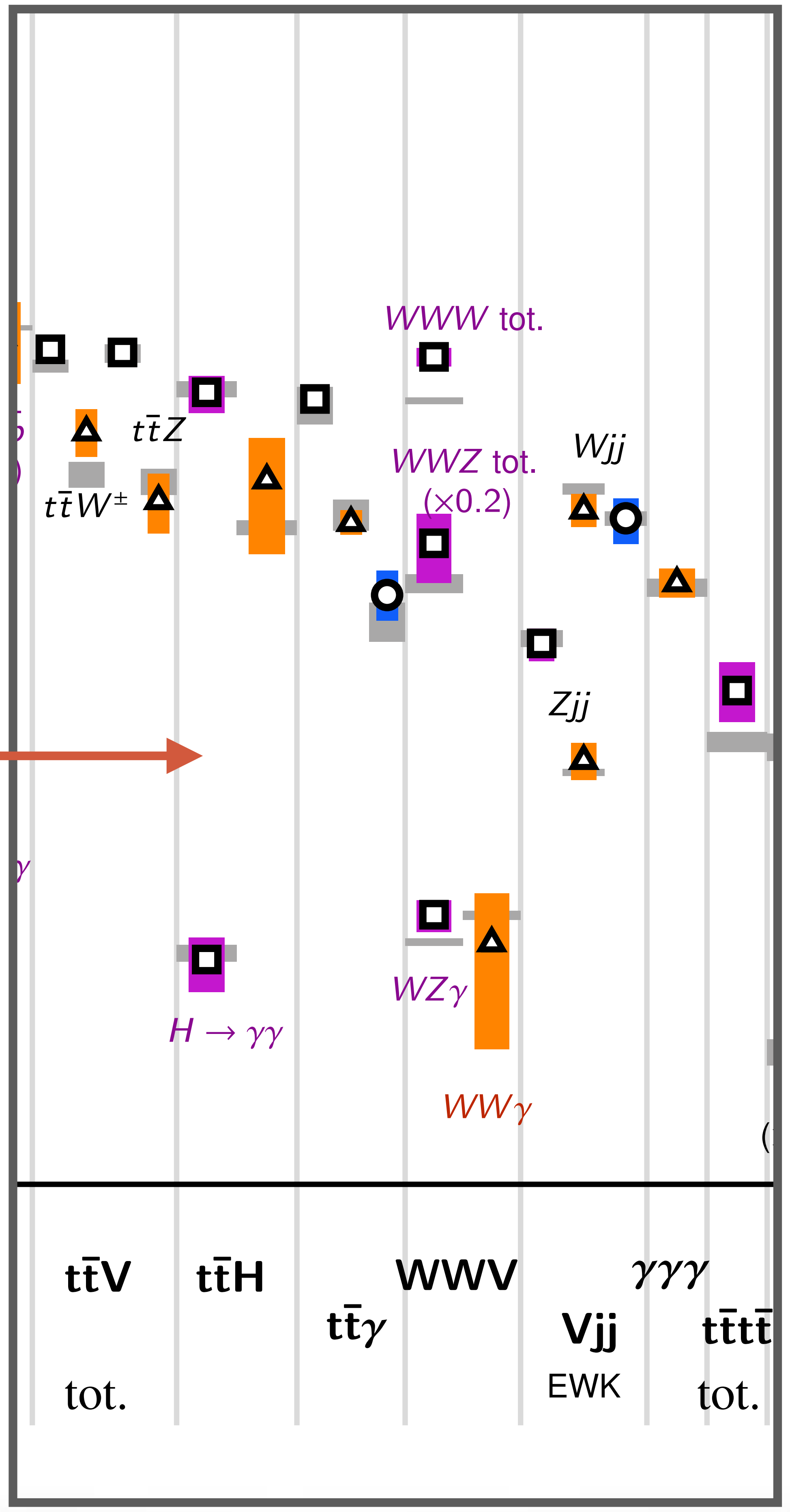
VBF (vector boson fusion)

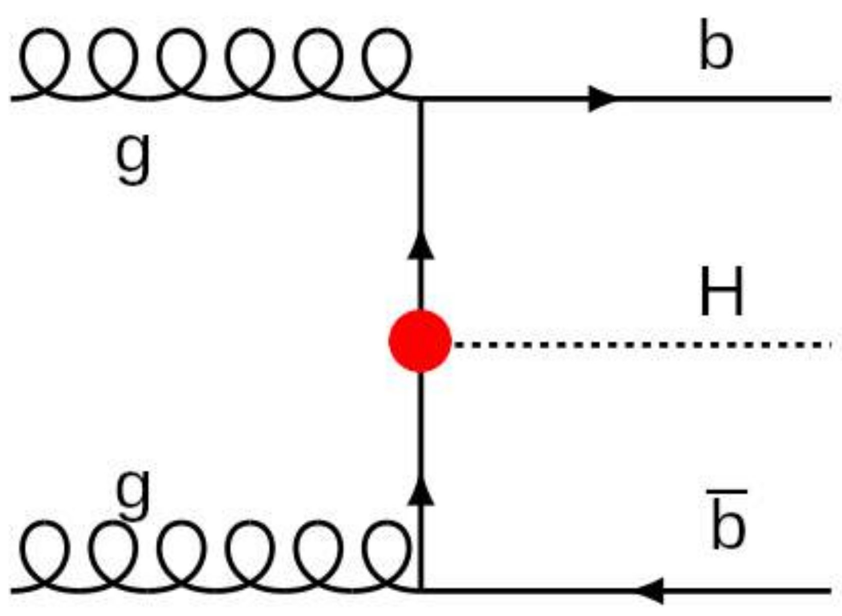


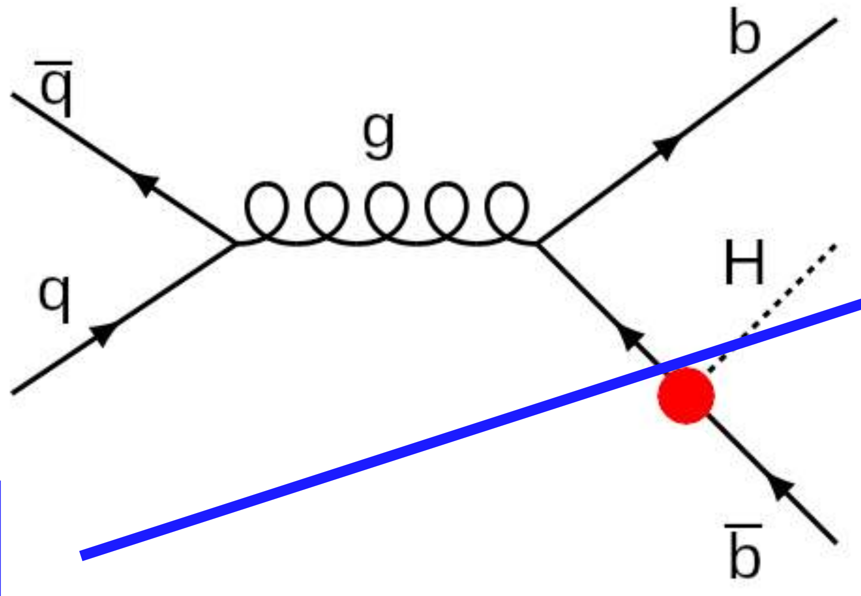
PROBING THE YUKAWA SECTOR

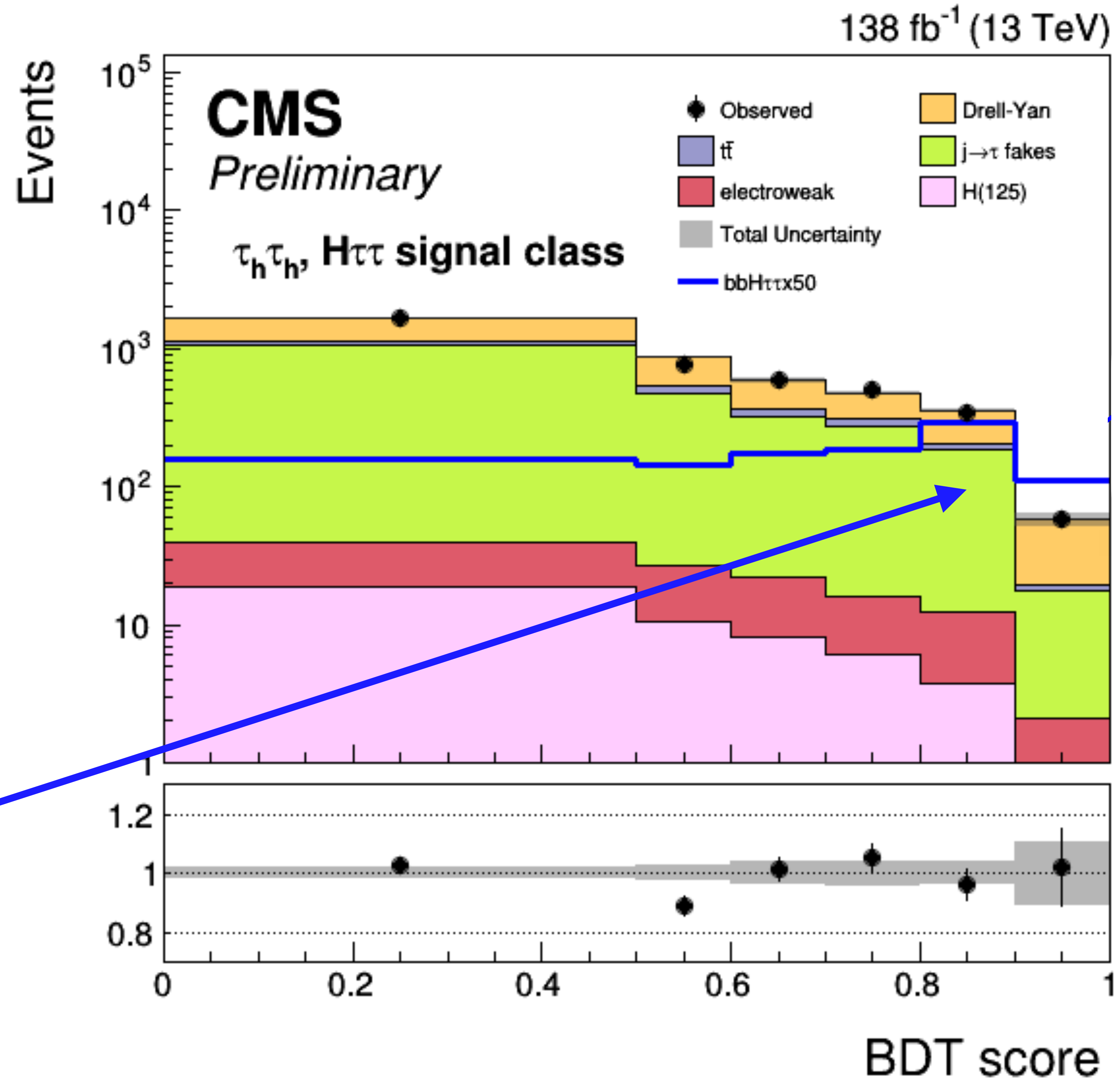


$t\bar{t}H$ production





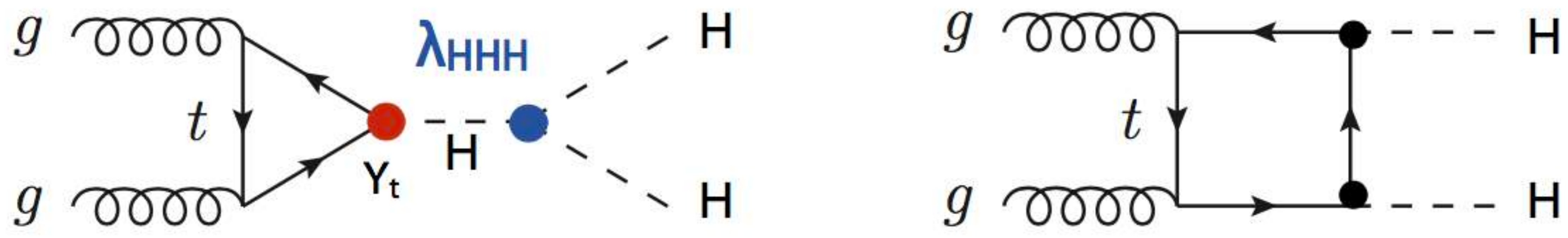




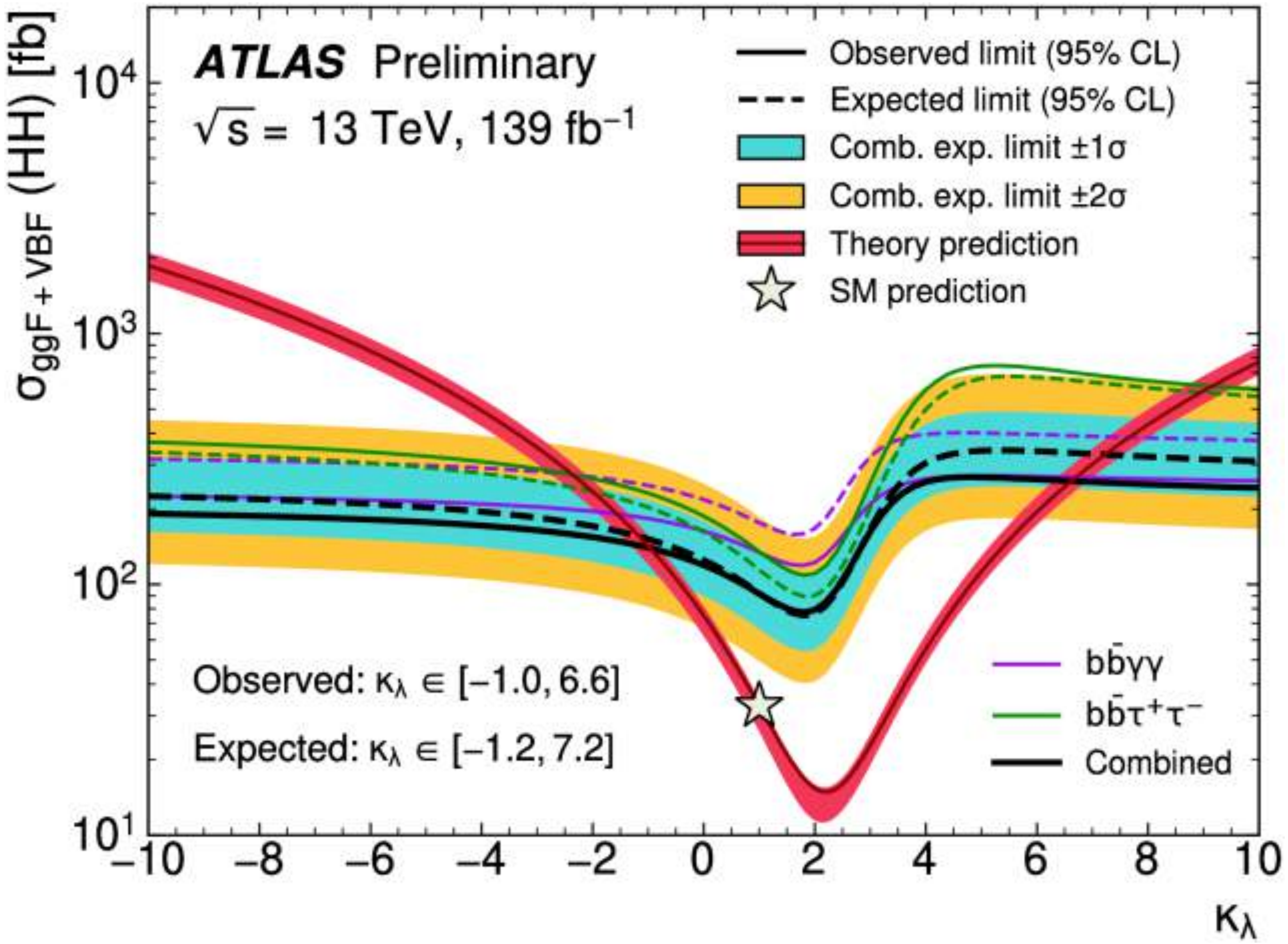
$b\bar{b}H$ production

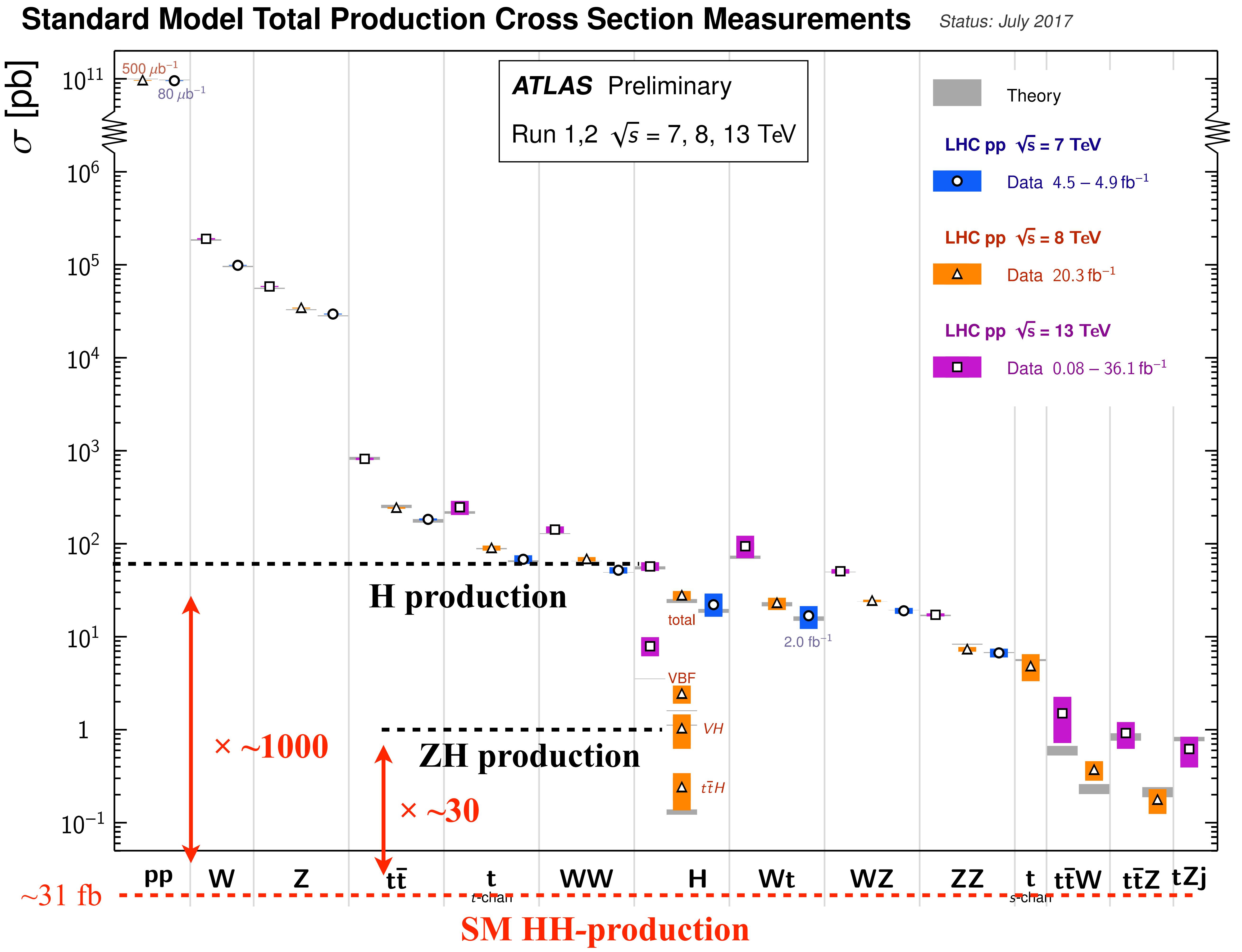
PROBING H SELF INTERACTION THE CHALLENGES AHEAD

Direct sensitivity in HH production: Progress, but extremely hard to measure even at (HL-)LHC



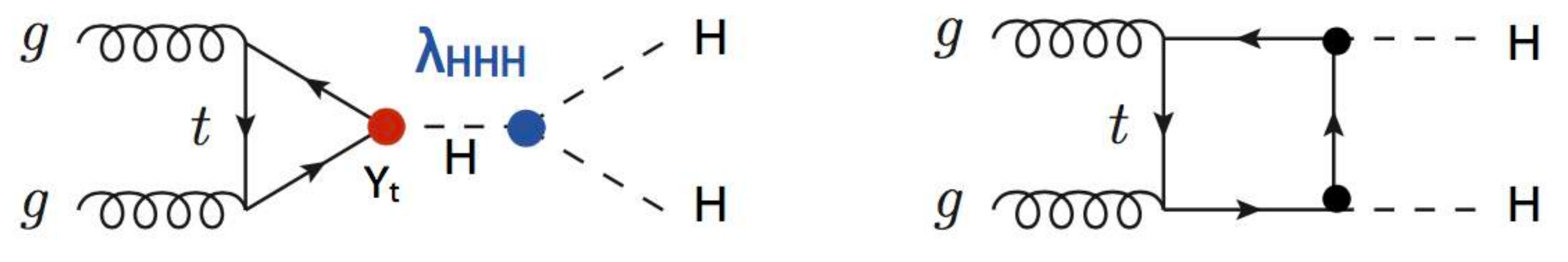
$b\bar{b}\tau\tau$ + $b\bar{b}\gamma\gamma$ + $b\bar{b}b\bar{b}$



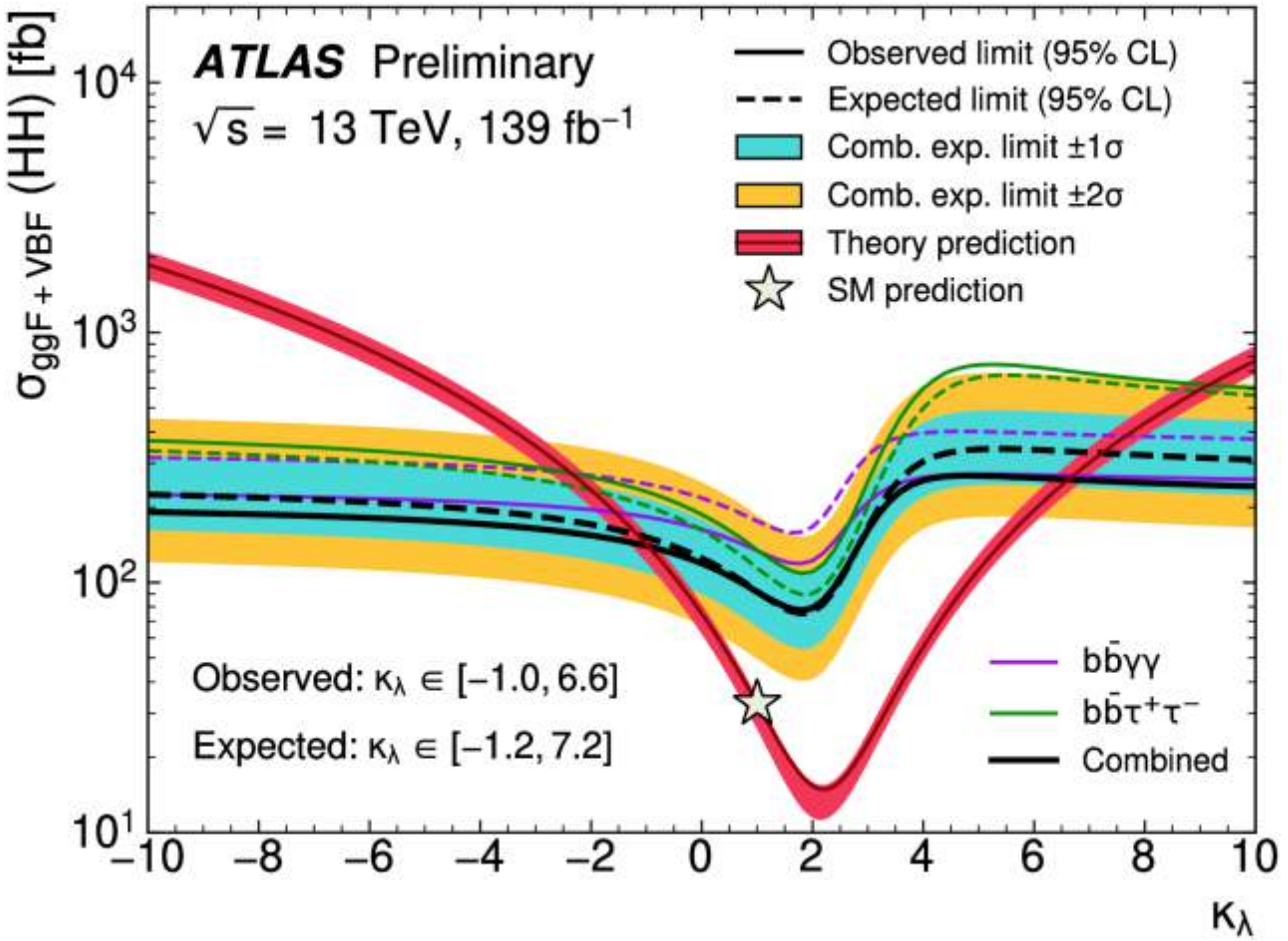


PROBING H SELF INTERACTION THE CHALLENGES AHEAD

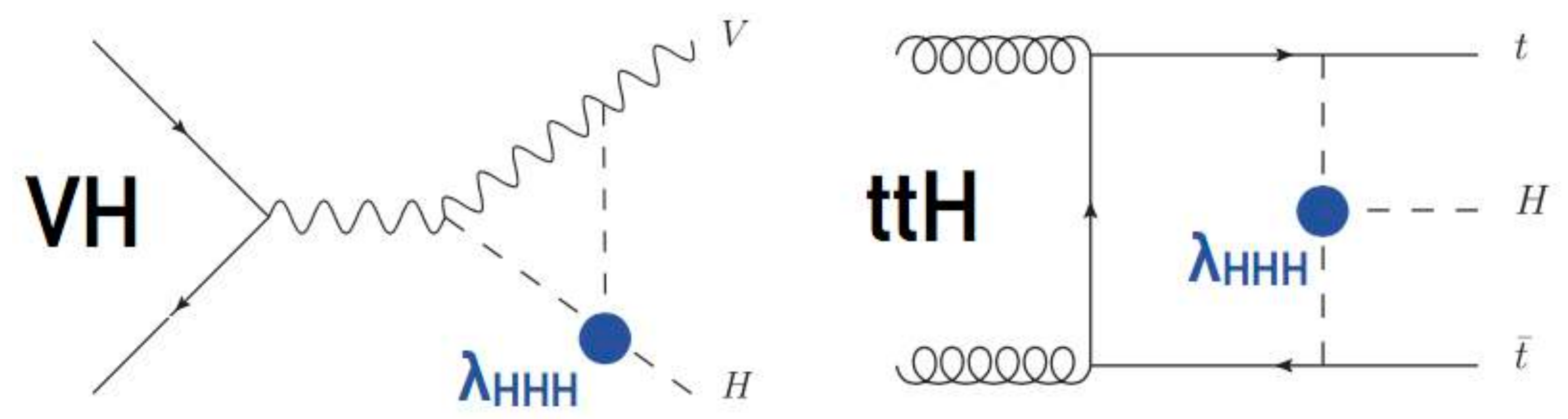
Direct sensitivity in HH production: Progress, but extremely hard to measure even at (HL-)LHC

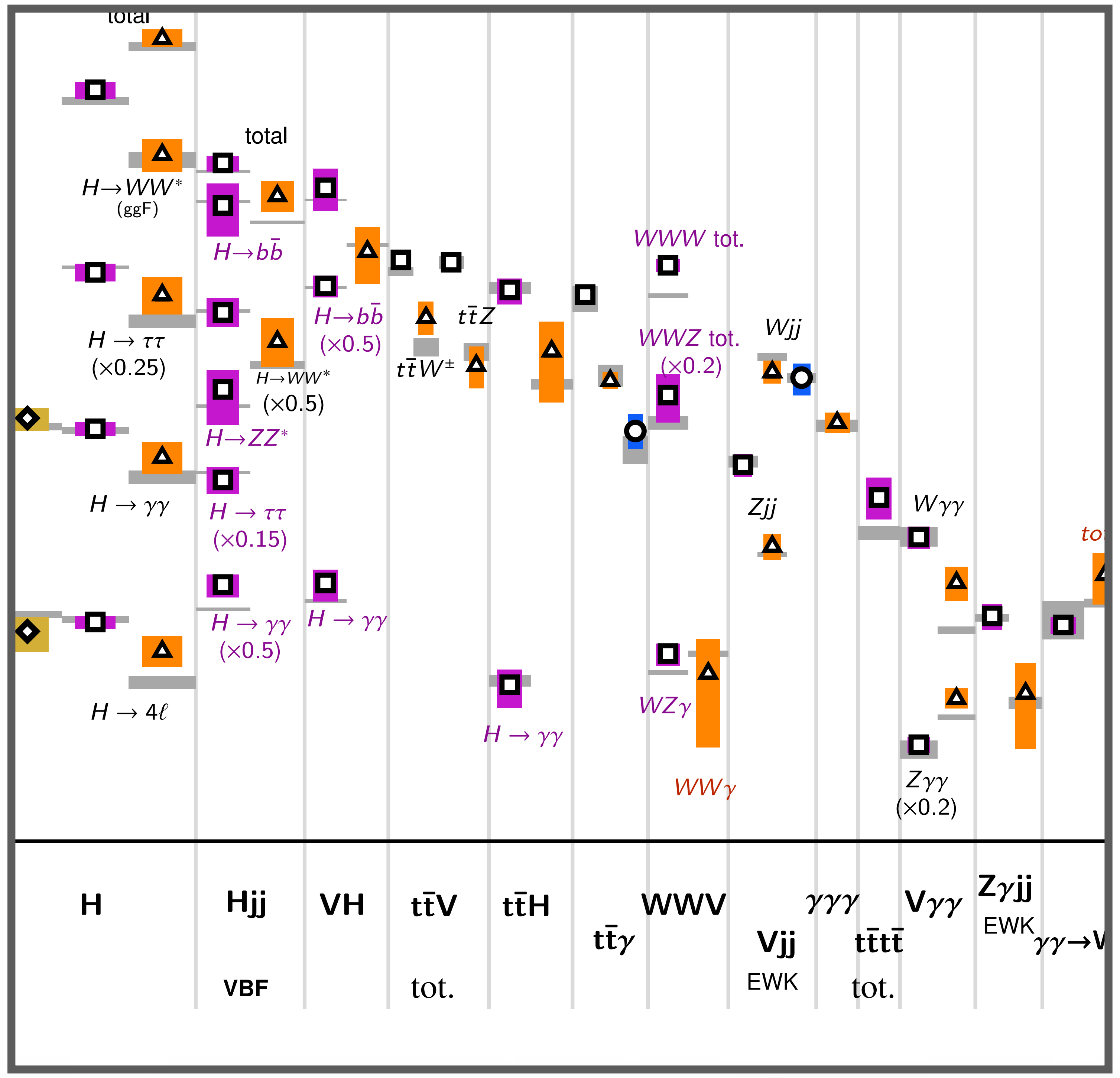


$b\bar{b}\tau\tau + b\bar{b}\gamma\gamma + b\bar{b}b\bar{b}$

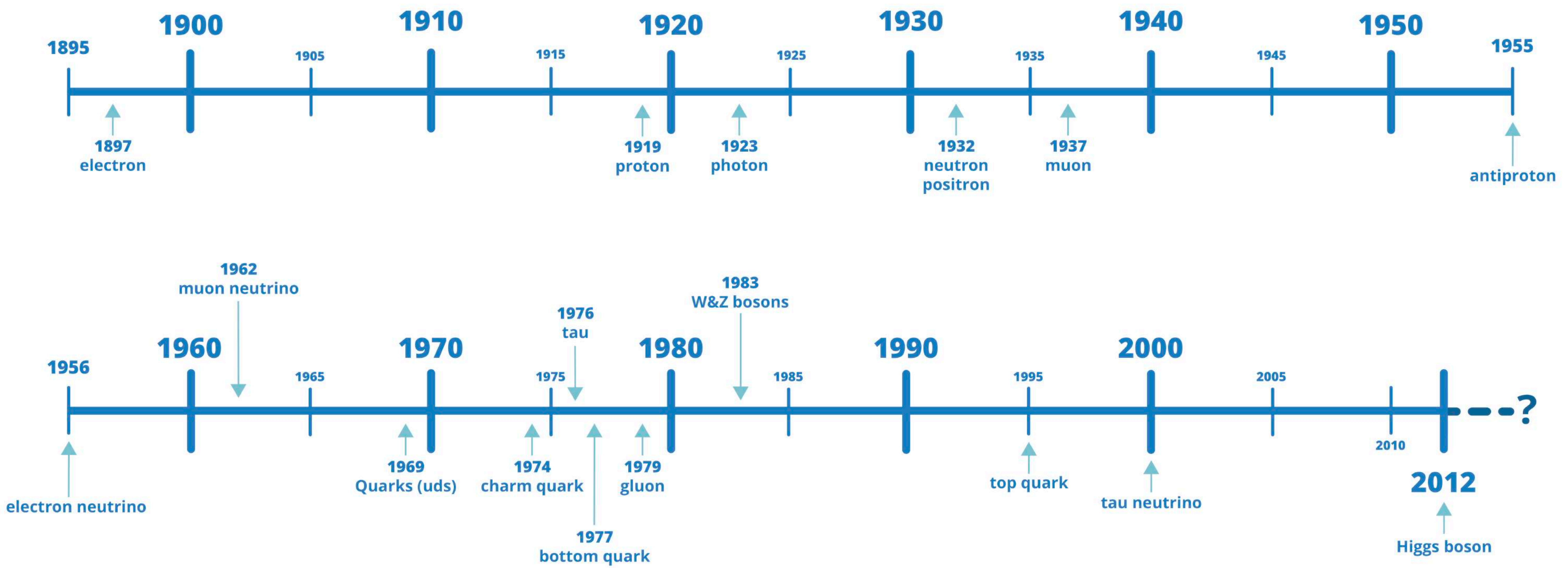


Indirect sensitivity through precision studies!





PLENTY OF DISCOVERY POTENTIAL AHEAD



For the first time in decades, we might not expect new particles ahead…

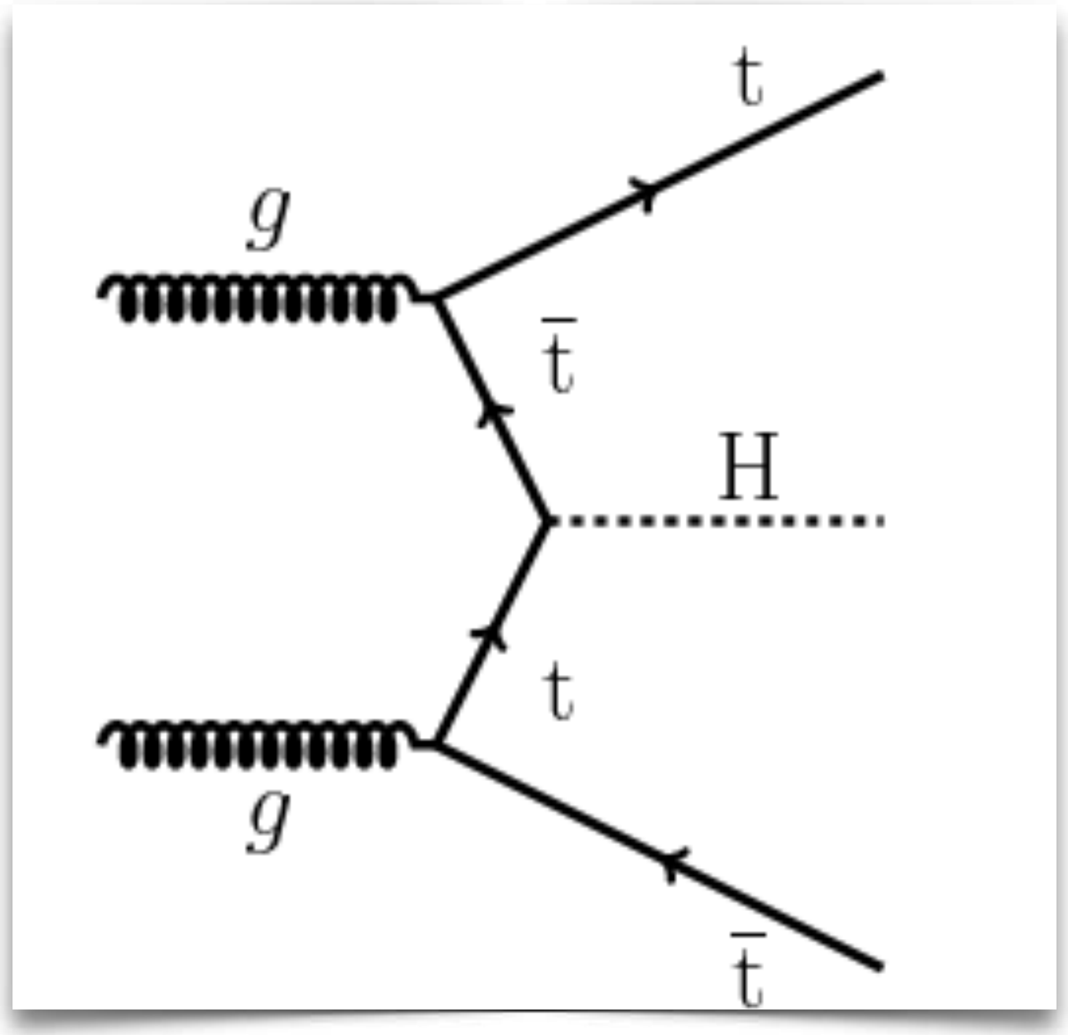
Still, thanks to % precision physics program at colliders, we have the chance to discover “new interactions”, and have the concrete opportunity to uncover details of new “Higgs” physics!

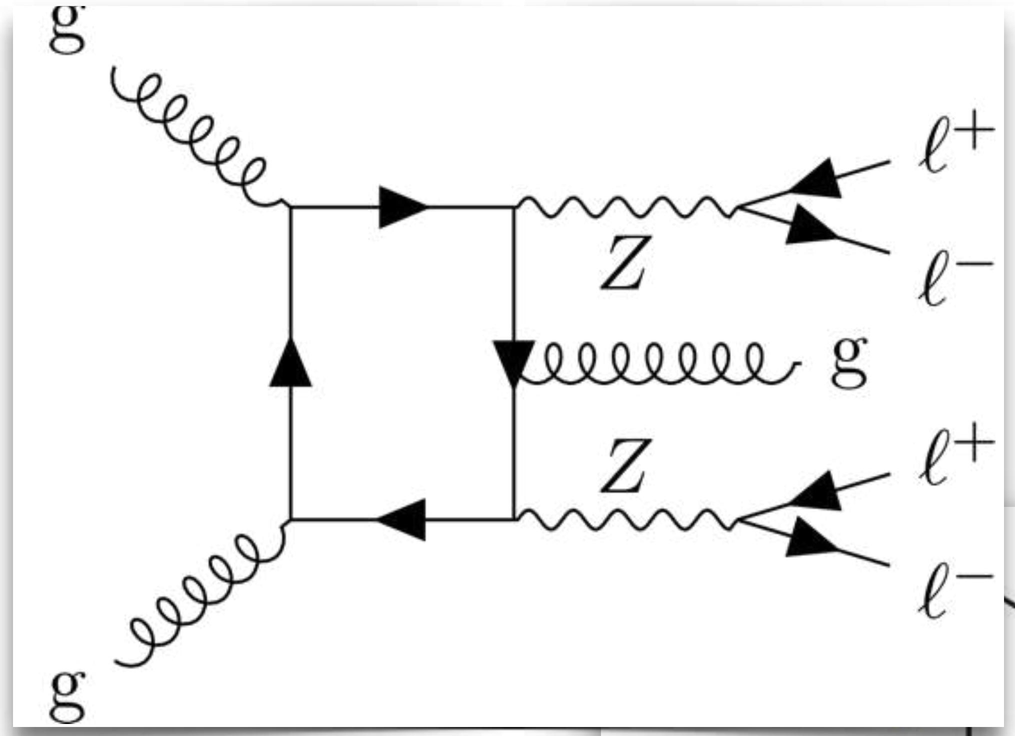
PRECISION STUDIES “OPPORTUNITIES” ALL OVER

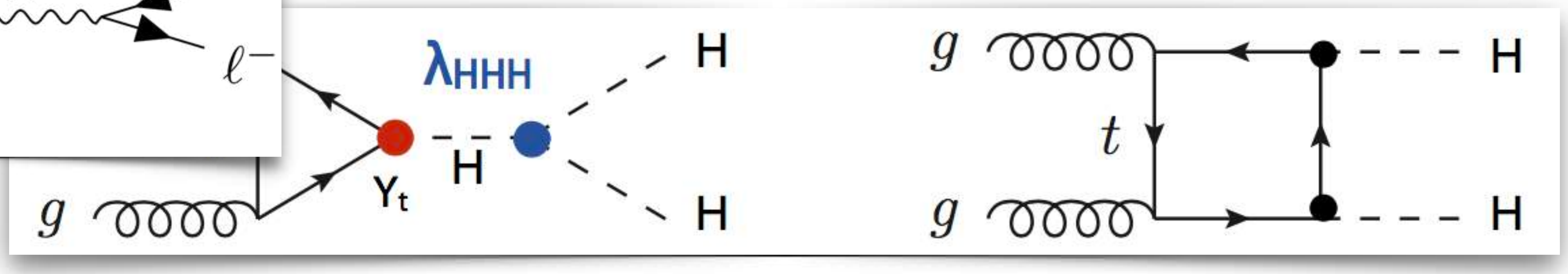
What do they have in common?

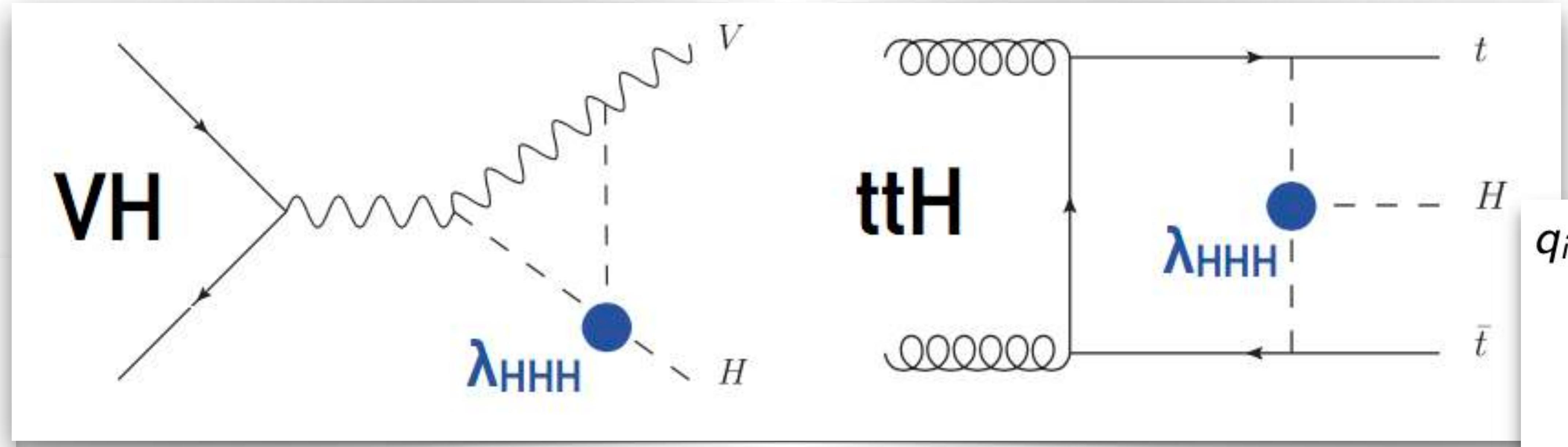
complex final states, QCD & EW corrections

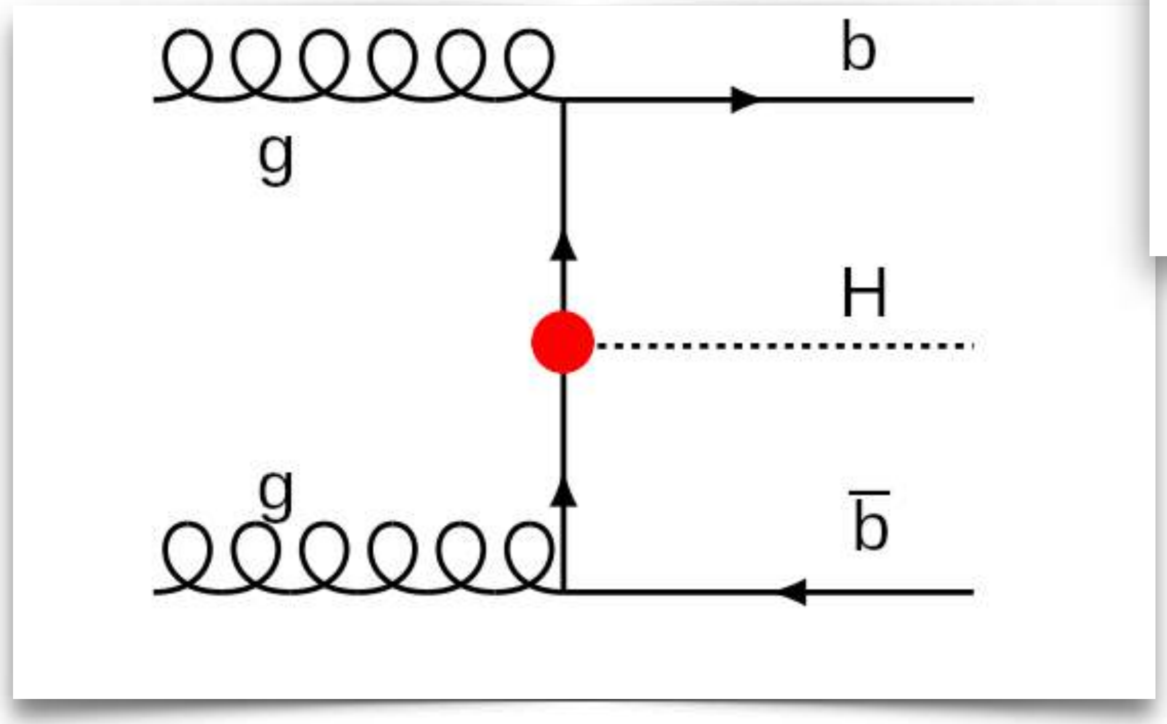
massive external and virtual states

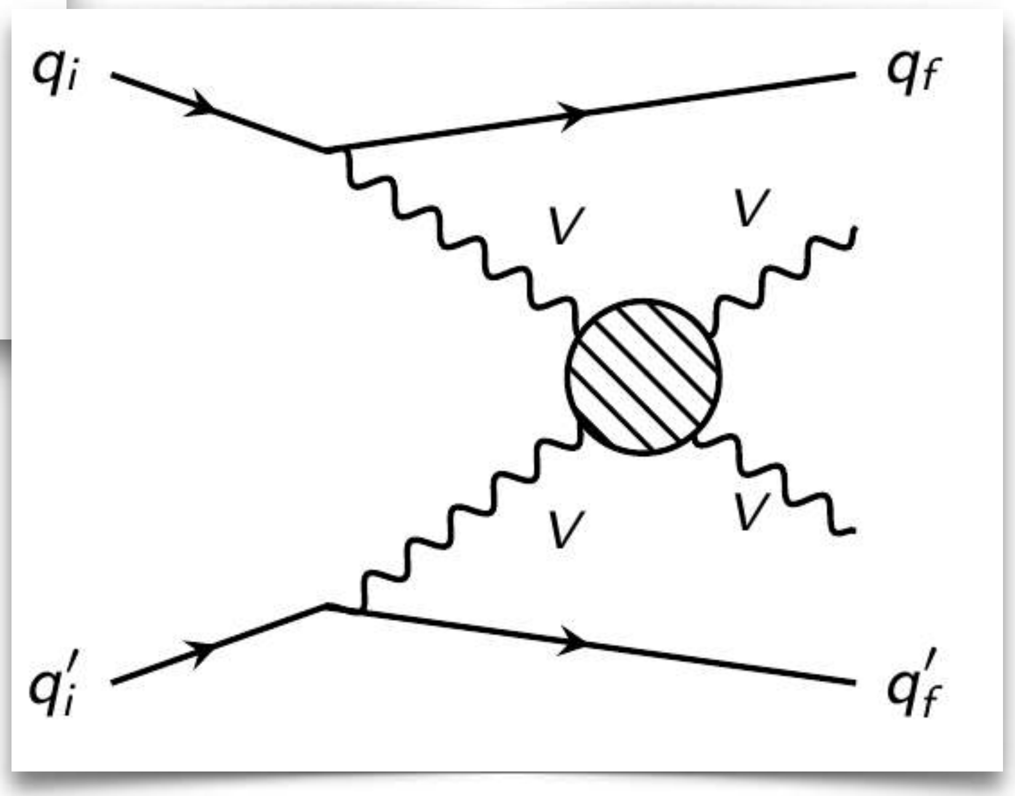






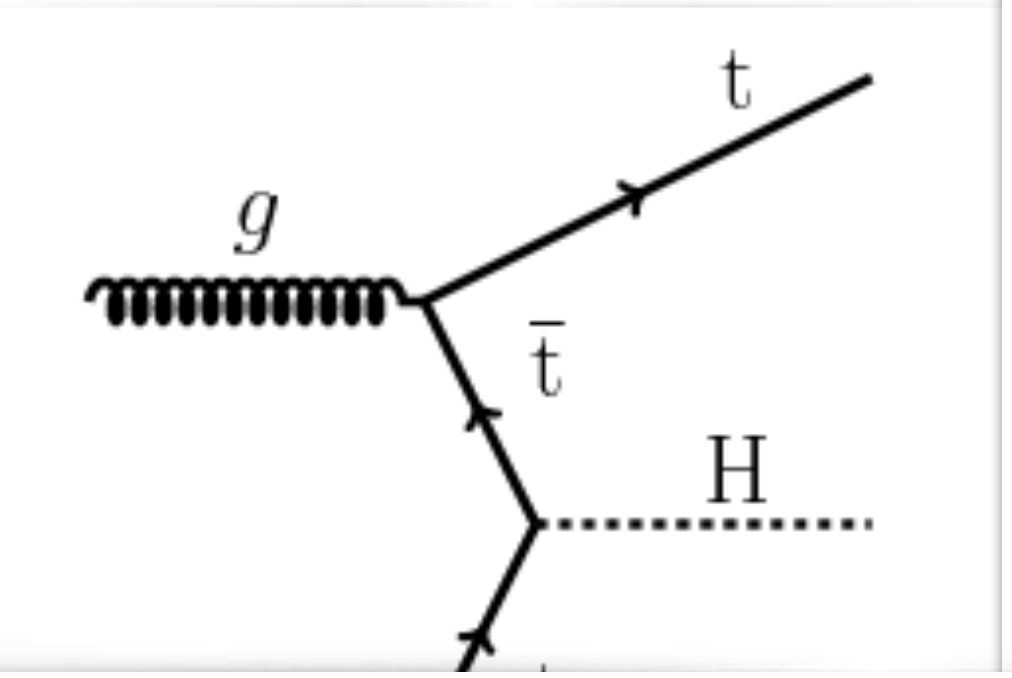


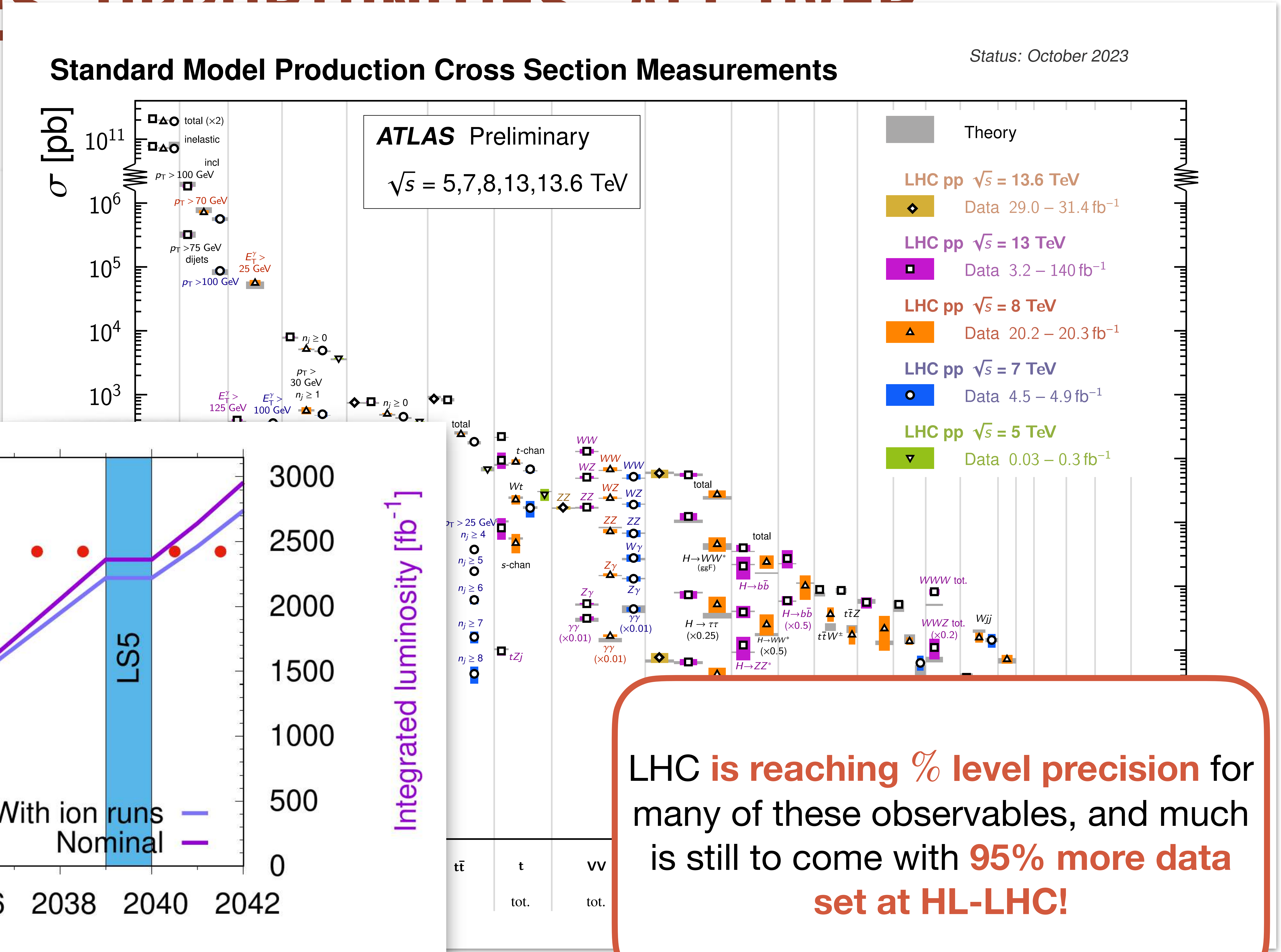


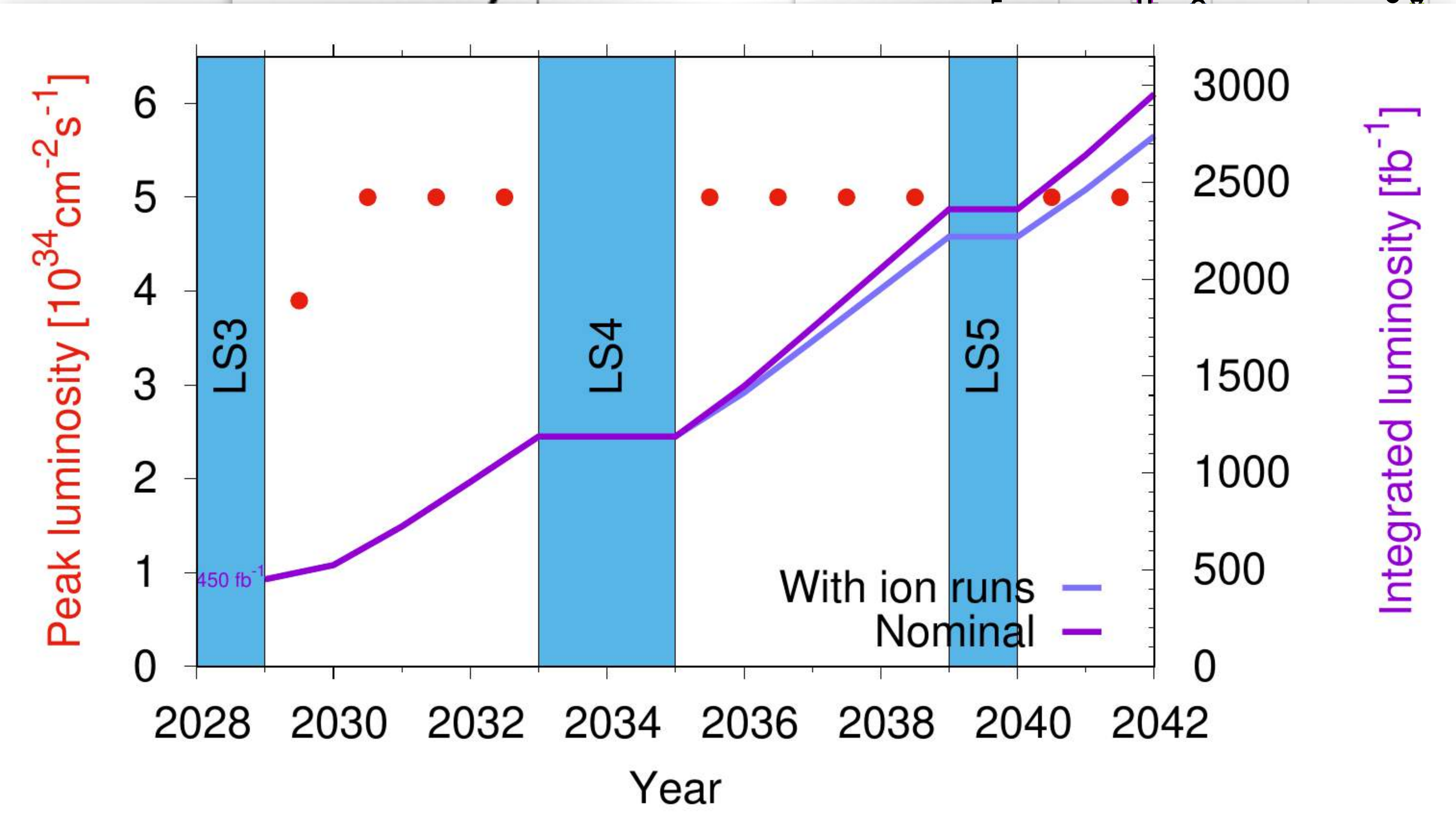


… and many others …

PRECISION STUDIES “OPPORTUNITIES” ALL OVER



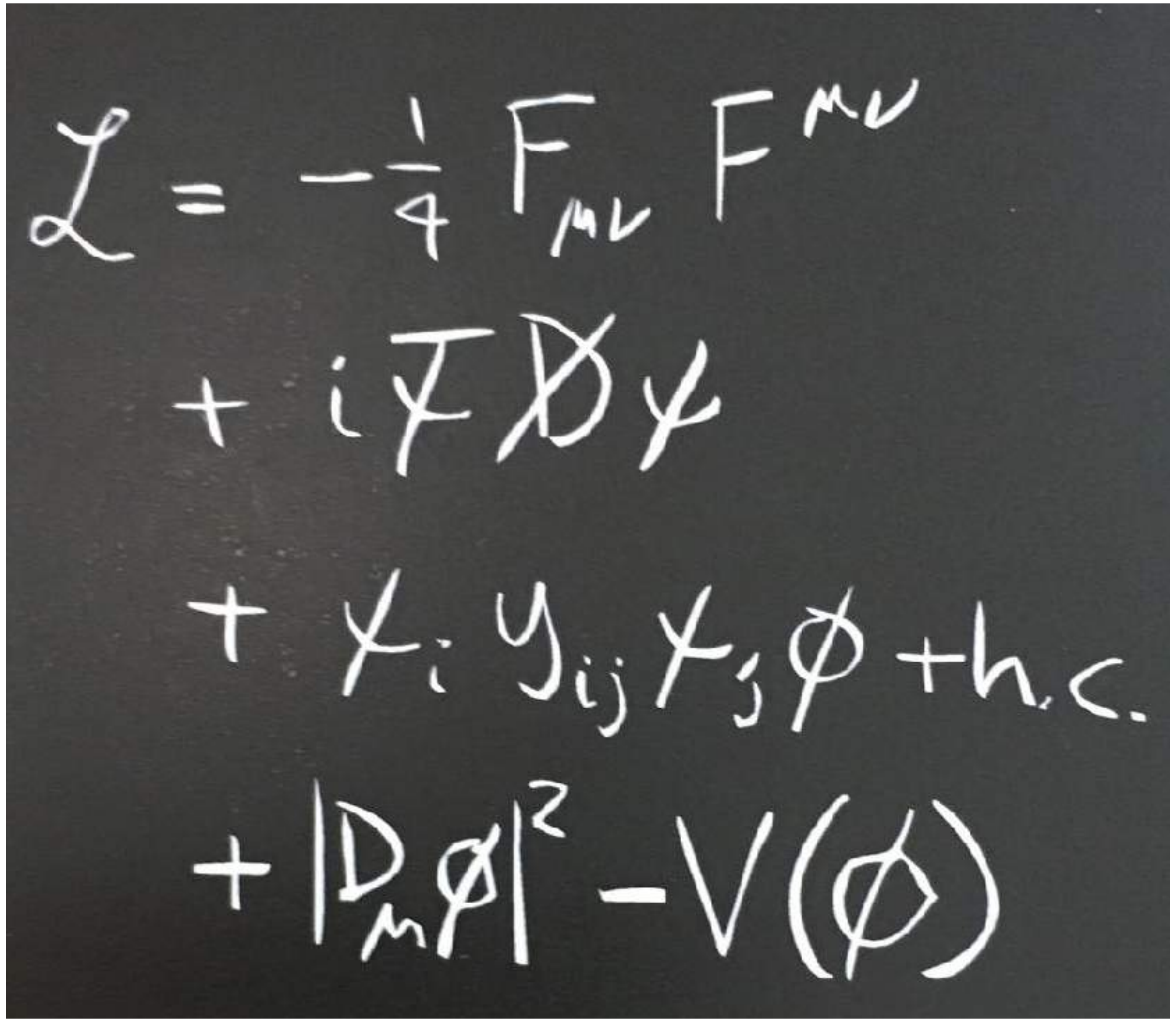




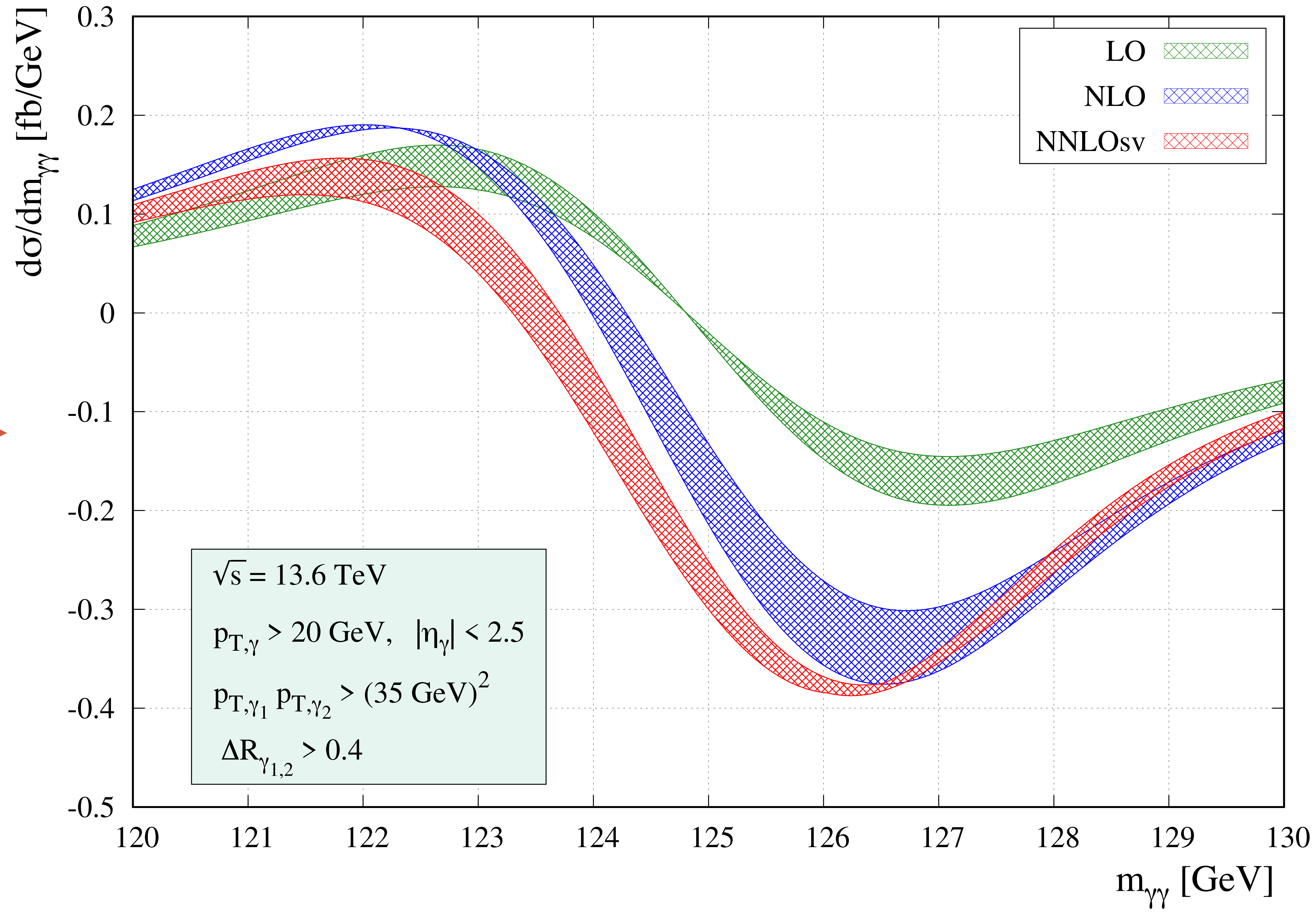
LHC **is reaching % level precision** for many of these observables, and much is still to come with **95% more data set at HL-LHC!**

% PRECISION, HOW DO WE GET THERE?

FROM THEORY TO THEORY PREDICTIONS IT'S A LONG WAY!



?

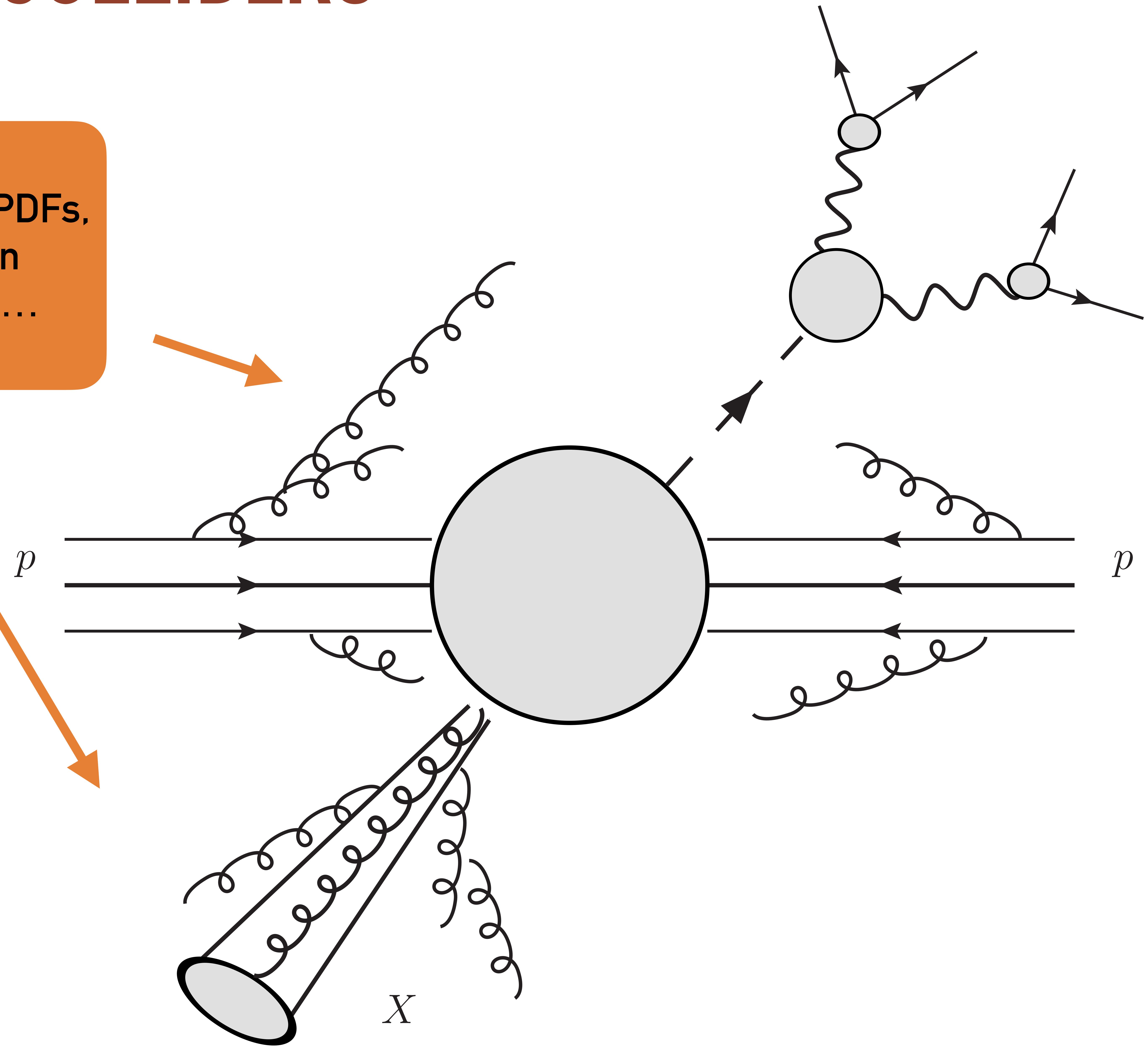


Signal to BKG interference for $gg \to H \to \gamma\gamma$

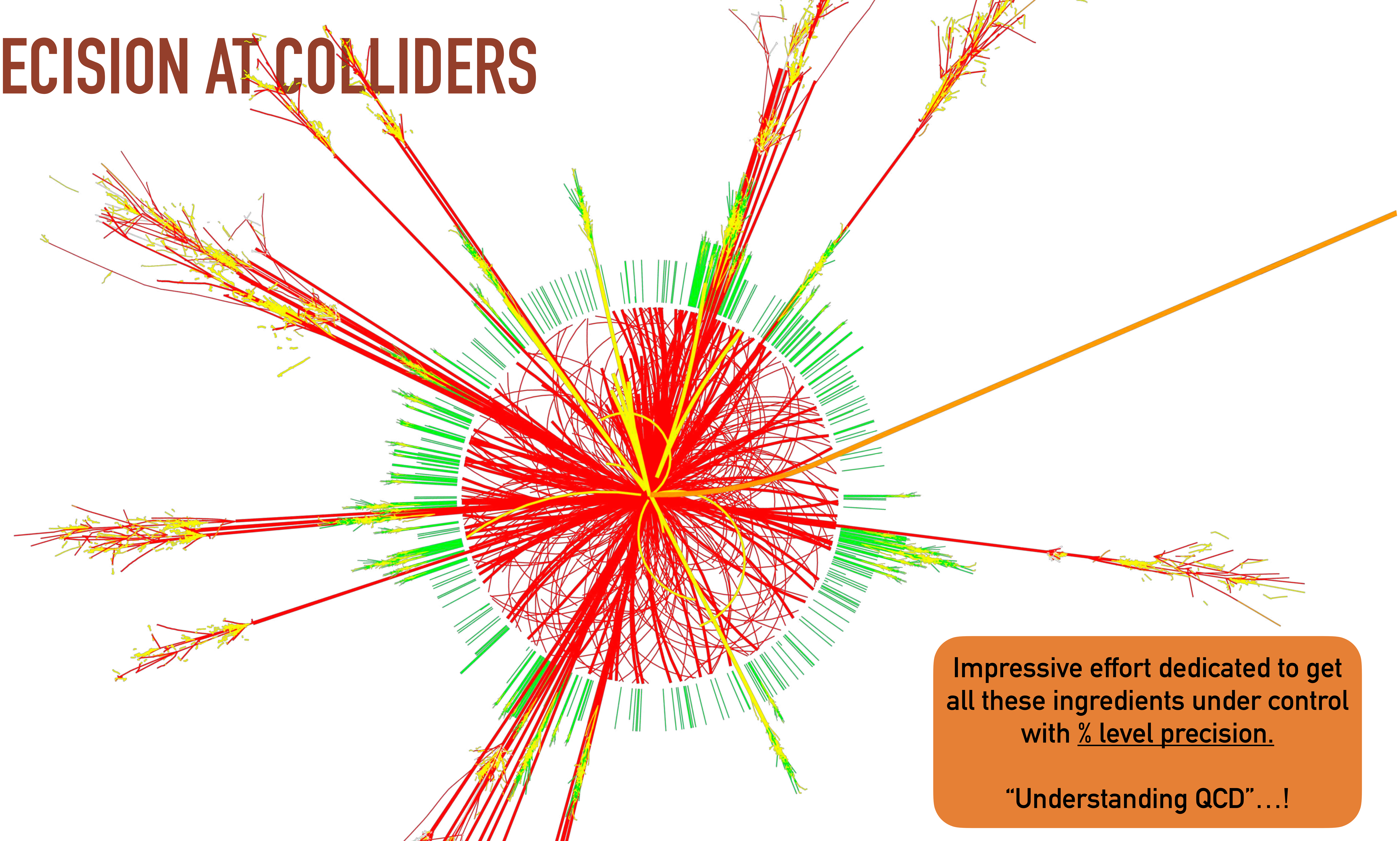
[Bargiela, Buccioni, Caola, Devoto, Manteuffel, Tancredi '22]

PRECISION AT COLLIDERS

"soft & collinear physics": PDFs, jet substructure, parton showers, hadronization…



PRECISION AT COLLIDERS

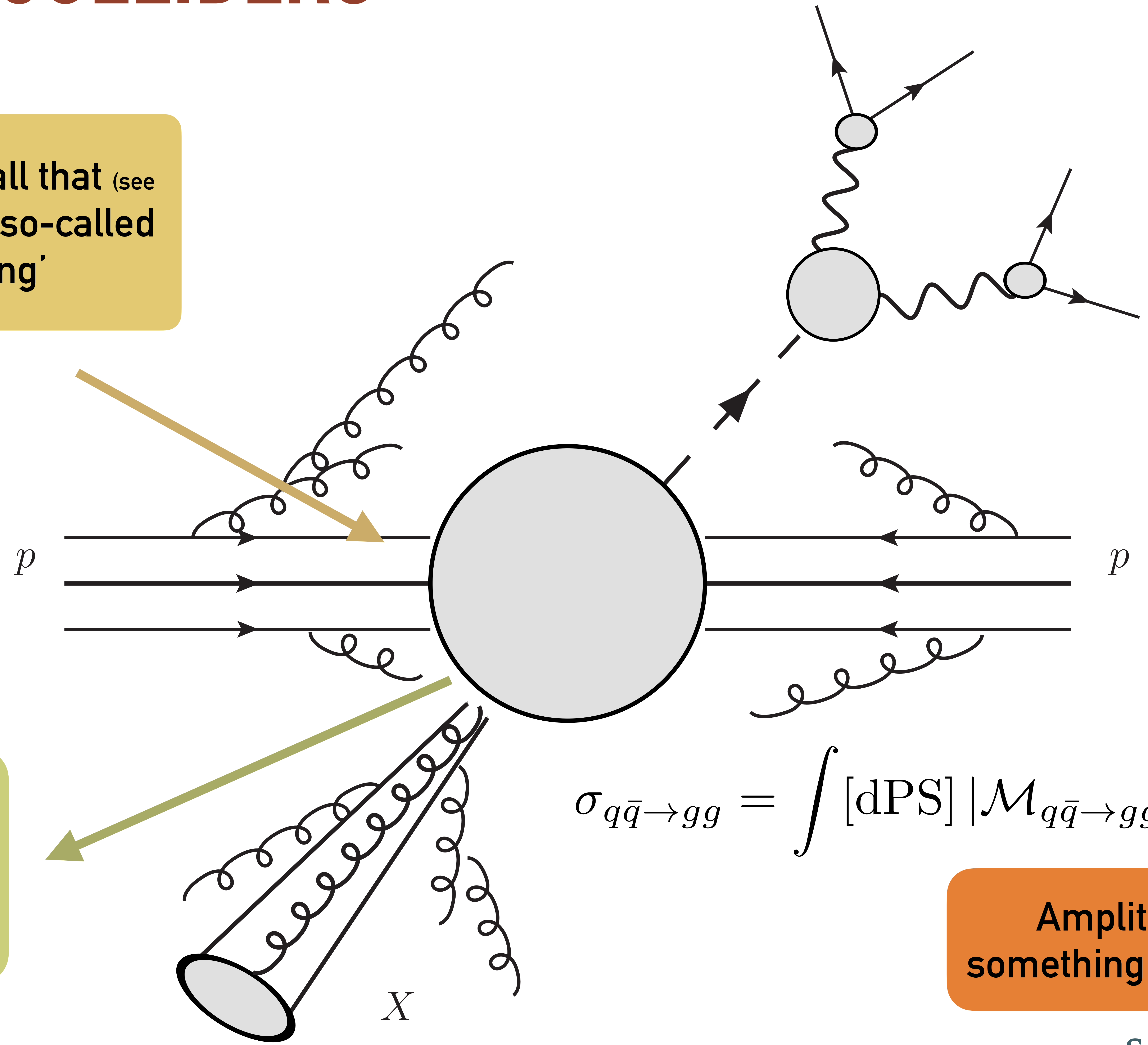


Impressive effort dedicated to get all these ingredients under control with % level precision.

"Understanding QCD"…!

PRECISION AT COLLIDERS

For now, we ignore all that (see later) and zoom in the so-called 'Hard Scattering'



Building blocks are "Scattering Amplitudes"

% precision possible?!

$$\sigma_{q\bar{q}\to gg} = \int [\mathrm{dPS}] \left|\mathcal{M}_{q\bar{q}\to gg}\right|^2$$

Amplitudes can tell us also something beyond perturbative HS!

See talk by F. Devoto

HARD SCATTERING

$$\sigma_{q\bar{q}\to gg} = \int [\mathrm{dPS}]\, |\mathcal{M}_{q\bar{q}\to gg}|^2$$

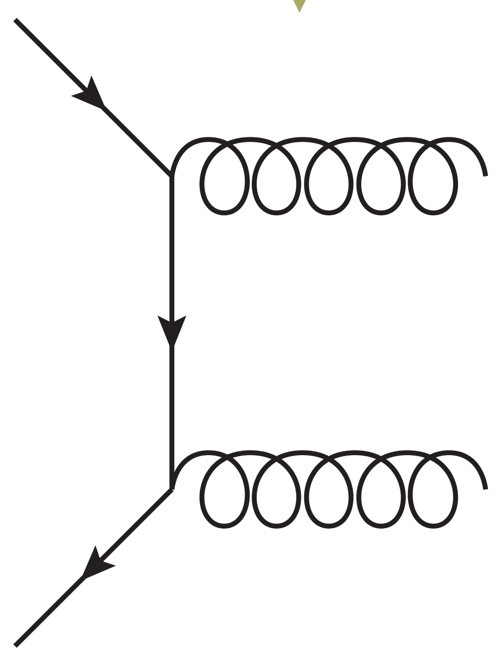
small "coupling constant" ~ 0.1

$$|\mathcal{M}_{q\bar{q}\to gg}|^2 = |\mathcal{M}^{LO}_{q\bar{q}\to gg}|^2 + \left(\frac{\alpha_s}{2\pi}\right) |\mathcal{M}^{NLO}_{q\bar{q}\to gg}|^2 + \left(\frac{\alpha_s}{2\pi}\right)^2 |\mathcal{M}^{NNLO}_{q\bar{q}\to gg}|^2 + \ldots$$

HARD SCATTERING

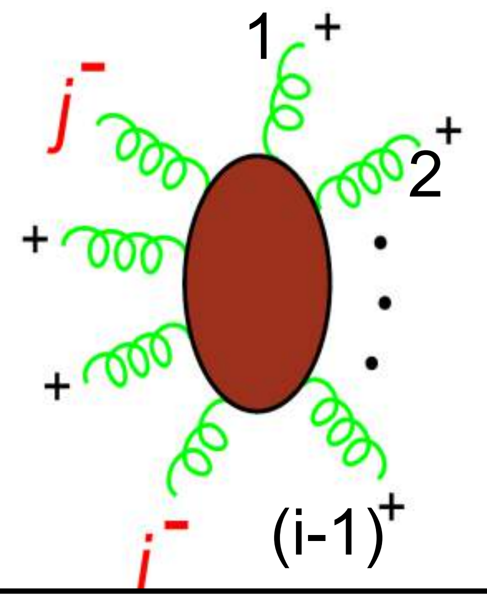
$$\sigma_{q\bar{q}\to gg} = \int [\mathrm{dPS}]\, \left|\mathcal{M}_{q\bar{q}\to gg}\right|^2$$

$$\left|\mathcal{M}_{q\bar{q}\to gg}\right|^2 = \left|\mathcal{M}^{LO}_{q\bar{q}\to gg}\right|^2 + \left(\frac{\alpha_s}{2\pi}\right)\left|\mathcal{M}^{NLO}_{q\bar{q}\to gg}\right|^2 + \left(\frac{\alpha_s}{2\pi}\right)^2\left|\mathcal{M}^{NNLO}_{q\bar{q}\to gg}\right|^2 + \ldots$$



~ O(100%-50%) precision

$$A_n^{ij,\ \mathrm{MHV}} = A_n^{\mathrm{tree}}(1^+, 2^+, \ldots, i^-, \ldots, j^-, \ldots, n^+)$$

$=$ 

$= \dfrac{\langle i\, j\rangle^4}{\langle 1\, 2\rangle\langle 2\, 3\rangle\cdots\langle n\, 1\rangle}$

Parke-Taylor formula (1986)

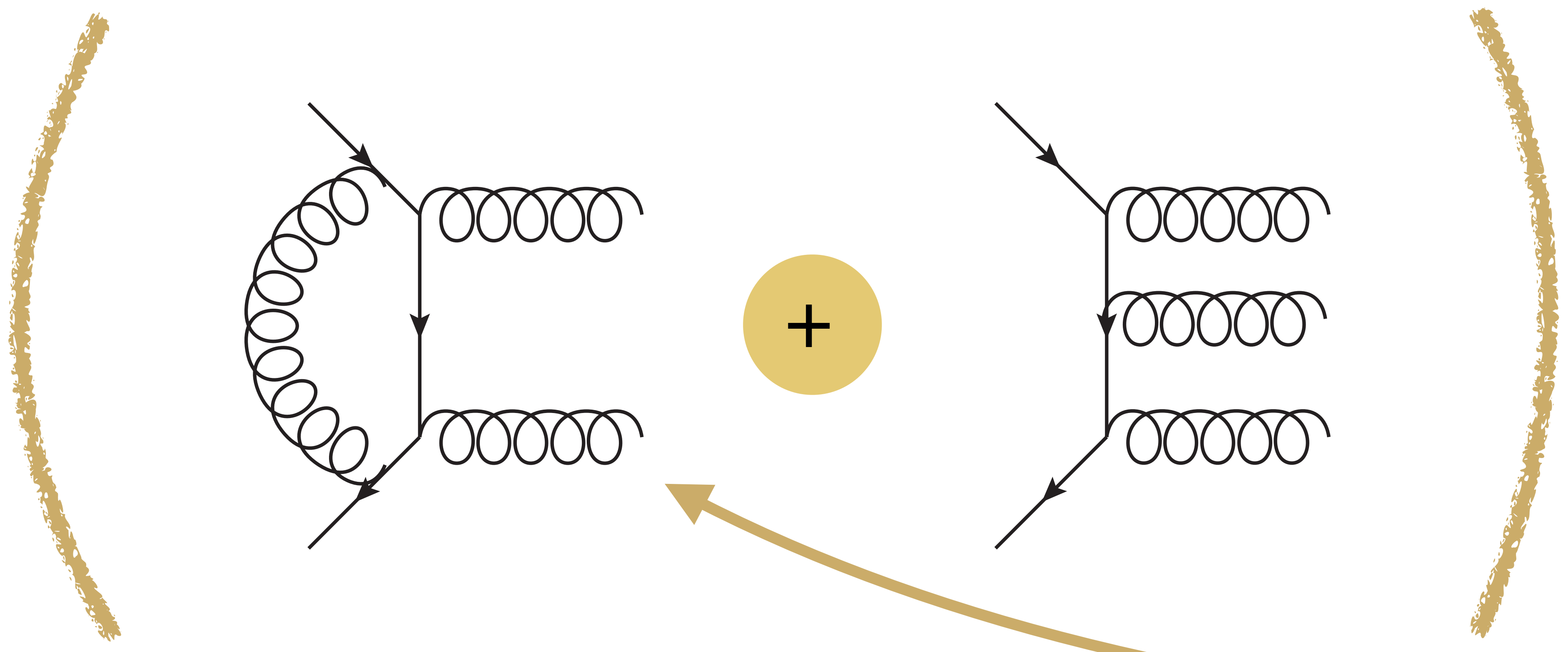
[slide from L. Dixon]

Tree-level Amplitudes

HARD SCATTERING

$$\sigma_{q\bar{q}\to gg} = \int [\mathrm{dPS}]\, \left|\mathcal{M}_{q\bar{q}\to gg}\right|^2$$

$$\left|\mathcal{M}_{q\bar{q}\to gg}\right|^2 = \left|\mathcal{M}^{LO}_{q\bar{q}\to gg}\right|^2 + \left(\frac{\alpha_s}{2\pi}\right)\left|\mathcal{M}^{NLO}_{q\bar{q}\to gg}\right|^2 + \left(\frac{\alpha_s}{2\pi}\right)^2\left|\mathcal{M}^{NNLO}_{q\bar{q}\to gg}\right|^2 + \ldots$$



Virtual Real

~ O(30%-10%) precision

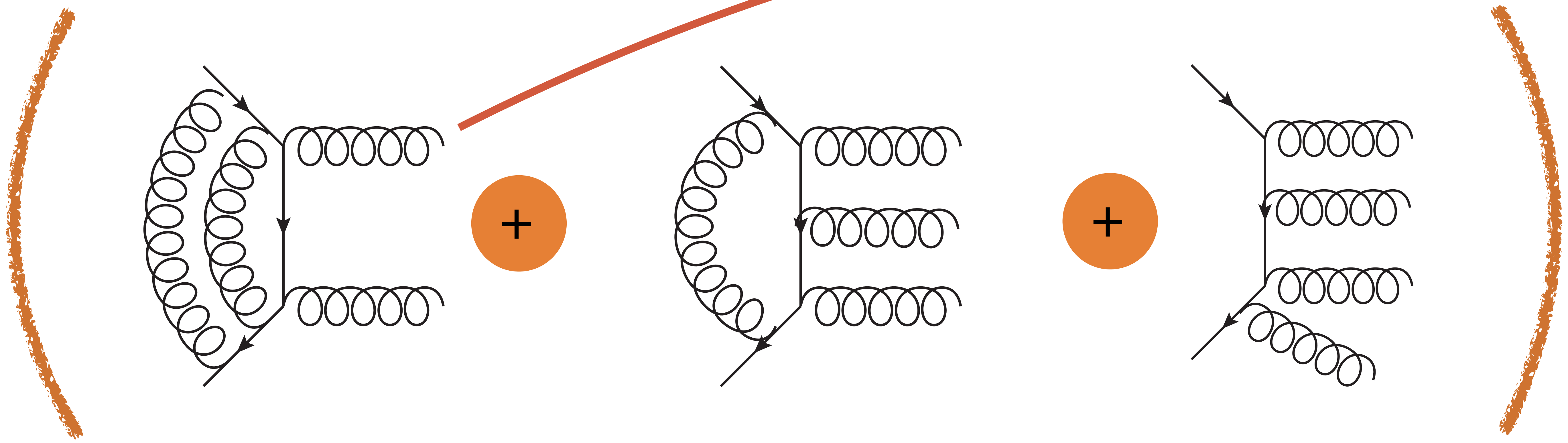
One-loop Amplitudes

HARD SCATTERING

$$\sigma_{q\bar{q}\to gg} = \int [\mathrm{dPS}] \left|\mathcal{M}_{q\bar{q}\to gg}\right|^2$$

$$\left|\mathcal{M}_{q\bar{q}\to gg}\right|^2 = \left|\mathcal{M}^{LO}_{q\bar{q}\to gg}\right|^2 + \left(\frac{\alpha_s}{2\pi}\right)\left|\mathcal{M}^{NLO}_{q\bar{q}\to gg}\right|^2 + \left(\frac{\alpha_s}{2\pi}\right)^2 \left|\mathcal{M}^{NNLO}_{q\bar{q}\to gg}\right|^2 + \dots$$

Two-loop amplitudes



Double Virtual

Real Virtual

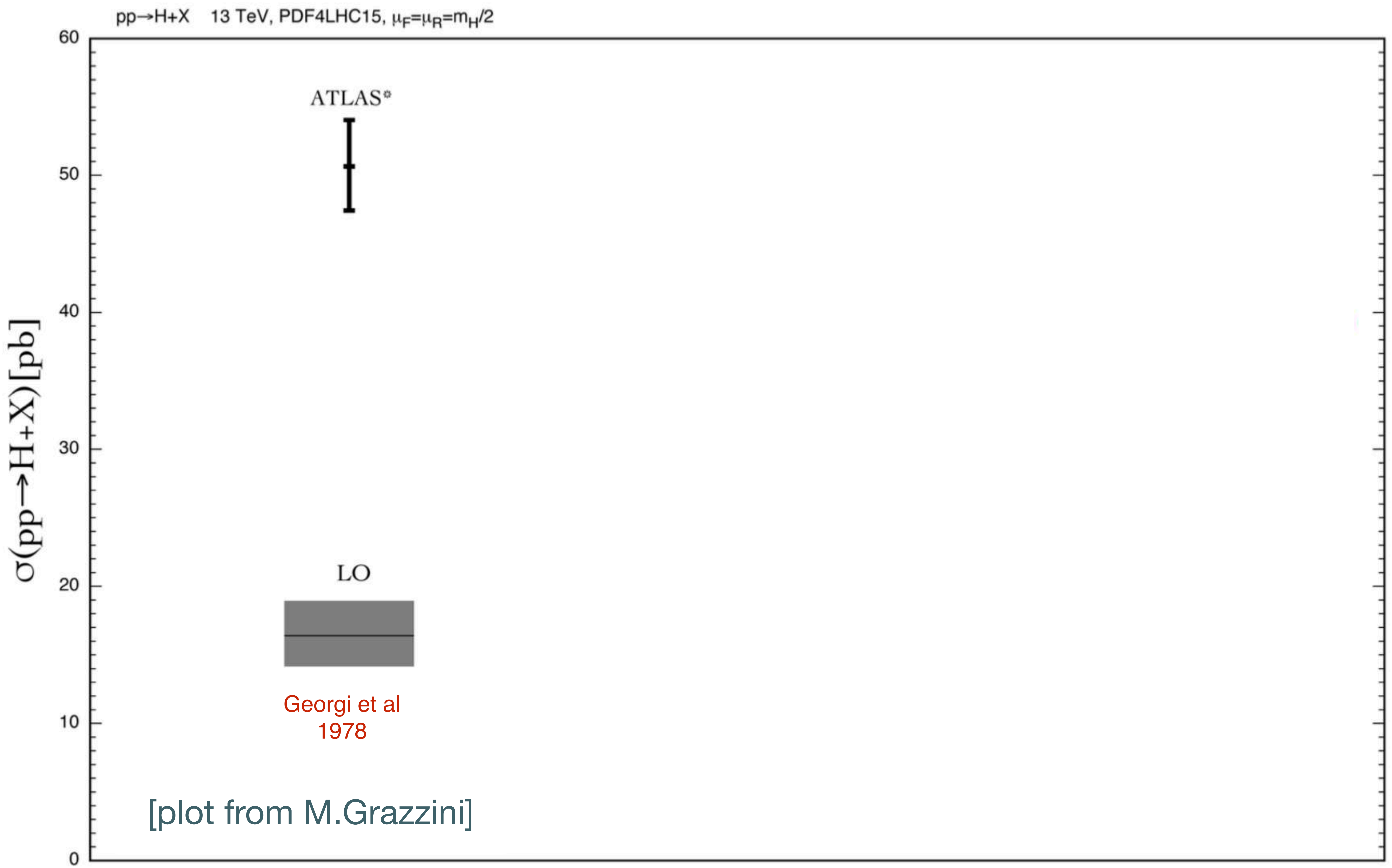
Double Real

~ O(5%) precision

Often not even enough!

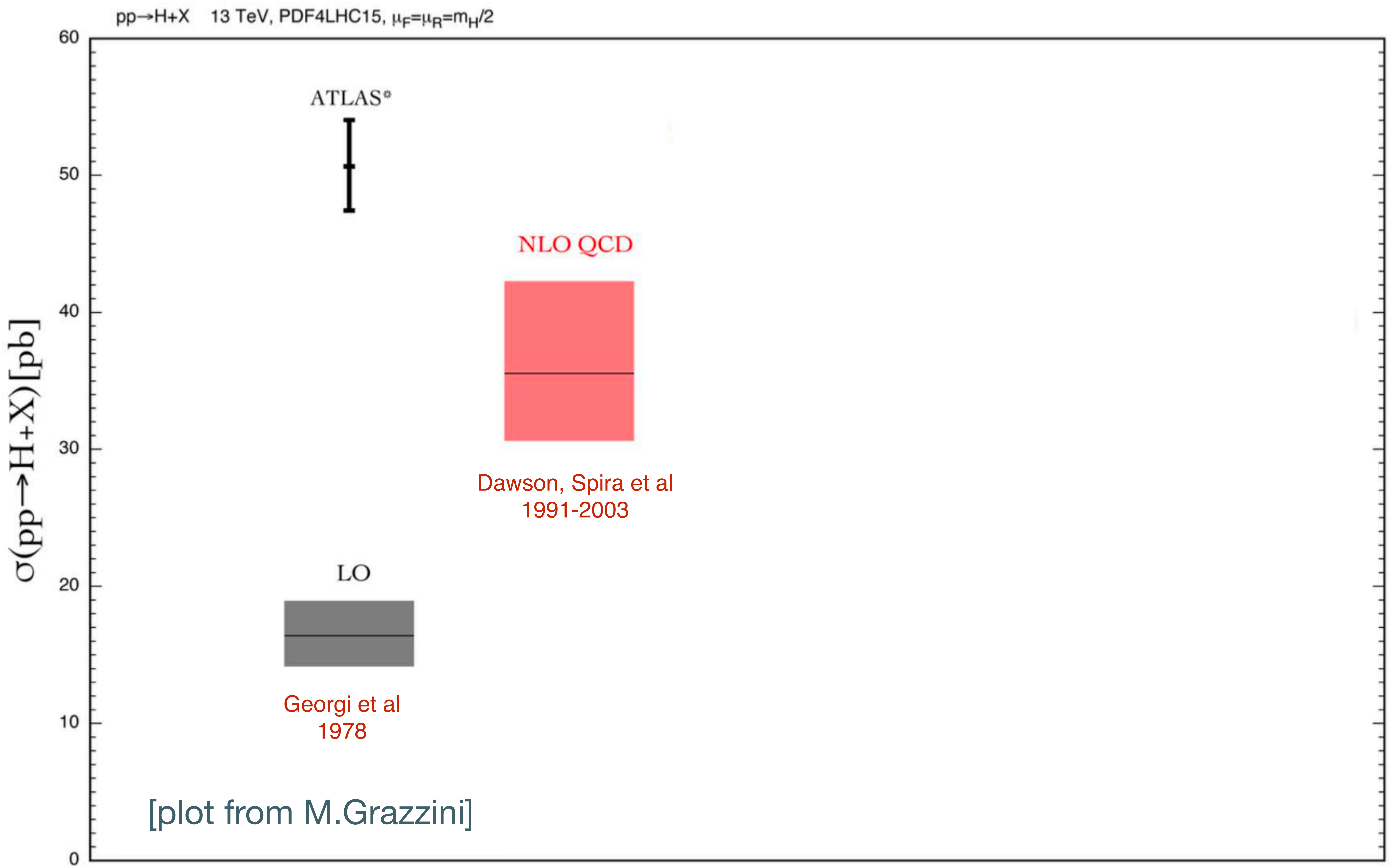
See talk by C. Signorile-Signorile

THE NEED OF PRECISION: TOWARDS THE % LEVEL

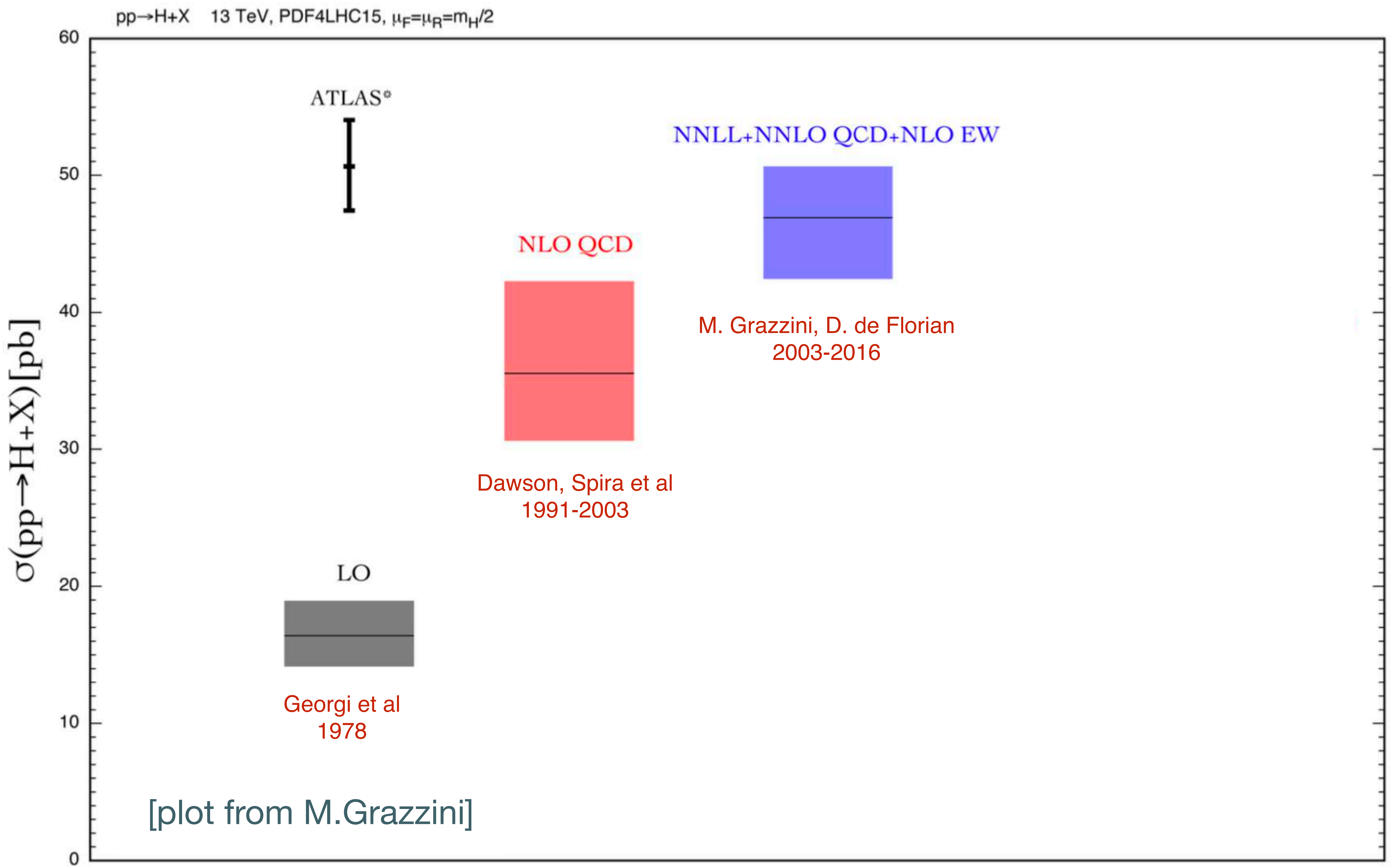


[plot from M.Grazzini]

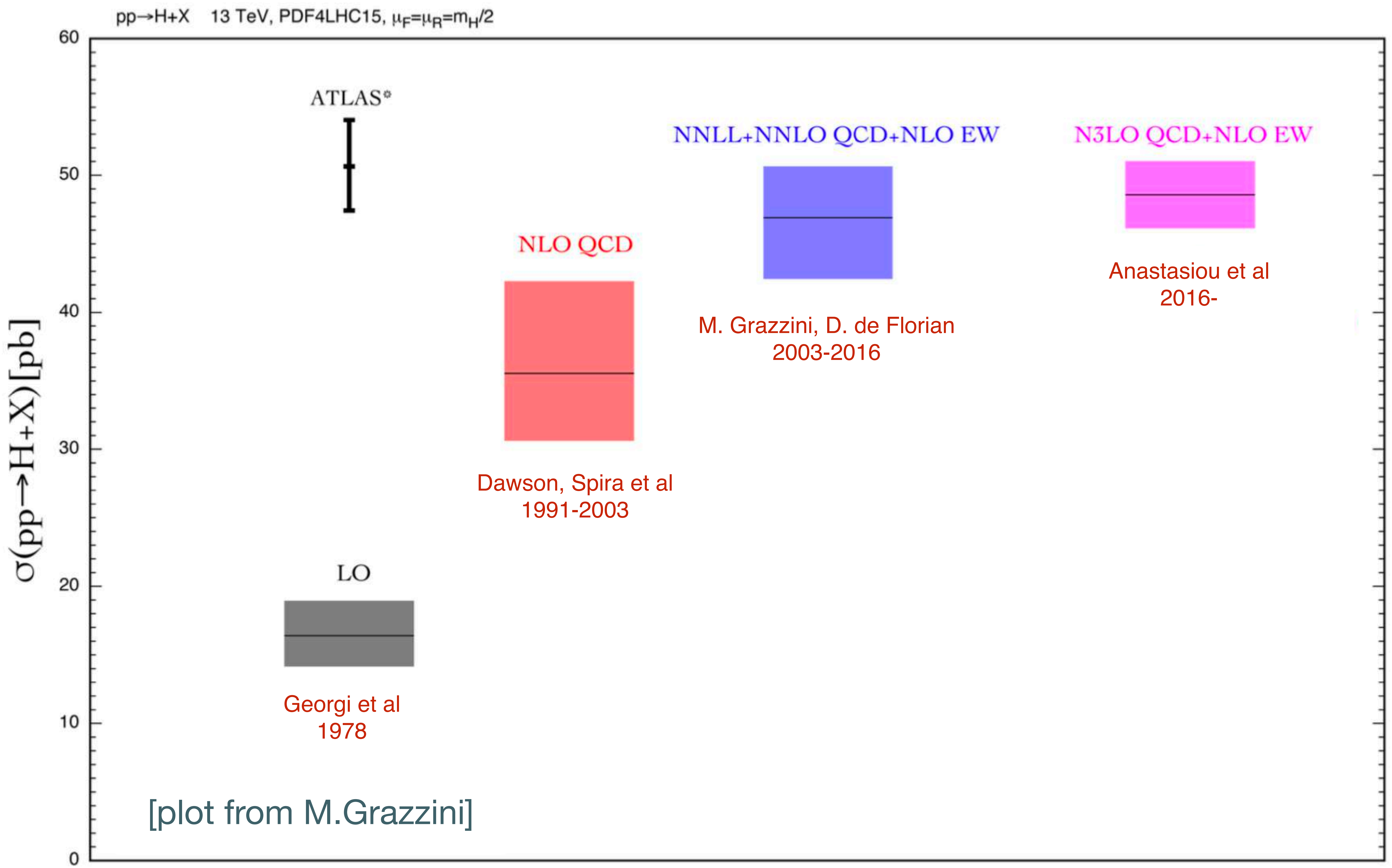
THE NEED OF PRECISION: TOWARDS THE % LEVEL



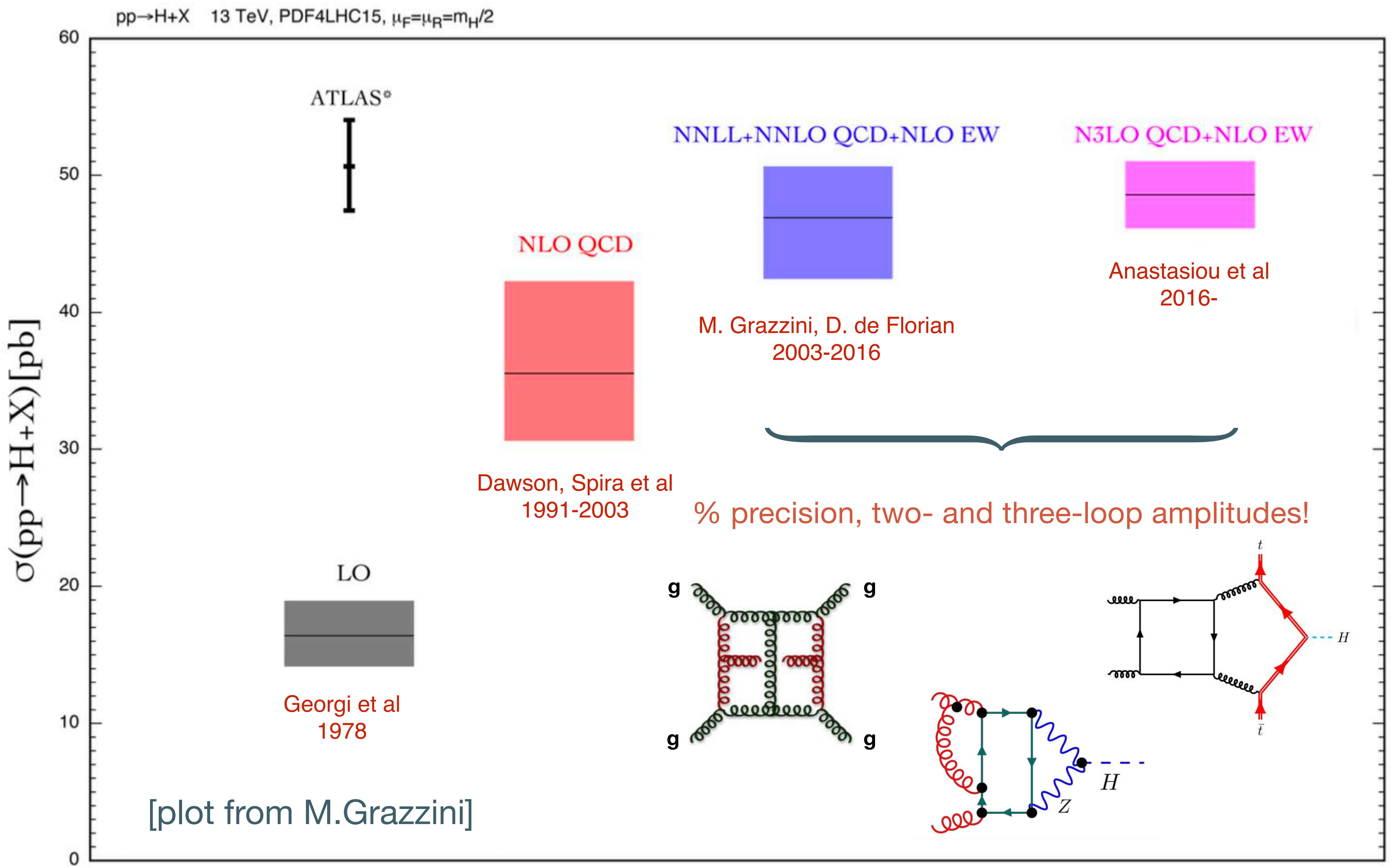
THE NEED OF PRECISION: TOWARDS THE % LEVEL



THE NEED OF PRECISION: TOWARDS THE % LEVEL



THE NEED OF PRECISION: TOWARDS THE % LEVEL

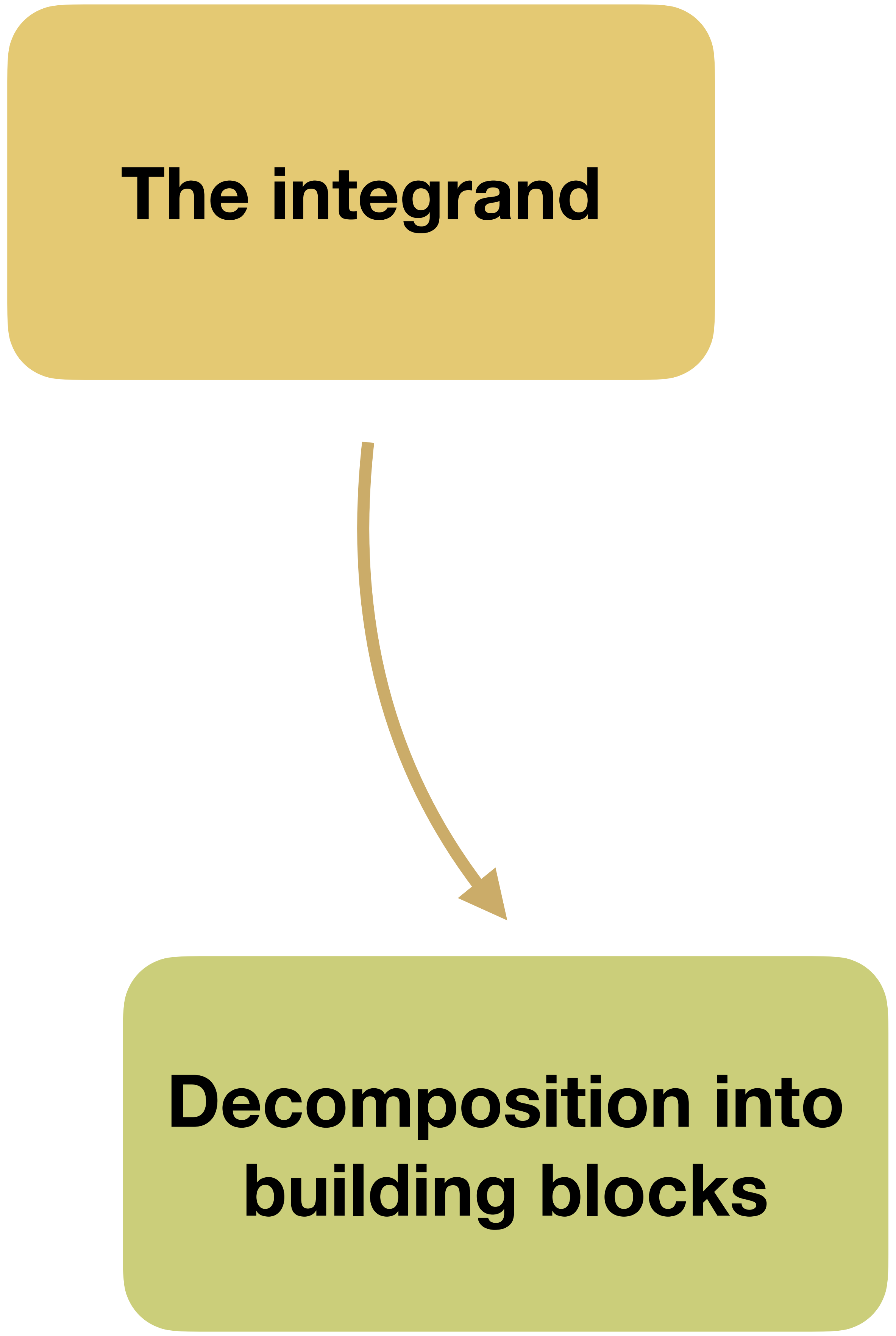


AMPLITUDES FOR COLLIDERS: HOW DO WE GO ABOUT THEM?

The integrand

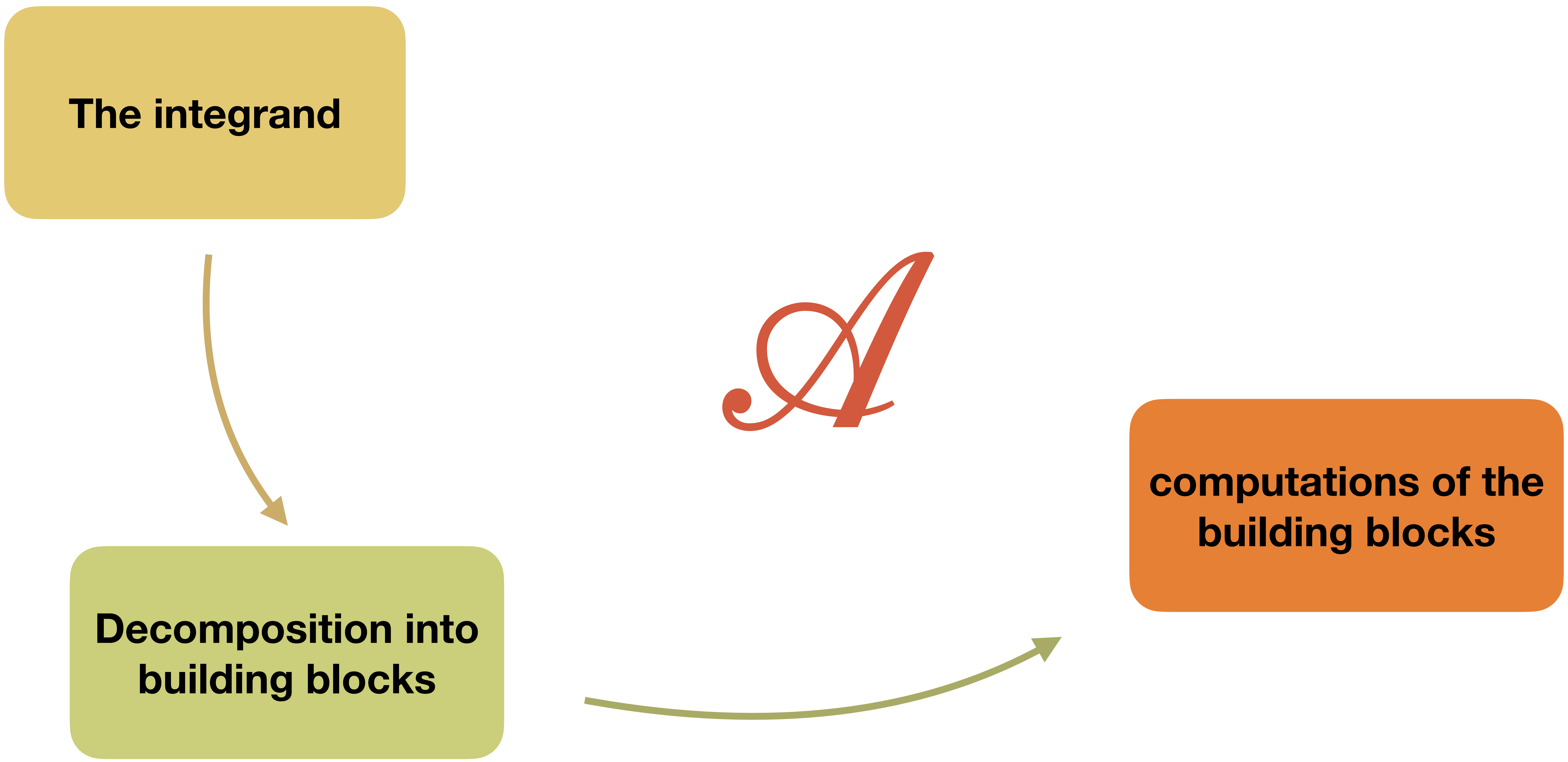
$$\mathcal{A}$$

AMPLITUDES FOR COLLIDERS: HOW DO WE GO ABOUT THEM?

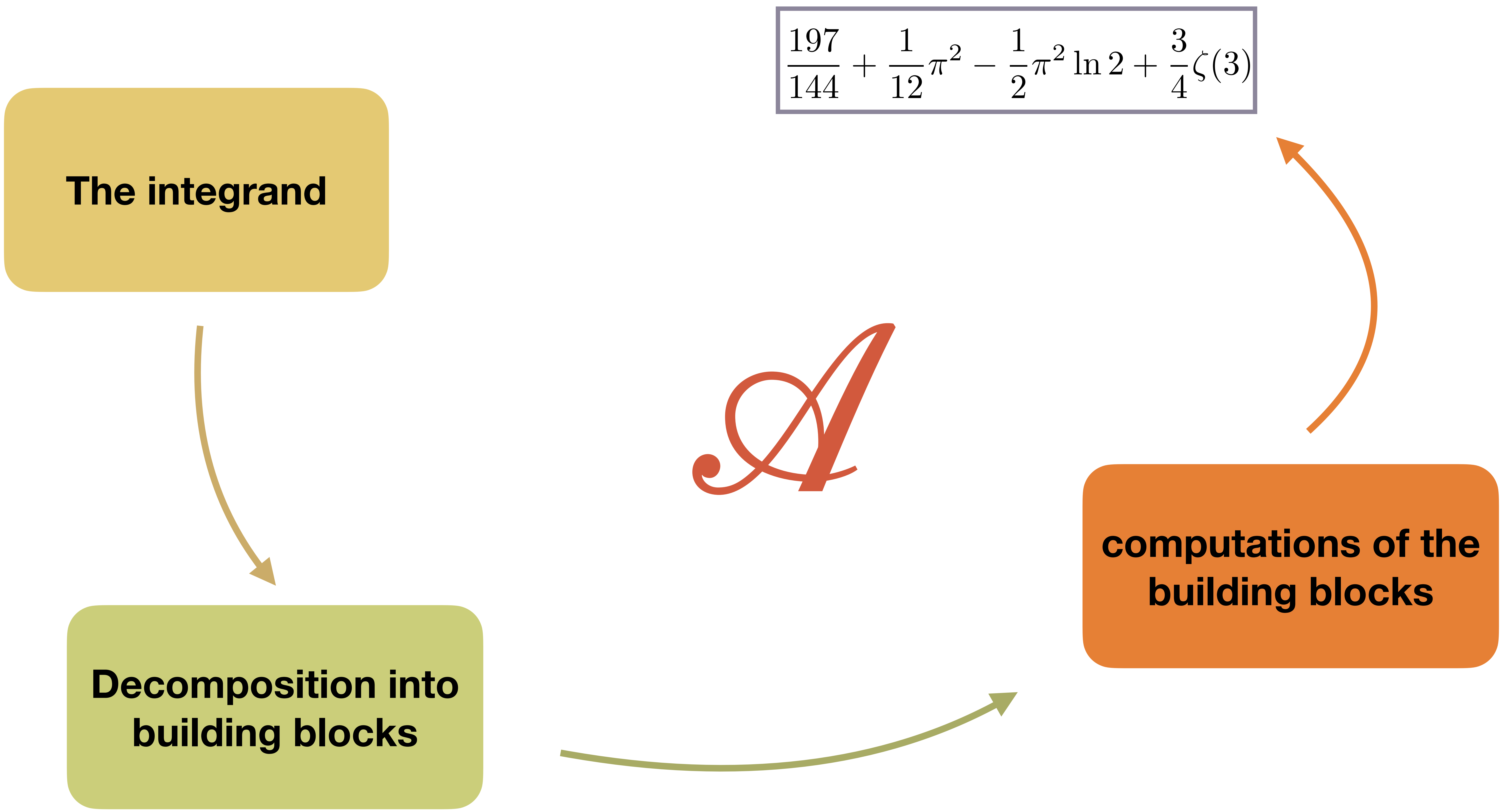


$$\mathcal{A}$$

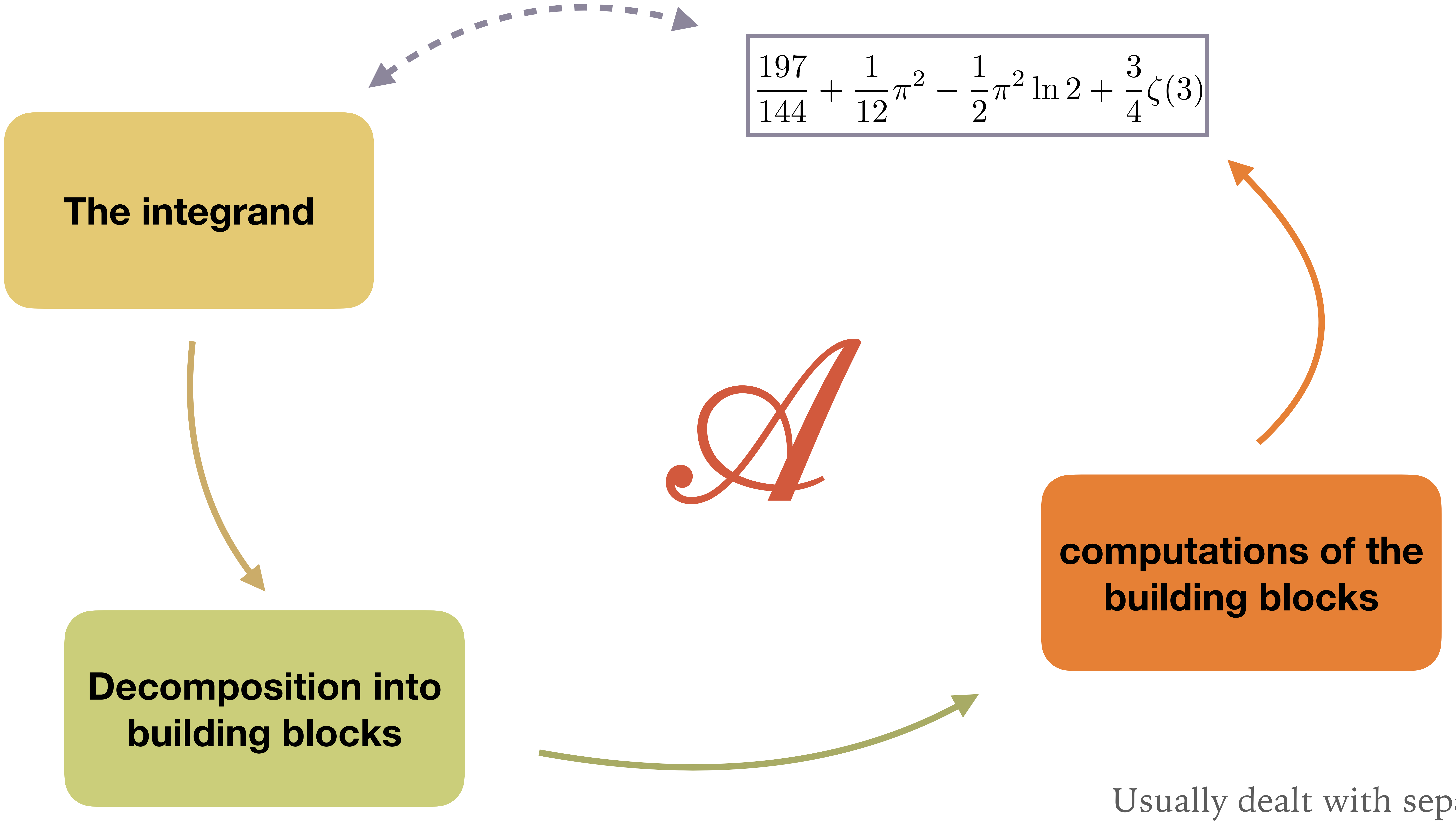
AMPLITUDES FOR COLLIDERS: HOW DO WE GO ABOUT THEM?



AMPLITUDES FOR COLLIDERS: HOW DO WE GO ABOUT THEM?



AMPLITUDES FOR COLLIDERS: HOW DO WE GO ABOUT THEM?



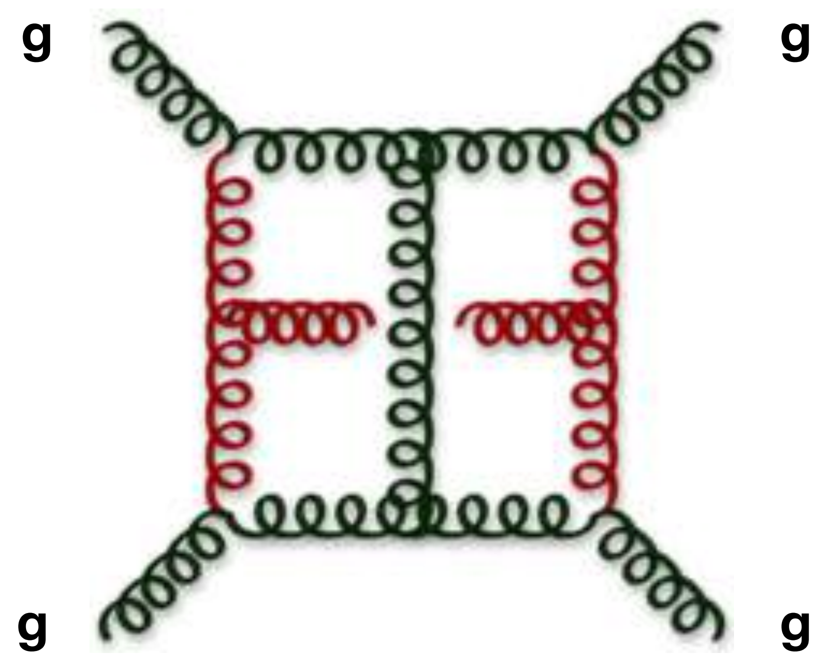
Usually dealt with separately

Connections among them, partly still to explore

ON THE INTEGRAND

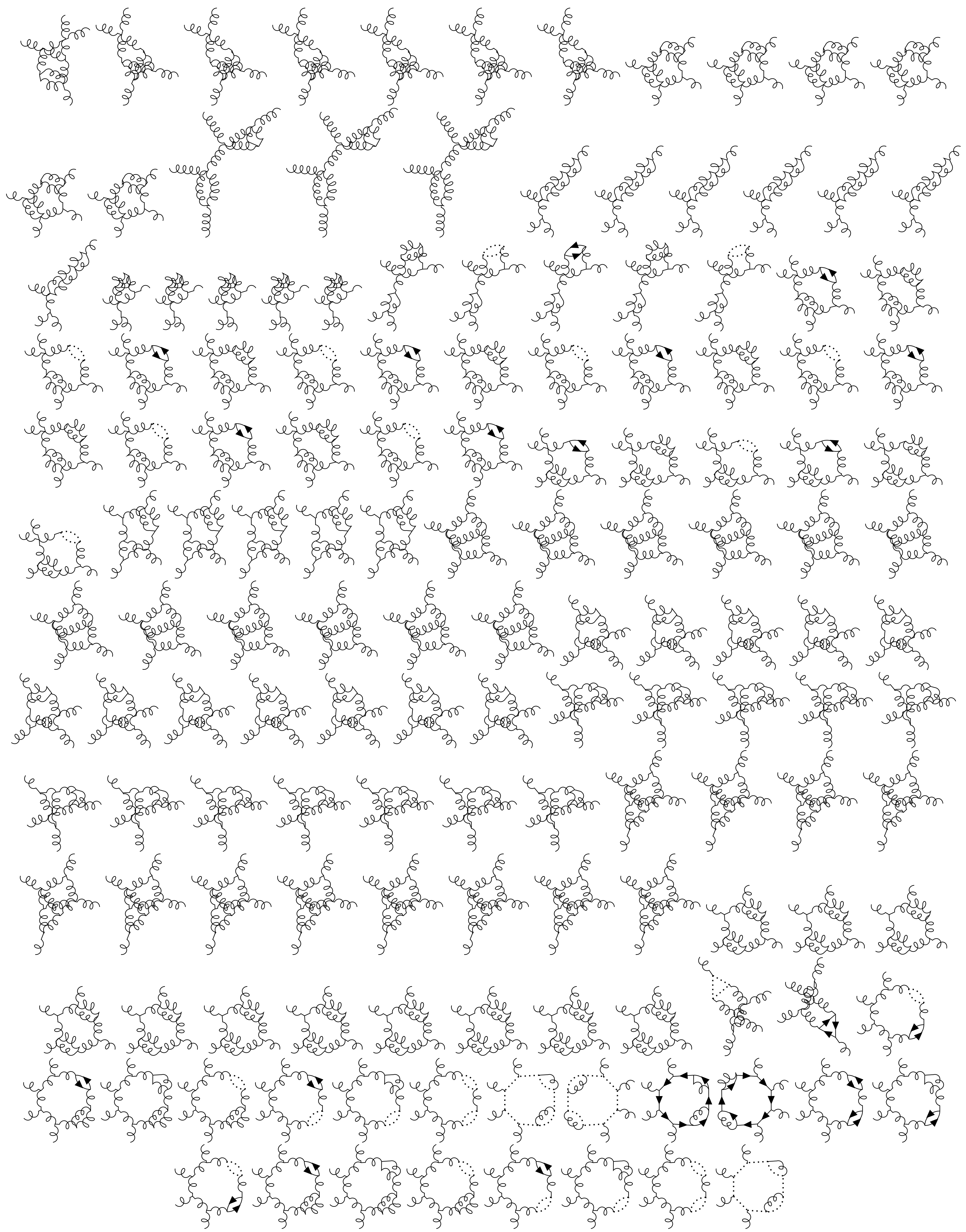
ON THE INTEGRAND: WHO IS AFRAID OF FEYNMAN DIAGRAMS?

$\mathcal{A} \longrightarrow$



“just a sum of Feynman diagrams”

ON THE INTEGRAND: WHO IS AFRAID OF FEYNMAN DIAGRAMS?



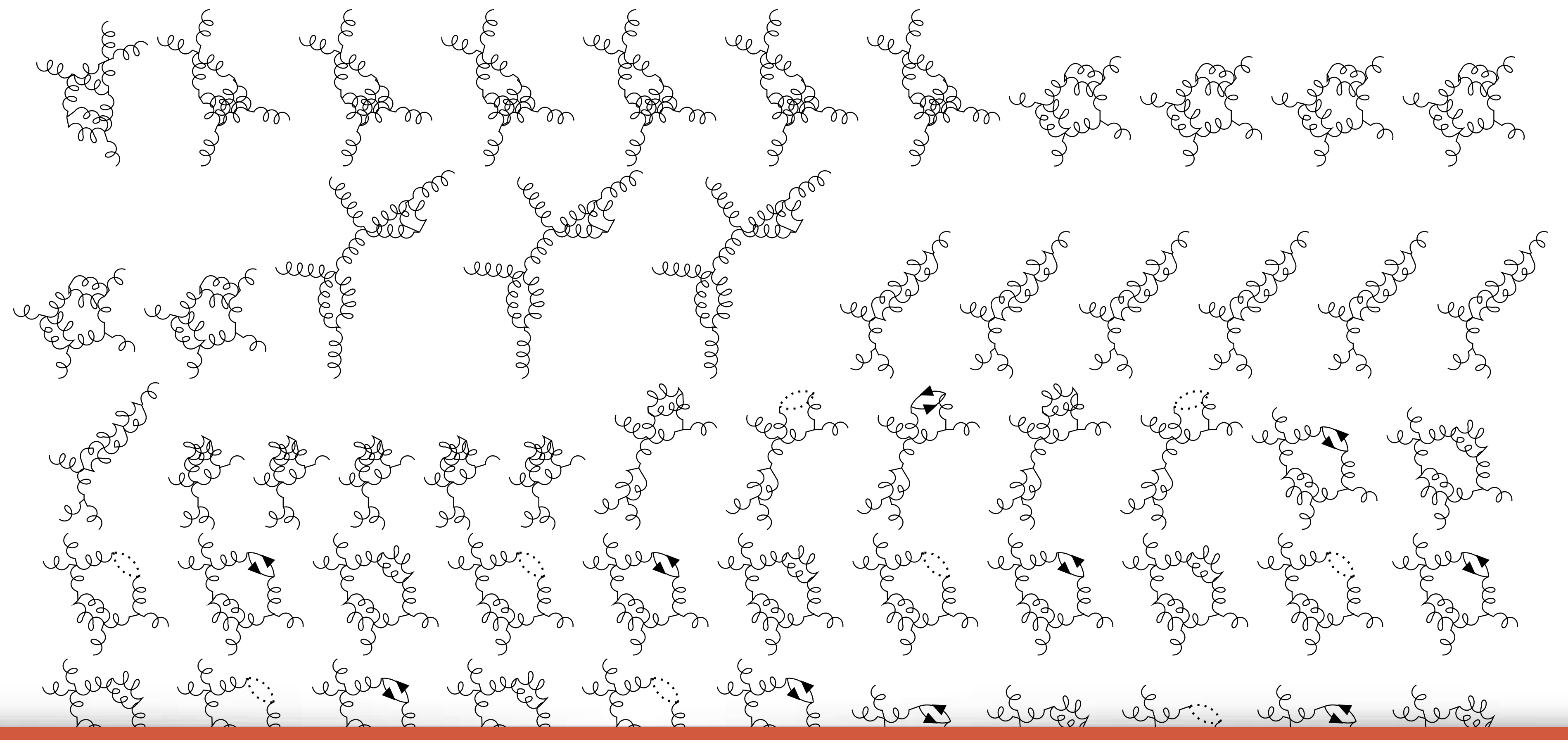
+ 500 more pages

(50000 Feynman diagrams)

Is this what scares us?

ON THE INTEGRAND: WHO IS AFRAID OF FEYNMAN DIAGRAMS?

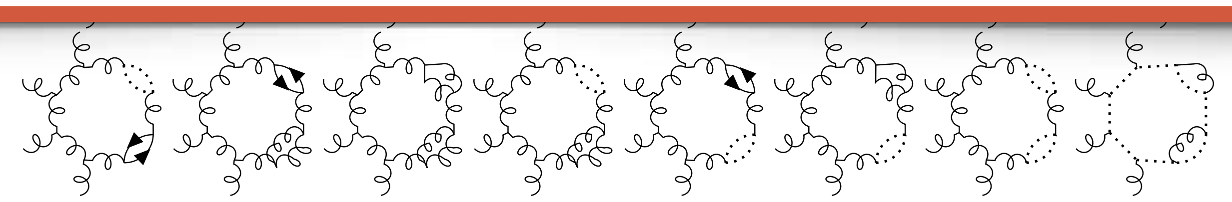
$\mathcal{A} \longrightarrow$



Computers and clever programming today can handle **hundreds of thousands of Feynman diagrams**

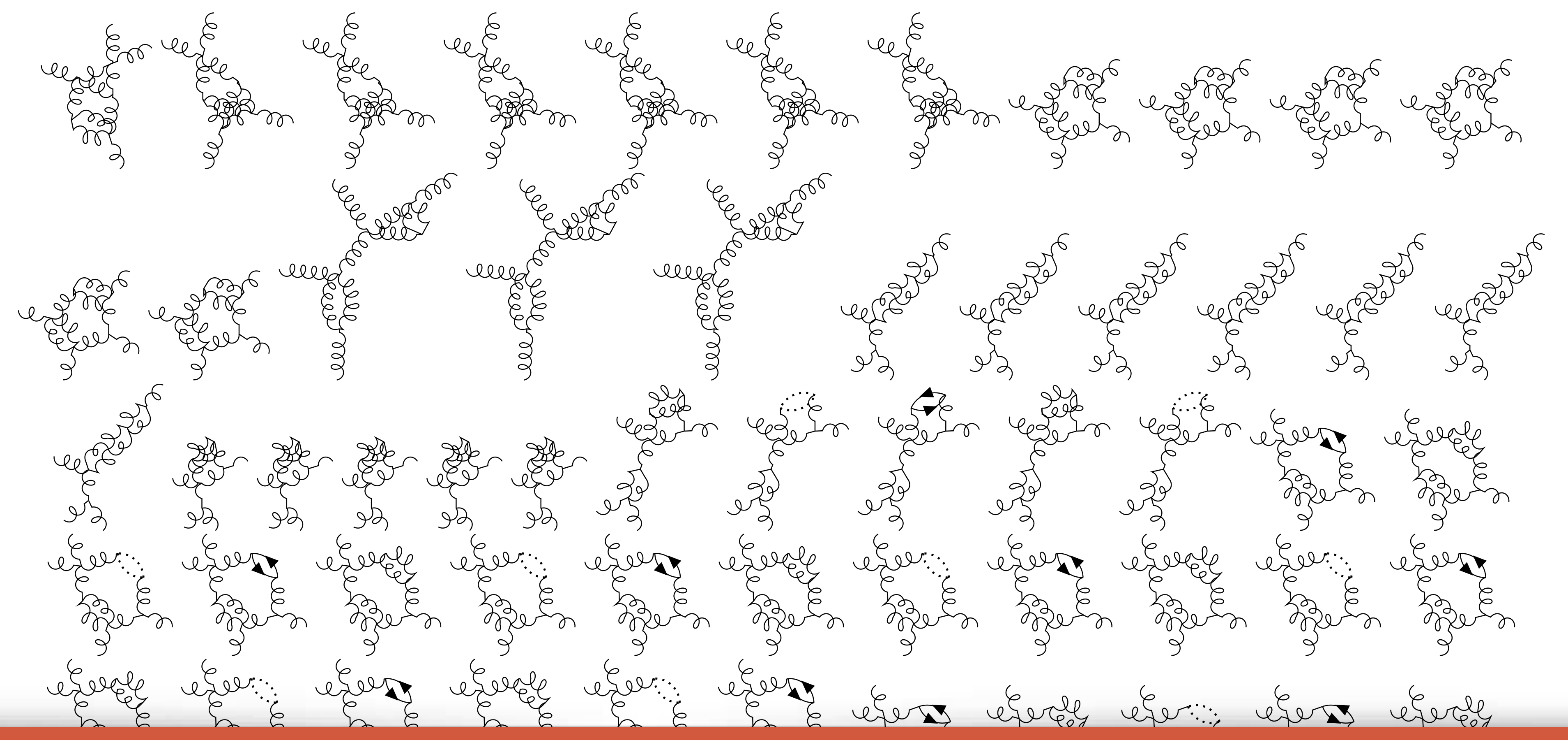
The real issue: **hidden simplicity**
Gauge symmetry, analytic structure (poles and branch cuts) etc are hidden in this decompositions

Starting from generic Feynman diagrams, things might look **much worse than what they really are**…



ON THE INTEGRAND: WHO IS AFRAID OF FEYNMAN DIAGRAMS?

$\mathscr{A} \longrightarrow$

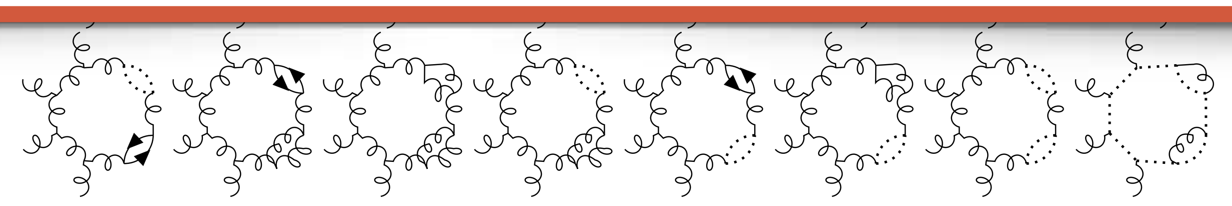


Where do we stand?

Tree-level: (On-shell) recursions: BCFW

One-loop: Unitarity (extended to higher loops in specific cases, no triangles/no bubbles etc)

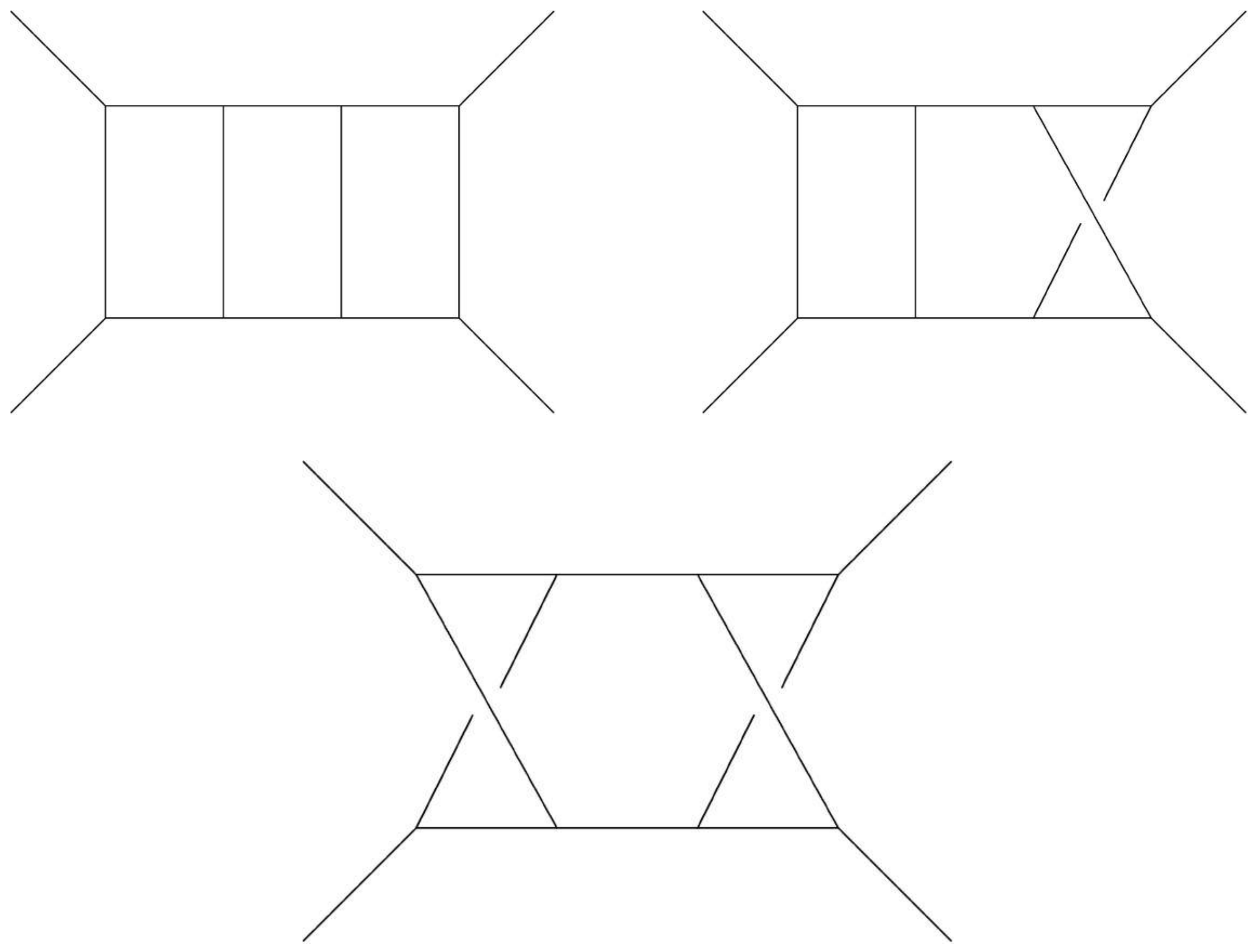
Higher loops: ??? **[…Arkani-Hamed, Frost, Salvatori, Plamondon, Thomas ’23 …]**



ON THE DECOMPOSITION

ON THE DECOMPOSITION

$$\mathcal{A} \longrightarrow \sum_i$$



If we can decompose the amplitude into a **(minimal) set of building blocks**, **problem solved***!

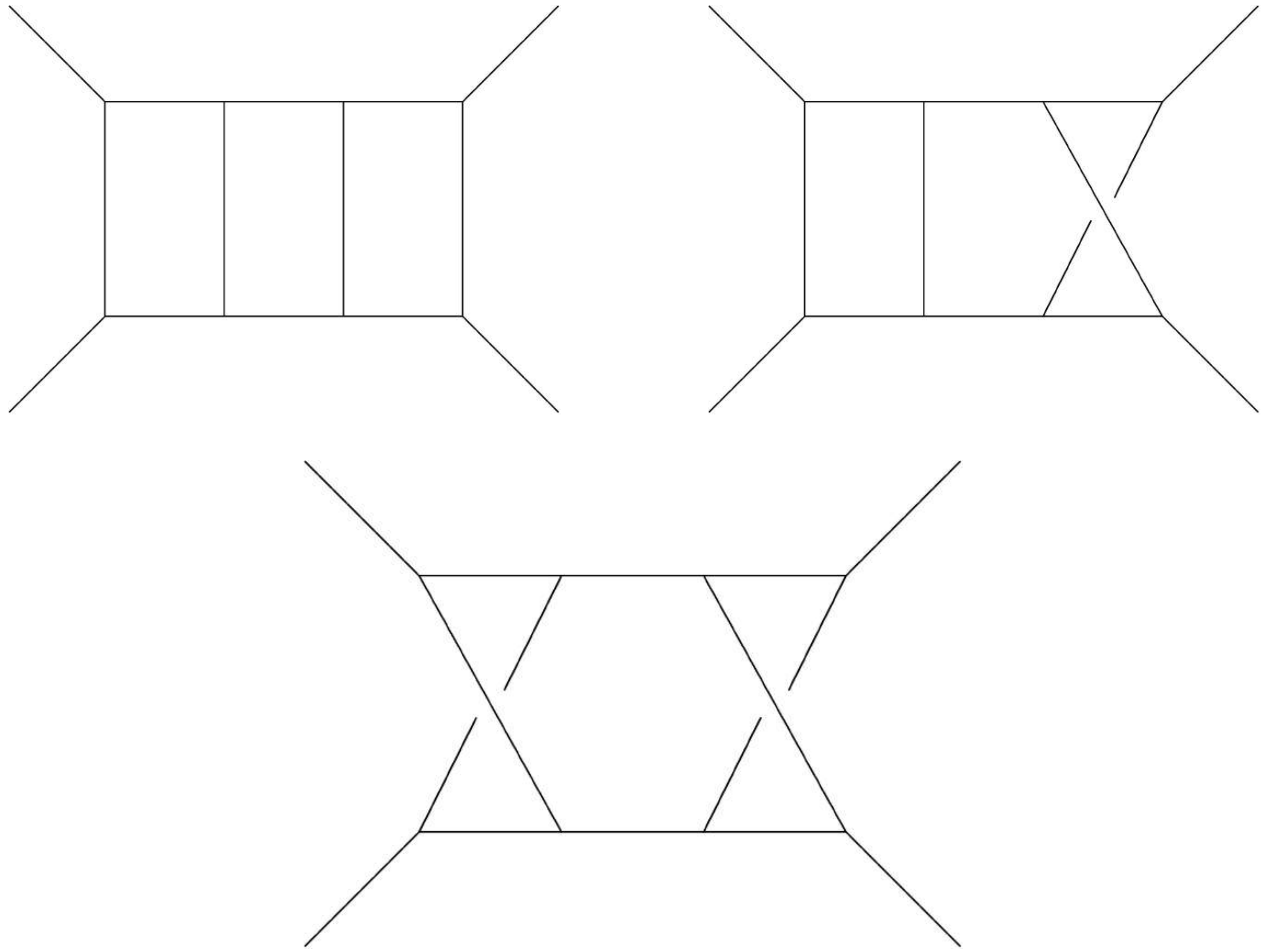
(all cancelations, structures, symmetries should become manifest…)

In practice, this can be achieved starting from any representation, but difficulty depends strongly on where we start

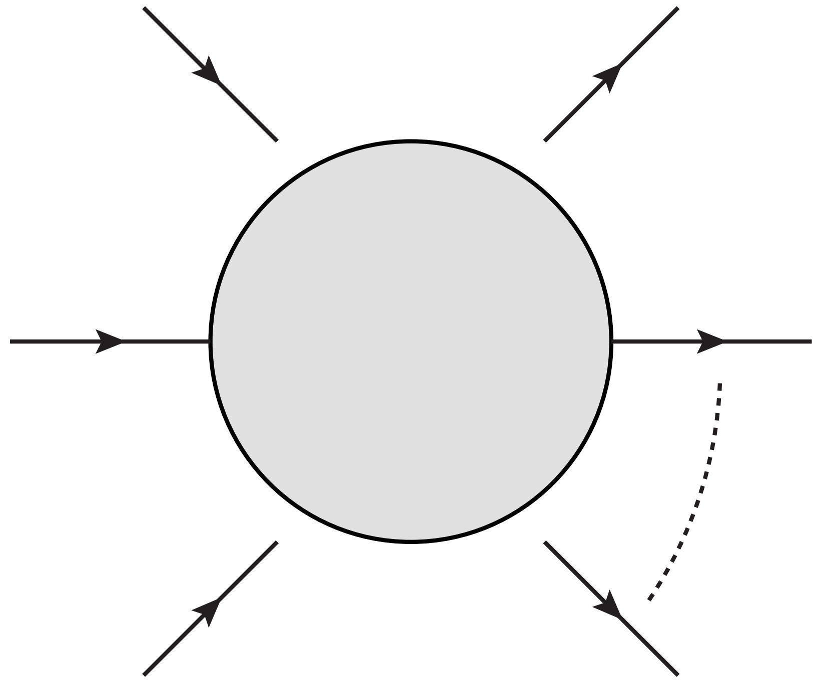
* more on this in a moment…

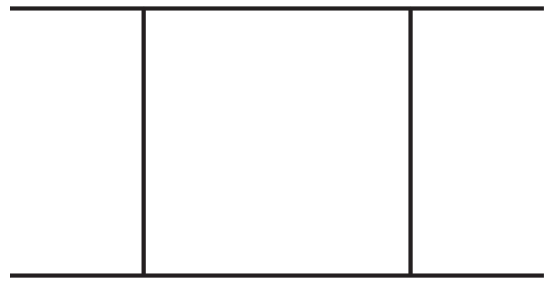
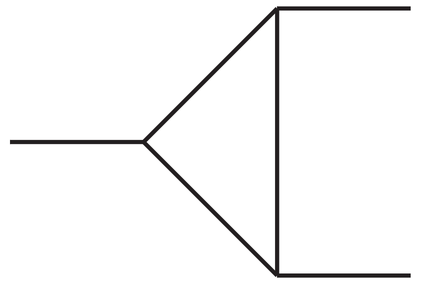
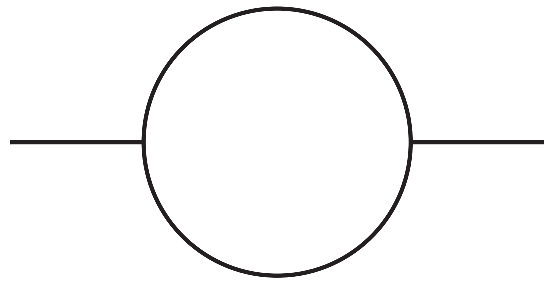
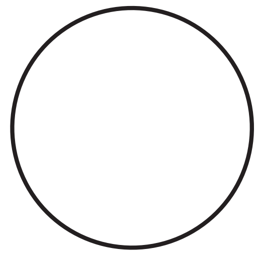
ON THE DECOMPOSITION: ONE LOOP AND NLO REVOLUTION

$$\mathcal{A} \longrightarrow \sum_i$$



One of the main reasons of the so-called **NLO revolution**



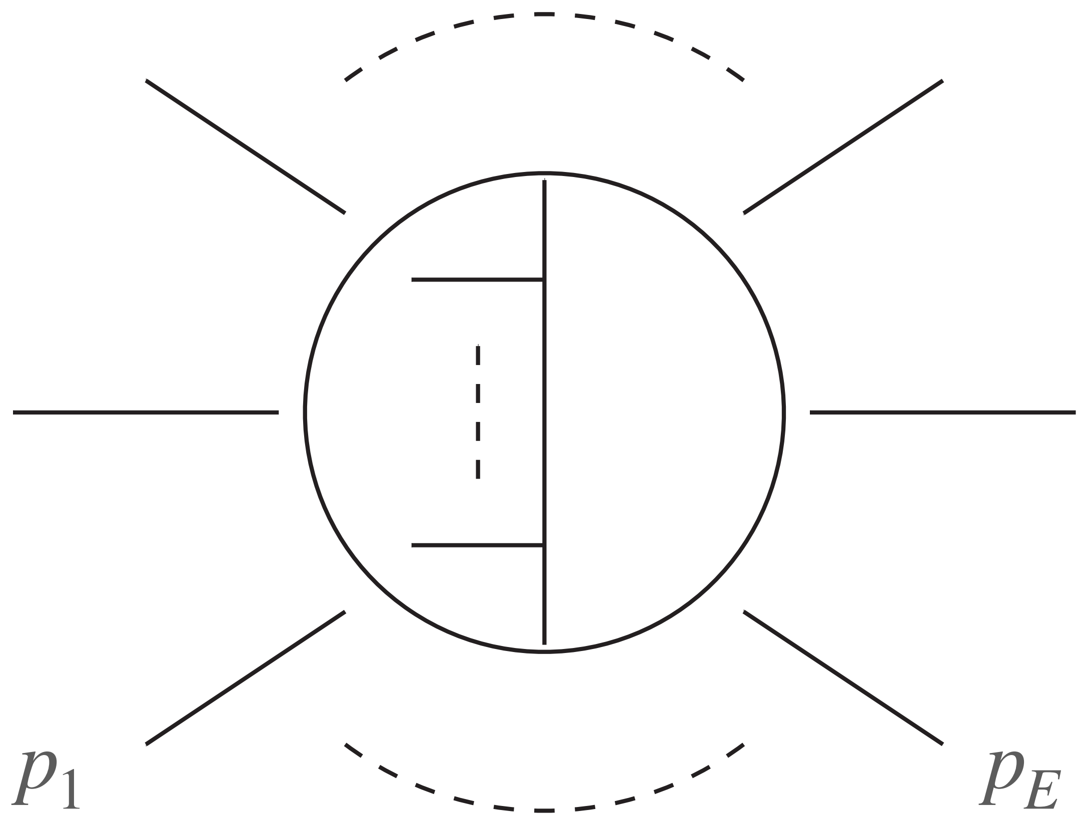
$= \sum_i C_i^4$  $+ \sum_i C_i^3$  $+ \sum_i C_i^2$  $+ \sum_i C_i^1$  $+ \mathcal{R}$

[Extremely efficient techniques to get the C_i → very efficient ones **ALSO based on Feynman diagrams**]

Blackhat, MadLoops, Openloops, Recola, GoSam, Ninja,...

ON THE DECOMPOSITION: THE STANDARD WAY AT ℓ LOOPS

$\mathscr{A}$ →



$$= \epsilon_1^{\mu_1} \cdots \epsilon_n^{\mu_n} \, \bar{v}(q) \, \Gamma_{\mu_1,\ldots,\mu_n} \, u(p)$$

$$\mathscr{I} = \int \prod_{l=1}^{L} \frac{d^D k_l}{(2\pi)^D} \frac{S_1^{b_1} \ldots S_m^{b_m}}{D_1^{a_1} \ldots D_n^{a_n}}$$

scalar Feynman integrals

$D_i = q_i^2 - m_i^2$

$S_i = \{\ell_j \cdot \ell_k , \ell_j \cdot p_k\}$

$$0 = \int \prod_{l=1}^{L} \frac{d^D k_l}{(2\pi)^D} \frac{\partial}{\partial \ell_k^\mu} \left[v^\mu \frac{S_1^{b_1} \ldots S_m^{b_m}}{D_1^{a_1} \ldots D_n^{a_n}} \right]$$

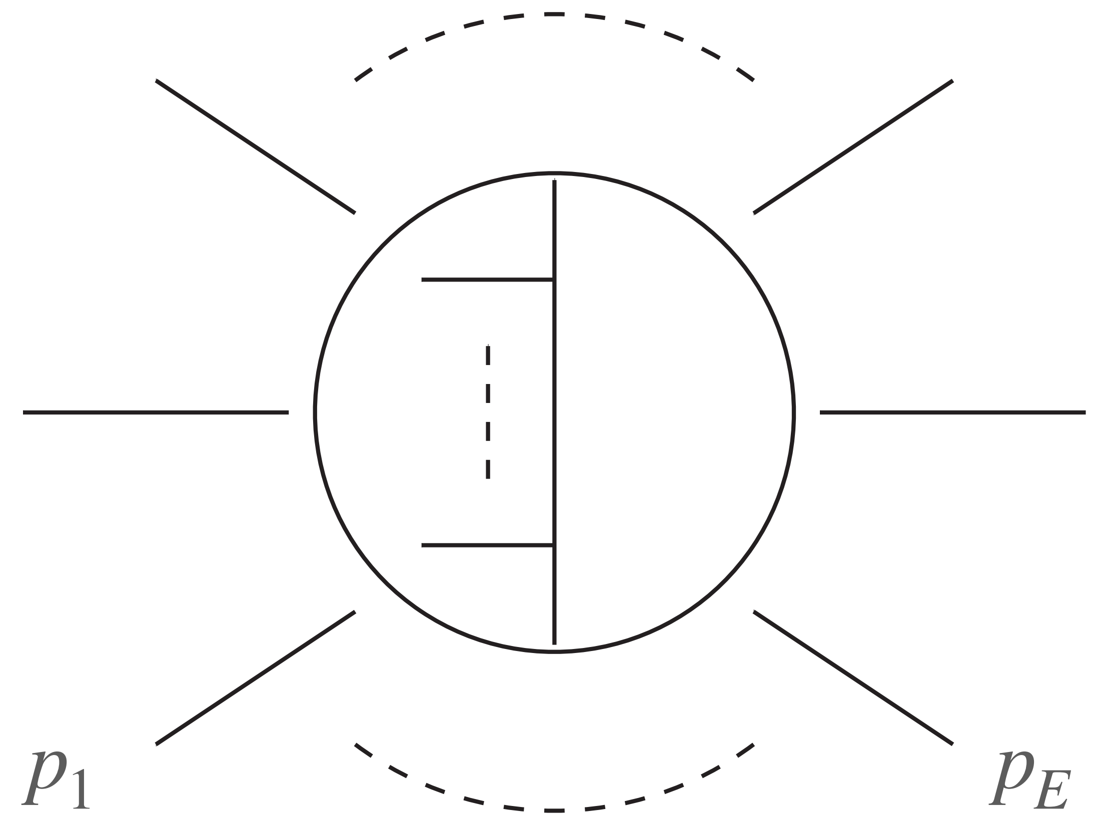
Integration by parts identities (IBPs)

(+ Symmetries, Lorentz ids and all that)

[Chetyrkin, Tkachov; Laporta; …]

ON THE DECOMPOSITION: IBPS AND MASTER INTEGRALS

$$\mathscr{A} \longrightarrow$$



IBPs $\longrightarrow$

Master Integrals (MIs)

$$\mathscr{I} = \int \prod_{l=1}^{L} \frac{d^D k_l}{(2\pi)^D} \frac{S_1^{b_1}\dots S_m^{b_m}}{D_1^{a_1}\dots D_n^{a_n}}$$

IBPs are extremely powerful, **both conceptually and practically!**

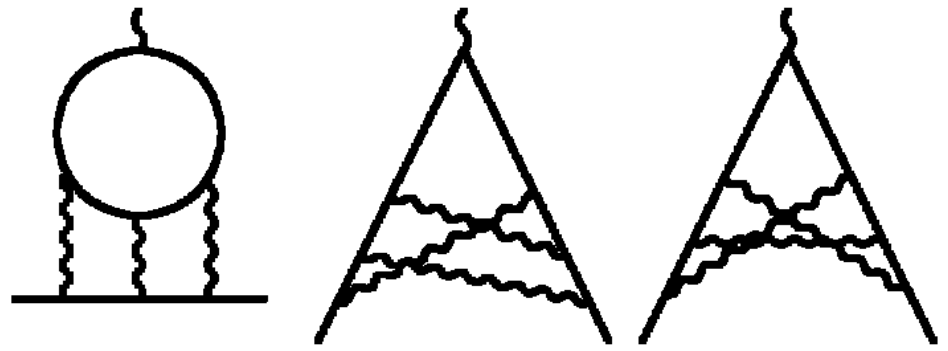
- One can prove that **MIs are in finite number**
- MIs **provide a basis** in space of all Feynman integrals → structure of a **vector space**
- Turn the decomposition problem into a **linear algebra problem**
- As any basis in any vector space, **some bases are better than others**
- VERY powerful bi-product: **the differential-equation method**

ON THE DECOMPOSITION: IBPS AND MASTER INTEGRALS

The "Laporta method", first applied* in a systematic way in 1997 to reduce **3loop g-2 to 17 MIs**

$$a_e^{QED} = C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + C_4\left(\frac{\alpha}{\pi}\right)^4 + C_5\left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

$C_3 =$



$$= \frac{83}{72}\pi^2\zeta(3) - \frac{215}{24}\zeta(5) + \frac{100}{3}\left[\left(\mathrm{Li}_4\left(\frac{1}{2}\right) + \frac{\ln^4 2}{24}\right) - \frac{\pi^2\ln^2 2}{24}\right]$$
$$- \frac{239}{2160}\pi^4 + \frac{139}{18}\zeta(3) - \frac{298}{9}\pi^2\ln 2 + \frac{17101}{810}\pi^2 + \frac{28259}{5184}$$

[Laporta, Remiddi '97]

* as far as I know…

ON THE DECOMPOSITION: IBPS AND MASTER INTEGRALS

The “Laporta method”, first applied* in a systematic way in 1997 to reduce **3loop g-2 to 17 MIs**

Since then, things have changed a lot!

Complexity **increases factorially** with **# of legs** and **# of loops**

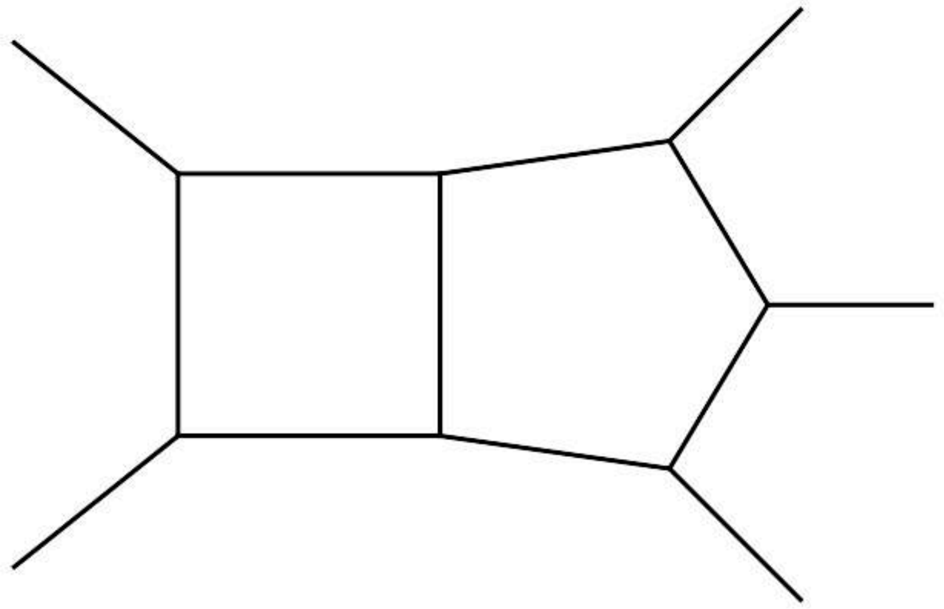
- **many scales** → huge rational functions to handle symbolically (typically TBs of RAM on large machines!)
- **many loops** → explosion in number of identities (typically $\geq 10^9$ for $2 \to 2$ at three loops, again TBs!)

* as far as I know…

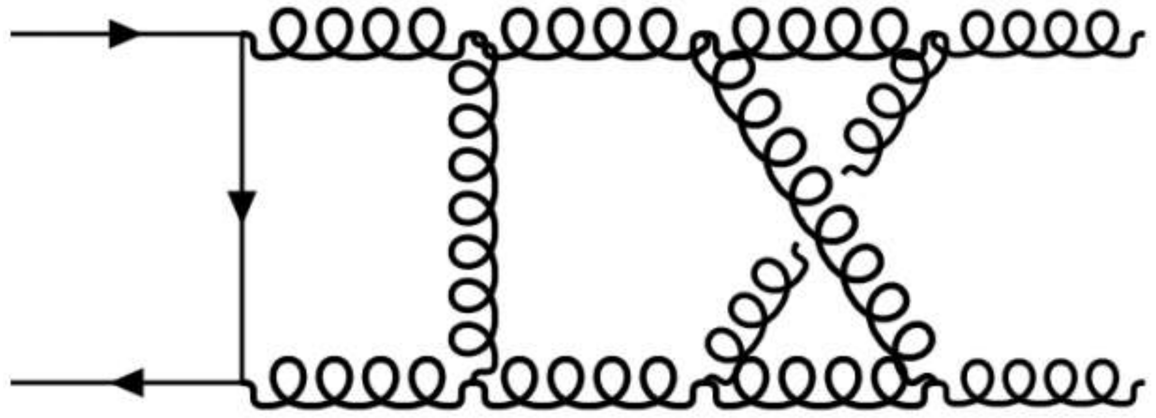
ON THE DECOMPOSITION: IBPS AND MASTER INTEGRALS

The "Laporta method", first applied* in a systematic way in 1997 to reduce **3loop g-2 to 17 MIs**

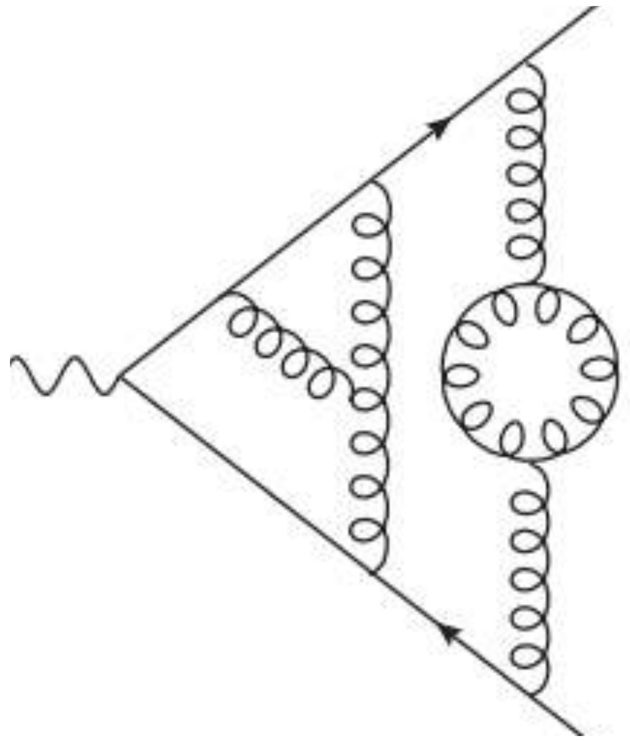
Since then, things have changed a lot! State-of-the-art: **2 loop 5 point - 3 loop 4 point - 4 loop 3 point**



Abreu, Agarwal, Badger, Buccioni, Chawdhry, Chicherin, Czakon, de Laurentis, Febres-Cordero, Gambuti, Gehrmann, Henn, Ita, Lo Presti, Manteuffel, Ma, Mitov, Page, Peraro, Pochelet, Schabinger, Sotnikov, Tancredi, Zhang, …

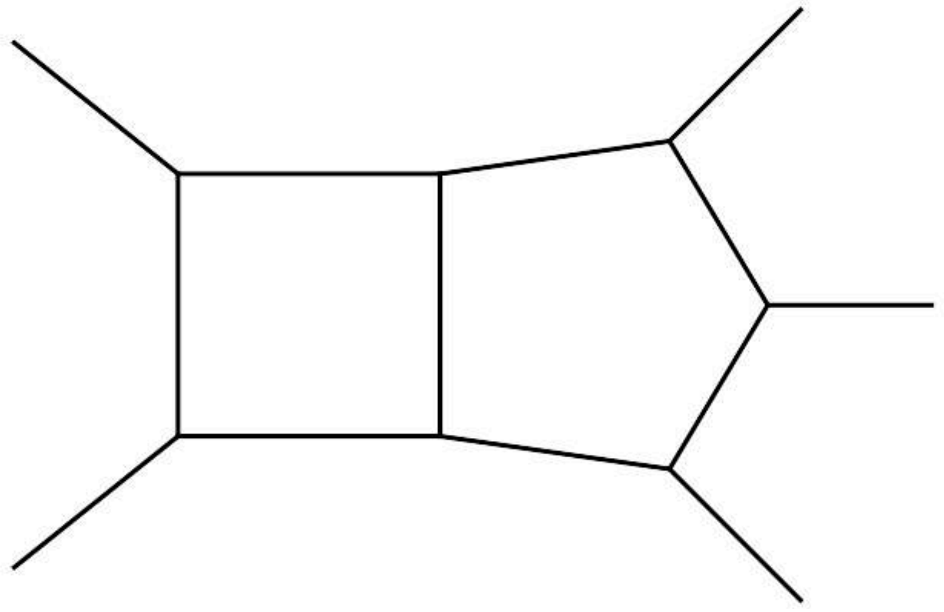


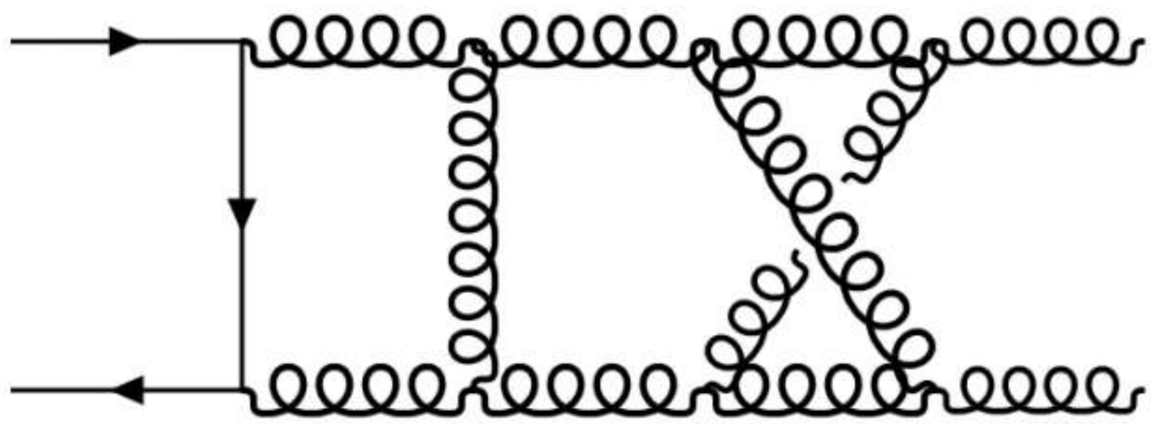
Bargiela, Bobadilla, Canko, Caola, Jakubcik, Gambuti, Gehrmann, Henn, Lim, Mella, Mistlberger, Wasser, Manteuffel, Syrrakos, Smirnov, Tancredi, …

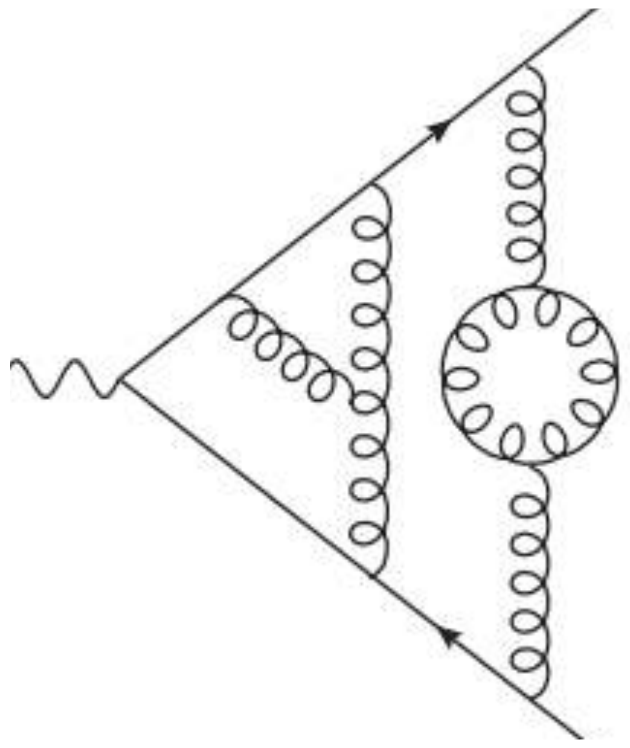


Henn, Lee, Manteuffel, Schabinger, Smirnov, Smirnov, Stainhauser,…

ON THE DECOMPOSITION: IBPS AND MASTER INTEGRALS







Finite-fields methods

Avoid intermediate expression swell

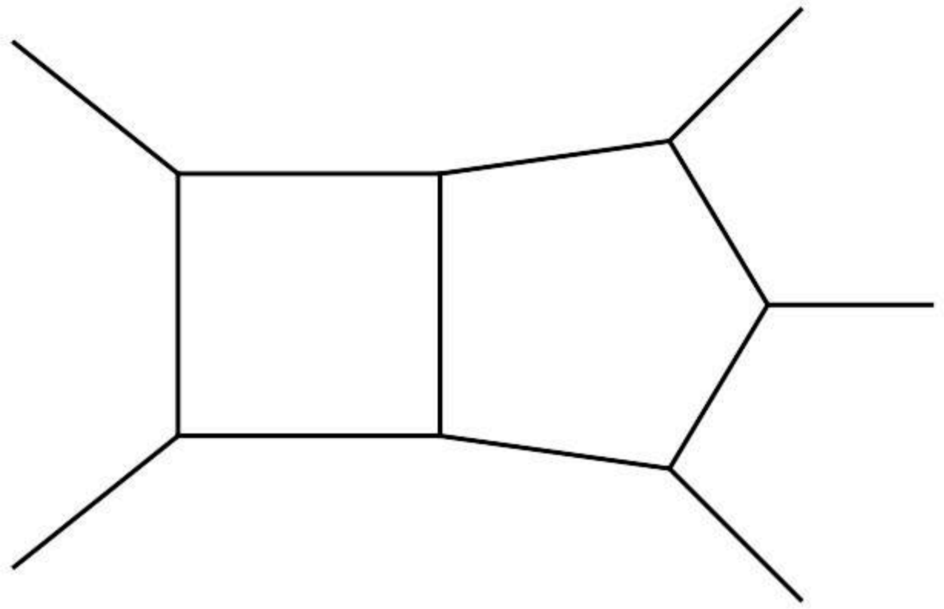
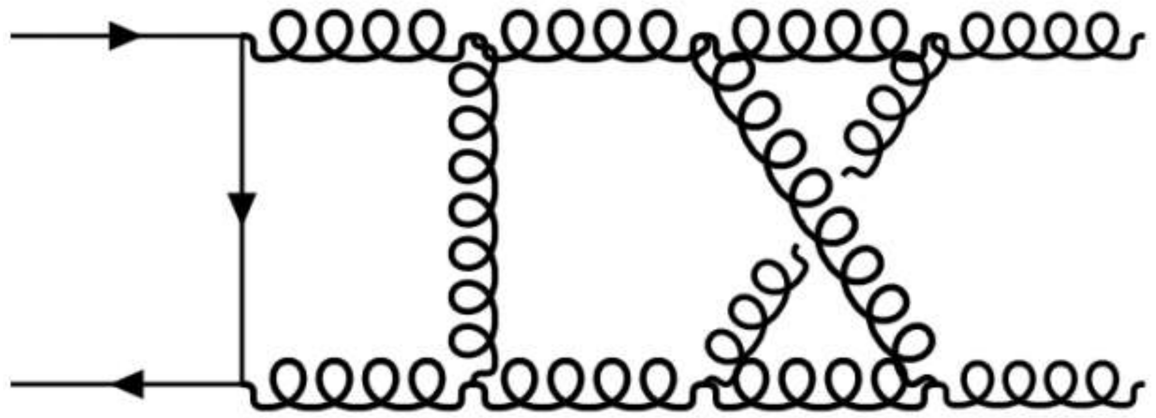
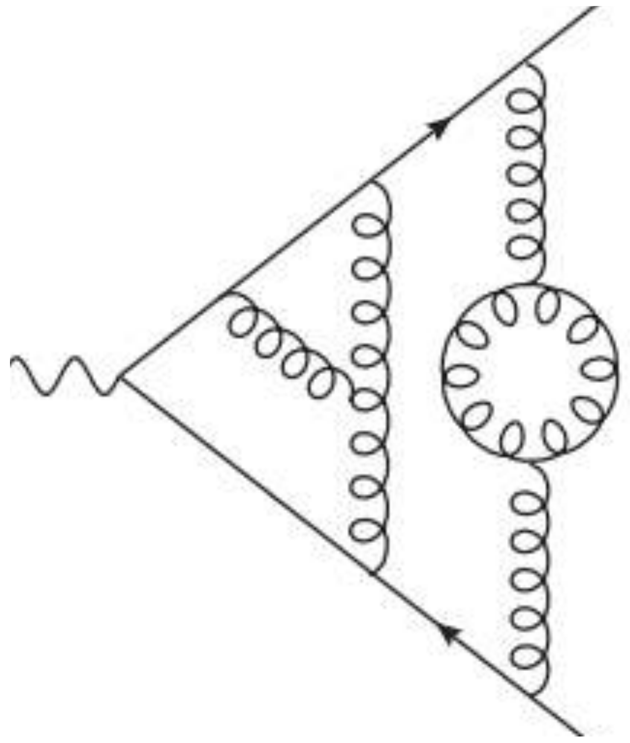
[von Manteufell, Schabinger, Peraro, Abreu, Page, Ita, Klappert, Lange,....]

Algebraic geometry methods

Reduce the number of IBPs generated

[Zhang, Bohem, Kosower, Peraro, Page, Abreu, Ita, von Manteuffel, Schabinger ...]

ON THE DECOMPOSITION: IBPS AND MASTER INTEGRALS

Finite-fields methods

Avoid intermediate expression swell

[von Manteufell, Schabinger, Peraro, Abreu, Page, Ita, Klappert, Lange,….]

Algebraic geometry methods

Reduce the number of IBPs generated

[Zhang, Bohem, Kosower, Peraro, Page, Abreu, Ita, von Manteuffel, Schabinger …]

intersection theory

$$\langle\varphi|\mathcal{C}] \; = \sum_{i,j,k,l=1}^{|\chi|} \langle\varphi|\varphi_j\rangle \, (\mathbf{C}^{-1})_{ji} \, \mathbf{P}_{il} \, (\mathbf{H}^{-1})_{lk} \, [\mathcal{C}_k|\mathcal{C}]$$

[Mizera, Mastrolia, Frellesvig, Brunello, Crisanti, Mattiazzi, Gasparotto, Smith, Chen, Feng, Yang, Xu, Pokraka, Caron-Huot, Giroux, Weinzierl, Fontana, Peraro…]

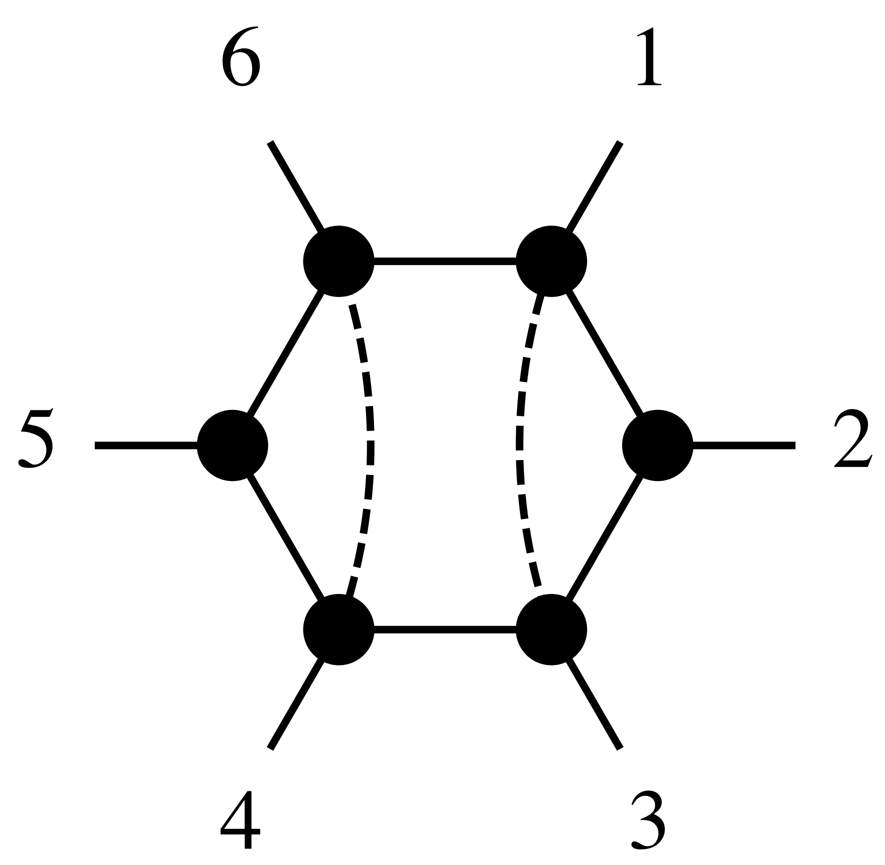
ON THE DECOMPOSITION: LEARNING FROM N=4 SYM

$\mathcal{A}$

Together with computational advances, the crucial question is, what we decompose onto?

Learning from N=4 important breakthroughs:

Local integrals ⟶ **Simplification happens in D=4**



encode (log) singularities of Amplitude locally
Pure and of Uniform Transcendental Weight

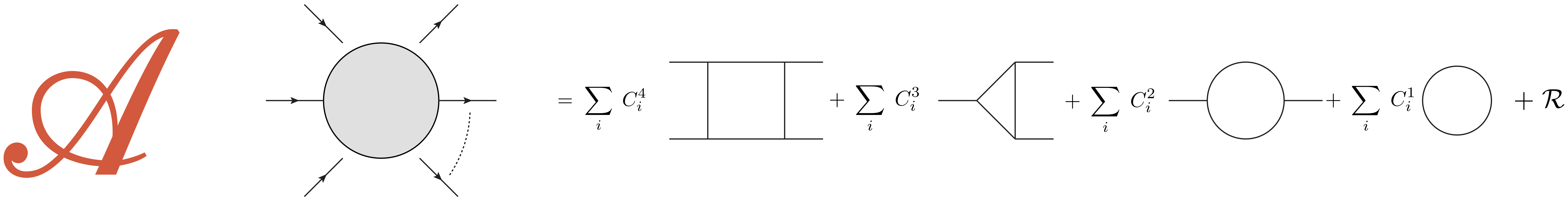
What about UV and IR divergences?

Dim Reg is good ? ↕ Dim Reg is evil ?

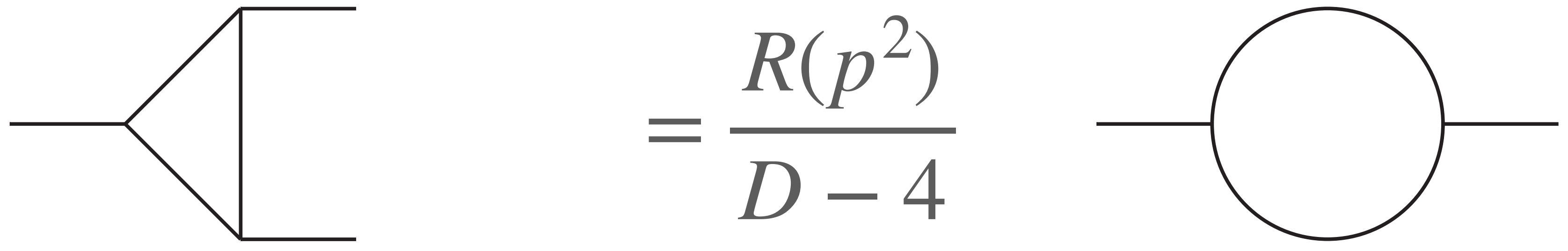
How do we keep dim reg, but also make simplicity in D=4 manifest?

How to define **finite remainders?**

ON THE DECOMPOSITION: OPEN ISSUES (& ONE-LOOP LESSONS)



One loop decomposition often non-minimal. Using IBPs (all massless):

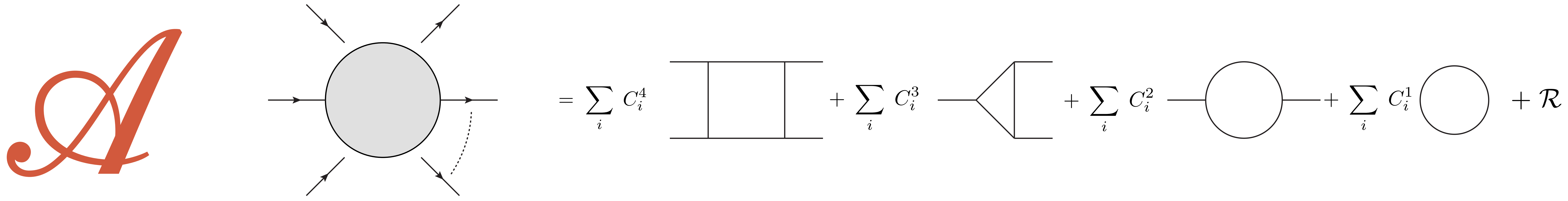


IR (soft+collinear) divergence $\sim \frac{1}{(D-4)^2}$

$\rightarrow \frac{1}{D-4} \times$ UV divergence

IBPs and dim-reg do "violence" on the Amplitude

ON THE DECOMPOSITION: OPEN ISSUES (& ONE-LOOP LESSONS)



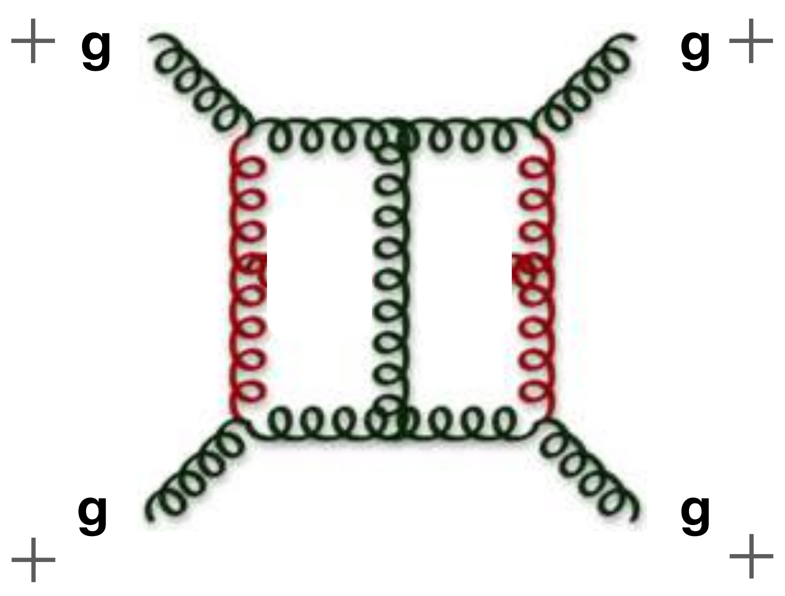
QCD is not N=4 SYM

QCD is not UT

QCD is not Pure

A good basis should encode **singularities of QCD Amplitudes**

Their richness indicates that we need to use an over-complete, non UT basis, which makes IR and UV divergences manifest (?)



$$= \frac{9}{4}\mathcal{I}^F + 2(d-4)\mathcal{I}^{\rm IR} + \frac{2}{3}(d-5)(d-1)\mathcal{I}_1^{\rm UV}$$

← not UT, not pure ints…

$$- \frac{(d-4)(d-2)}{3}\left[\left(\frac{1}{2}+\frac{s_{23}}{s_{12}}\right)\mathcal{I}_2^{\rm UV} - (1-\frac{s_{23}}{s_{12}})\mathcal{I}_3^{\rm UV} + \mathcal{I}_4^{\rm UV}\right]$$

$$+ (1 \to 2 \to 3 \to 4 \to 1)$$

Result by G. Gambuti

See talk by D. Kosower

ON THE INTEGRALS

ON THE INTEGRALS: THE GEOMETRY OF FEYNMAN INTEGRALS

$\mathcal{I}$

Two points of view on Feynman integrals

ON THE INTEGRALS: THE GEOMETRY OF FEYNMAN INTEGRALS

$\mathscr{I}$

Two points of view on Feynman integrals

1. Direct Integration

$$\mathfrak{I}(a_1, \ldots, a_n) = \frac{(-1)^{\omega+d}\Gamma(d/2)}{\Gamma((L+1)d/2-\omega)} \left(\prod_{k=1}^{n} \int_0^\infty \frac{x_k^{a_k-1} dx_k}{\Gamma(a_k)} \right) \left[\mathscr{G}(x_i, s_{ij}, m_i^2) \right]^{-d/2}$$

[Lee, Pomeransky '13]

$$\mathscr{G}(x_i, s_{ij}, m_i^2) = 0$$

Determines an **algebraic variety**

its **first de Rham cohomology** fixes numbers and functions that will appear

ON THE INTEGRALS: THE GEOMETRY OF FEYNMAN INTEGRALS

$\mathscr{I}$

Two points of view on Feynman integrals

2. Differential Equations

$d\mathscr{I} = A(D, s_{ij})\mathscr{I}$ "trivial" consequence of IBPs [Kotikov '93; Remiddi '97; Gehrmann, Remiddi '99]

$A(D, s_{ij})$ Matrix of rational functions in D, kinematical invariants and masses

$A(D \to 4, s_{ij})$ Homogeneous solution(s) close to $D \to 4$ $(2n)$ determine geometry **(periods of algebraic varieties)** [Frellesvig, Papadopoulos, Primo, Tancredi, Weinzierl, Zhang, ...]

ON THE INTEGRALS: THE RIEMANN SPHERE

$\mathscr{I}$

All meromorphic functions defined on RS are **rational functions** $\dfrac{1}{(x-a)^k} \quad a \in \mathbb{C}$

If we integrate a rational function on $\mathbb{CP}^1$

Only non-trivial thing:

$$\log(1 - x/a) = \int_0^x \frac{dt}{t-a}$$

Generalisation: **Multiple PolyLogarithms (MPLs)**

$$G(c_1, c_2, \ldots, c_n, x) = \int_0^x \frac{dt_1}{t_1 - c_1} G(c_2, \ldots, c_n, t_1)$$
$$= \int_0^x \frac{dt_1}{t_1 - c_1} \int_0^{t_1} \frac{dt_2}{t_2 - c_2} \ldots \int_0^{t_{n-1}} \frac{dt_n}{t_n - c_n}$$

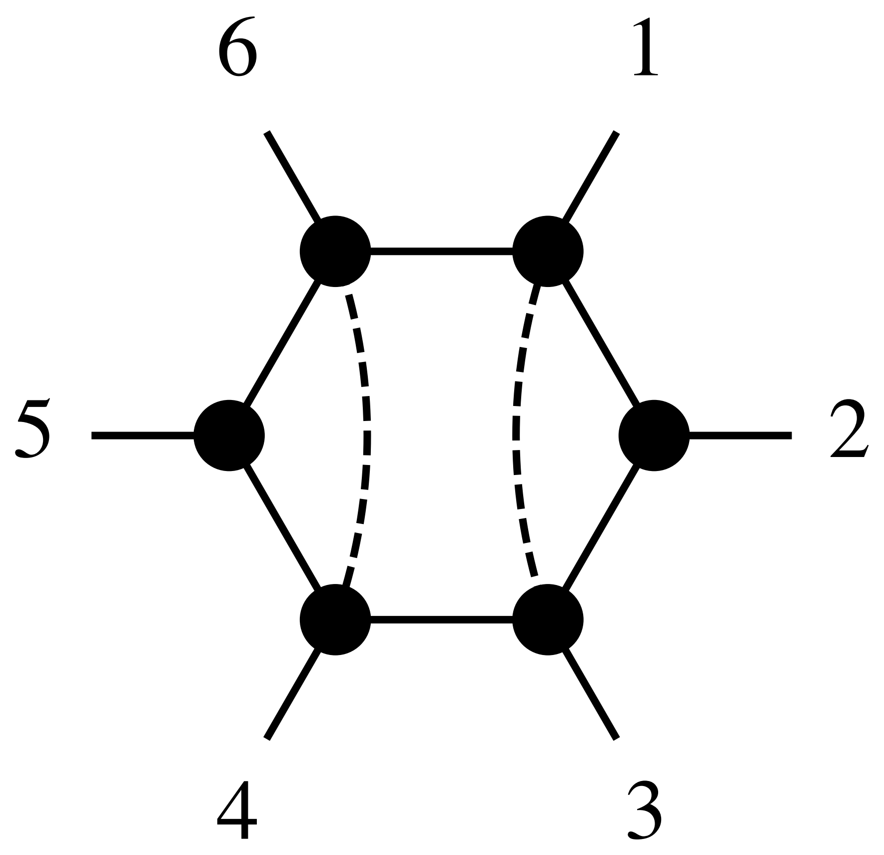
ON THE INTEGRALS: THE RIEMANN SPHERE & LOCAL INTEGRALS

$\mathcal{I}$

Further generalization:

Chen iterated integrals over dlog forms

$$\int_\gamma d\log f_1 \wedge d\log f_2 \wedge \ldots \wedge d\log f_n$$



Local integrals fulfil **canonical diff-equations**

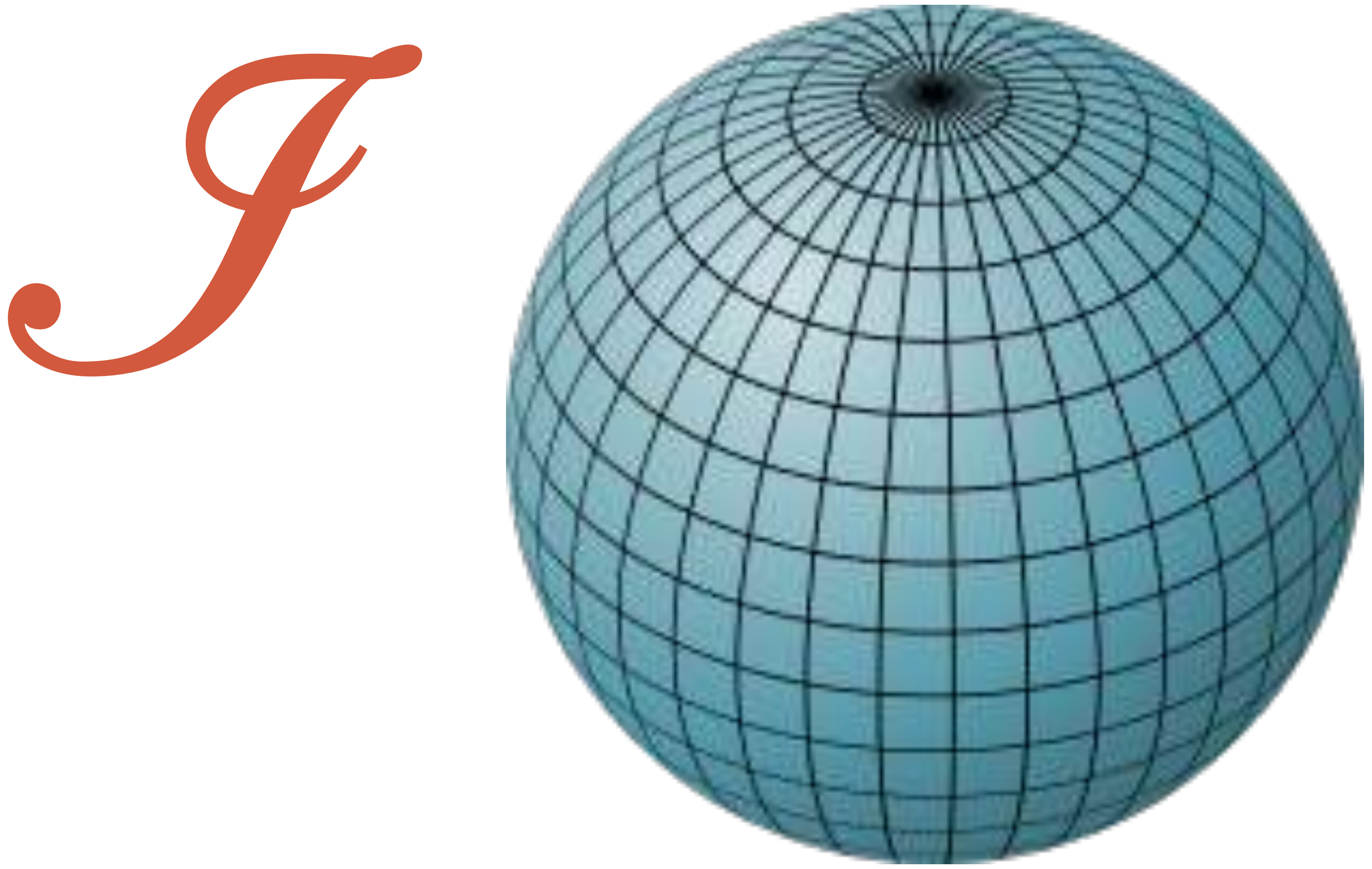
$$d\mathcal{I} = (D-4)dA(s_{ij})\mathcal{I}$$

dA **in** $d\log$**-form** → naturally expressed as **Chen iterated integrals**

[Henn; Kotikov; Lee ...]

See talk by S. Abreu

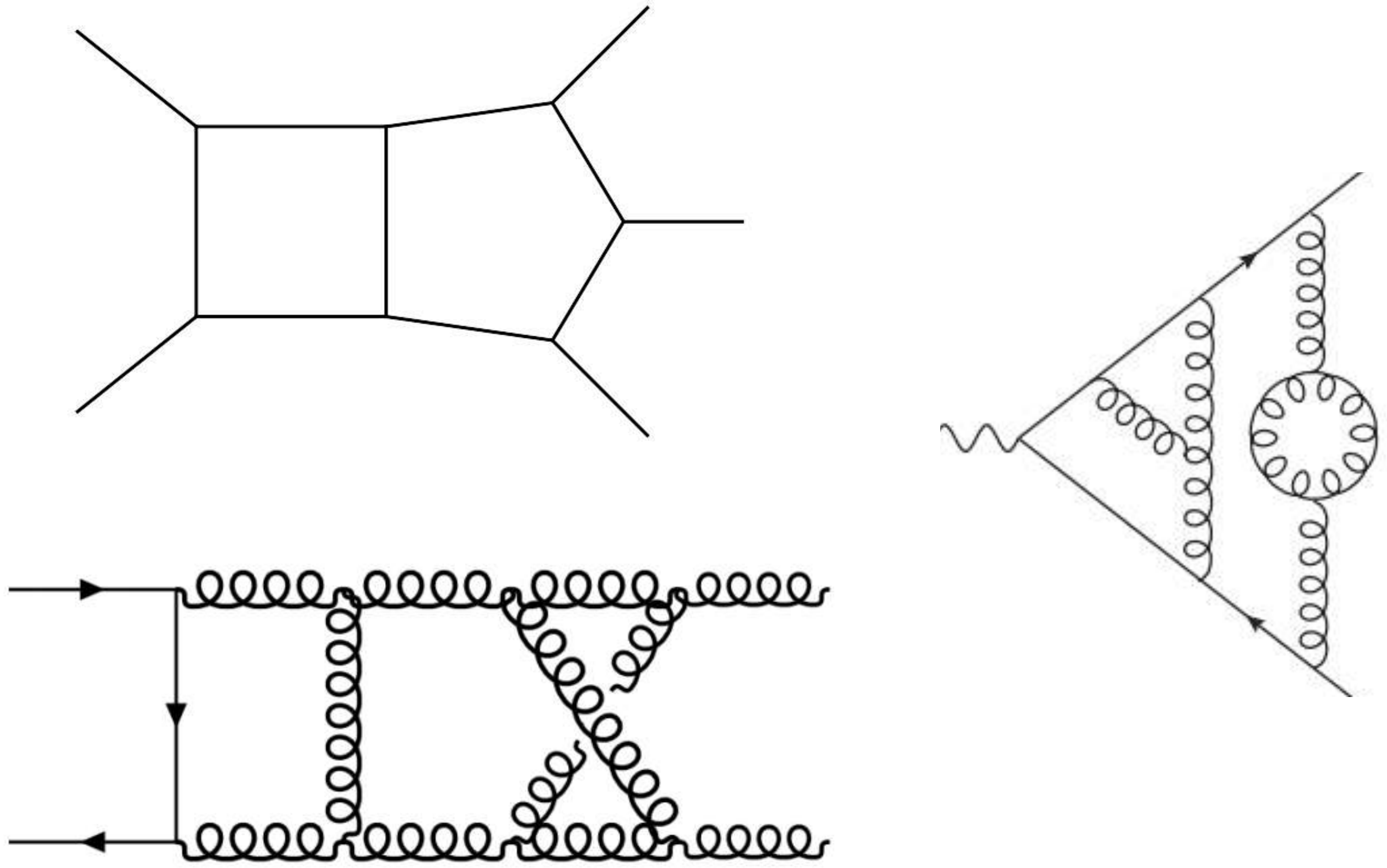
ON THE INTEGRALS: THE RIEMANN SPHERE & LOCAL INTEGRALS



Further generalization:

Chen iterated integrals over dlog forms

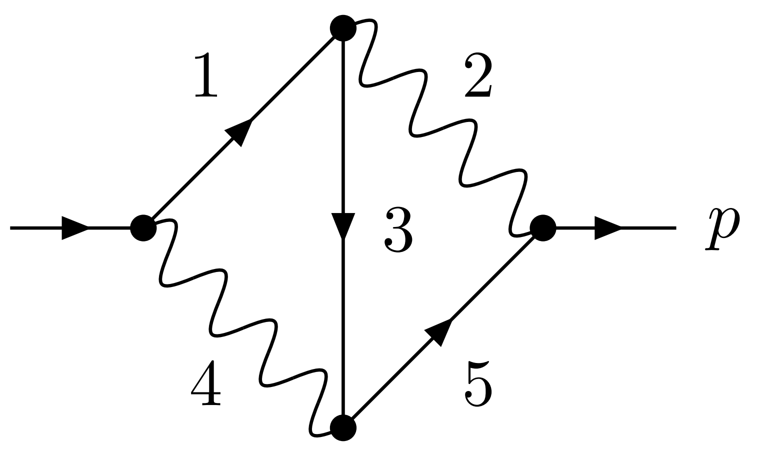
$$\int_{\gamma} d\log f_1 \wedge d\log f_2 \wedge \ldots \wedge d\log f_n$$



Massless QCD profited enormously from MPLs and "iterated integrals of dlog-forms"

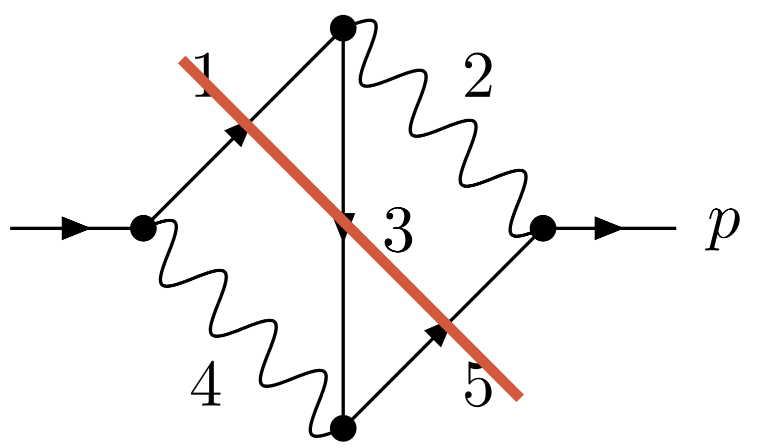
See talk by S. Abreu

ON THE INTEGRALS: GENUS ONE

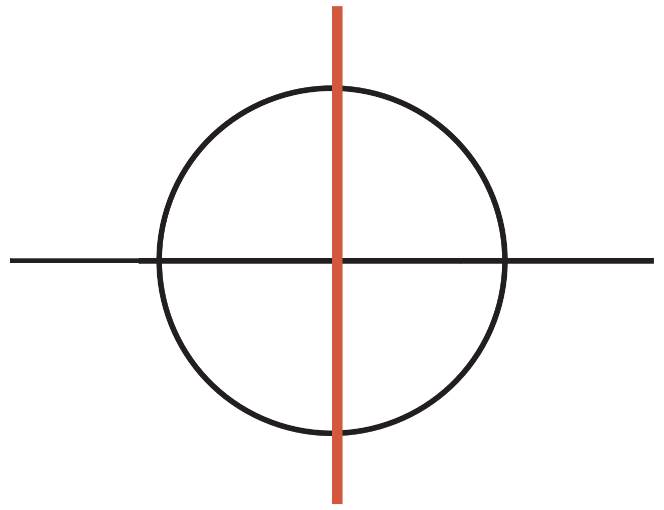


The electron propagator in QED; A. Sabri 1962

ON THE INTEGRALS: GENUS ONE



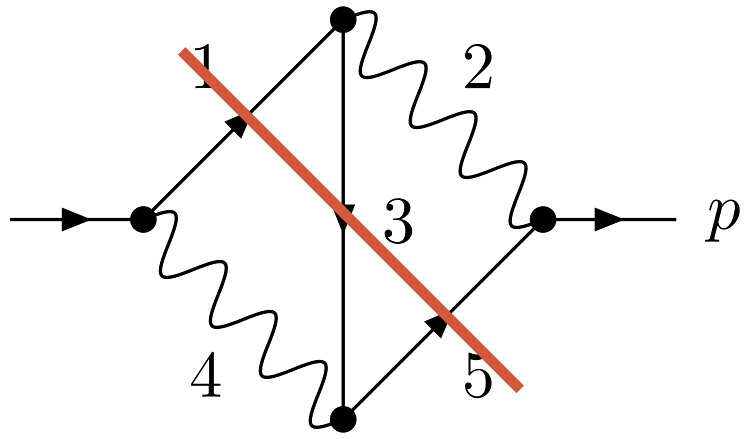
The electron propagator in QED; A. Sabri 1962



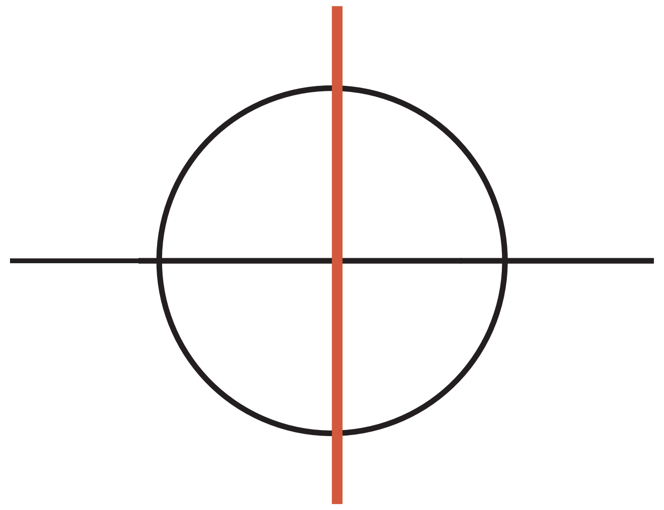
$$= \frac{1}{\sqrt{(3m-\sqrt{s})(\sqrt{s}+m)^3}} \mathrm{K}\left(\frac{16m^3\sqrt{s}}{(3m-\sqrt{s})(\sqrt{s}+m)^3}\right)$$

The sunrise integral

ON THE INTEGRALS: GENUS ONE



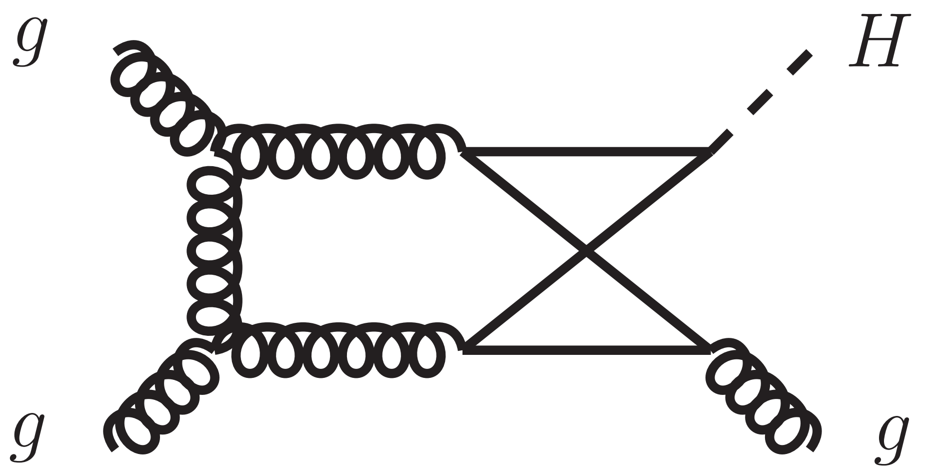
The electron propagator in QED; A. Sabri 1962

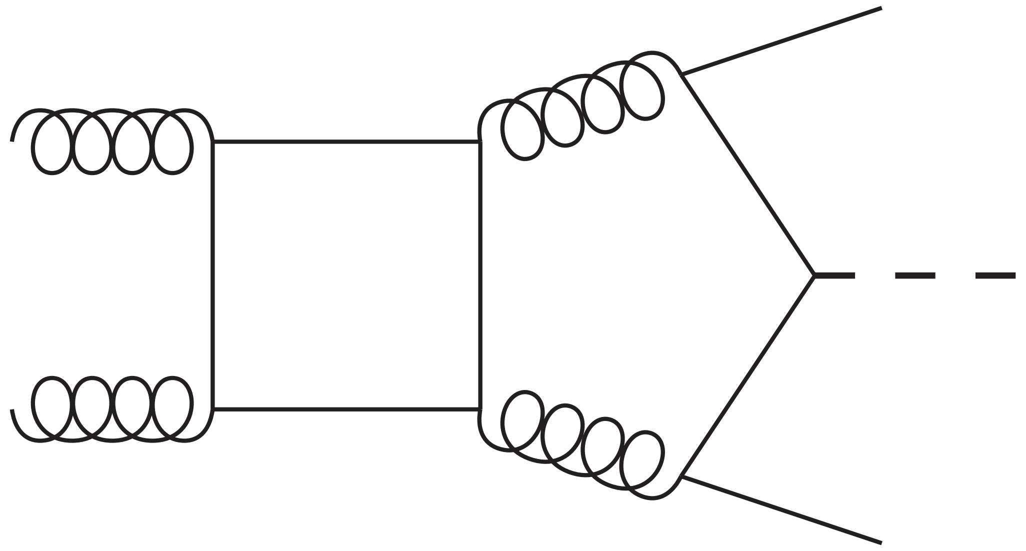


$$= \frac{1}{\sqrt{(3m-\sqrt{s})(\sqrt{s}+m)^3}}\,\mathrm{K}\left(\frac{16m^3\sqrt{s}}{(3m-\sqrt{s})(\sqrt{s}+m)^3}\right)$$

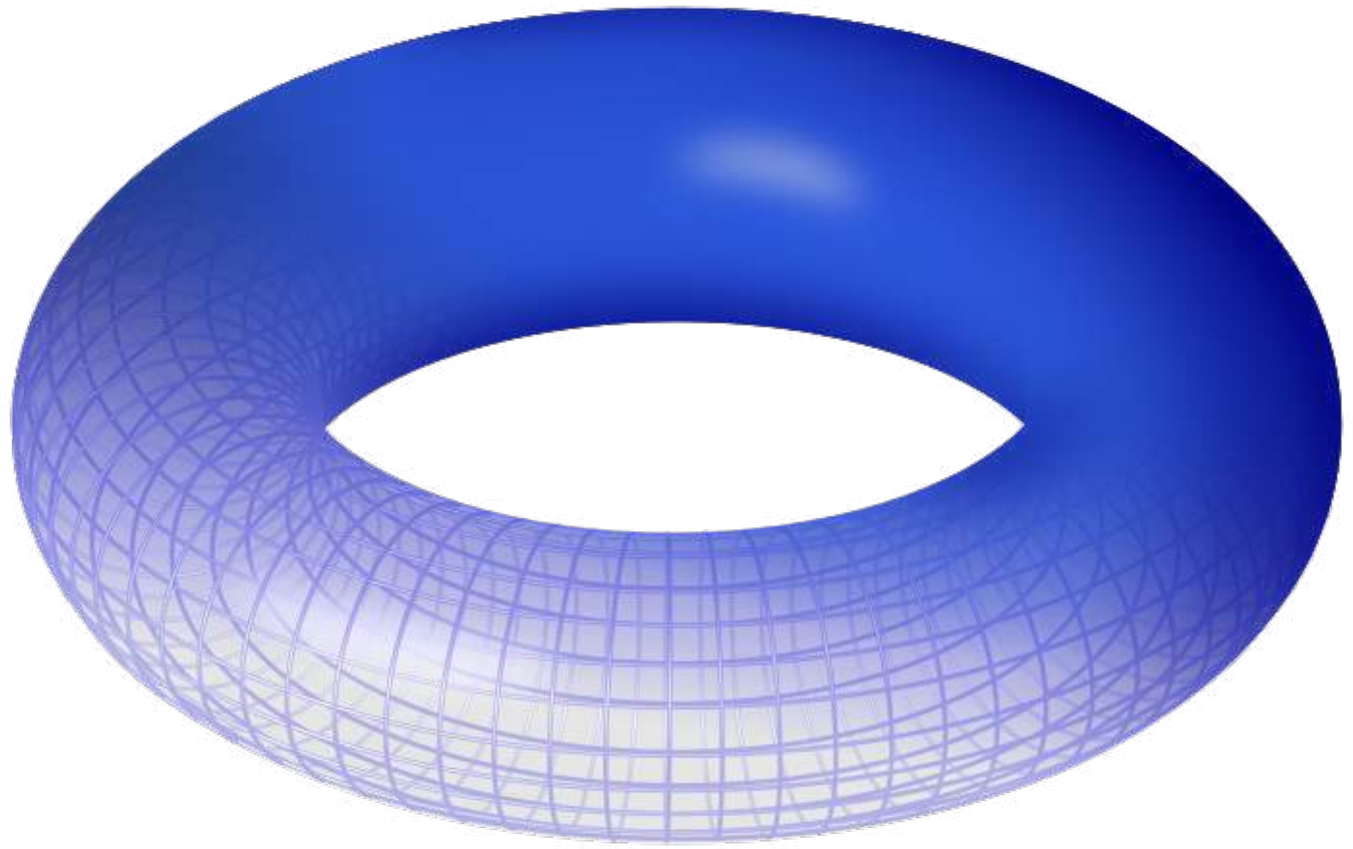
The sunrise integral

In QCD this happens all the time, especially when **masses cannot be neglected**

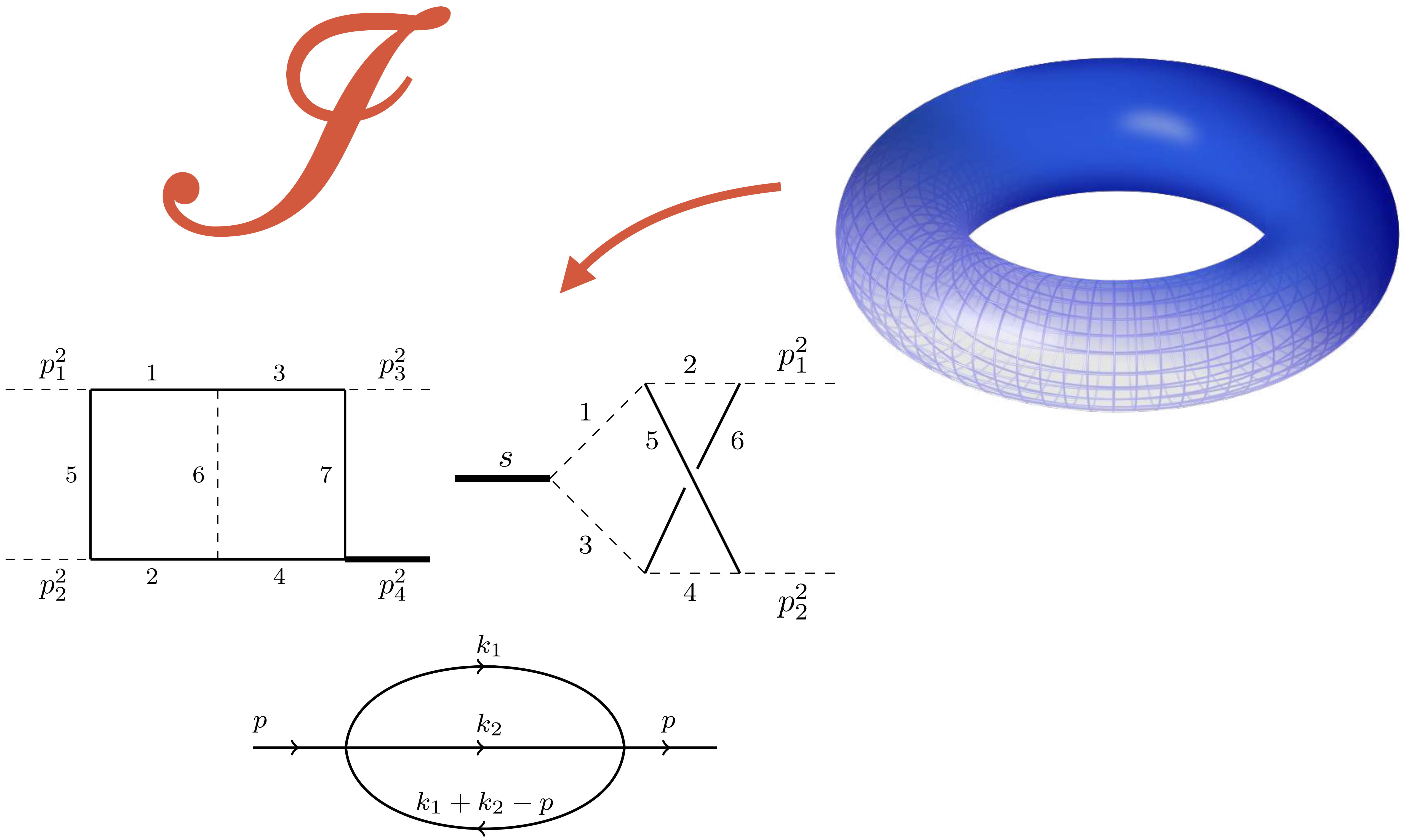




See talk by F. Porkert



ON THE INTEGRALS: BEYOND IN DIMENSION AND IN GENUS



Higgs physics

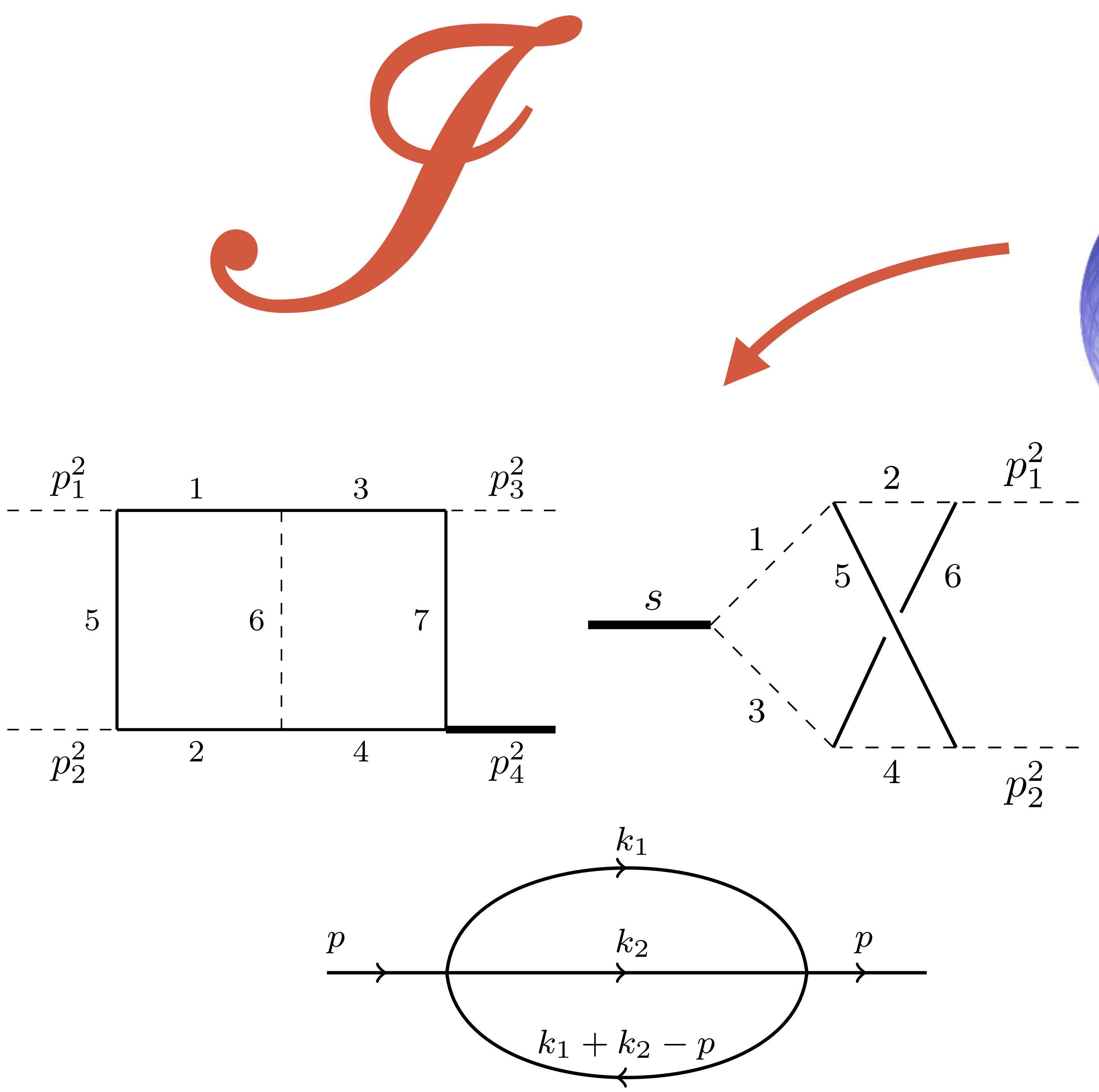
Top physics

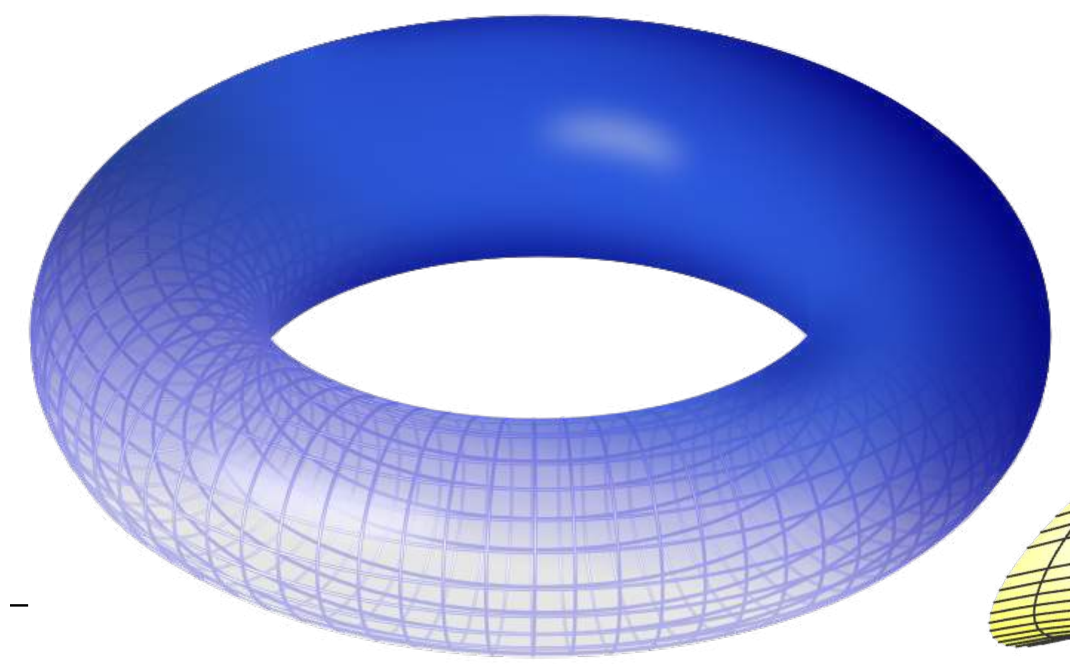
EW physics

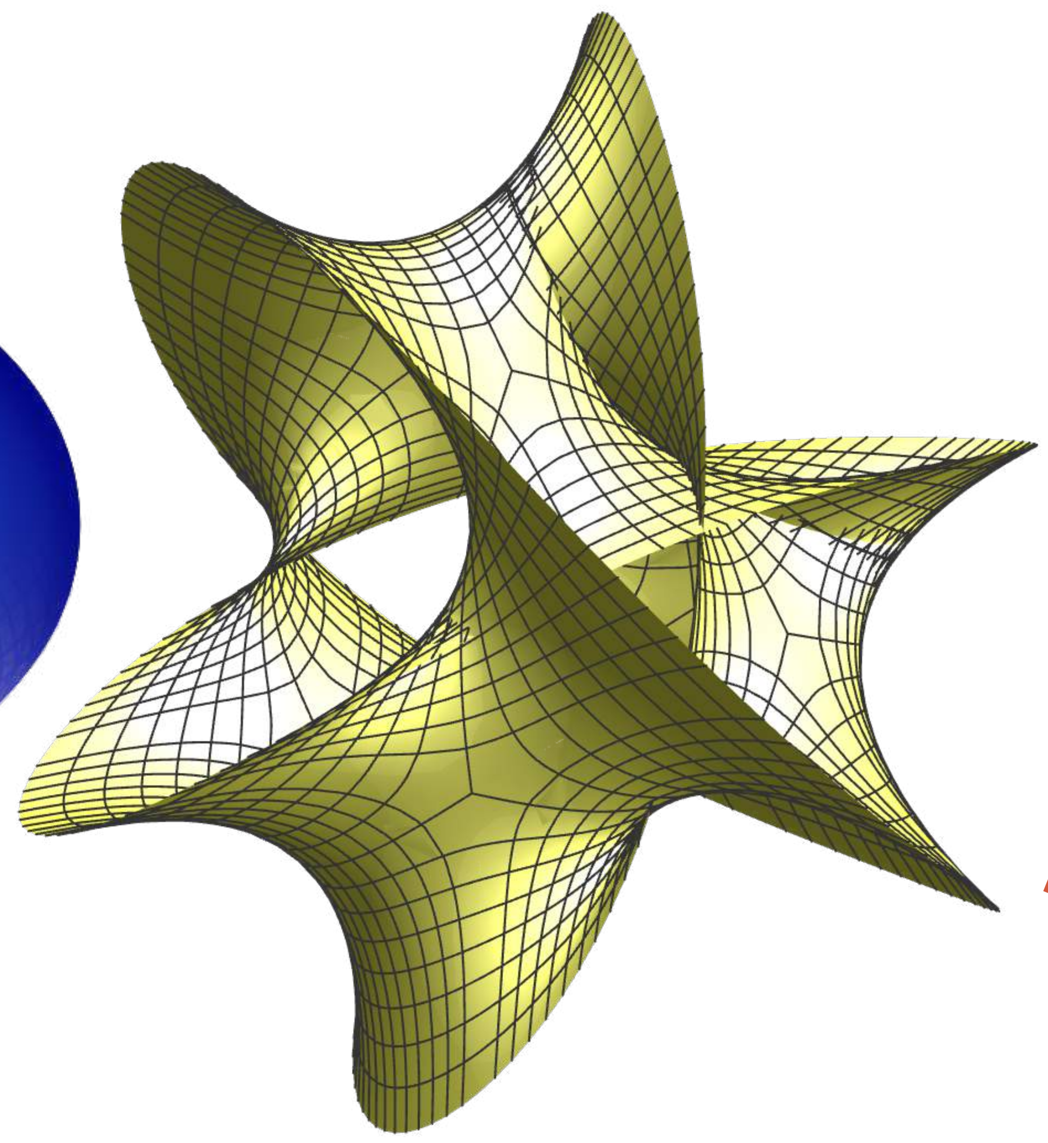
ON THE INTEGRALS: BEYOND IN DIMENSION AND IN GENUS

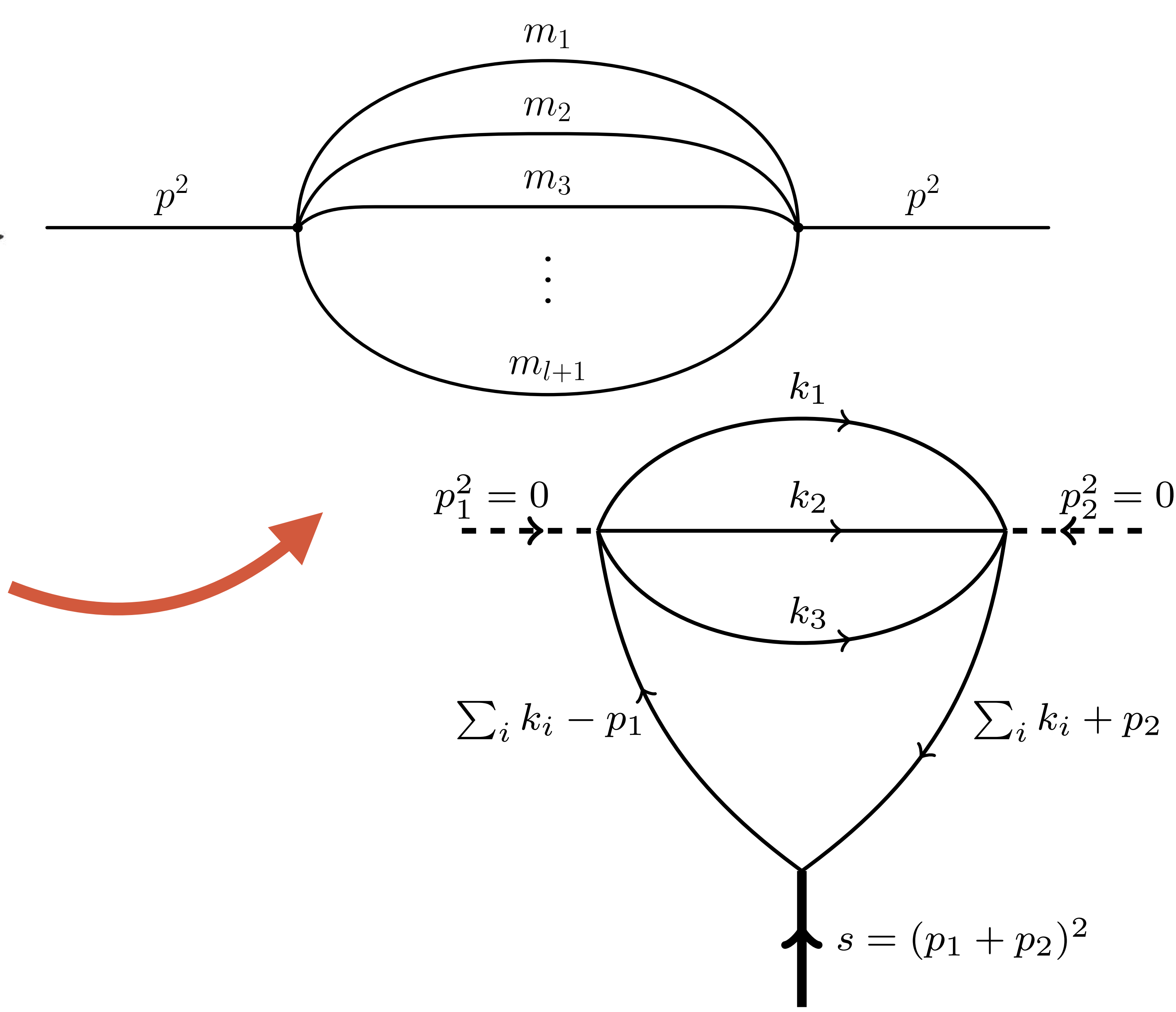
QED propagator

EW corrections









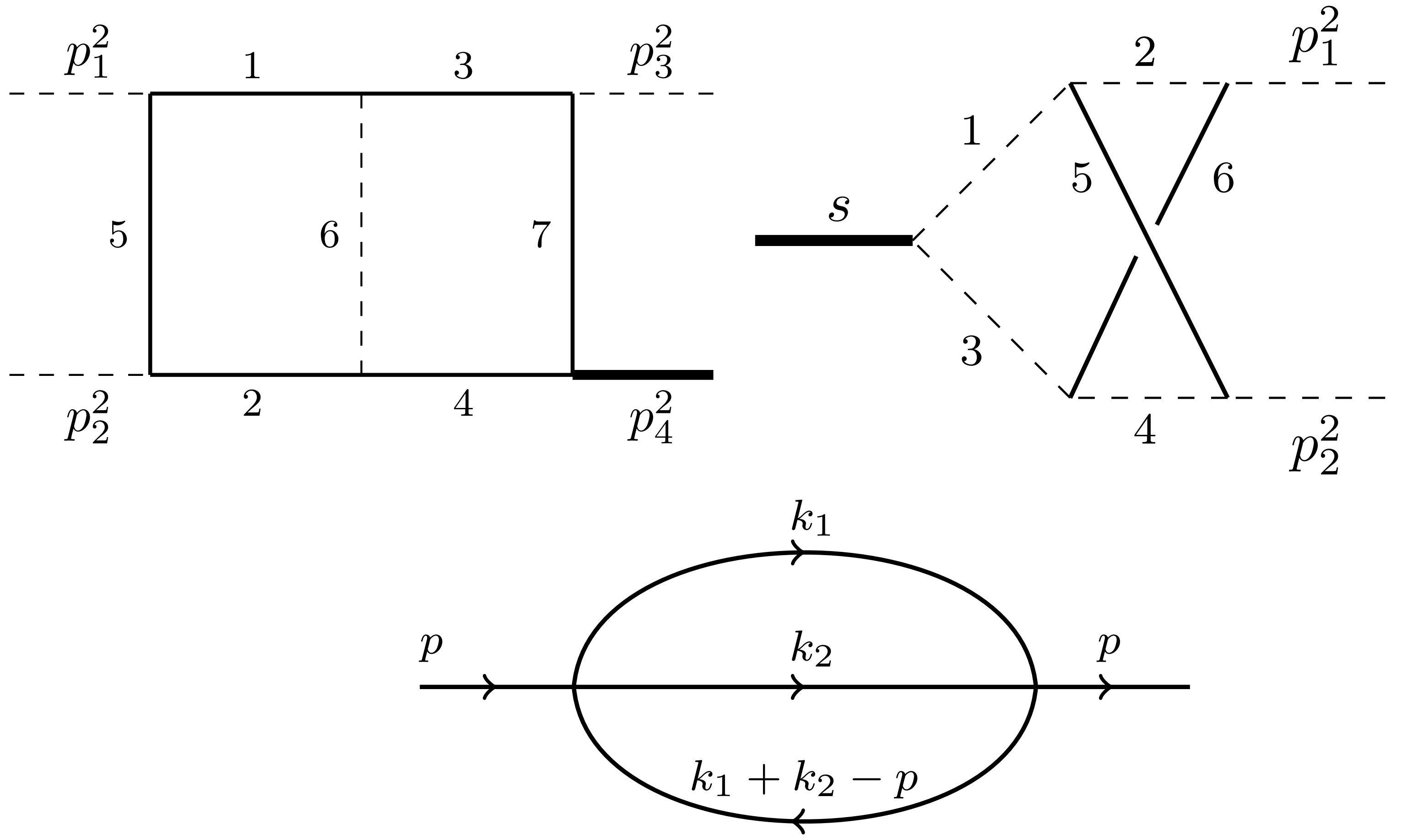
Higgs physics

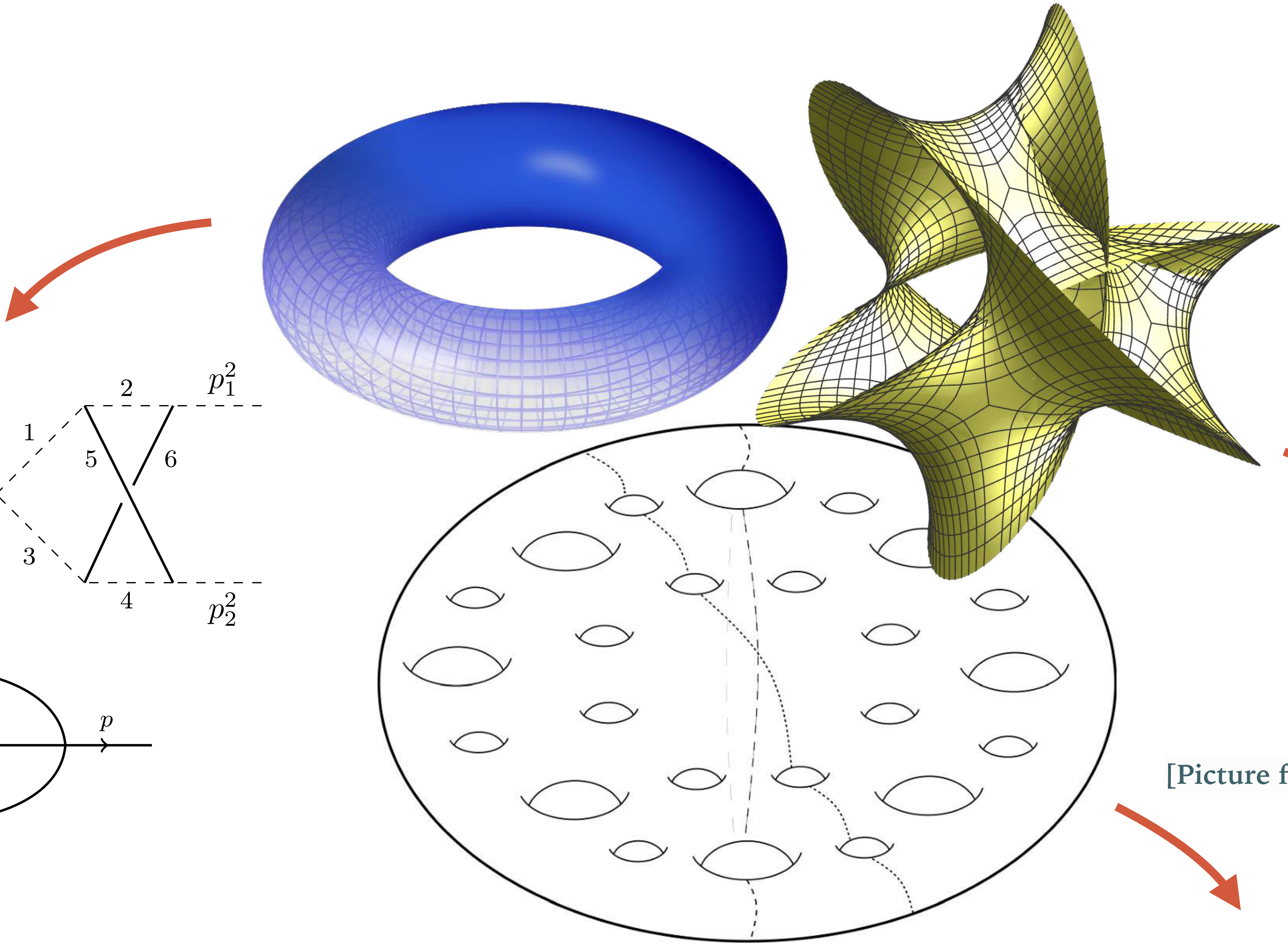
Top physics

EW physics

ON THE INTEGRALS: BEYOND IN DIMENSION AND IN GENUS

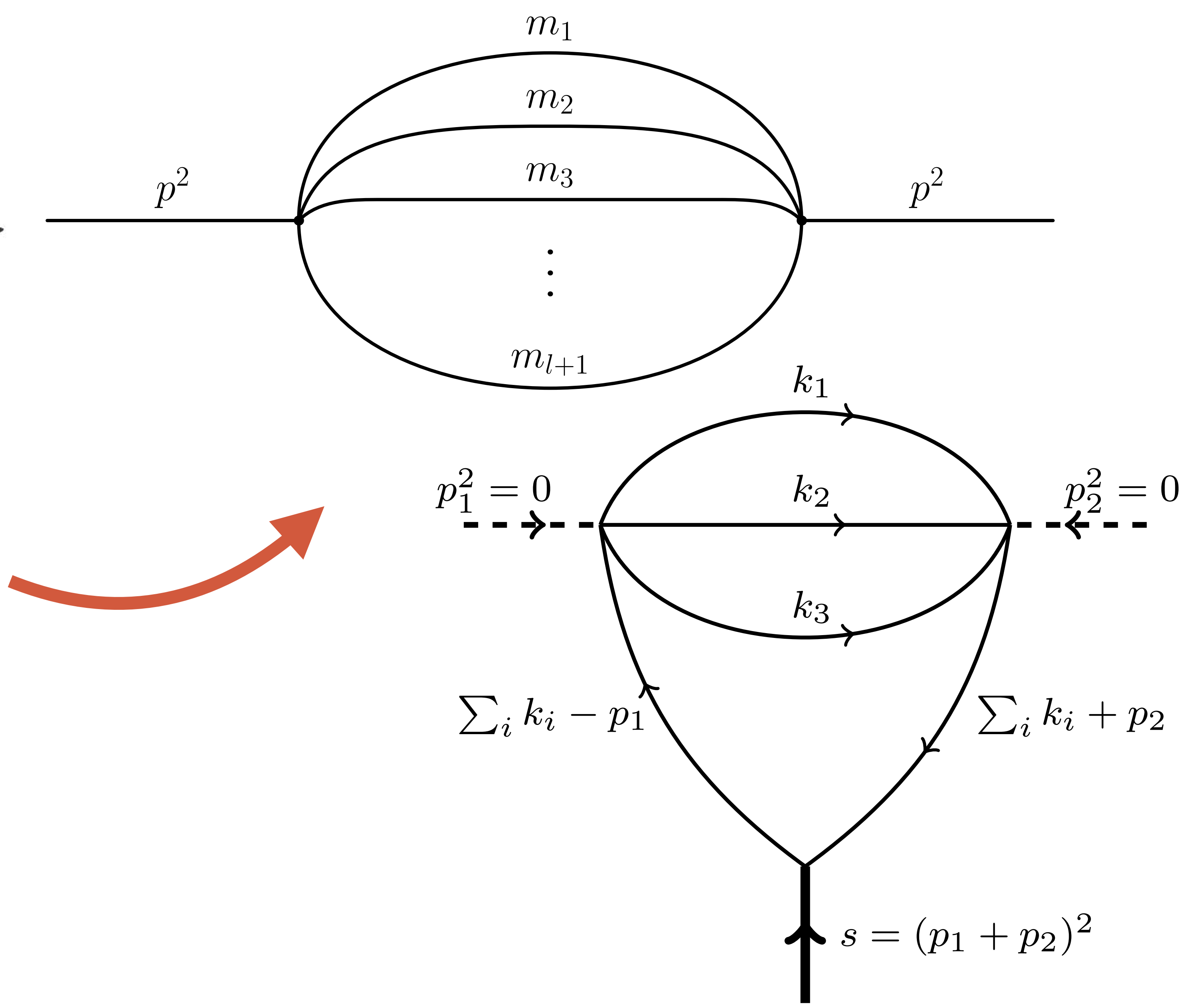
$\mathcal{J}$





QED propagator

EW corrections



[Picture from: Kozłowska-Walania ‘20]

Higgs physics

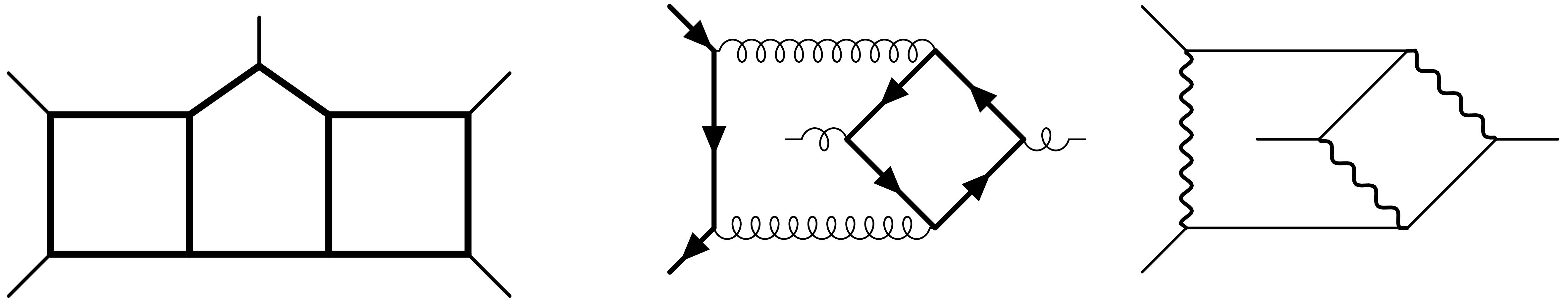
Top physics

EW physics

EW corrections

multi-particle scattering

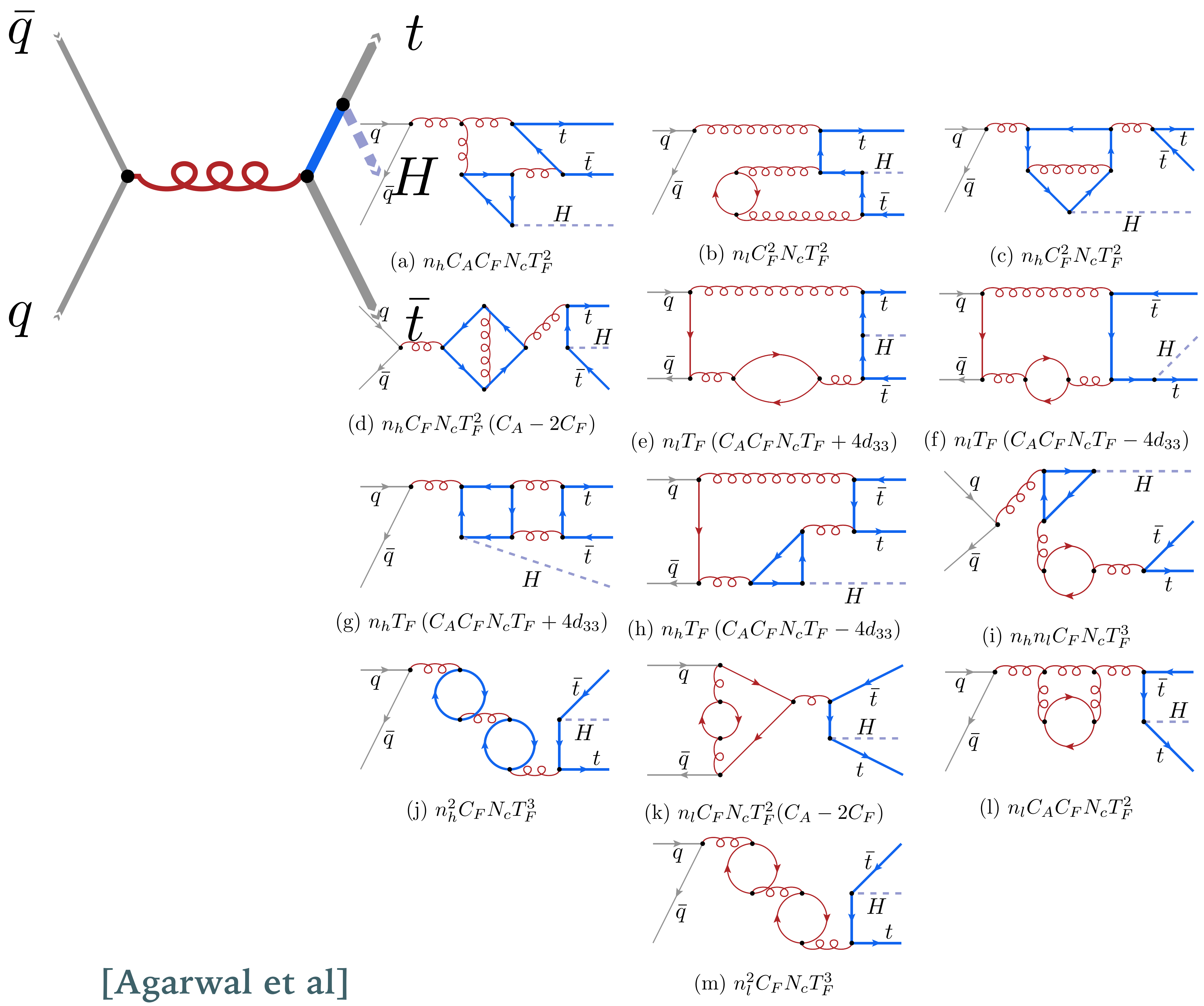
$t\bar{t} + X$ production



THE PRACTICAL APPROACH: NUMERICAL SOLUTIONS

Methods for direct numerical integration

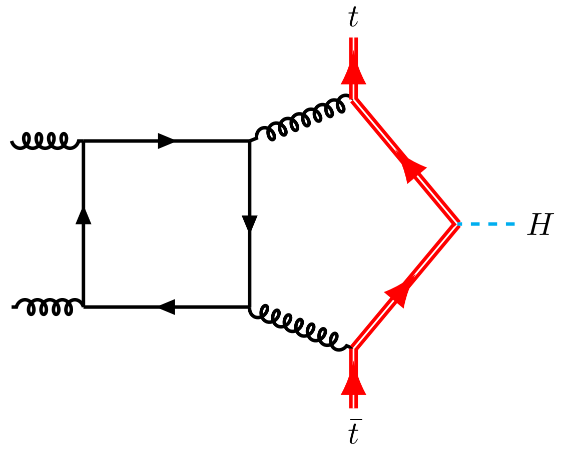
$q\bar{q} \to t\bar{t}H$ n_f enhanced contributions with sector decomposition



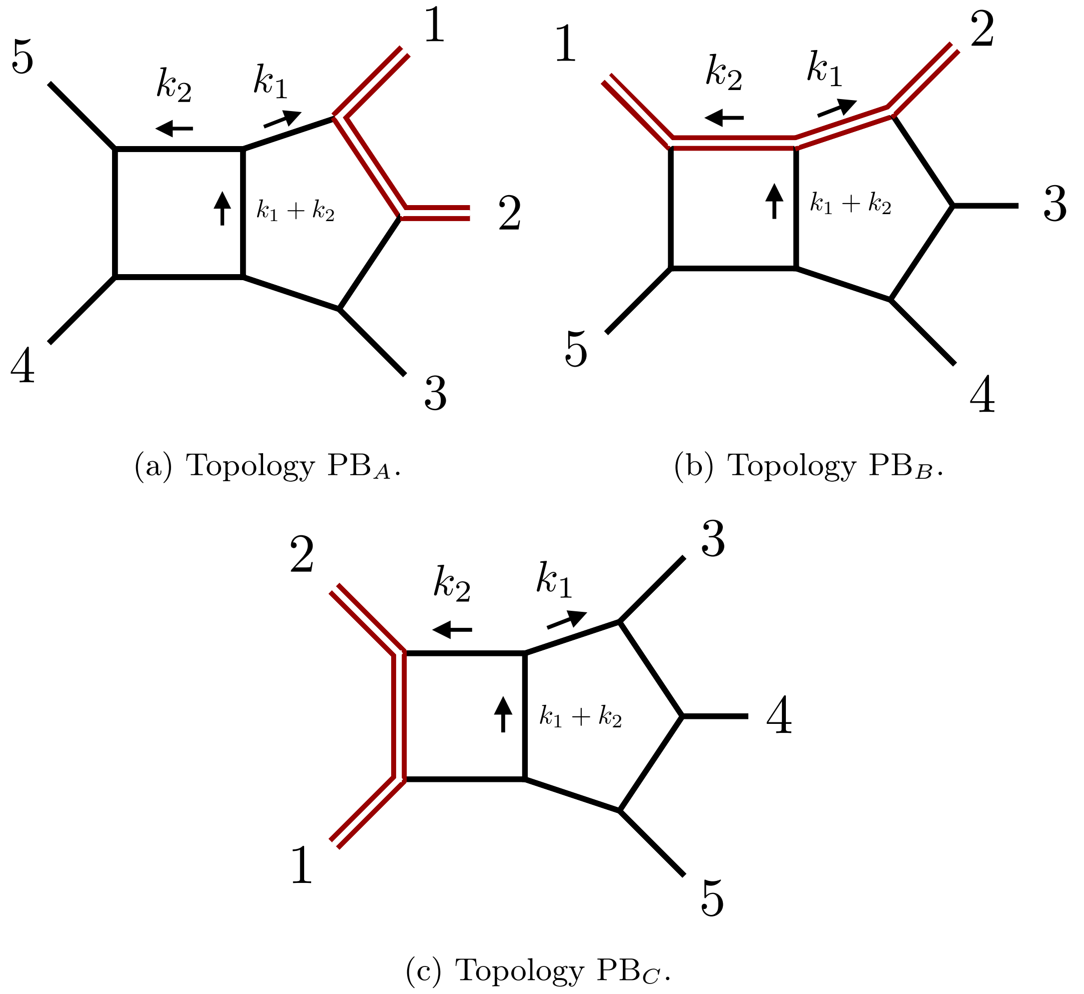
[Agarwal et al]

Numerical solution of differential equations

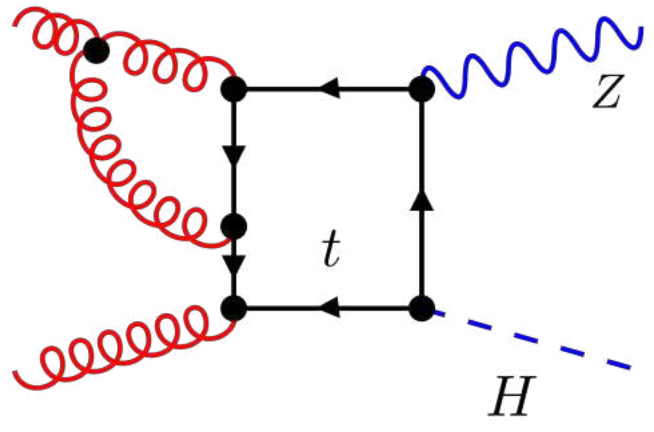
series expansions a la Frobenius (many applications)



ttH [Cordero, Figueiredo, Kraus, Page, Reina]



ttj [Badger, Becchetti, Giraudo, Zoia]



[Hasselhuhn, Luthe, Steinhauser]
[Wang, Xu, Xu, Yang]
[Chen, Davies, Jones, Kerner]
[Degrassi, Gröber, Vitti, Zhao]

Many other examples of problems successfully solved numerically ($t\bar{t}$@$NNLO$, HH@NLO, Hj@NLO, $\gamma\gamma$@NLO …)

ON THE INTEGRALS: OPEN QUESTIONS

Whether we use numerical or analytical methods to evaluate the integrals, a question remains central:

What are **good integrands** that give rise to **nice integrals?** See also work by J. Bourjaily, Caron-Huot, …

↓

ϵ factorised bases of differential equations?

for elliptics, Calabi-Yaus and beyond

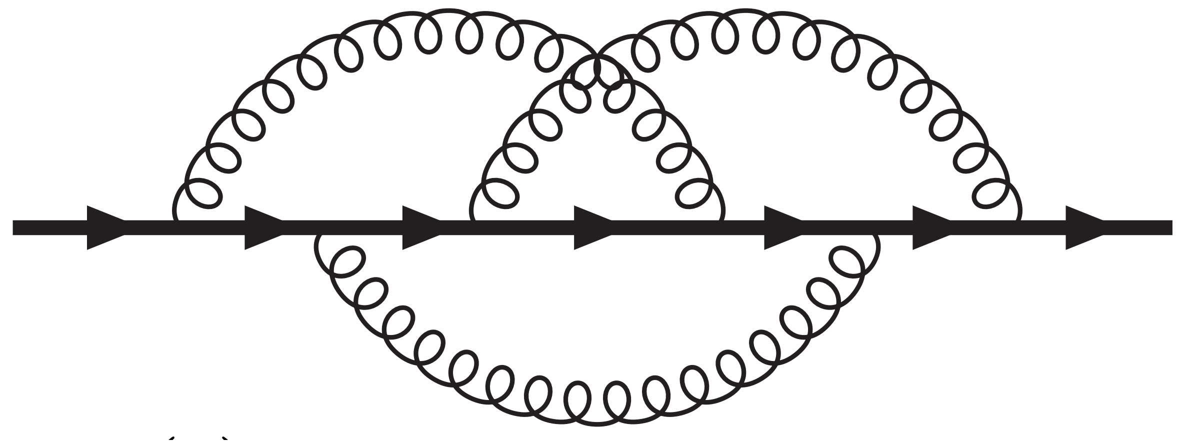
Progress: **Dlapa, Henn, Wagner; Pögel, Wang, Weinzierl; Frellesvig; Görges, Nega, Tancredi, Wagner; …**

Cohomology beyond Riemann sphere **requires higher poles**

Space of functions must *(in some form)* encode higher singularities

Conspiracy at the level of physical amplitudes for their cancellation — make it explicit?

AN EXAMPLE CALCULATION: THE THREE-LOOP QED SELF ENERGY



$$\hat{p}\,\Sigma_V(p^2,m^2) + m\,\Sigma_S(p^2,m^2)$$

Σ_V & Σ_S expressed in terms of $\mathcal{O}(50)$ Masters Integrals

There is a bunch of **elliptic sectors**

For each such sector:

$$y = \sqrt{P_3(x)}$$

Forms of first kind

No poles $\omega \sim \int \frac{dx}{y}$

Forms of third kind

single poles $G \sim \int \frac{dx}{(x-c_i)y}$

Forms of second kind

double poles $\eta \sim \int \frac{dx\,x}{y}$

Only integrals involving forms of first and third kind show up at order $\mathcal{O}(\epsilon^0)$ — second kind suppressed by $\mathcal{O}(\epsilon)$!

[Duhr, Gasparotto, Nega, Tancredi, Weinzierl] to appear soon

Similar structure in other elliptic amplitudes…!

CONCLUSIONS

1. Colliders remain some of the most flexible (multi-purpose) experiments to investigate fundamental questions in physics

2. Next generation colliders have the guaranteed outcome of **discovering the Higgs self-interactions** and **measuring the Higgs potential**

3. Problems in Collider physics and Scattering Amplitudes are **tightly intertwined**

4. These furnish motivation to **solve challenging problems for Amplitudes community**

5. Exporting from N=4 SYM, String theory, CFT to QCD provides us the **chance to address deep problems in QFT** (IR divergences, high-energy Regge limit, structure of singularities etc)

Let's keep exploring!


THANK YOU !