Hidden Amplitude Zeros and the Double Copy

Shruti Paranjape

University of California, Davis
Center for Quantum Mathematics and Physics

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Based on arXiv:2403.10594 and arXiv:2305.05688 with C Bartsch, T Brown, K Kampf, U Oktem and J Trnka

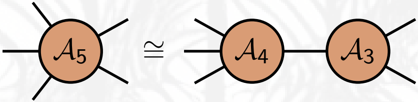
Tree Amplitudes

At tree-level, the amplitude is a rational function of kinematic variables,

$$\mathcal{A}_n = \frac{\text{numerator}}{\text{denominator}}$$

Additionally, a lot is known about the singularity structure from:

- Locality: Poles and branch cuts of the amplitudes
  $\lim_{p_I^2 \to 0} A_n \sim \frac{1}{p_I^2}$
- Unitarity: Factorization into lower-point amplitudes
  $\operatorname*{Res}_{p_I^2=0} A_n = A_L \times A_R$



What About Numerators?

Unitarity

$\downarrow$

n-point numerator at singular kinematic points

$\downarrow$

Determines the amplitude up to “off-pole” contributions

Zeros and poles fully determine any rational function $\Rightarrow$ do amplitudes have zeros?

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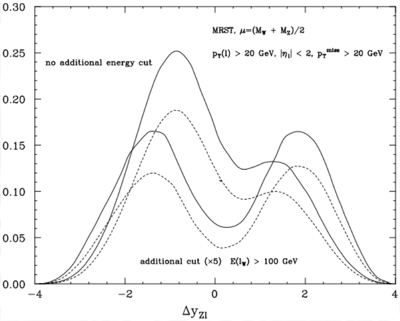
Zeros and poles fully determine any rational function $\Rightarrow$ do amplitudes have zeros?

Symmetry	Zero	Example Theory
$\phi \to \phi + c$	$\mathcal{A}_n \xrightarrow{p_i \to 0} \mathcal{O}(p_i)$	$\frac{SU(N)\times SU(N)}{SU(N)}$ NLSM
$\phi \to \phi + c + a^\mu x_\mu$	$\mathcal{A}_n \xrightarrow{p_i \to 0} \mathcal{O}(p_i^2)$	DBI brane scalar
$\phi \to \phi + c + a^\mu x_\mu + s^{\mu\nu} x_\mu x_\nu$	$\mathcal{A}_n \xrightarrow{p_i \to 0} \mathcal{O}(p_i^3)$	special Galileon theory

Example of a Zero

- Amplitudes of pions have an Adler zero i.e. they vanish in the limit of vanishing external pion momentum.
- Radiation zeros exist at tree-level in standard model processes like $q_1\bar{q}_2 \to W^{\pm}Z$ at specific angles.



[Dixon, Kunszt, Signer]

Hidden Zeros

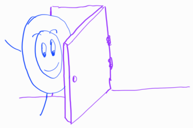
Recently, hidden zeros were discovered in partial amplitudes of a certain class of theories e.g. NLSM, Yang-Mills and $\mathrm{Tr}(\phi^3)$.

[Arkani-Hamed, Cao, Dong, Figueiredo, He]

In d-dimensional scalar theories, these zeros are reached by sending a specific set of Mandelstam invariants $s_{ij} = (p_i + p_j)^2$ to zero,

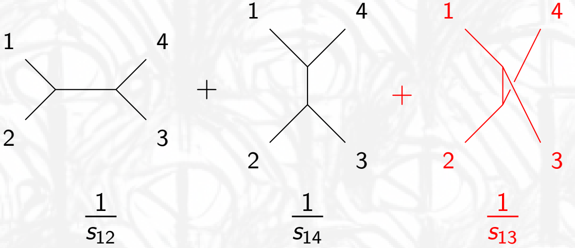
4-point	$s_{13} = 0$	
5-point	$s_{13} = s_{14} = 0$	
6-point	$s_{13} = s_{14} = s_{15} = 0$	C_2
	$s_{14} = s_{15} = s_{24} = s_{25} = 0$	C_3

Note: The first type of zero is one Mandelstam away from being an Adler zero $p_1 \to 0$.



Example in $\mathrm{Tr}(\phi^3)$

$A_4^{\mathrm{Tr}(\phi^3)}[1234] =$



We only include diagrams compatible with color ordering 1234:

$$A_4^{\mathrm{Tr}(\phi^3)}[1234] = \frac{1}{s_{12}} + \frac{1}{s_{14}} = -\frac{s_{13}}{s_{12}s_{14}} \xrightarrow{s_{13}\to 0} 0$$

Hidden zeros do not occur diagram by diagram.

Examples in SU(N) NLSM

4-point is trivial:

$$A_4^{\text{NLSM}}[1234] = s_{13} \xrightarrow{s_{13}\to 0} 0$$

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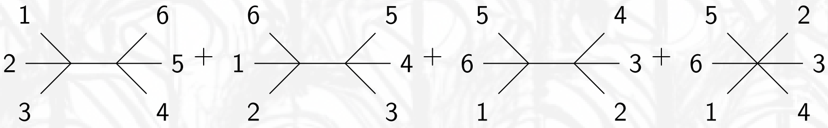

$$A_6^{\text{NLSM}}[123456] = \frac{s_{13}s_{46}}{s_{123}} + \frac{s_{35}s_{26}}{s_{126}} + \frac{s_{15}s_{24}}{s_{156}} - s_{135}$$

Again, the zeros are not manifest.

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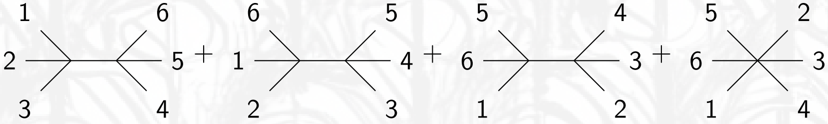

$$A_6^{\text{NLSM}}[123456] = \frac{s_{13}s_{46}}{s_{123}} + \frac{s_{35}s_{26}}{s_{26}} + \frac{s_{15}s_{24}}{s_{156}} - s_{13} - s_{15} - s_{35}$$

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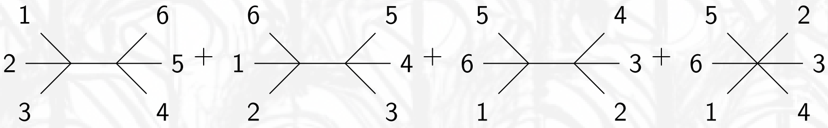


$$A_6^{\text{NLSM}}[123456] = \frac{s_{13}s_{46}}{-s_{126}} + \frac{s_{35}s_{26}}{s_{126}} + \frac{s_{15}s_{24}}{s_{156}} - s_{13} - s_{15} - s_{35}$$

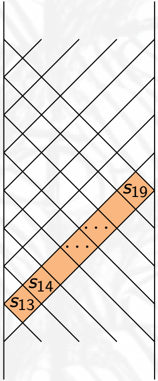
Again, the zeros are not manifest.

General n-Point Example

For scalar theories like NLSM and $\mathrm{Tr}(\phi^3)$, $A_n|_{C_m} = 0$ where the zero condition is given by

$$C_m = \begin{cases} s_{1m+1} = s_{1m+2} = \cdots = s_{1n-1} = 0 \\ s_{2m+1} = s_{2m+2} = \cdots = s_{2n-1} = 0 \\ \vdots \\ s_{m-1m+1} = \cdots = s_{m-1n-1} = 0 \end{cases}$$

where $m = 2, \cdots, \lfloor \frac{n}{2} \rfloor$.



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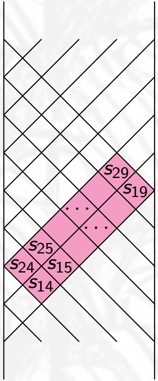

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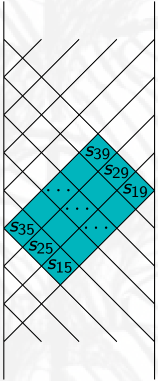

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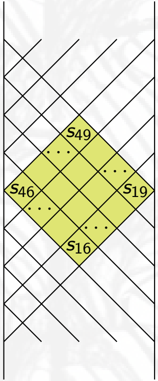

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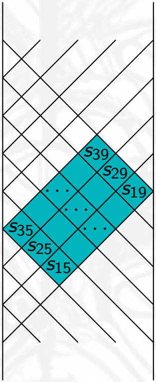
Factorization Near Zeros

Like near poles, the amplitude factorizes near zeros into lower-point amplitudes, when all-but-one Mandelstam in C_m vanishes.



Unlike near poles, there is no physical principle that tells us why this should be the case.

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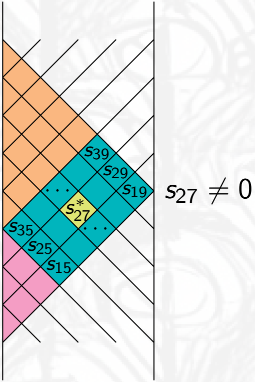

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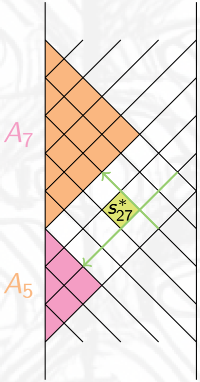

$$A_{10} \xrightarrow{C_4, s^*_{27}} \left(\frac{1}{s_{1234}} + \frac{1}{s_{3456789}} \right) \times A_5 \times A_7$$

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[Arkani-Hamed, Cao, Dong, Figueiredo, He]

Spinning Zeros

For gluons, we need to extend the zero conditions slightly to now include polarization vectors as well i.e.

$$\text{if } s_{ij} = 0 \text{ on } C_m\,,$$
$$(p_i \cdot p_j) = (\varepsilon_i \cdot p_j) = (p_i \cdot \varepsilon_j) = (\varepsilon_i \cdot \varepsilon_j) = 0 \text{ on } C_m^{\text{spinning}}$$

4-point YM has contact+pole terms:

$$(\varepsilon_1 \cdot \varepsilon_3)(\varepsilon_2 \cdot \varepsilon_4) \xrightarrow{(\varepsilon_1 \cdot \varepsilon_3) \to 0} 0$$

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4-point YM has contact+pole terms:

$$(\varepsilon_1 \cdot \varepsilon_3)(\varepsilon_2 \cdot \varepsilon_4) \xrightarrow{(\varepsilon_1 \cdot \varepsilon_3) \to 0} 0$$

$$\frac{1}{s_{12}} \left((\varepsilon_1 \cdot \varepsilon_2) p_1^\mu + (\varepsilon_1 \cdot p_2) \varepsilon_2^\mu + (\varepsilon_2 \cdot p_1) \varepsilon_1^\mu \right)$$
$$\times \left((\varepsilon_3 \cdot \varepsilon_4) p_3^\nu + (\varepsilon_3 \cdot p_4) \varepsilon_4^\nu + (\varepsilon_4 \cdot p_3) \varepsilon_3^\nu \right) \eta_{\mu\nu} + \text{cyc} \xrightarrow{C_2} 0$$

Can this be seen from any of the many constructions we have for YM amplitudes?

Theories with Hidden Zeros

So far, we've seen that the following theories have hidden zeros:

- Tr(ϕ^3) theory of adjoint scalars
- SU(N) non-linear sigma model
- Yang-Mills theory
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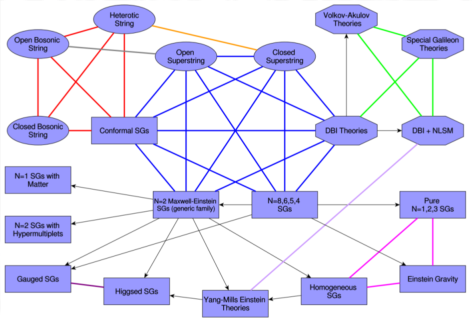
They all "play a role" in the double copy.

Web of Theories



[Bern, Carrasco, Chiodaroli, Johansson, Roiban]

For example:

$$\text{BI} = \text{YM} \otimes \text{NLSM} \quad \text{e.g.} \quad \mathcal{M}_4^{\text{BI}}(1234) = \frac{us}{t} A_4^{\text{YM}}[1234] A_4^{\text{NLSM}}[1234]$$

All theories with hidden zeros are related to this map, including

$$A_n^{\phi^3_{a\tilde{a}}}[\alpha|\alpha] = A_n^{\text{Tr}(\phi^3)}[\alpha]$$

This Talk

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BCJ Relations at 6-Point

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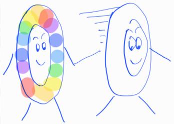

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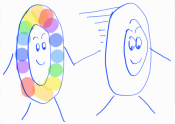
$$C_2 = \{s_{13} = s_{14} = s_{15} = 0\}$$



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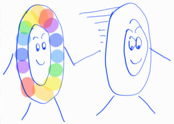

BCJ Relations for Tr(ϕ^3)

$$
\begin{aligned}
&A_6[123456|123456] \\
&\quad = \frac{1}{s_{12}s_{123}s_{56}}\Big[s_{13}s_{25}(s_{56}-s_{24})\ A_6[162543|123456] \\
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\end{aligned}
$$

The amplitudes on the RHS are doubly color-ordered bi-adjoint scalar amplitudes, while the one on the LHS is that of Tr(ϕ^3).

2-Particle Poles?

- NLSM+h.d.: Only 4-particle interactions i.e. 3-particle poles
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- YM: The amplitude factorizes on 2-particle poles into

$$A_n \xrightarrow{p_1 \cdot p_2} A_3(p_1, p_2, -(p_1+p_2)) \times A_{n-1}$$

$$\Big[(\varepsilon_1 \cdot \varepsilon_2)(\varepsilon_3 \cdot p_1) + (\varepsilon_2 \cdot \varepsilon_3)(\varepsilon_1 \cdot p_2) + (\varepsilon_3 \cdot \varepsilon_1)(\varepsilon_2 \cdot p_3)\Big] \times A_{n-1}$$

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- $(DF)^2 + h.d.$: Non-local extension of YM also has an A_3 that cancels 2-particle poles.
- $(F)^3 + h.d.$: Higher-derivative extension of YM also has an A_3 that cancels 2-particle poles.

BCJ Relation at n-Point

$$A_n[123\cdots n] = (-1)^n \sum_{\sigma(3\ldots n-1)} A_n[1n2\sigma] \times \prod_{k=3}^{n-1} \frac{\mathcal{F}_k[2\sigma 1]}{s_{kk+1\ldots n}}$$

[Bern, Carrasco, Johansson]

The factors $\mathcal{F}_k$ are given by

$$\mathcal{F}_k[\rho] = \left\{ \begin{array}{ll} \sum_{l=t_k}^{n-1} \mathcal{S}_{k,\rho_l} & \text{if } t_k > t_{k+1} \\ -\sum_{l=1}^{t_k} \mathcal{S}_{k,\rho_l} & \text{if } t_k < t_{k+1} \end{array} \right\} + \begin{cases} s_{kk+1\ldots n} & \text{if } t_{k-1} > t_k > t_{k+1} \\ -s_{kk+1\ldots n} & \text{if } t_{k-1} < t_k < t_{k+1} \\ 0 & \text{else,} \end{cases}$$

where t_k is the position of leg k in the ordered list $\rho = \{2\sigma 1\}$ and ρ_l denotes its l-th element and

$$t_2 = 0, \qquad t_n = t_{n-2}$$

$$\mathcal{S}_{i,j} = \begin{cases} s_{ij} & \text{if } i > j \text{ or } j = 1, 2 \\ 0 & \text{else} \end{cases}$$

BCJ + absence of 2-particle poles $\Rightarrow$ Hidden zeros

[Bartsch, Brown, Kampf, Oktem, SP, Trnka]

1. Do the BCJ relations guarantee the presence of hidden zeros?
2. What are the relative strengths of these conditions in an EFT expansion?
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Comparing Constraints in NLSM

Take a bootstrap approach: Can I construct a local 6-point NLSM amplitude with a particular mass dimension that satisfy the constraints?

$\mathcal{O}(p^{\#})$	2	4	6	8	10	12	14
Alder zero	1	2	10	29	78	203	461
Hidden zeros	1	1	5	13	41	112	282
BCJ satisfying	1	0	1	1	2	4	7

$$(\text{BCJ satisfying}) \subset (\text{Hidden zeros}) \subset (\text{Adler zero})$$

- After imposing factorization near zeros, (BCJ satisfying) is still a subset of (Hidden zeros)
- Adler $\not\Rightarrow$ C_2 zero
  Exception: 4 dimensions due to the Gram determinant

$$G(12345) \xrightarrow{C_2} s_{12}^2 s_{34} s_{35} s_{45}$$

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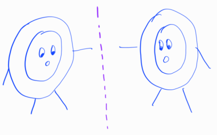

Double Zero Six

Remember that the KLT relation at 6-point:

$$\mathcal{M}_6 = \sum_{\alpha\beta} A_6[162\alpha(345)]\ S[\alpha|\beta]\ A_6[1\beta(345)26]$$

where $S[\alpha|\beta]$ is the KLT kernel. Consider the last row of the matrix

$$\begin{bmatrix} s_{13}(s_{14}+s_{34})(s_{15}+s_{35}+s_{45}) & s_{13}(s_{14}+s_{34})(s_{15}+s_{35}) & s_{13}s_{14}(s_{15}+s_{35}+s_{45}) & s_{13}s_{14}(s_{15}+s_{45}) & s_{13}s_{15}(s_{14}+s_{34}) & s_{13}s_{14}s_{15} \end{bmatrix}$$

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$$\begin{bmatrix} s_{13}(s_{14}+s_{34})(s_{15}+s_{35}+s_{45}) & s_{13}(s_{14}+s_{34})(s_{15}+s_{35}) & s_{13}s_{14}(s_{15}+s_{35}+s_{45}) & s_{13}s_{14}(s_{15}+s_{45}) & s_{13}s_{15}(s_{14}+s_{34}) & s_{13}s_{14}s_{15} \end{bmatrix}$$

$C_2 = \{s_{13} = s_{14} = s_{15} = 0\}$

Double Zero Six

Remember that the KLT relation at 6-point:

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Thus it is important to choose the correct KLT basis in order to manifest a particular zero.

Still, the BCJ relations imply basis independence and so we expect the zeros to survive through the double copy.

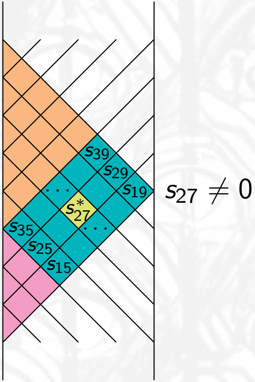
Special Galileon Example

As before, 4-point is trivial,

$$\mathcal{M}_4^{\text{sGal}} = A_4^{\text{NLSM}} \otimes A_4^{\text{NLSM}} = s_{12} s_{13} s_{23} \xrightarrow{s_{13}\to 0} 0$$

At 6-point, it is entirely surprising,

$$\mathcal{M}_6^{\text{sGal}} = A_6^{\text{NLSM}} \otimes A_6^{\text{NLSM}}$$
$$= \frac{s_{12} s_{13} s_{23} s_{45} s_{46} s_{56}}{s_{123}} + \text{perms} - \frac{1}{2} G(12345)$$



The zeros C_2 and C_3 are not manifest, but exist.

Additionally, the amplitude factorizes near these zeros!

$$\mathcal{M}_6^{\text{sGal}}\Big|_{C_3} \xrightarrow{s^*_{25}} \left(\frac{1}{s_{123}} + \frac{1}{s_{345}}\right) \mathcal{M}_4^B \times \mathcal{M}_4^T$$

KLT Relation at n-Point

At n-point,

$$\mathcal{M}_n = \sum_{\alpha\beta} A_n[1mn\alpha]\ S[\alpha|\beta]\ A_n[1\beta mn]$$

where the kernel is

$$S_[i_1, \cdots i_m | j_1 \cdots j_m]_p = \left(\frac{1}{2}\right)^{-m} \prod_{t=1}^{m} \left(p \cdot k_{i_t} + \sum_{q>t}^{m} \theta(i_t, i_q) k_{i_t} \cdot k_{i_q} \right)$$

where $\theta(i_t, i_q) = 1$ if the ordering of i_t and i_q is the opposite in $\{i_1, \cdots i_m\}$ and $\{j_1, \cdots j_m\}$ and 0 if the same.

This manifests the m^{th} type of zero i.e.
KLT + absence of 2-particle poles $\Rightarrow$ hidden zeros

[Bartsch, Brown, Kampf, Oktem, SP, Trnka]

Special Galileon Theory

Since NLSM amplitudes are free of 2-particle poles, amplitudes of the double copy i.e. special Galileon amplitudes

$$\mathcal{M}_n^{\mathrm{sGal}} = A_n^{\mathrm{NLSM}} \otimes A_n^{\mathrm{NLSM}}$$

also have the hidden zeros despite having full permutational symmetry.

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Comparing 6-point constraints in special Galileon theory,

$\mathcal{O}(p^{\#})$	10	12	14	16	18	20	22
$\mathcal{O}(t^3)$ soft behavior	1	0	1	3	10	23	49
Hidden zeros	1	0	1	1	4	6	14
Generated from KLT	1	0	1	1	3	5	10

KLT amp = BCJ amp $\otimes$ BCJ amp

KLT generated $\subset$ Hidden zeros $\subset$ Enhanced soft behaviour

Does the factorization near zeros survive the double copy?

1. Do the BCJ relations guarantee the presence of hidden zeros?
2. What are the relative strengths of these conditions in an EFT expansion?
3. Do these zeros double copy?
4. What about factorization?

Factorization Revisited

Recall that the factorization rule involved a pre-factor:

$$A_4^{\text{Tr}(\phi^3)} = \frac{1}{s_{\{i\}}} + \frac{1}{s_{\{j\}}}$$

Interestingly, for an EFT we get the expected modified rule

$$\mathcal{M}_n^{\text{sGal+h.d.}}\Big|_{C_m} \xrightarrow{s_{ij}^*} A_4^{\text{Tr}(\phi^3)\text{+h.d.}} \times \mathcal{M}_{m+1}^{\text{sGal+h.d.}} \times \mathcal{M}_{n-m+1}^{\text{sGal+h.d.}}$$

where the pre-factor is a particular correction to $\text{Tr}(\phi^3)$,

$$A_4^{\text{Tr}(\phi^3)\text{+h.d.}} = \Lambda(\frac{1}{s} + \frac{1}{t}) - \frac{1}{\Lambda}t + \frac{1}{\Lambda^2}t^2 + \frac{1}{\Lambda^3}t(7t^2 - us) + \cdots$$

[Brown, Kampf, Oktem, SP, Trnka][Chi, Elvang, Herderschee, Jones, SP]

n-point Factorization and the CHY Formalism

Since the factorization rule changes, we need a double copy formalism that differentiates between leading order theories and EFTs.

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Since the factorization rule changes, we need a double copy formalism that differentiates between leading order theories and EFTs.

CHY integral representation of tree-level amplitudes does this.

One can show that the special Galileon CHY integrand factorizes $\Rightarrow$ the amplitude does too.

Further discussions on splitting, scattering equations and CHY are in a recent paper.

[Cao, Dong, He, Shi, Zhu]

Preliminary Results: Spinning Double Copies

- BI+h.d. = NLSM + h.d. $\otimes$ YM + h.d. : A_3^{YM} vanishes on the polarization conditions, leading to no 2-particle poles

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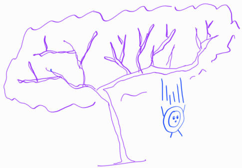
- Dilaton gravity = YM $\otimes$ YM: Only graviton polarizations of the type $\varepsilon_{\mu\nu} = \varepsilon_\mu \varepsilon_\nu$ satisfy the polarization conditions.
- Conformal gravity = $(DF)^2 \otimes (DF)^2$: Same polarization selection rule
- Higher-derivative gravity = YM+h.d. $\otimes$ YM + h.d.: Same polarization selection rule



Next Questions

- To what extent do gravitational theories exhibit zeros and factorization?
- Does including factorization constraints into the double copy bootstrap change the results?
- To what extent do 4-dimensional theories exhibit zeros and factorization? BCFW?
- Can we use these constraints to build amplitudes recursively?
- Do hidden zeros in double copy theories also have geometric origin?

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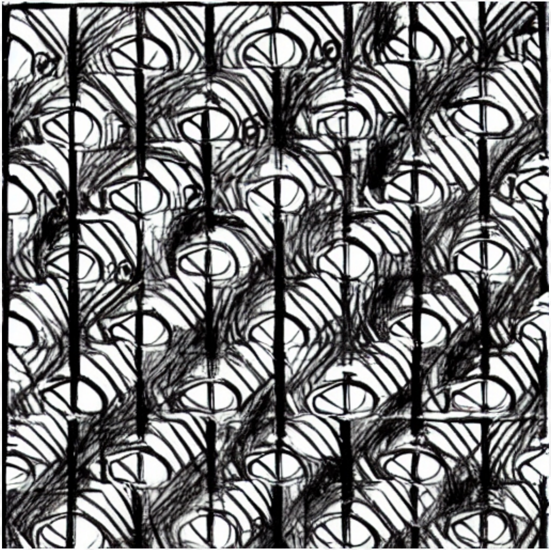
- Can we use these constraints to build amplitudes recursively?
- Do hidden zeros in double copy theories also have geometric origin?
- Loops?

Summary

- The BCJ relations guarantee the presence of hidden zeros in single-copy amplitudes.
- These are strong constraints but are generically weaker than BCJ itself.
- The KLT relations guarantees the presence of hidden zeros in double-copy amplitudes.
- The amplitudes factorize near these zeros despite no physical principle requiring it.
- This should allow us to find new zeros of gravitational amplitudes.



Thank You