Soft Collinear Effective Theory & Collider Physics

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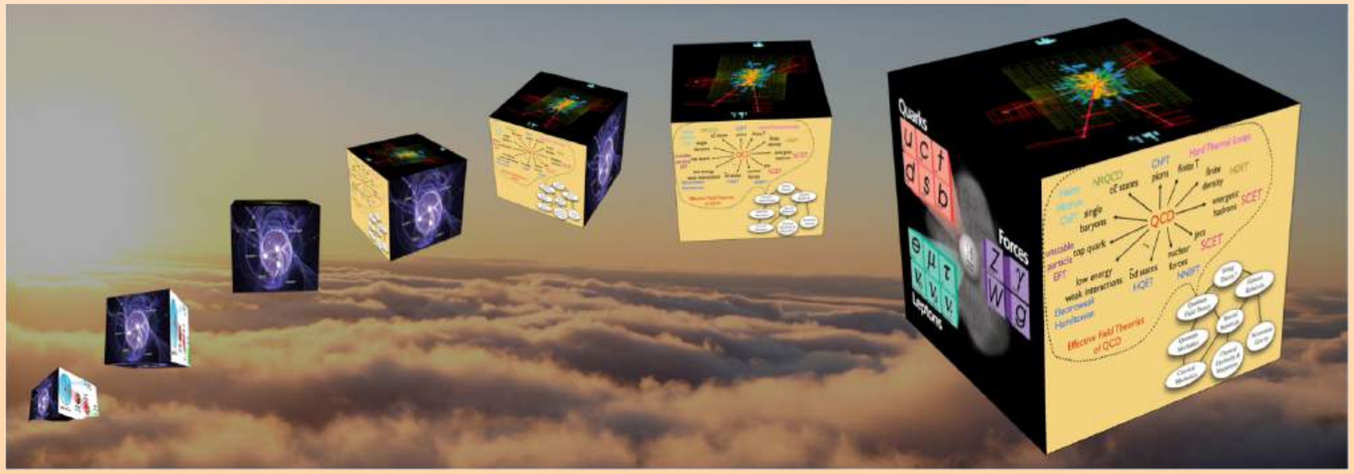
MIT

Amplitudes
IAS
June 11, 2024



Massachusetts Institute of Technology

Soft Collinear Effective Theory (SCET)

"EFT for Collider Physics"

EFT for hard interactions which produce energetic (collinear) and soft particles.

Bauer, Fleming, Luke, Pirjol, IS `00, `01

Higgs production, DY, …

Jet Physics

Jet Substructure

B-Decays and CP violation

Quarkonia Production

TMDs / Nuclear Physics

(Heavy Ion collisions)

Infrared Structure of Gauge Theory

Factorization for Collider Processes

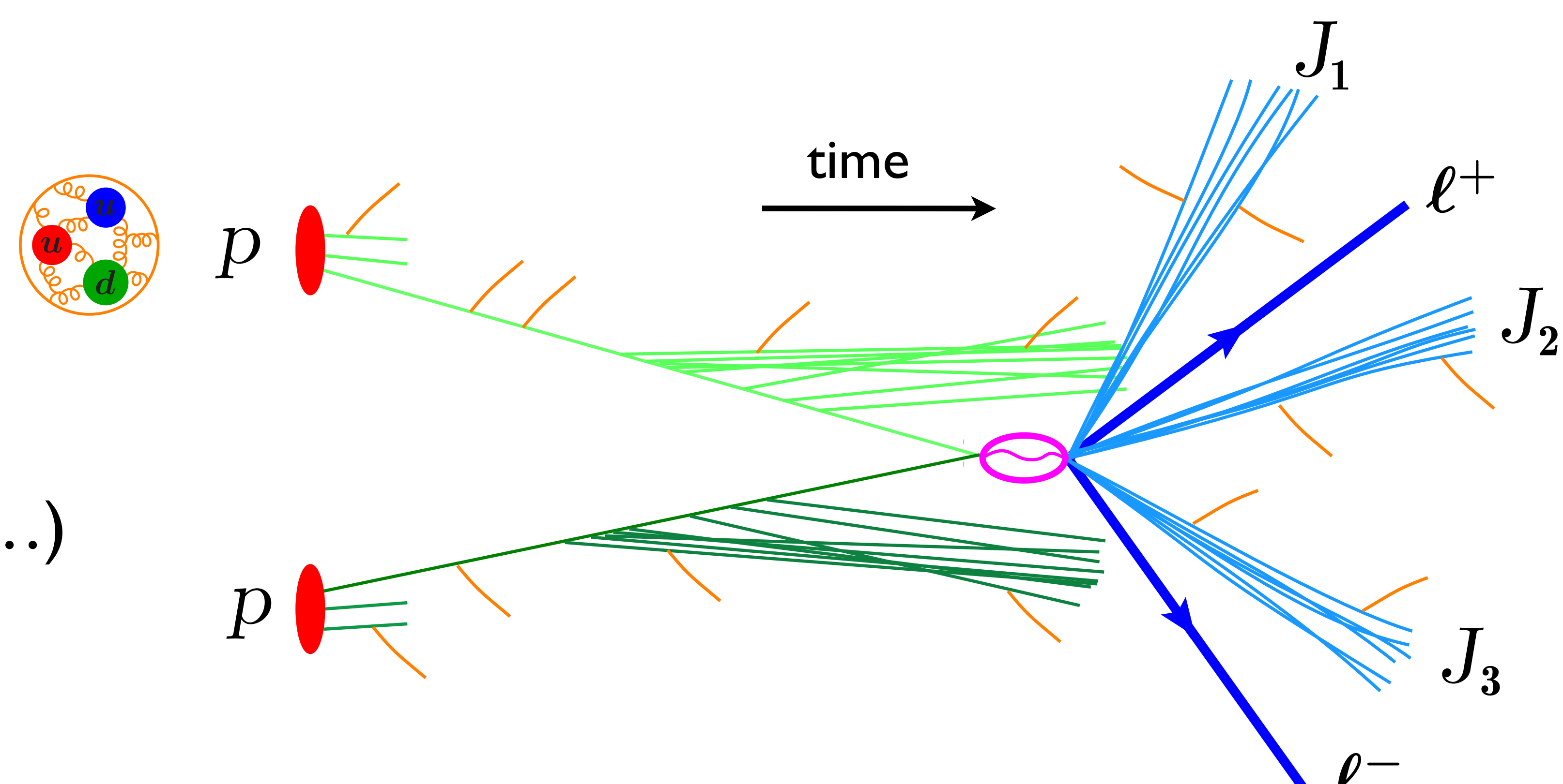
Higher order Resummation

Gauge theory at Subleading Power

Subtractions for Fixed Order QCD

High Energy Limit / Regge phenomena

builds on extensive past literature
(CSS factorization, exclusive fact, …)



Outline

SCET Formalism:

- Introduction to SCET & Factorization
- Wilson Lines, Large Logs and Renormalization Group
- Forward Scattering & Factorization Violation

Collider Physics Applications:

- High Precision Resummation e^+e^-
- High Precision Resummation pp
- Power Corrections
- Amplitudes in the Regge Limit

Non-perturbative Factorization:

parton distributions

$$d\sigma = f_a f_b \otimes \hat{\sigma} \otimes F$$

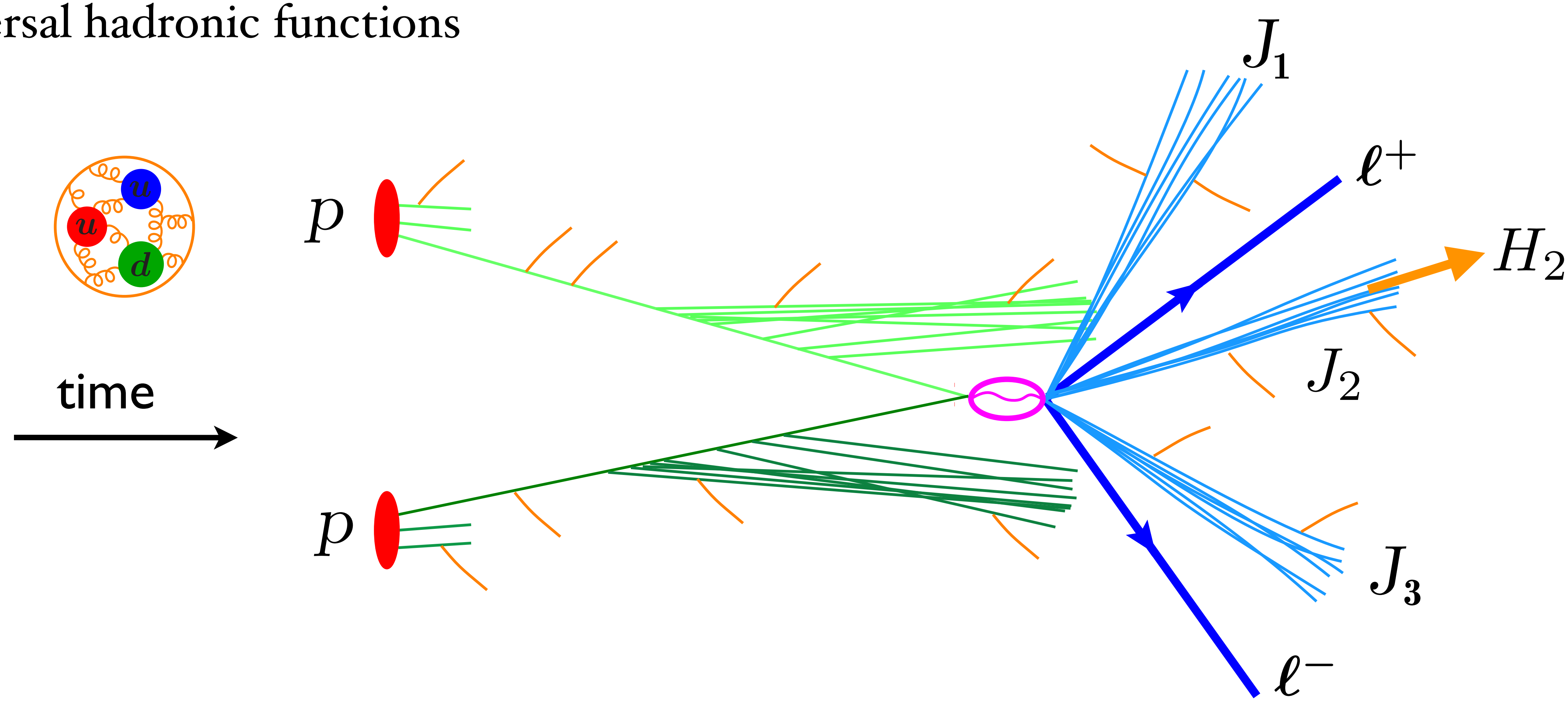
hadronization:

fragmentation fns.,
soft hadronization, ...

(QFT operators)

perturbative cross section

universal hadronic dynamics
via
universal hadronic functions



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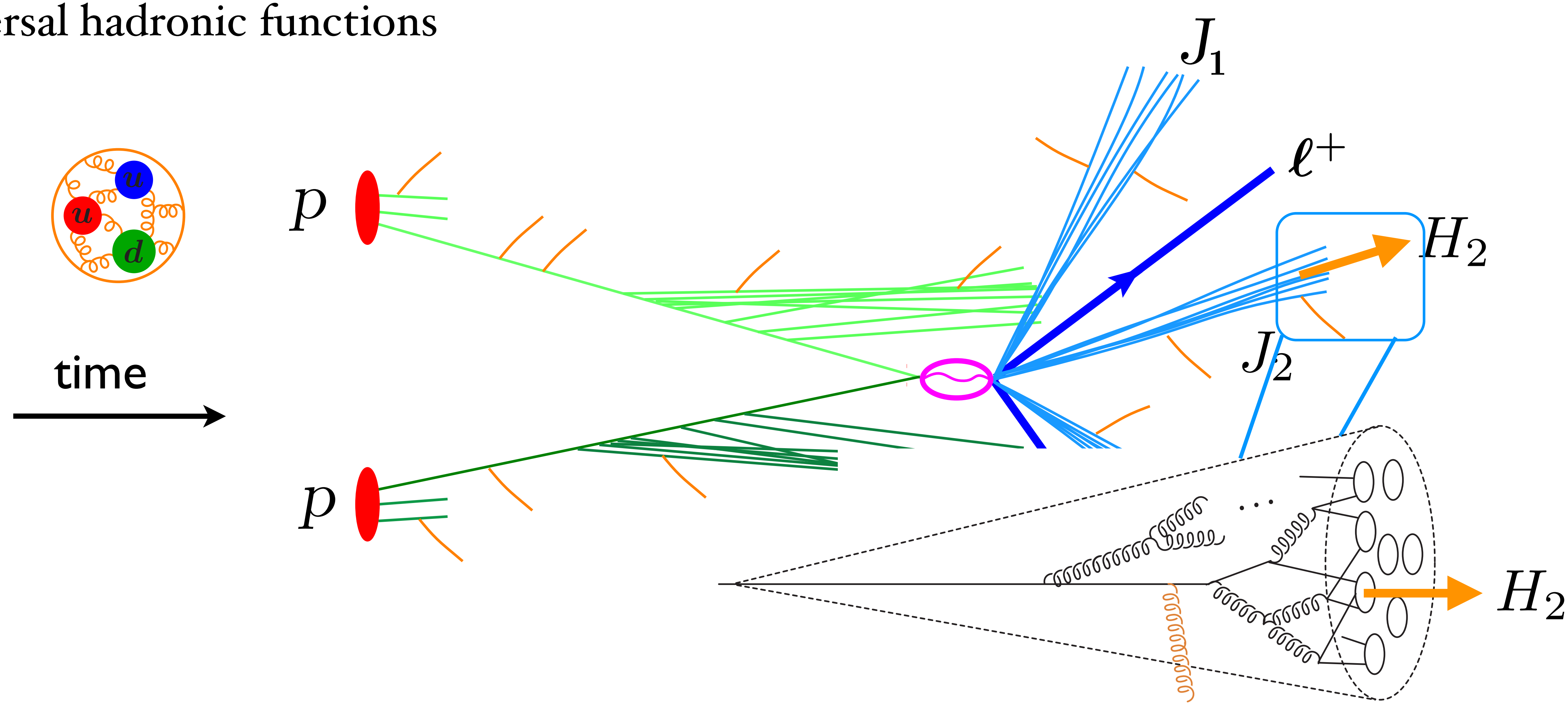
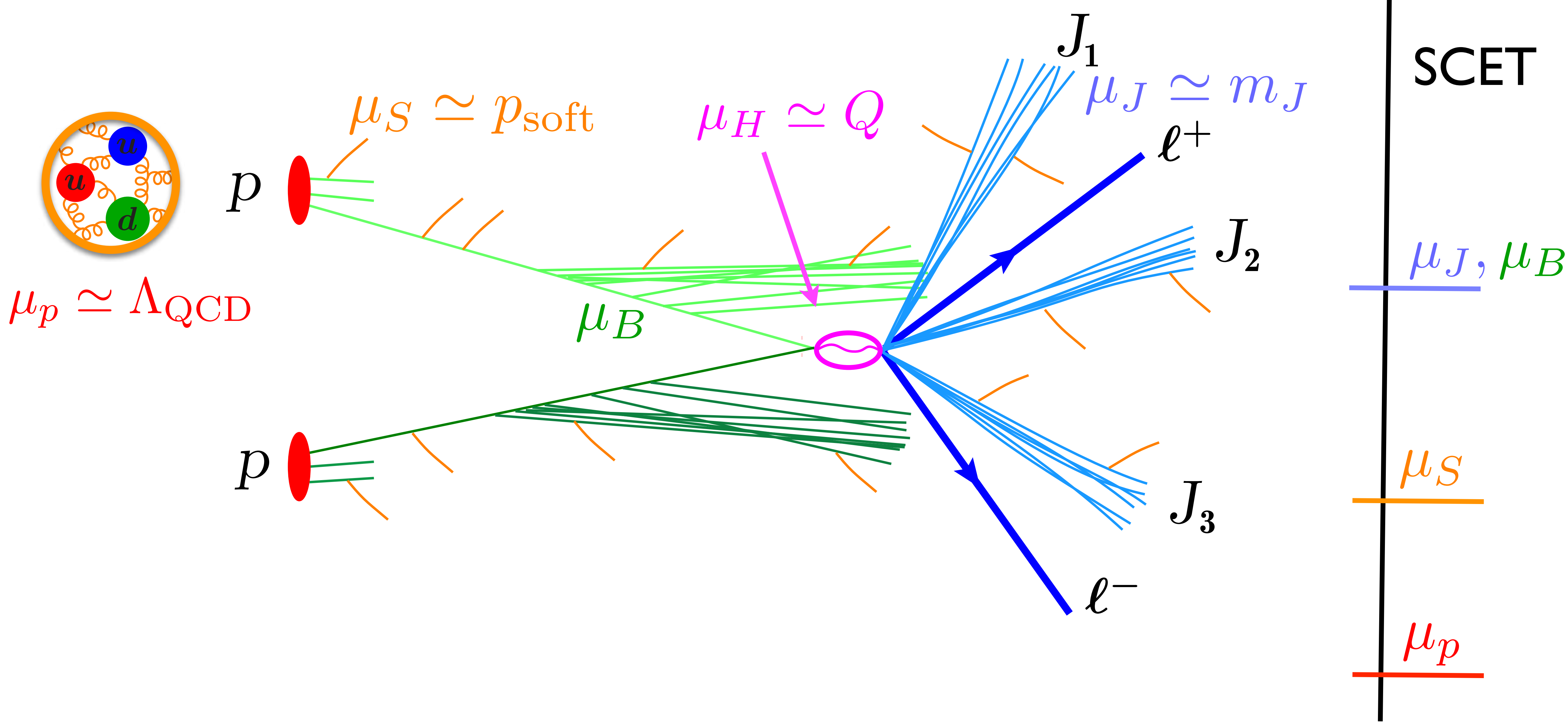


Perturbative Factorization: for multi-scale problems with N jets

beam hard jet pert. soft

$$\hat{\sigma}_{\rm fact} = \mathcal{I}_a \mathcal{I}_b \otimes H \otimes \prod_i J_i \otimes S$$

μ_B μ_H μ_J μ_S

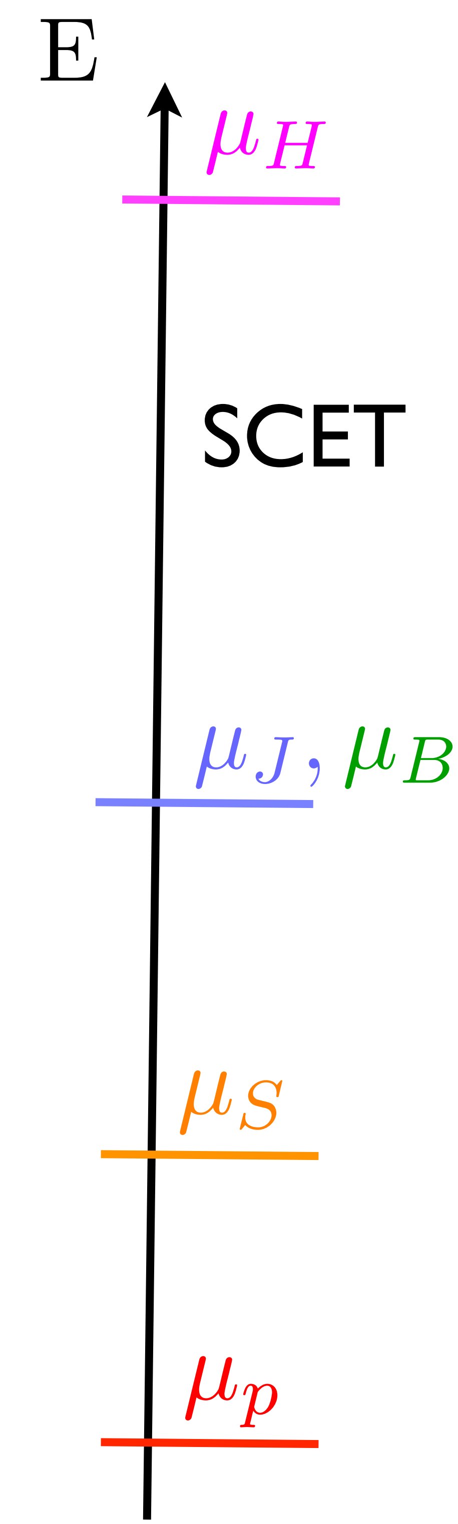


Perturbative Factorization: for multi-scale problems with fixed # jets

beam hard jet pert. soft

$$\hat{\sigma}_{\text{fact}} = \mathcal{I}_a \mathcal{I}_b \otimes H \otimes \prod_i J_i \otimes S$$

μ_B μ_H μ_J μ_S



Perturbative Universality

- H determined by hard process, independent of jet radius, etc.
- J_i , $\mathcal{I}_{a,b}$ splitting and virtual effects for parton i, encode jet dynamics, independent of H — universal collinear dynamics
- S soft radiation, all partons contribute, eikonal Feynman rules — universal soft dynamics

Scale dependence $\longleftrightarrow$ RGE sums up logarithms $\log\left(\frac{\mu_H}{\mu_S}\right), \ldots$

Perturbative QCD Results:

fixed order:

$$\hat{\sigma} = \sigma_0 \left[1 + \alpha_s + \alpha_s^2 + \ldots\right]$$
$$= \mathrm{LO} + \mathrm{NLO} + \mathrm{NNLO} + \ldots$$

SCET anomalous dimensions:

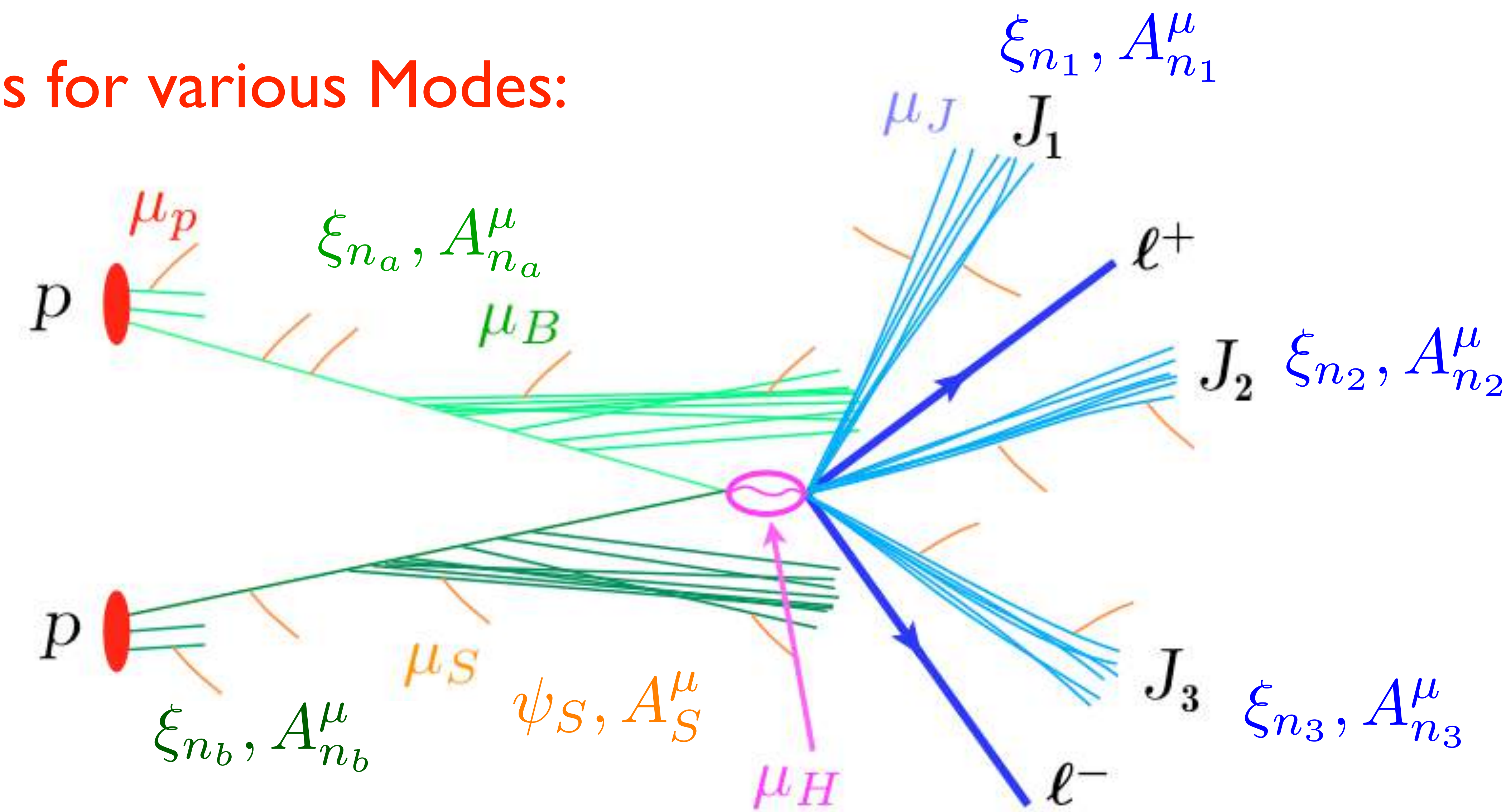
resummation of large (double) logs $L = \log(...)$

$\log\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)$, $\log\left(\frac{p_T}{Q}\right)$, ...

$$\ln \hat{\sigma}(y) = \sum_k L(\alpha_s L)^k + \sum_k (\alpha_s L)^k + \sum_k \alpha_s (\alpha_s L)^k + \sum_k \alpha_s^2 (\alpha_s L)^k + \ldots$$
$$= \mathrm{LL} + \mathrm{NLL} + \mathrm{NNLL} + \mathrm{N^3LL} + \ldots$$

Soft Collinear Effective Theory

Fields for various Modes:



dominant contributions from isolated regions of momentum space

power counting $\lambda \ll 1$

n-collinear

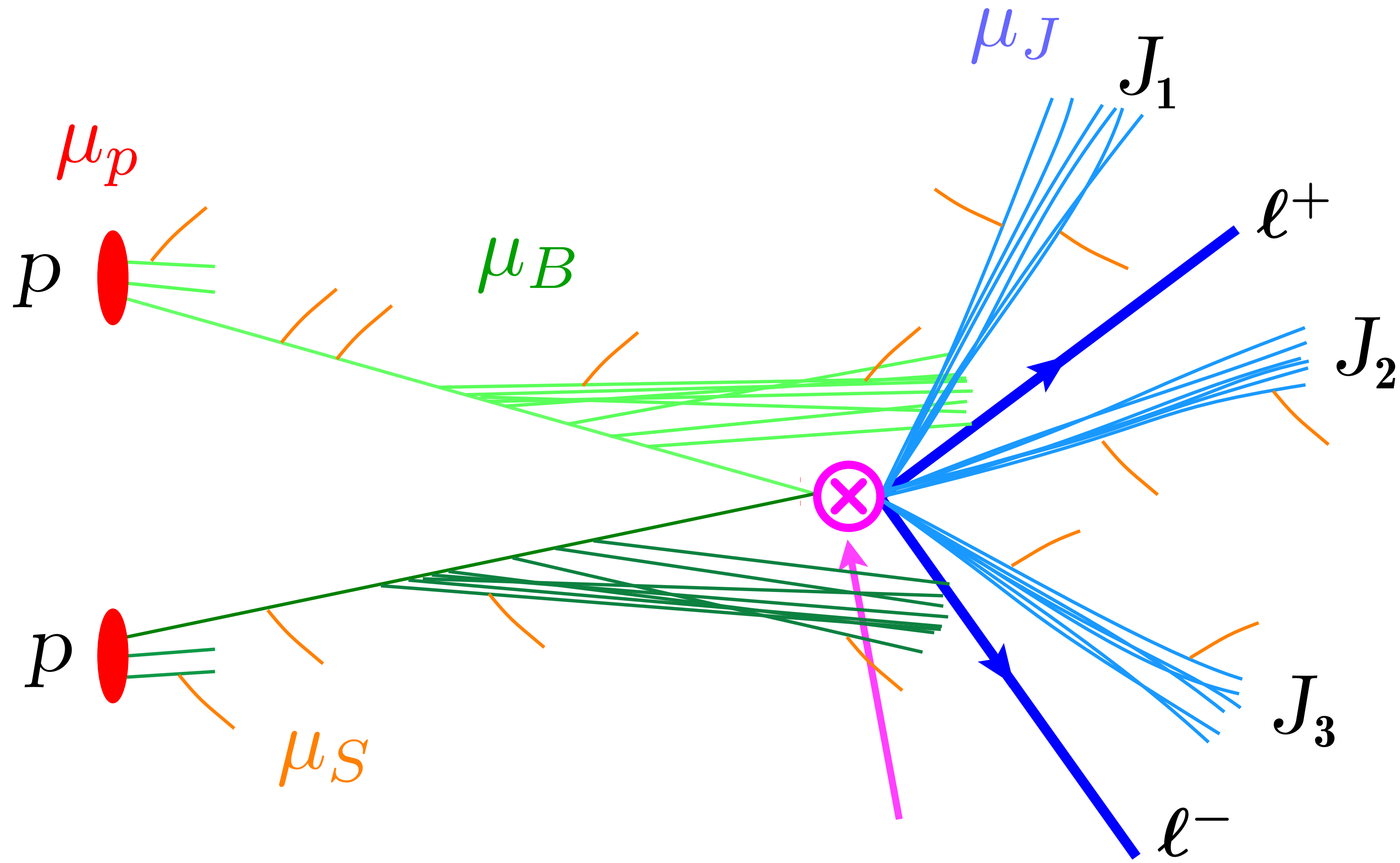
$$(n \cdot p, \bar{n} \cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$$

soft

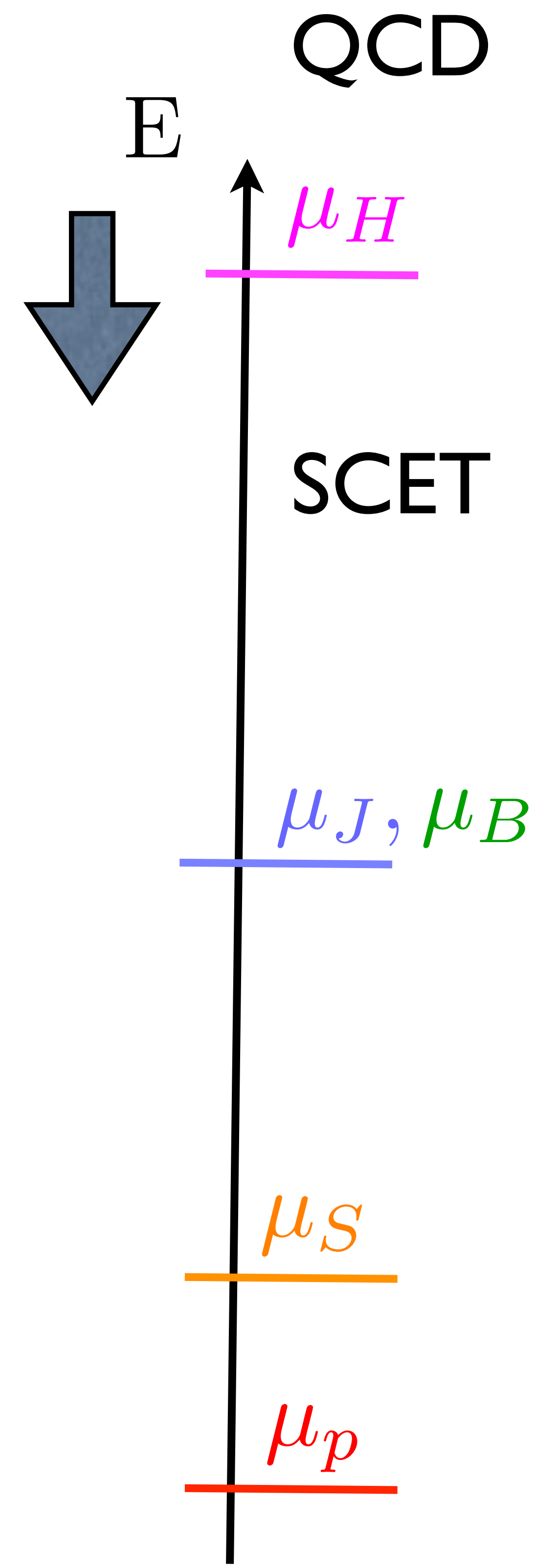
$$(n \cdot p, \bar{n} \cdot p, p_\perp) \sim Q\lambda^k$$

$k \geq 1$

Key Simplifying Principle is to Exploit the Hierarchy of Scales



Wilson coefficients
+ operators at μ_H

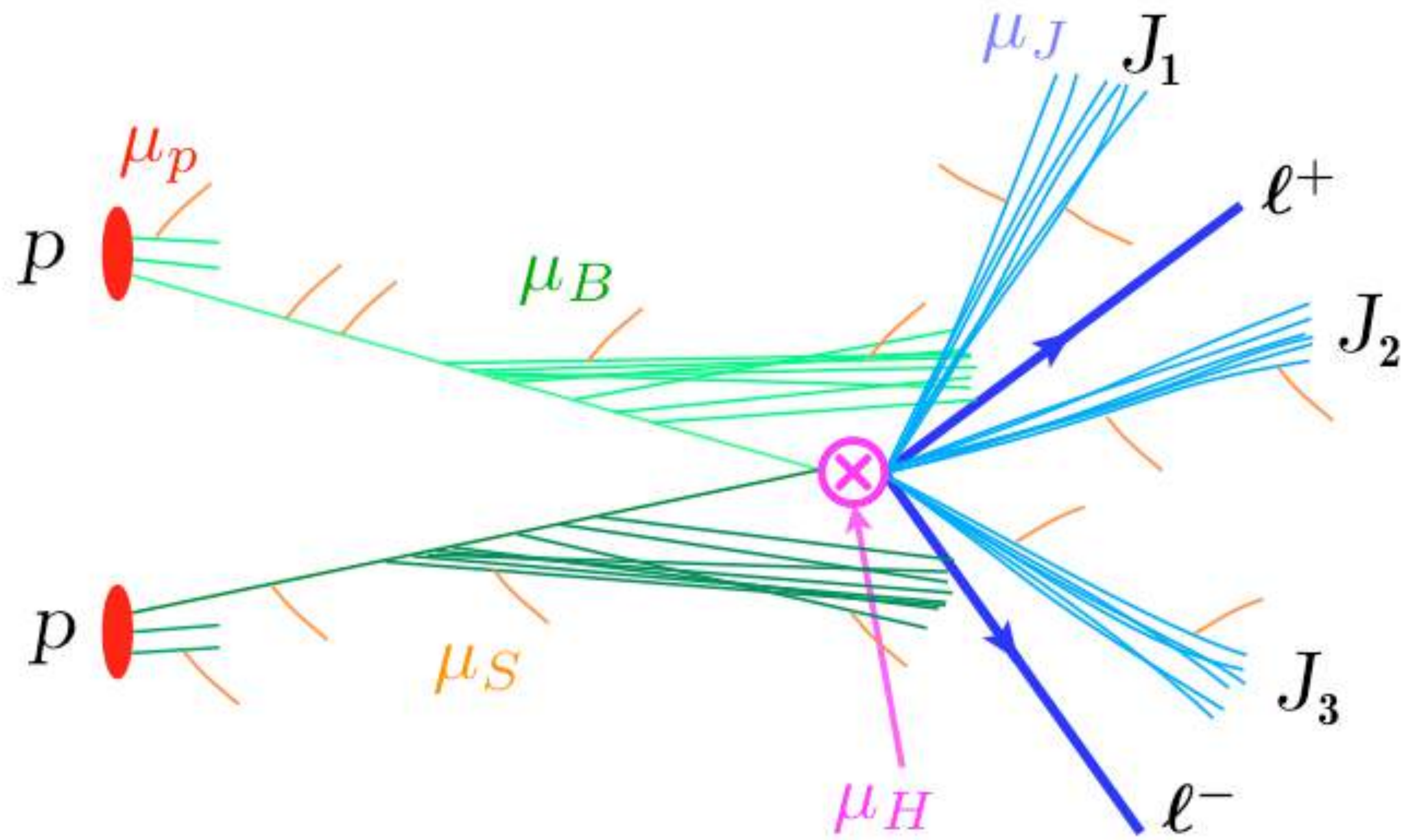


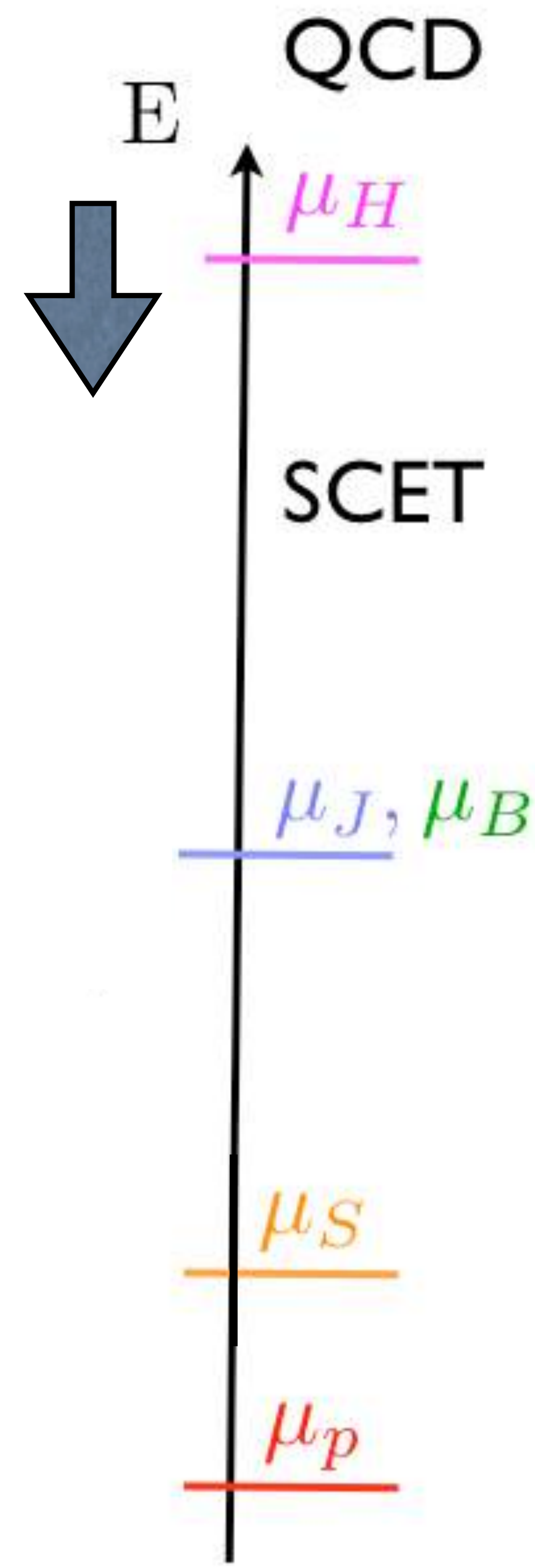
$$\mathcal{L} = \sum_i C_i O_i$$

Amplitudes!

$$d\sigma = \int (\text{phase space}) \left| \sum_i C_i \langle O_i \rangle \right|^2 = \sum_j H_j \otimes (\text{longer distance dynamics})_j$$

Hard-collinear factorization



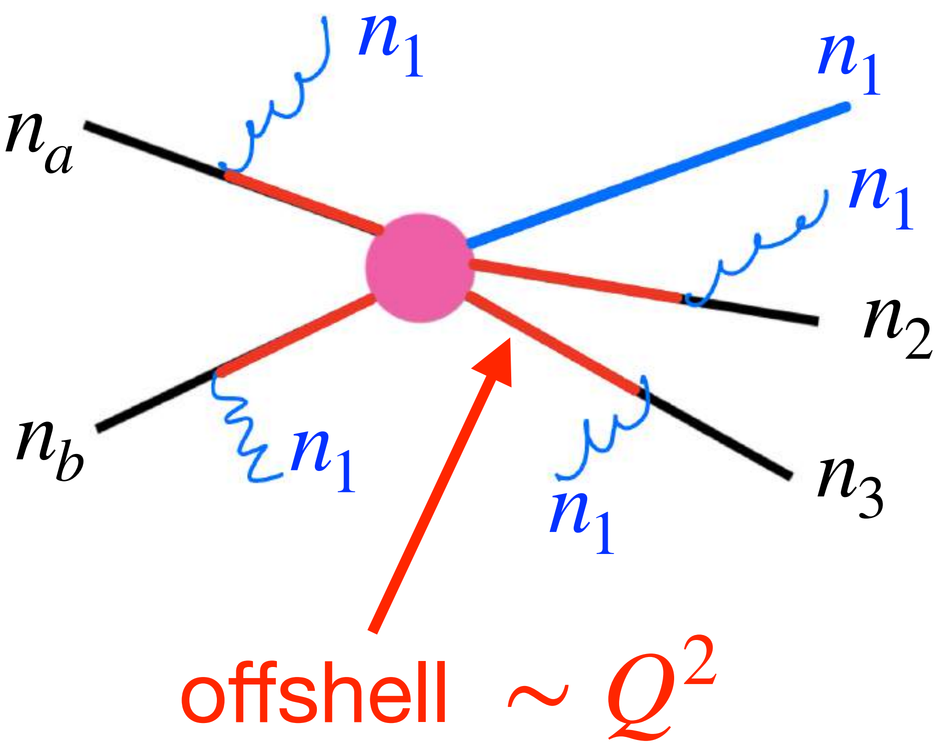


Hard scale operators from building block fields:

$$\mathcal{O} = (\mathcal{B}_{n_a\perp})(\mathcal{B}_{n_b\perp})(\mathcal{B}_{n_1\perp})(\bar{\chi}_{n_2})(\chi_{n_3})$$

"quark jet" $\chi_n = (W_n^\dagger \xi_n)$

"gluon jet" $\mathcal{B}_{n\perp}^\mu = \frac{1}{g}[W_n^\dagger i D_\perp^\mu W_n]$ or $\mathcal{B}_{n\perp}^{A\mu} = \frac{1}{g}\frac{1}{\bar{n}\cdot\partial_n}\bar{n}_\nu G_n^{B\nu\mu}\mathcal{W}_n^{BA}$



attachments of n_1-collinear gluons to $n_{i\neq 1}$

Wilson line

$$W^{\dagger}_{n_1} = P\exp\left(ig\int_0^{\infty} ds\, \bar{n}_1\cdot A_{n_1}(\bar{n}_1 s)\right)$$

$$n_1\cdot\bar{n}_1 = 2$$

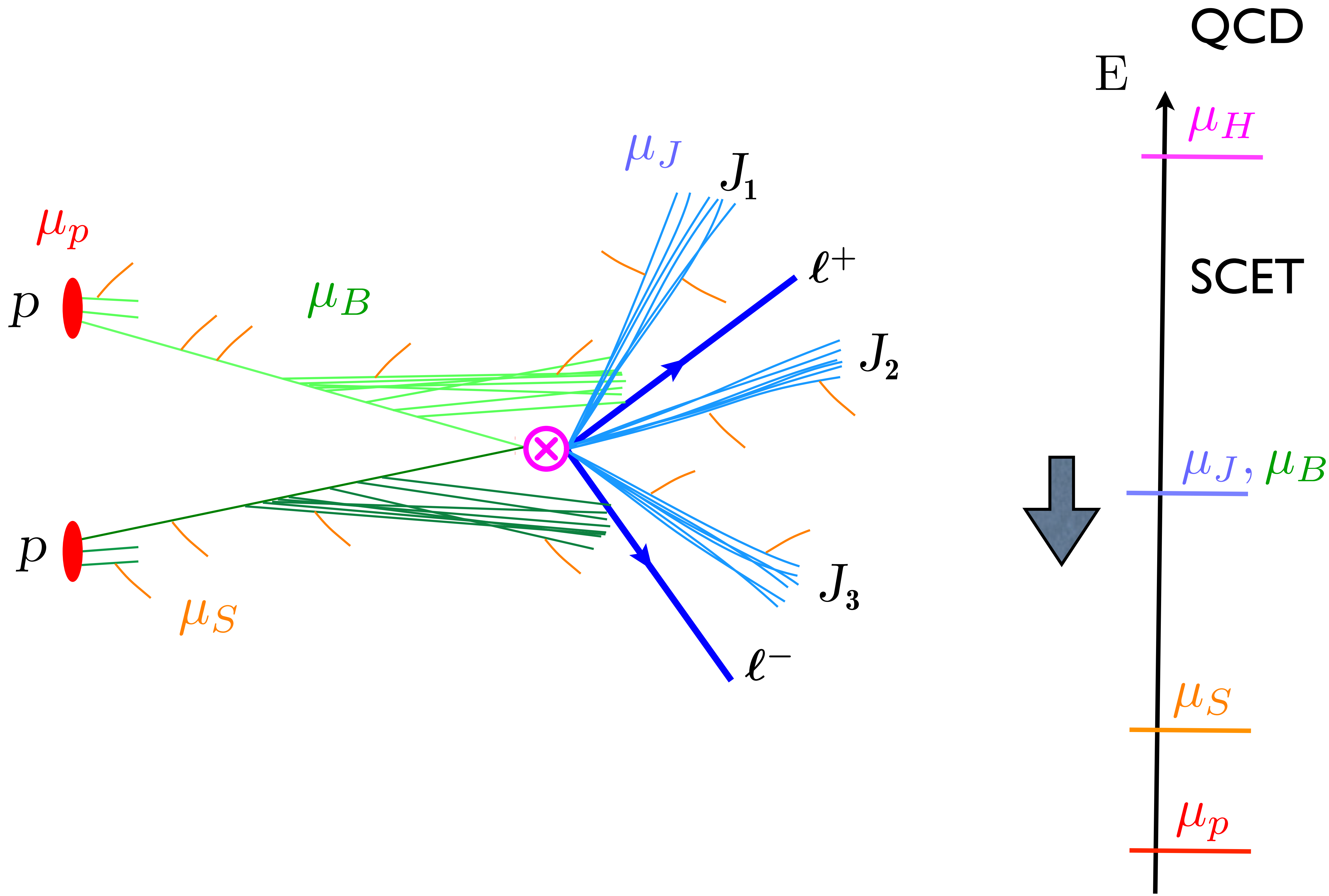
building block

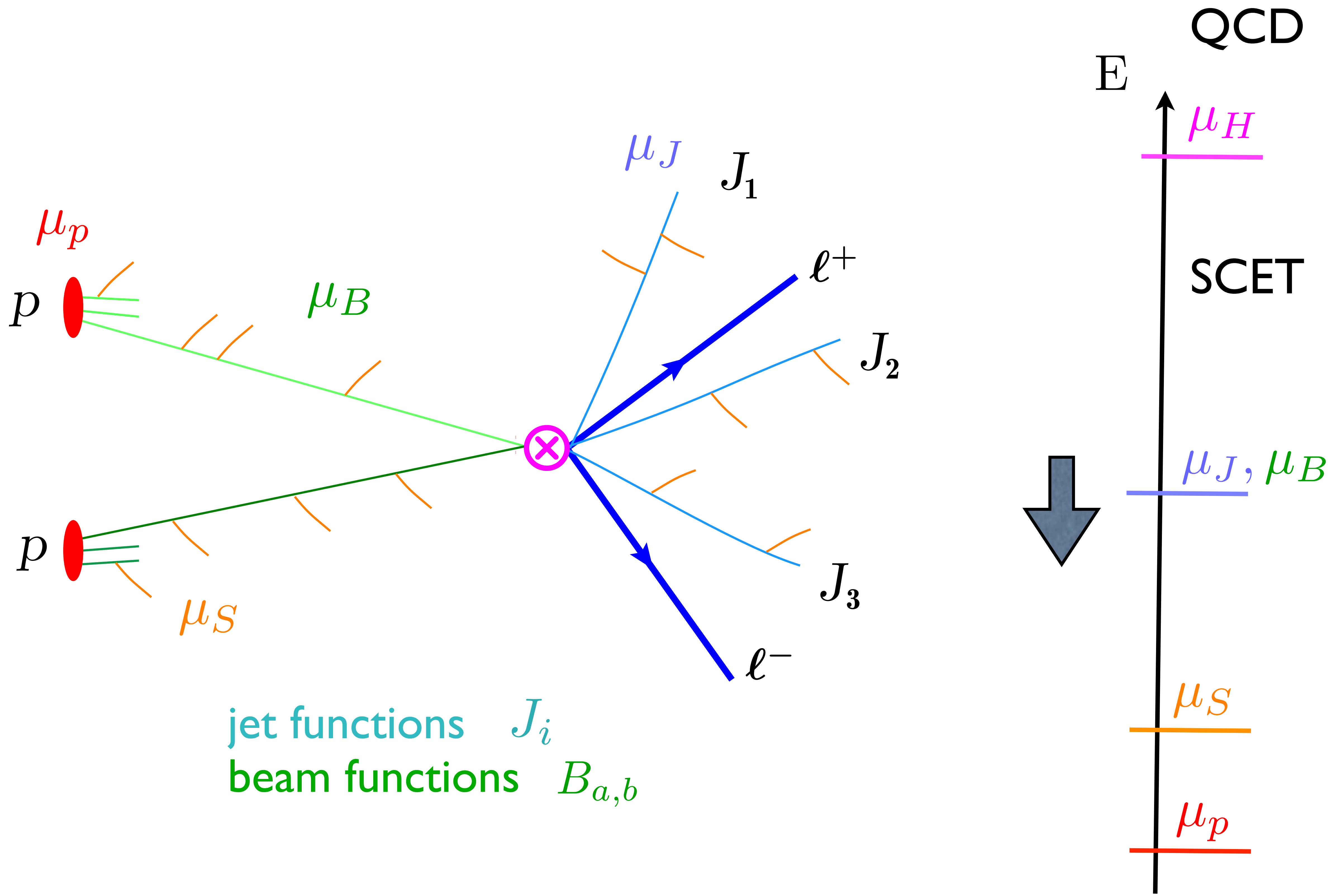
$$\chi_{n_1} = W^{\dagger}_{n_1}\xi_{n_1}$$

Often convenient to use helicity basis for building blocks to make it easier to match to amplitude calculations

$\mathcal{B}^{\pm}_{n\perp}$ $\qquad$ $J^{\pm}_{n\bar{n}}$

see 1508.02397

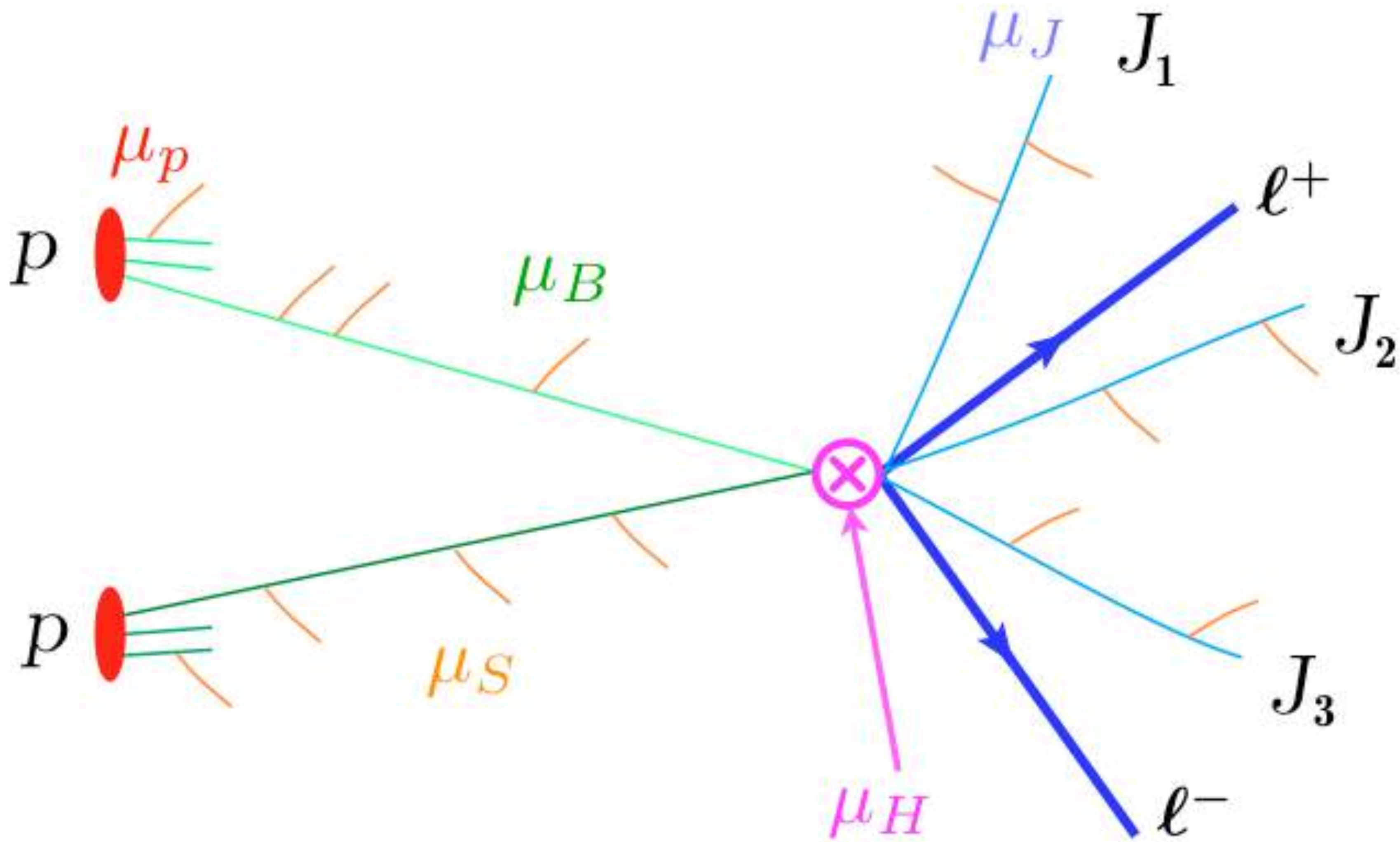

QCD
E
μ_H
μ_J
J_1
μ_p
p
μ_B
ℓ^+
SCET
J_2
μ_J, μ_B
p
J_3
μ_S
ℓ^-
μ_S
μ_p

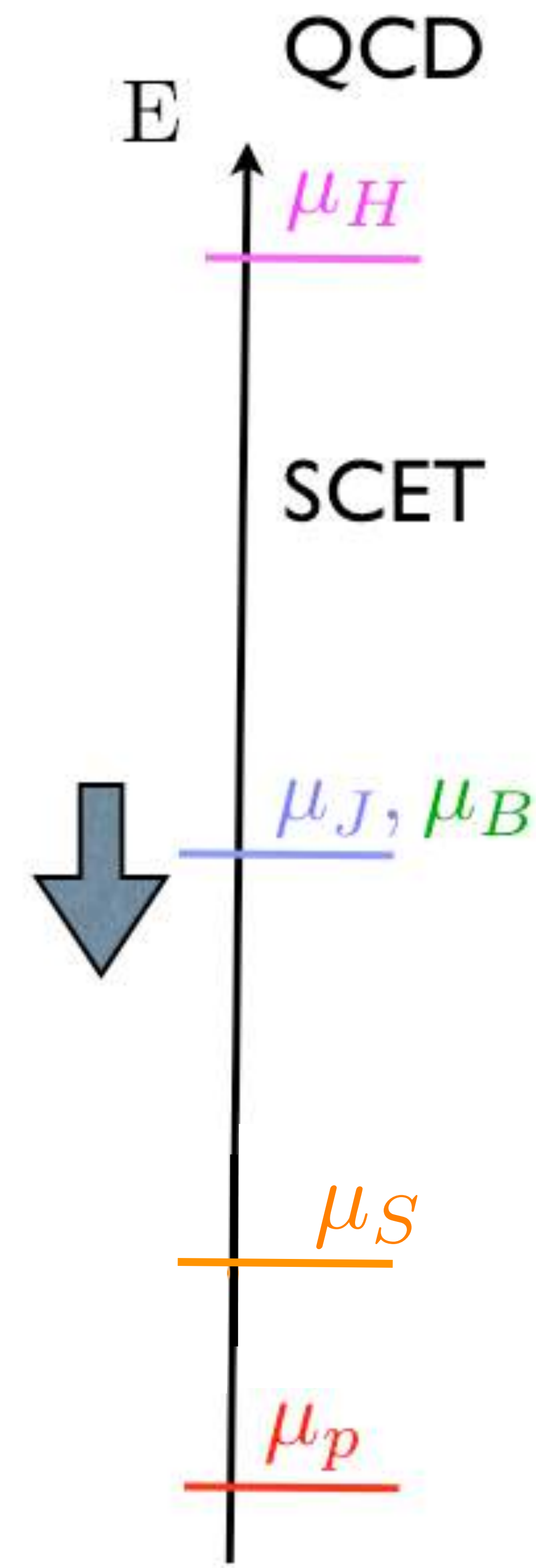


$$d\sigma = B_{a,b} \otimes H_j \otimes \prod_i J_i \otimes (\text{longer distance dynamics})$$

Soft-collinear factorization

Soft radiation knows only about bulk properties of radiation in the jets (color & direction)





Soft Wilson lines: $\left(S_{n_a} S_{n_b} S_{n_1} S_{n_2} S_{n_3}\right)$

Soft function S =
Matrix Elements of Soft Wilson Lines

Leading Power Glauber Lagrangian:

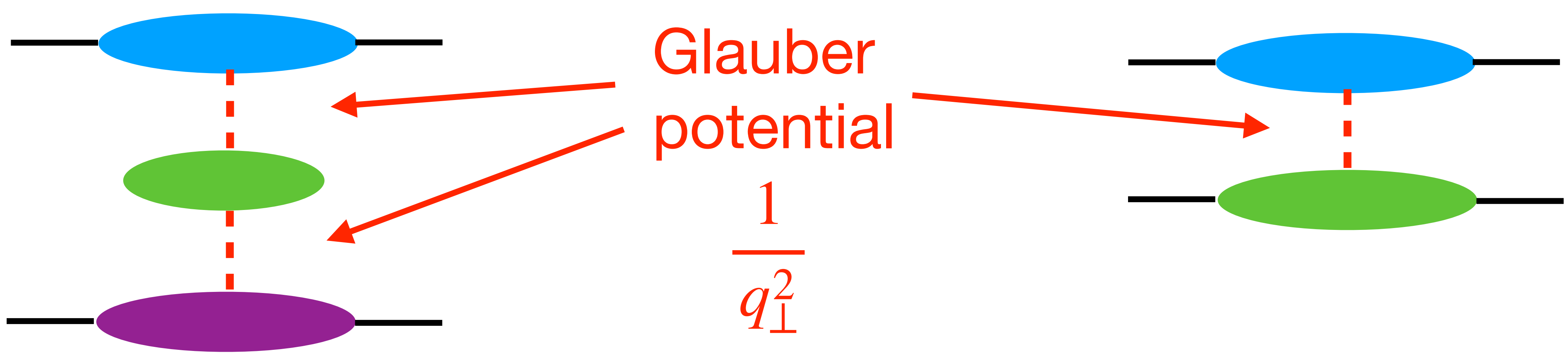
Rothstein, IS (2016)

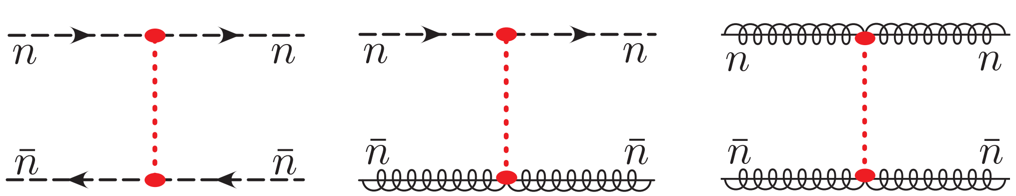
$$\mathcal{L}_G^{(0)} = \sum_{n,\bar{n}} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jC} + \sum_{n} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B}$$

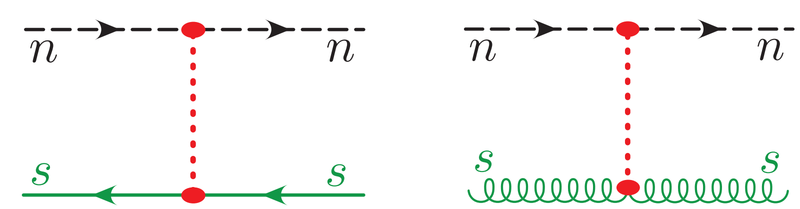
(3 rapidity sectors) (2 rapidity sectors)

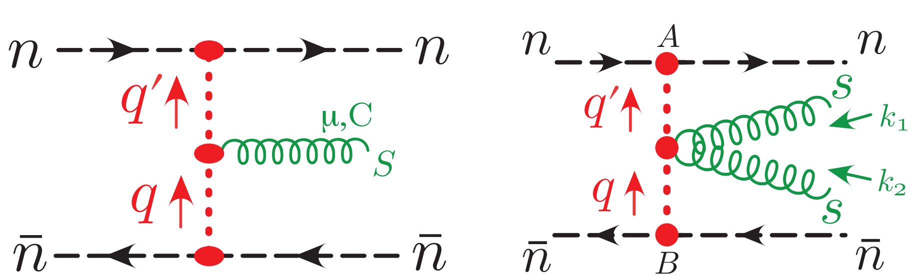
n-$\bar{n}$ fwd. scattering

n-s fwd. scattering









...

...

$s \gg |t|$

constructed from top-down matching: QCD → SCET

Lipatov vertex

Leading Power Glauber Lagrangian:

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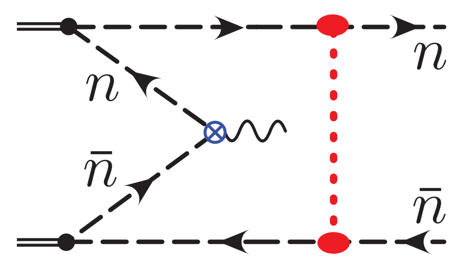

In Hard Scattering

- Glauber Lagrangian can spoil factorization by coupling sectors in a non-factorizable manner. (Describes ONLY non-trivial fact. violation.)
- Its effects often cancel due to unitarity (summing over inclusive enough final states) or by exponentiating into an unobservable phase.
- Lagrangian can be used to systematically study non-factorizable Collider physics phenomena. (eg. super leading logs, "underlying event")

In Forward Scattering $s \gg |t|$

- Describes the leading scattering process. Old and well studied limit.
- SCET provides top-down EFT description, new tools

SCET Lagrangian at leading power

$$\mathcal{L} = \mathcal{L}^{(0)}_{\rm dyn} + \mathcal{L}^{(0)}_{\rm hard} + \mathcal{L}^{(0)}_{\rm G}$$

Dynamics of infrared modes

Hard Scattering operators (typically once)

Glauber gluon exchange (only factorization violating term)

- $\mathcal{L}^{(0)}_{\rm hard} = \sum_i C_i^{(0)} \mathcal{O}_i^{(0)}$ Leading operators for a given process
- $\mathcal{L}^{(0)}_{\rm dyn} = \sum_n \mathcal{L}^{(0)}_n + \mathcal{L}^{(0)}_{\rm soft}$ Collinear and Soft dynamics (Factorizes after soft-collinear decoupling)

$\cancel{\mathcal{L}^{(0)}_{\rm G}}$ ➡ Copies of QCD* give dynamics in different sectors, with hard operators providing the only connection between sectors

SCET Lagrangian at leading power

$$\mathcal{L} = \mathcal{L}^{(0)}_{\rm dyn} + \mathcal{L}^{(0)}_{\rm hard} + \mathcal{L}^{(0)}_{\rm G}$$

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Factorization →

$$d\sigma = f_a f_b \otimes \hat{\sigma} \otimes F$$

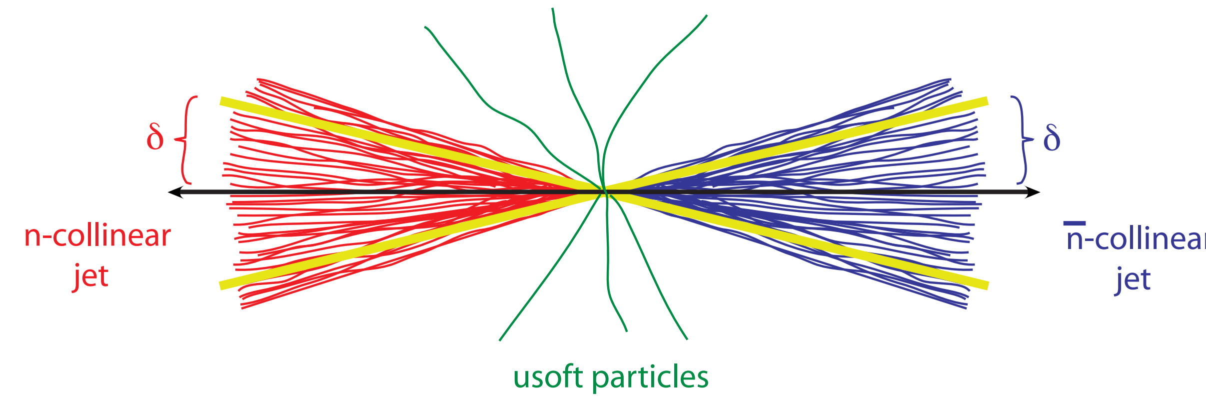
$$\hat{\sigma}_{\rm fact} = \mathcal{I}_a \mathcal{I}_b \otimes H \otimes \prod_i J_i \otimes S$$

Applications

Dijet production $e^+e^- \to 2$ jets

thrust $\tau = 1 - T$

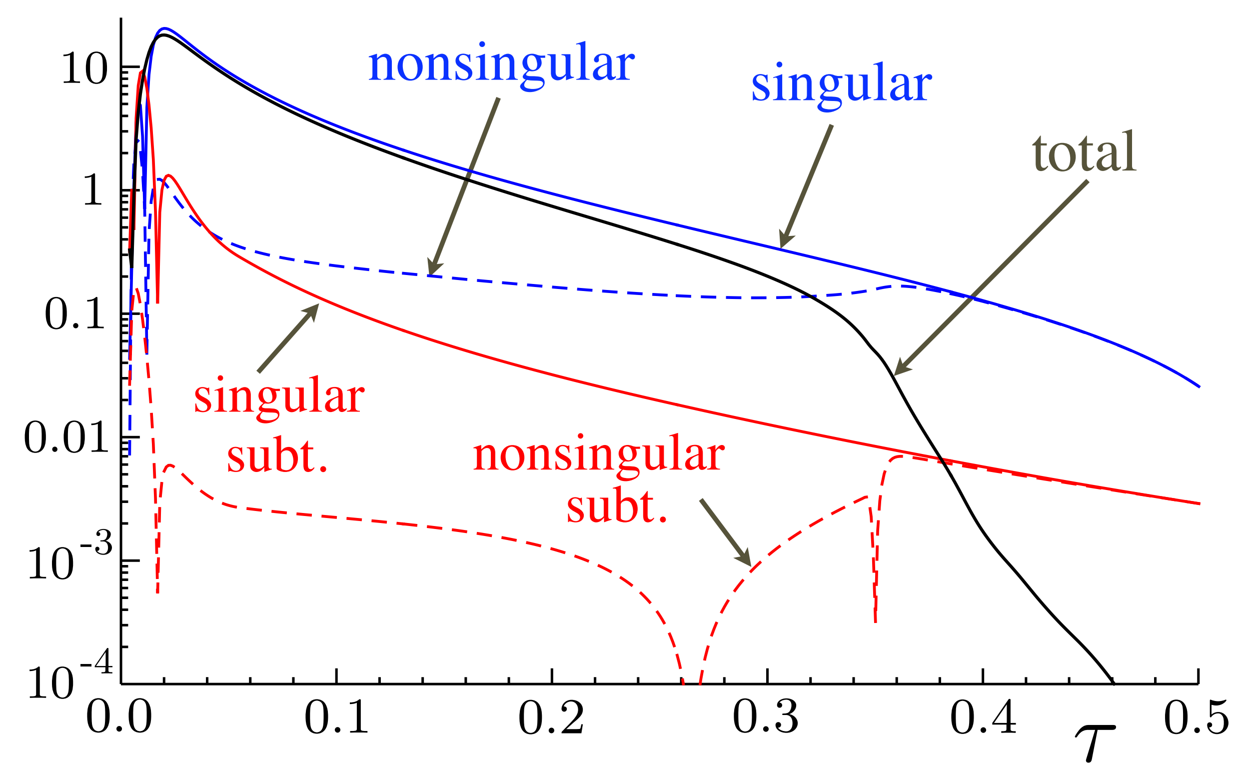
$\tau \ll 1$



$$\frac{d\sigma}{d\tau} = \sigma_0 H(Q,\mu)\, Q \int d\ell\, d\ell'\, J_T(Q^2\tau - Q\ell, \mu)\, S_T(\ell - \ell', \mu)\, F(\ell')$$

hard function | jet functions (combined) | perturbative soft function | non-perturbative soft function

$$+\frac{d\sigma^{\rm nonsingular}}{d\tau}$$



$\alpha_s(m_Z)$ from Thrust

$e^+e^- \to$ jets

Aim at 1% precision

Becher, Schwartz `09

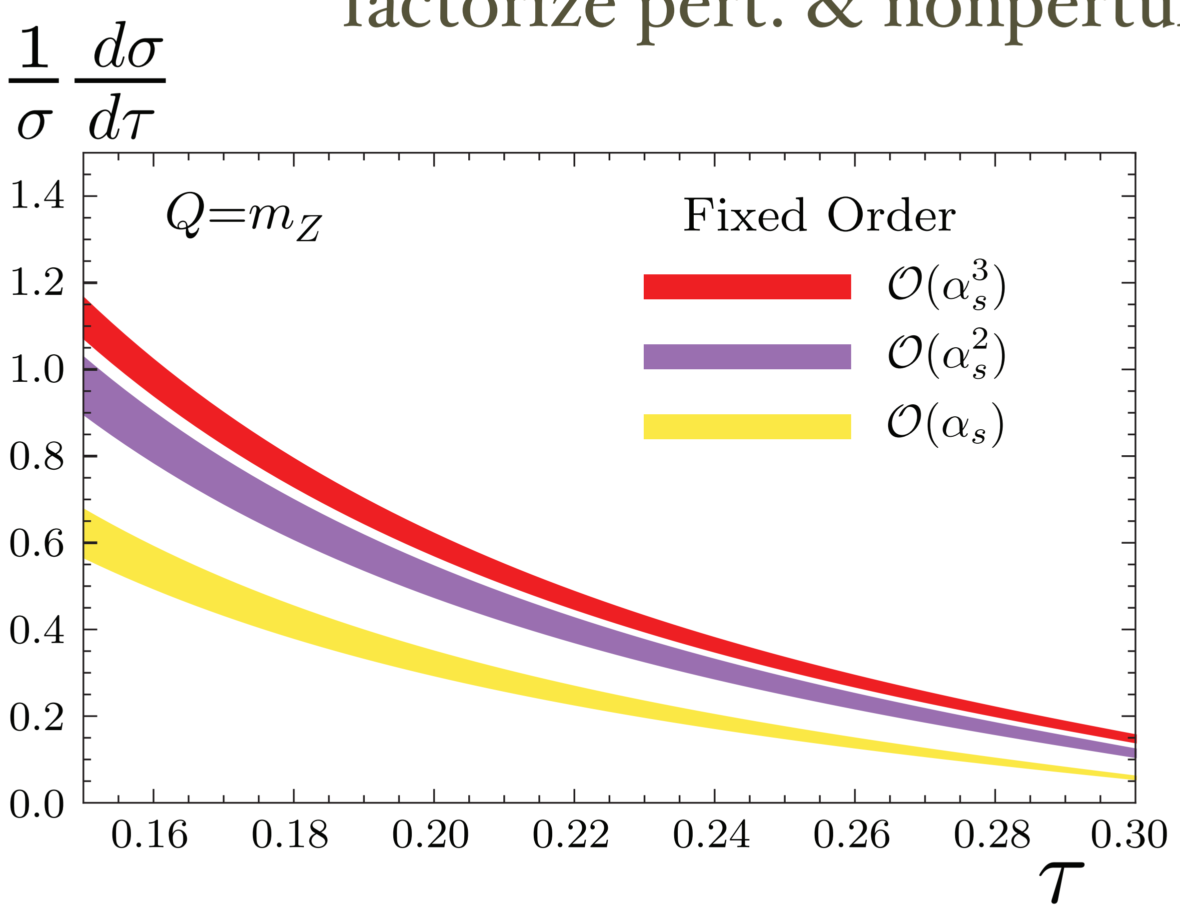
Abbate, Fickinger, Hoang, Mateu, I.S. `10

- $\mathcal{O}(\alpha_s^3)$ + $\mathrm{N^3LL}$ + $\frac{\Omega_1}{Q\tau}$ power correction + renormalon subtractions, R-RGE

  + full treatment of {peak, tail, multijet} + QED effects + b-mass effects + global fit, various Q's

with $\mathcal{O}(\alpha_s^3)$ from Gehrmann-De Ridder et al. & Weinzierl (`07-`09)

factorize pert. & nonperturbative soft effects: $S = S^{\mathrm{pert}} \otimes S^{\mathrm{mod}}$



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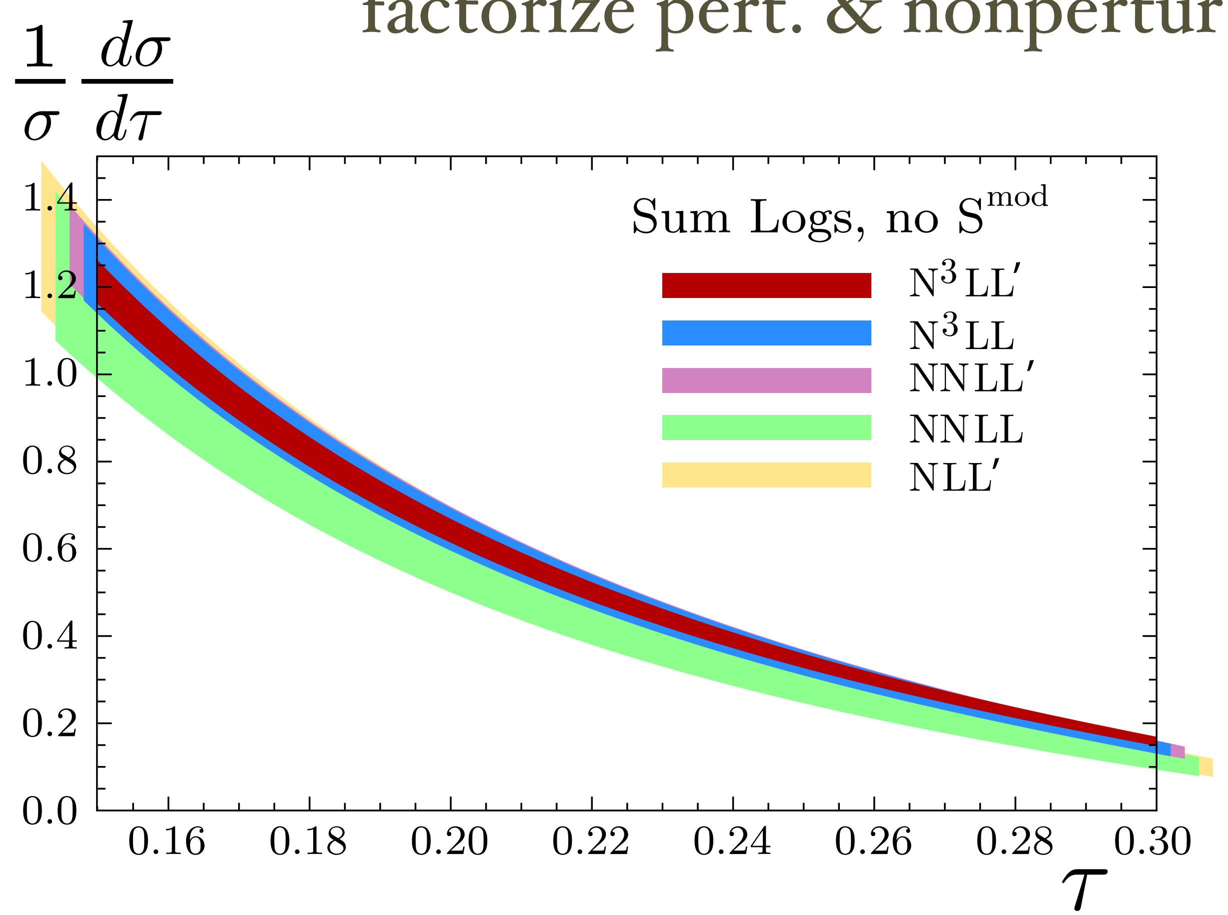

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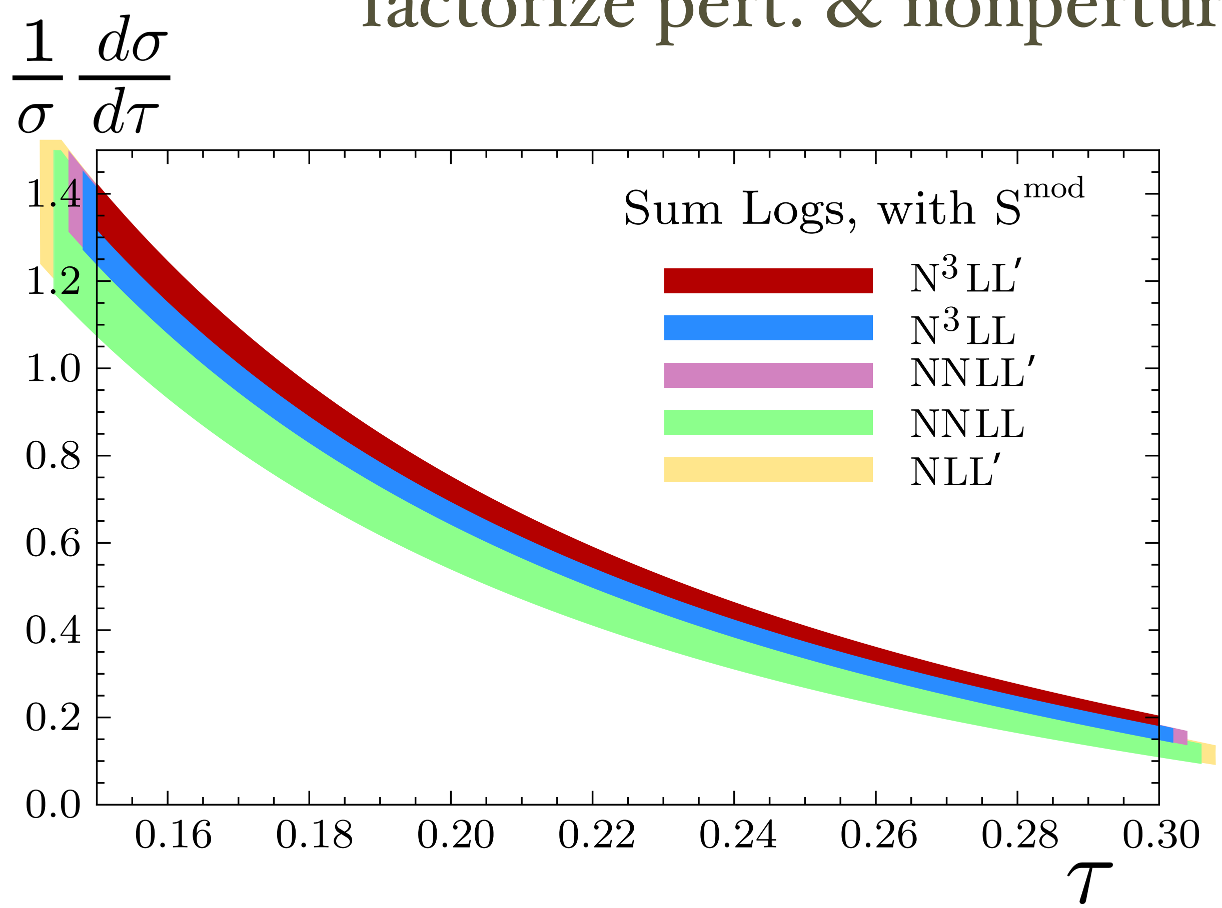

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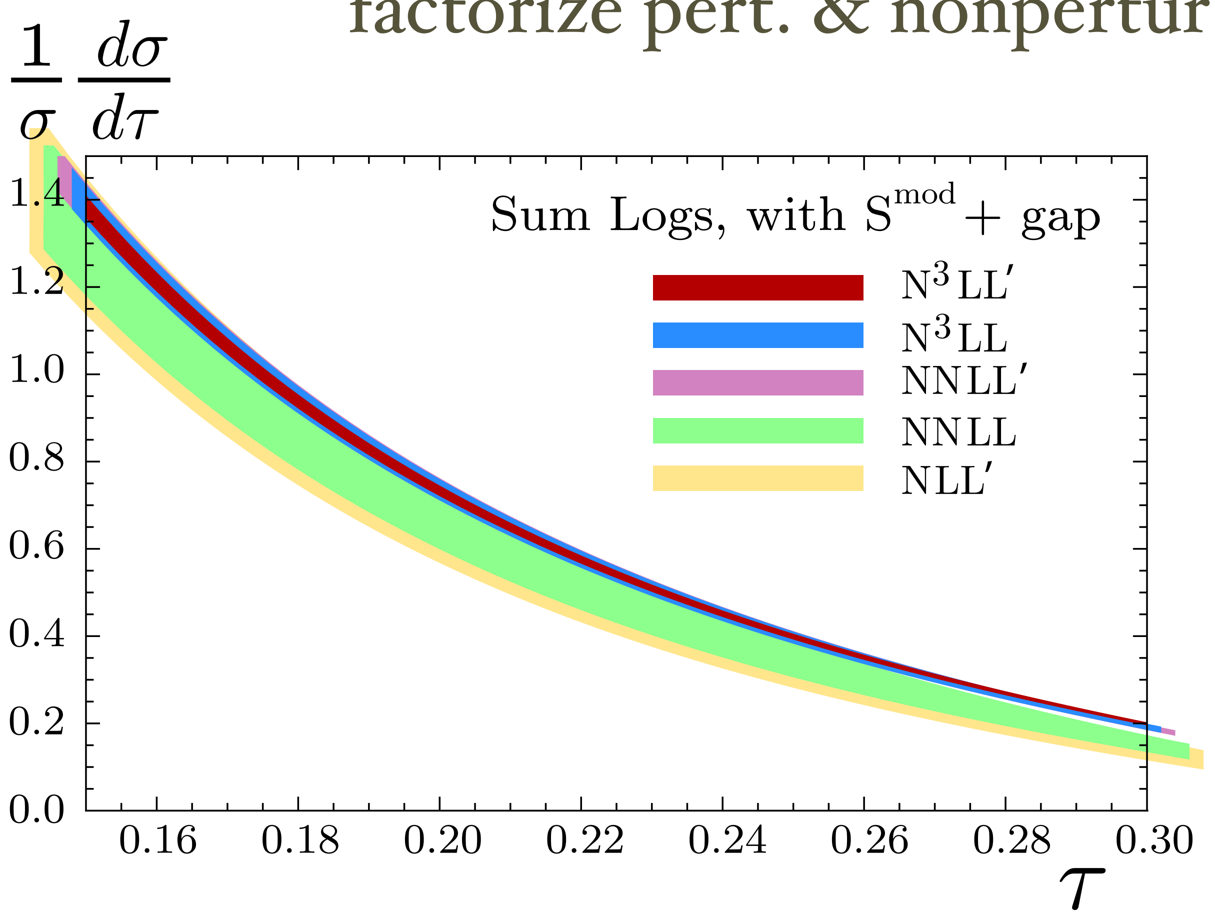

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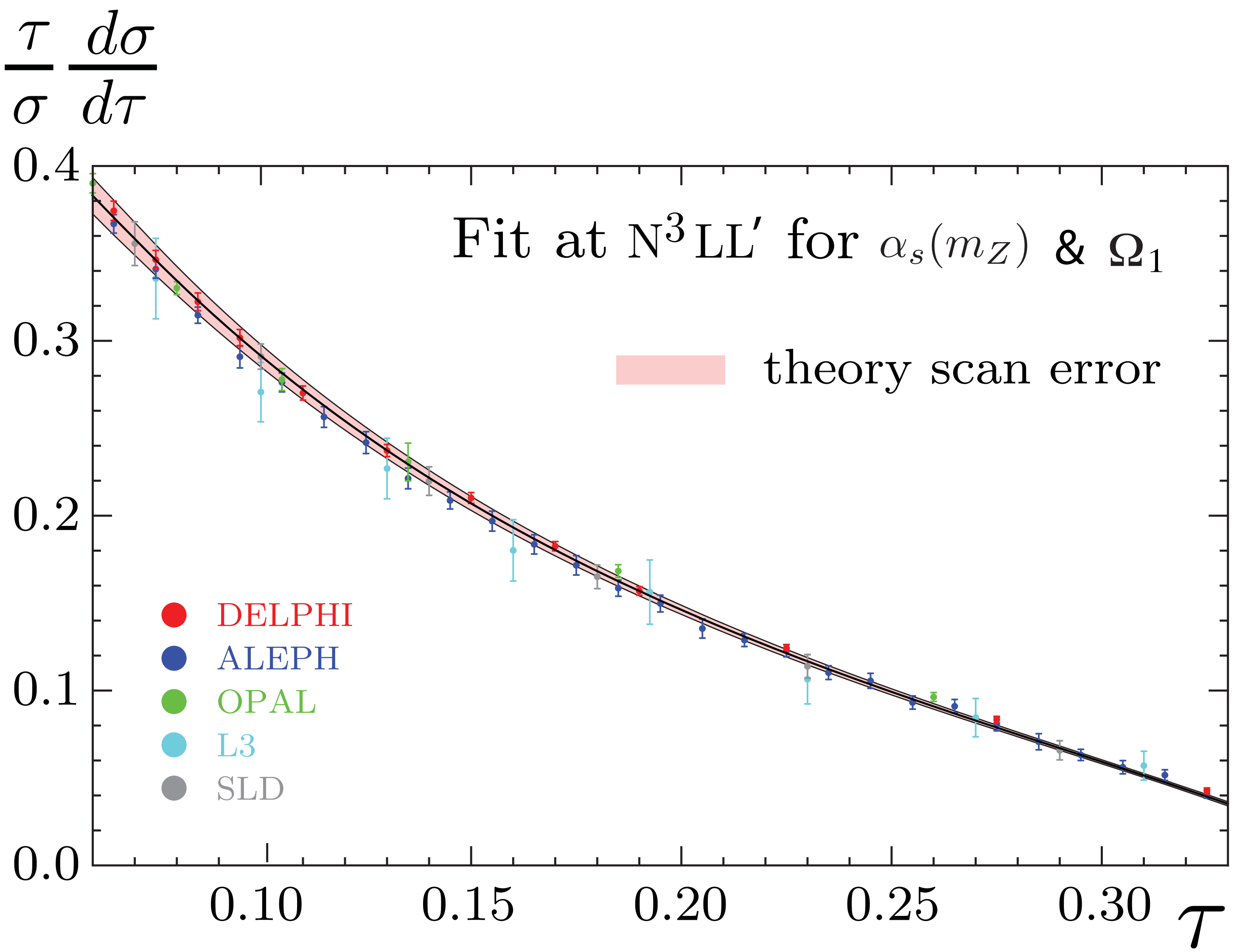
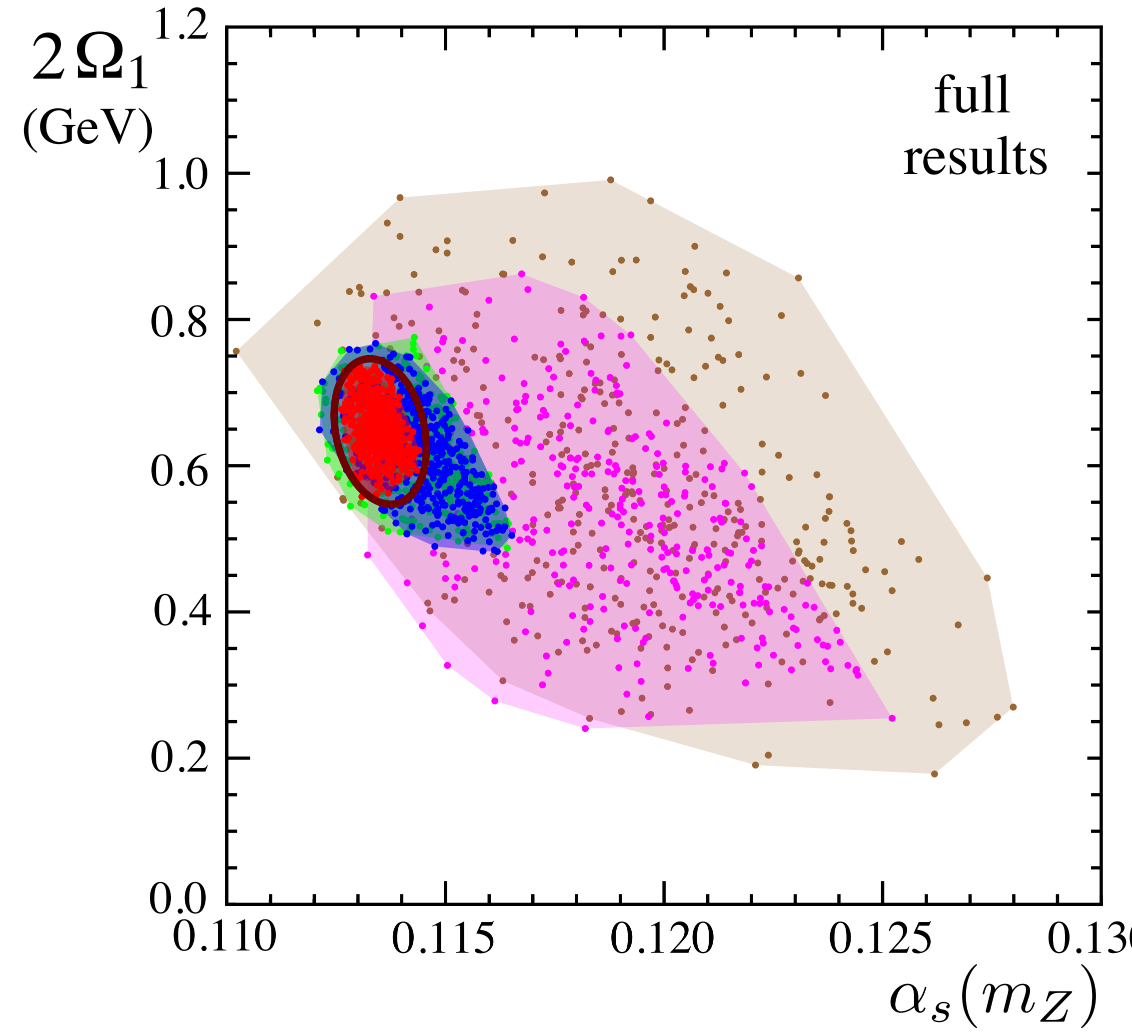


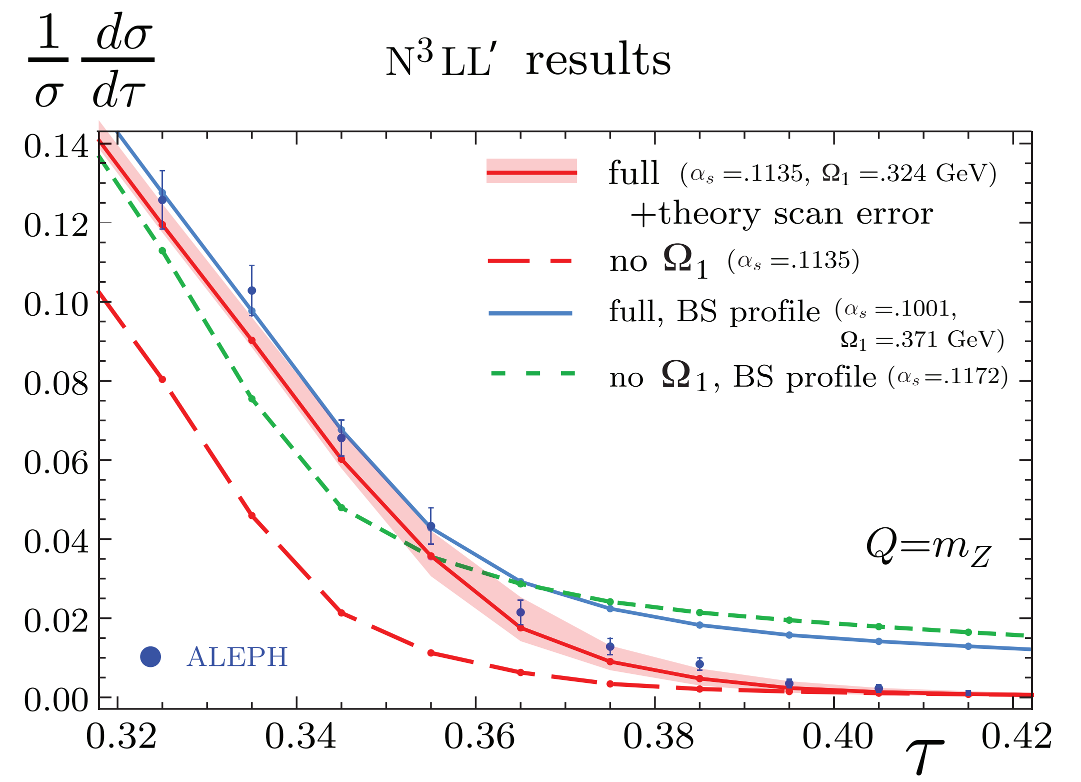


Consistency checks

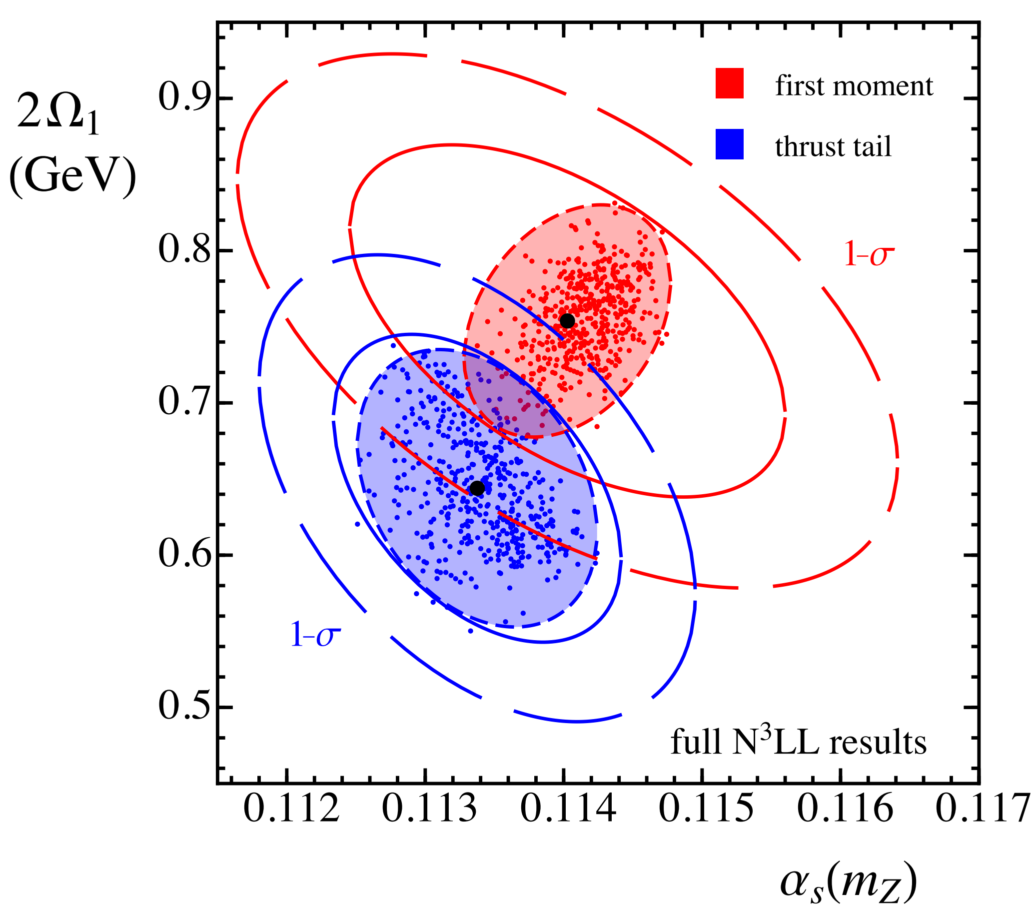
QED & b-mass effects small

$$\Delta\alpha_s(m_Z) = -0.0005$$

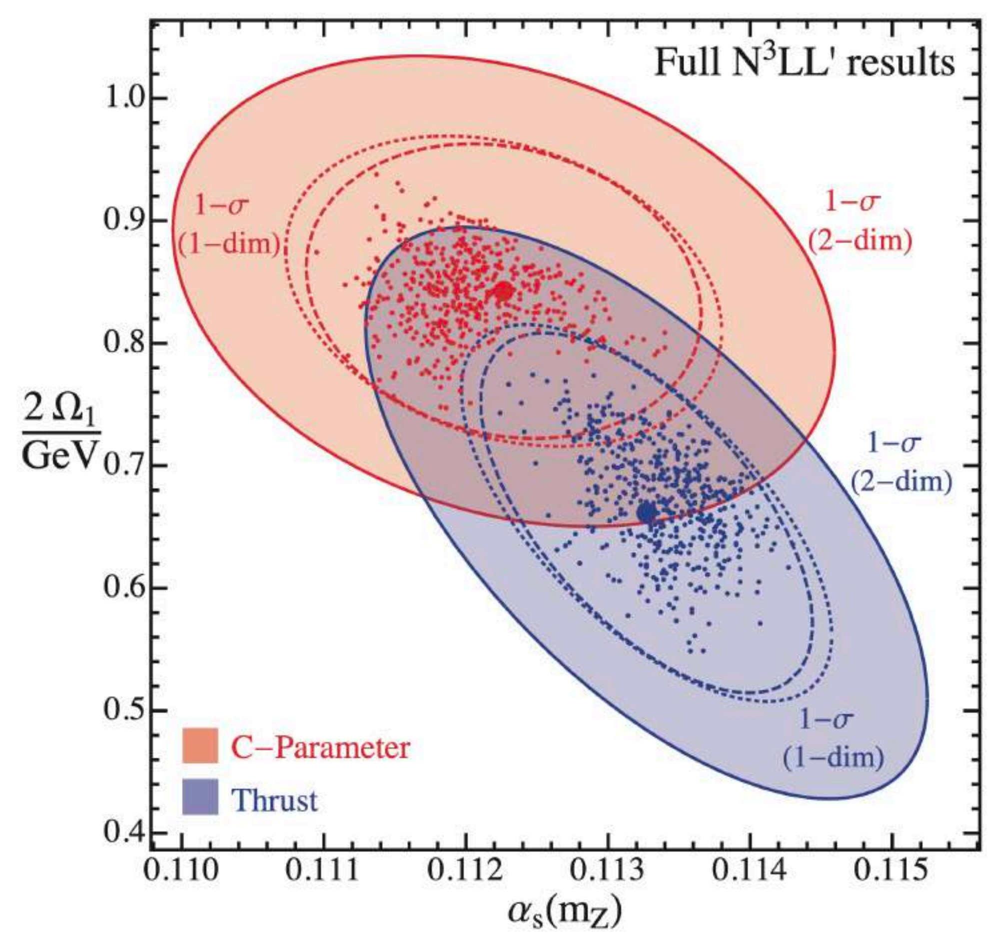
Agreement beyond the fit region



Thrust vs. thrust moments



Thrust vs. C-parameter



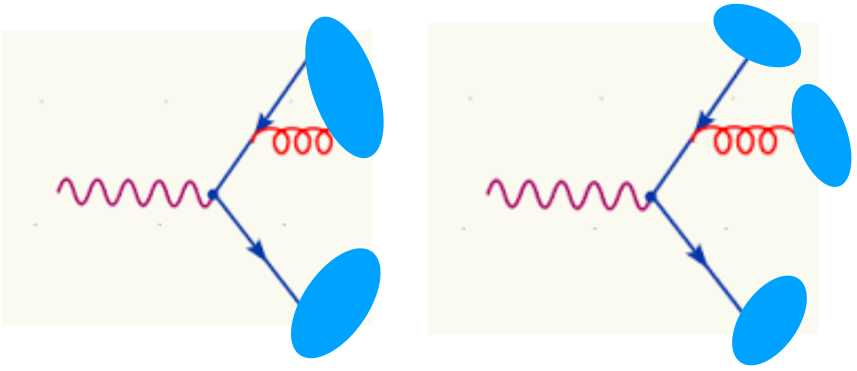
Small $\alpha_s(m_Z)$?

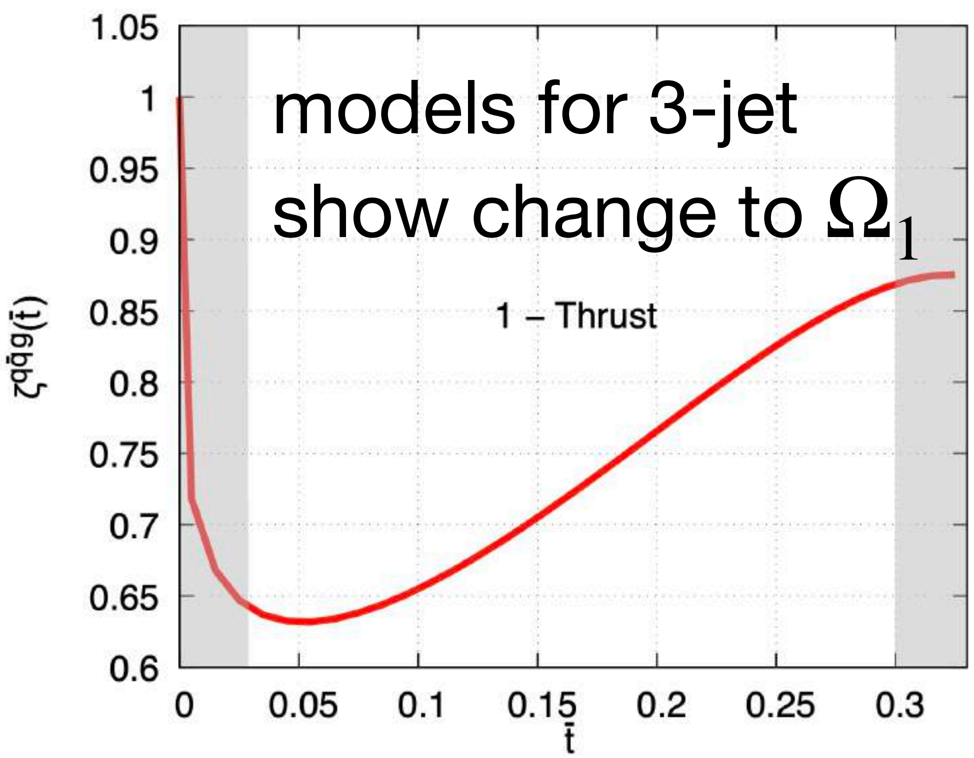
thrust 2010: $\alpha_s(m_Z) = 0.1135 \pm 0.0011$
PDG 2023: $\alpha_s(m_Z) = 0.1180 \pm 0.0009$

thrust 2023 reanalysis: Bell, Lee, Makris, Talbert, Yan (2023), also small α_s

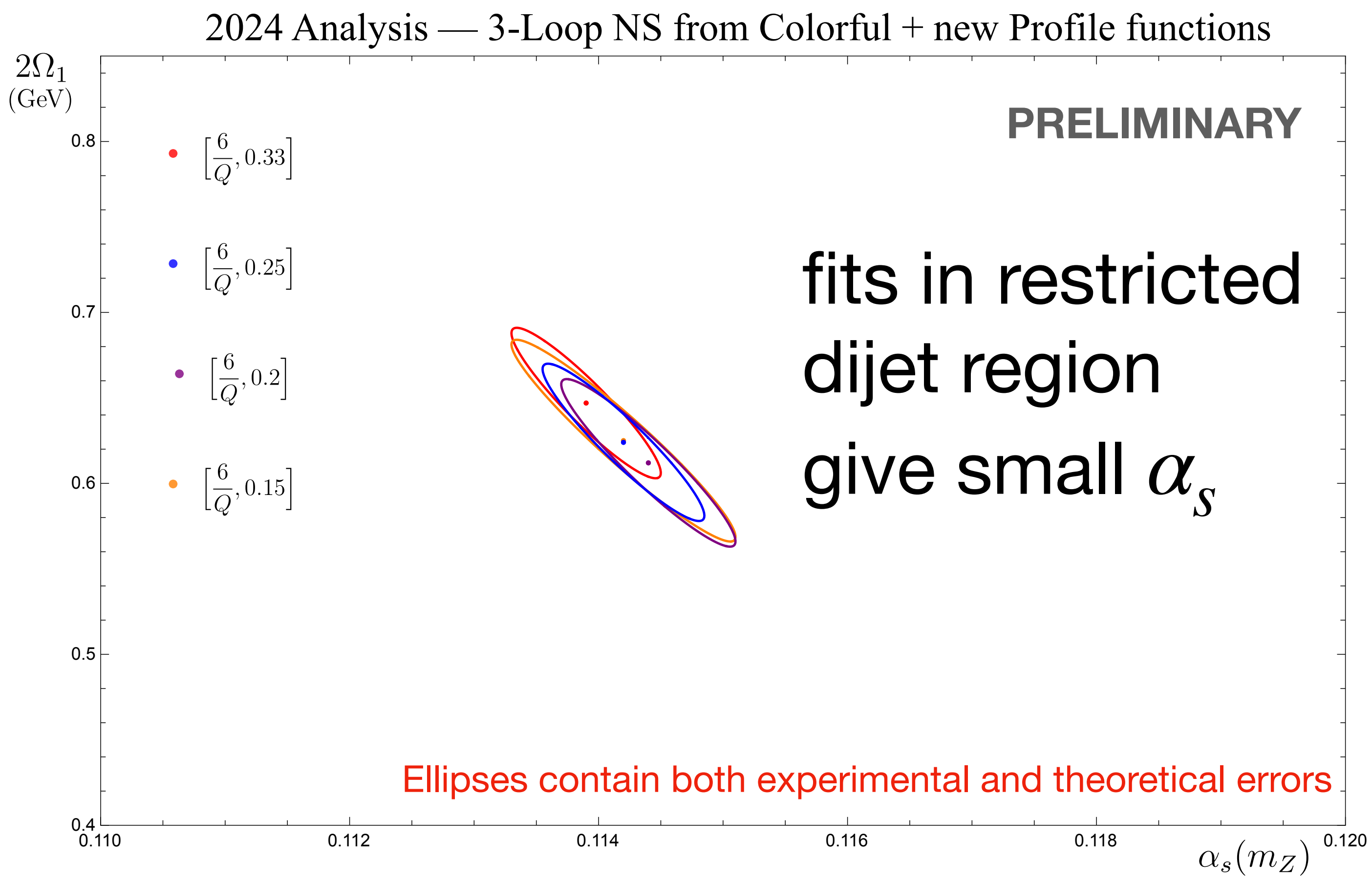
? Power corrections for 2-jets (Ω_1) versus 3-jets ($\neq \Omega_1$)

Luisoni, Monni, Salam (2021)
Caola, Ravasio, Limatola, Melnikov, Nason, Ozcelik (`21-22)
Nason, Zanderighi (2023)
Benitez-Rathgeb, Hoang, Mateu, IS, Vita (2024)





models for 3-jet show change to Ω_1



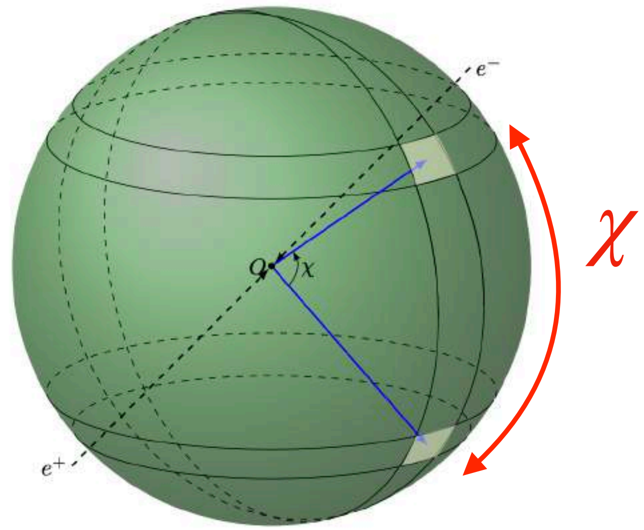
fits in restricted dijet region give small α_s

Energy Energy Correlators and power corrections

here e^+e^-

see talk by Ian Moult

Exciting class of observables for collider physics (both theoretically and experimentally)



$$\frac{d\Sigma}{d\chi} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta(\chi - \theta_{ij})$$

perturbative QCD

$\sum_n c_n(\chi, \mu/Q)\alpha_s^n(\mu)$

universal power correction describing hadronization

Korchemsky, Sterman (1999)

$$\frac{1}{\sigma_0}\frac{d\Sigma}{d\chi} = \frac{1}{\sigma_0}\frac{d\hat{\Sigma}}{d\chi} + \frac{2}{\sin^3\chi}\frac{\bar{\Omega}_1}{Q}$$

$$\Omega_1 \equiv \frac{1}{N_c}\langle 0 | \operatorname{tr} \overline{Y}_{\bar{n}}^\dagger Y_n^\dagger \mathcal{E}_T(0) Y_n \overline{Y}_{\bar{n}} | 0 \rangle$$

modified perturbative QCD

$\sum_n c_n(\chi, \mu/Q)\alpha_s^n(\mu)$ + d_n-series

scheme change to remove leading renormalon

$$\Omega_1(R) = \bar{\Omega}_1 - R \sum_n d_n(\mu/R)\alpha_s^n(\mu)$$

$$\frac{1}{\sigma_0}\frac{d\Sigma}{d\chi} = \frac{1}{\sigma_0}\frac{d\Sigma^R}{d\chi} + \frac{2}{\sin^3\chi}\frac{\Omega_1(R)}{Q}$$

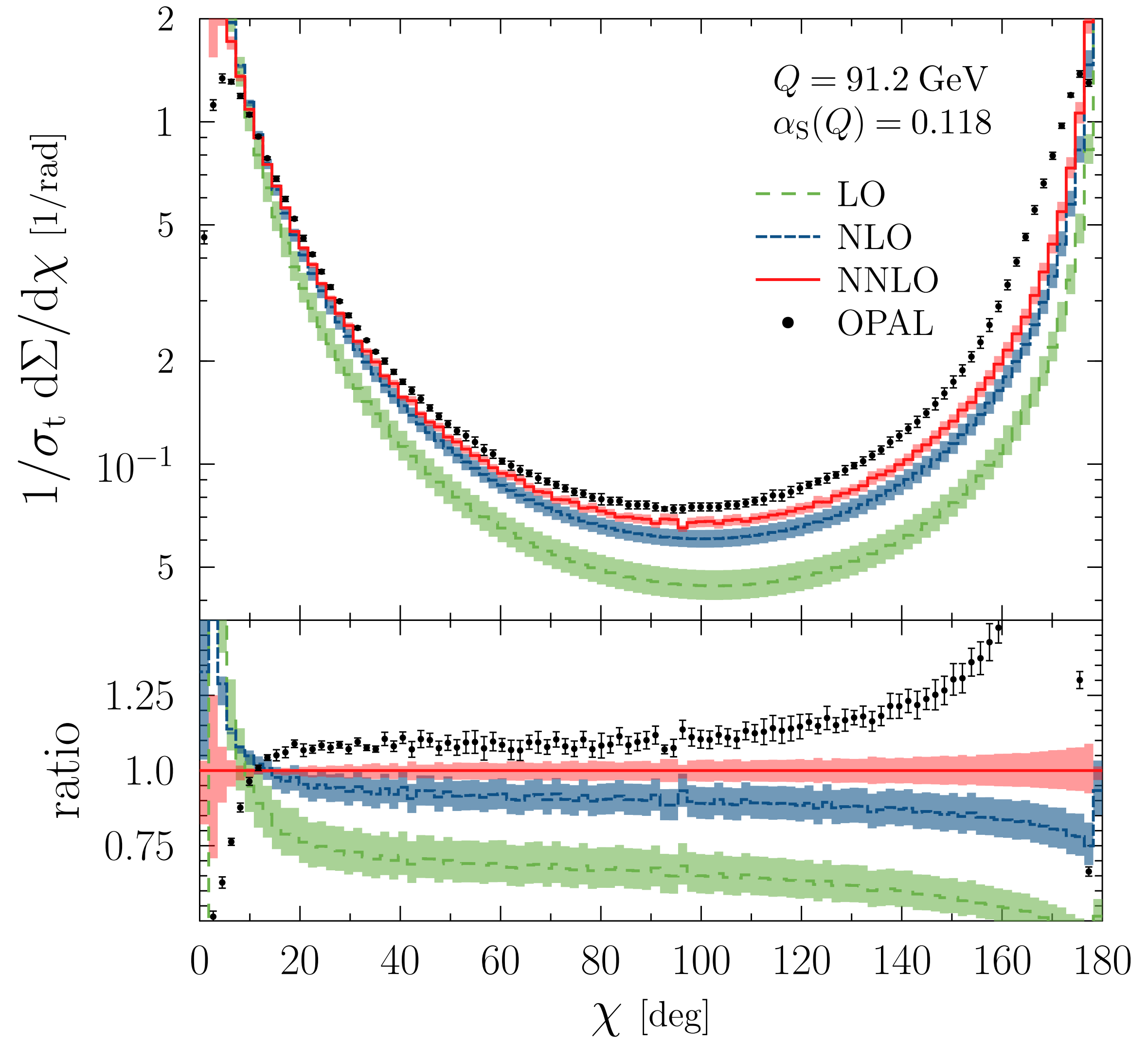
$\overline{\text{MS}}$ scheme $\Longrightarrow$ R scheme

Hoang, I.S.(2007); Hoang Kluth(2008); Schindler, Sun, I.S. (2023)

Perturbative Energy Energy Correlators

NLO (analytic): Dixon, Luo, Shtabovenko, Yang, Zhu (2018)

NNLO (CoLoRFul): Del Duca, Duhr, Kardos, Somogyi, Trócsányi (2017); Tulipánt, Kardos, Somogyi (2018)

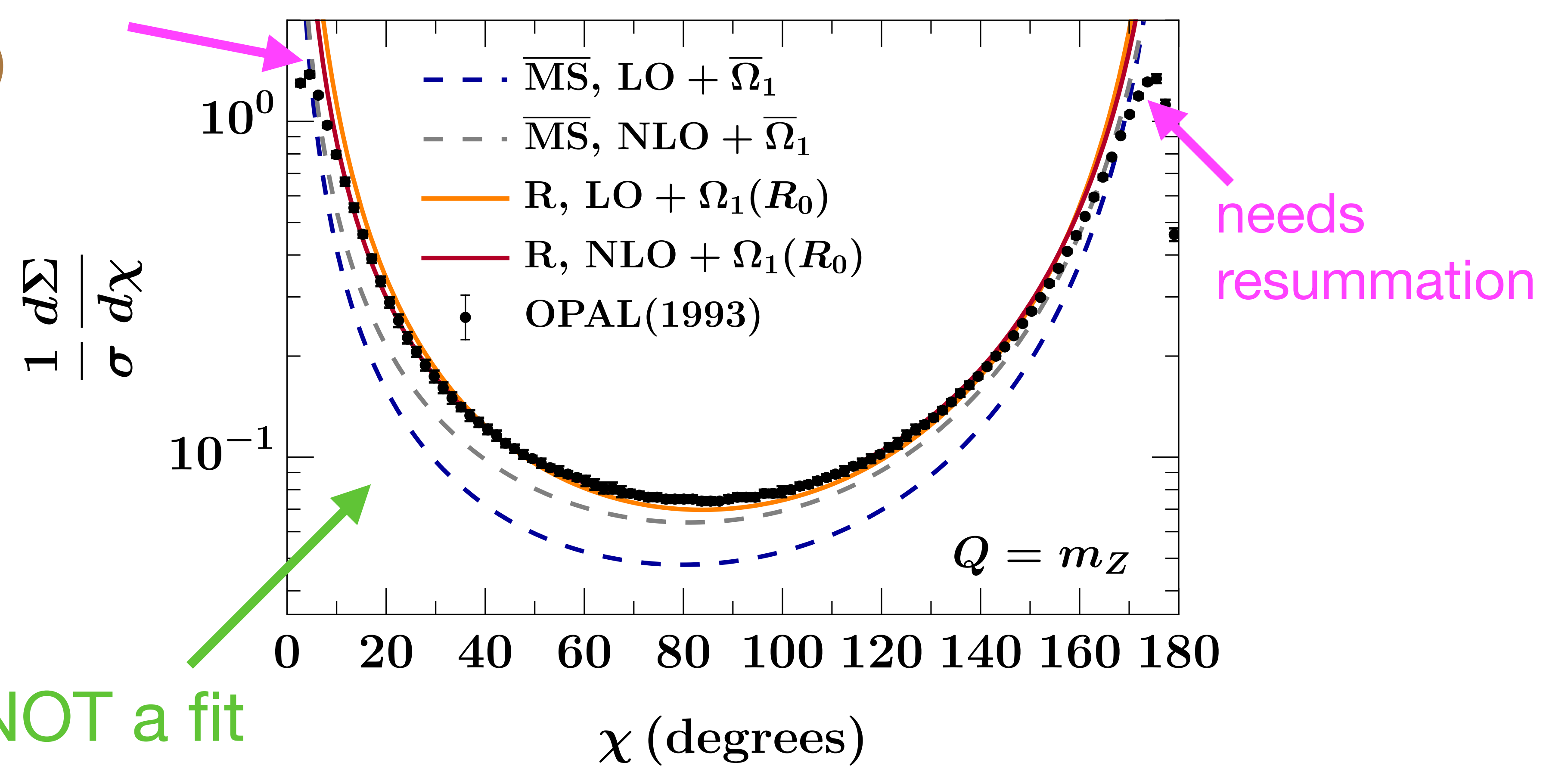


EEC With Power Corrections

Schindler, Sun, I.S. (2023)

R scheme: Ω_1, α_s from thrust fit

- Better convergence
- Agrees with data!
- Confirms Ω_1 universality



Projected N-point Energy Correlators

e^+e^-

Ω_1, α_s from thrust fit

Chen, Moult, Zhang, Zhu (2020)

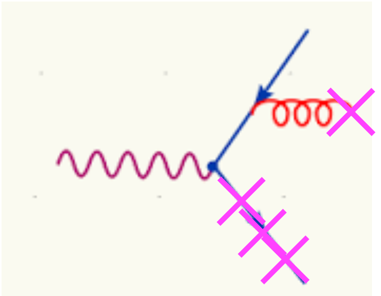
$$\langle \mathcal{E}_1 \mathcal{E}_2 \cdots \mathcal{E}_N \rangle$$

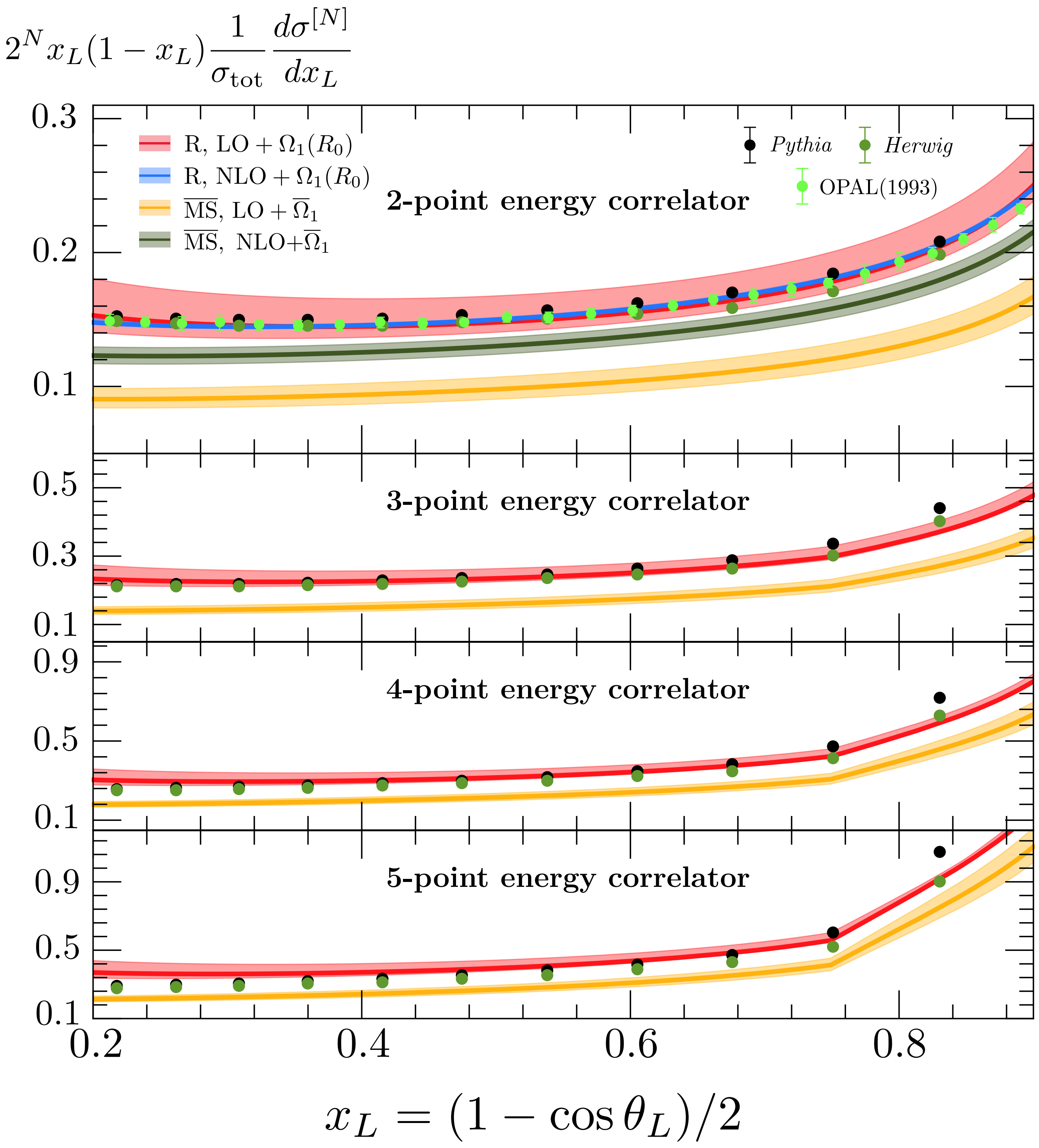
$$\theta_L = \max(\theta_{ij})$$

Power Corrections

Lee, Pathak, I.S., Sun (2024)

$$\frac{1}{\sigma}\frac{d\sigma^{[N]}}{d\theta_L} = \frac{1}{\sigma}\frac{d\hat{\sigma}^{[N]}}{d\theta_L} + \frac{4N}{2^N}\frac{\overline{\Omega}_1}{Q\sin^3\theta_L}$$





Projected N-point Energy Correlators

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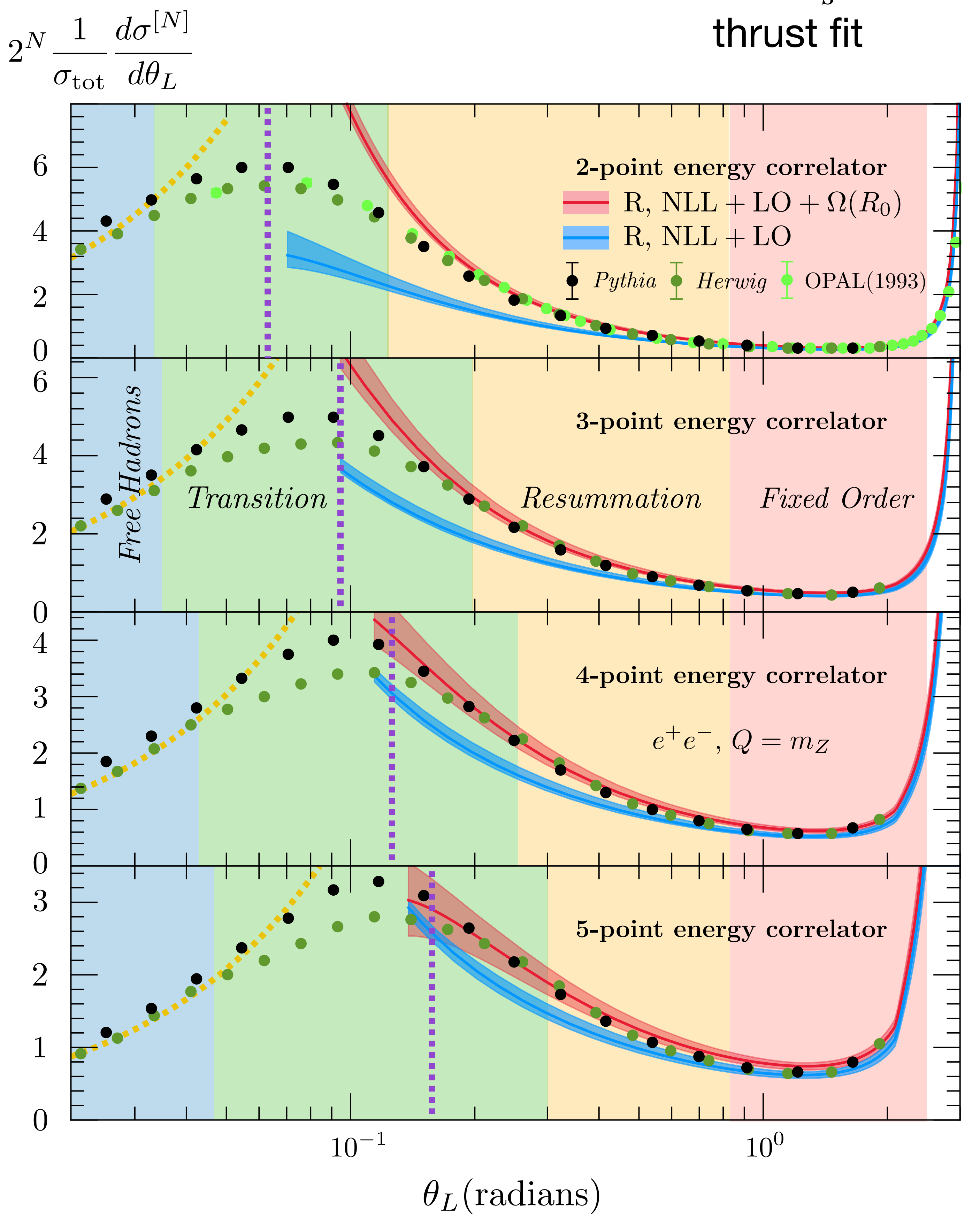
$$\langle \mathcal{E}_1 \mathcal{E}_2 \cdots \mathcal{E}_N \rangle$$

$$\theta_L = \max(\theta_{ij})$$



Power Corrections

Lee, Pathak, I.S., Sun (2024)

$$\frac{1}{\sigma}\frac{d\sigma^{[N]}}{d\theta_L} = \frac{1}{\sigma}\frac{d\hat{\sigma}^{[N]}}{d\theta_L} + \frac{4N}{2^N}\frac{\overline{\Omega}_1}{Q\sin^3\theta_L}$$

Resummation $\theta_L \ll 1$

Dixon, Moult, Zhu (2019)

Chen, Moult, Zhang, Zhu (2020)

$$\frac{1}{\sigma}\frac{d\sigma^{[N]}}{d\theta_L} \sim \int dx\, x^N \vec{J}^{[N]} \cdot \vec{H}$$

EEC in back-to-back limit

$\mathrm{N^4LL}$ **Duhr, Mistlberger, Vita (2022)**

using factorization: Moult, Zhu (2018)

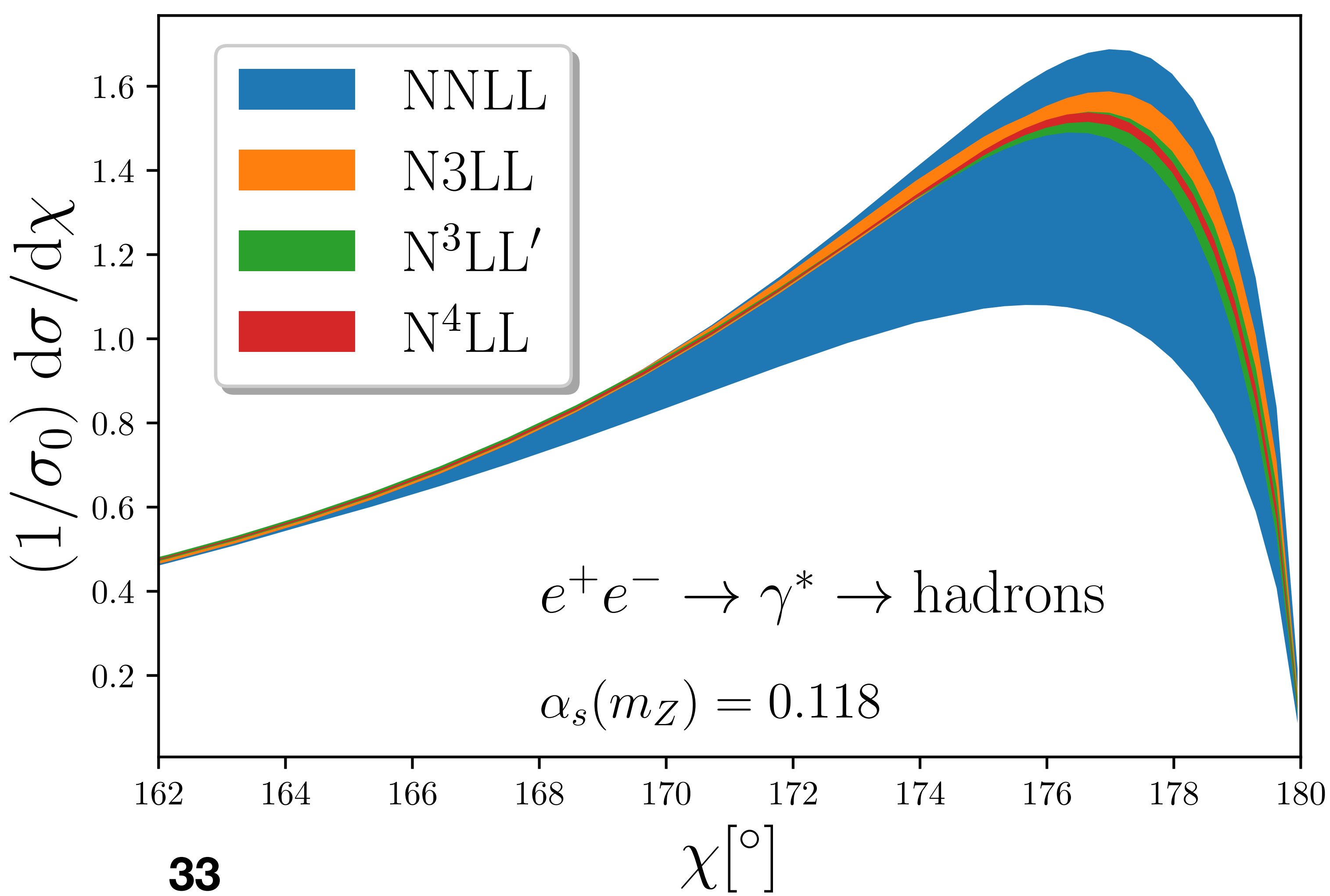
Accuracy	$H,\ \mathcal{J}$	$\Gamma_{\rm cusp}(\alpha_s)$	$\gamma_H^q(\alpha_s)$	$\gamma_r^q(\alpha_s)$	$\beta(\alpha_s)$
LL	Tree level	1-loop	–	–	1-loop
NLL	Tree level	2-loop	1-loop	1-loop	2-loop
NLL′	1-loop	2-loop	1-loop	1-loop	2-loop
NNLL	1-loop	3-loop	2-loop	2-loop	3-loop
NNLL′	2-loop	3-loop	2-loop	2-loop	3-loop
$\mathrm{N^3LL}$	2-loop	4-loop	3-loop	3-loop	4-loop
$\mathrm{N^3LL'}$	3-loop	4-loop	3-loop	3-loop	4-loop
$\mathrm{N^4LL}$	3-loop	5-loop	4-loop	4-loop	5-loop
$\mathrm{N^4LL'}$	4-loop	5-loop	4-loop	4-loop	5-loop

Key new ingredients:

- OPE for TMD PDFs and FFs to 3-loops (all channels)
  Ebert, Mistlberger, Vita (2020)
  Luo, Yang, Zhu, Zhu (2020)

$$f_{i/h}^{\rm pert}(x, b_T, \mu, \zeta) = \sum_j \int \frac{dy}{y} C_{ij}(x/y, b_T, \mu, \zeta) f_{j/h}(y, \mu)$$

Resummation $\chi \to \pi$



- CS kernel to 4-loops
  Duhr, Mistlberger, Vita (2022)
  Moult, Zhu, Zhu (2022)

$$\gamma_\zeta^q[\alpha_s] = \alpha_s\, \gamma_\zeta^{q(1)} + \alpha_s^2\, \gamma_\zeta^{q(2)} + \alpha_s^3\, \gamma_\zeta^{q(3)} + \alpha_s^4\, \gamma_\zeta^{q(4)} + \dots$$

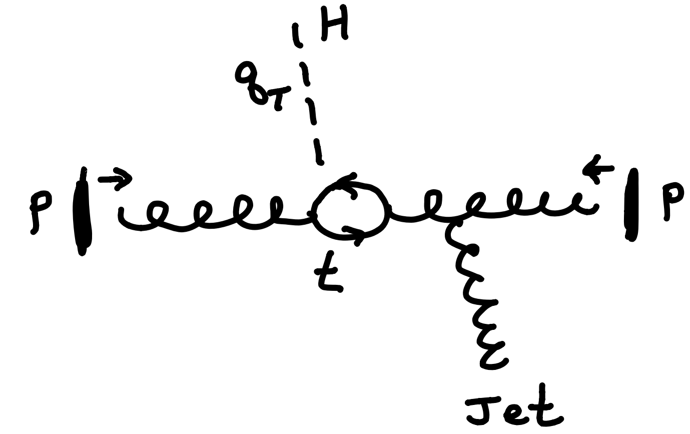
(3-loop result: Li, Zhu 2016; Vladimirov 2016)

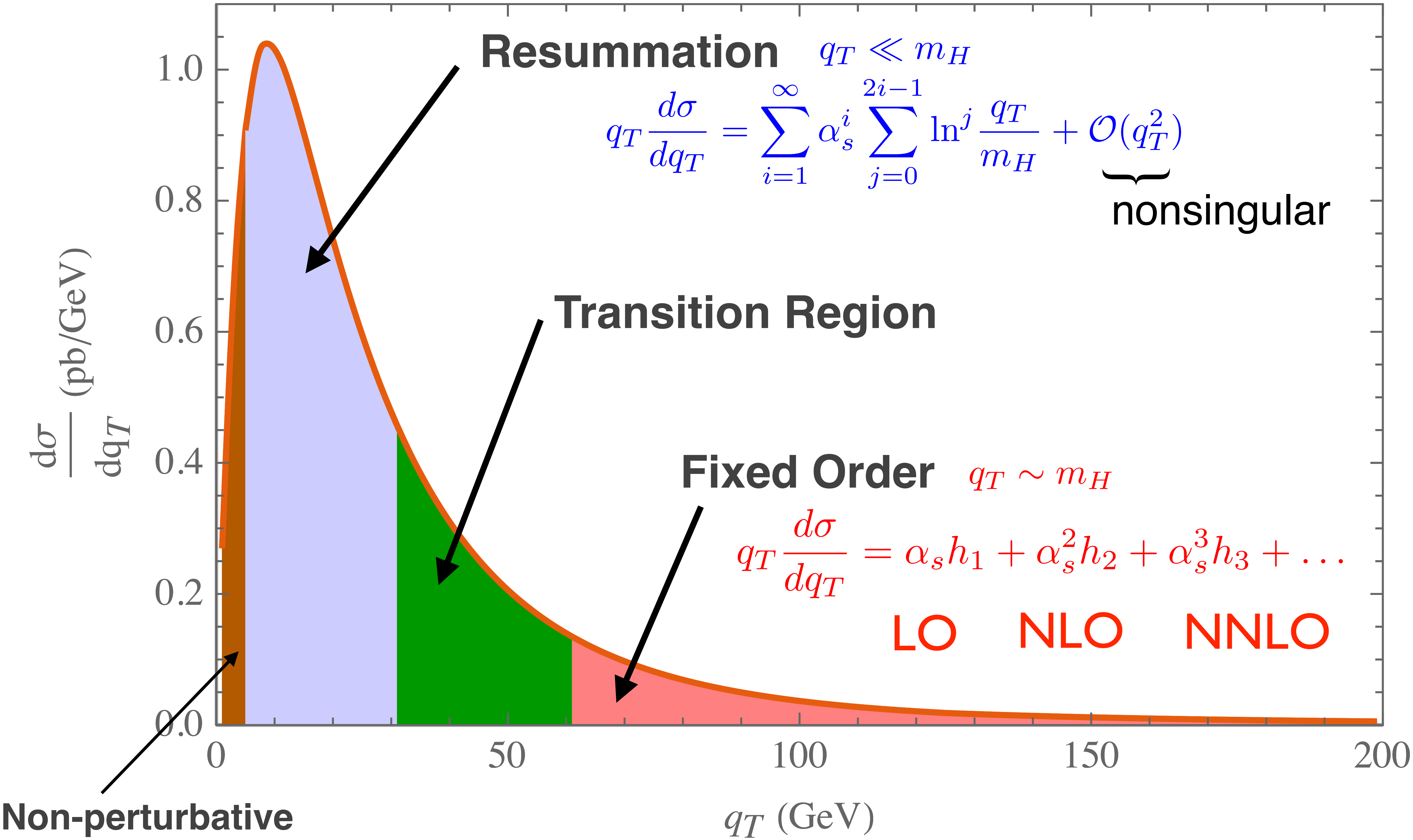
High Precision Resummation *pp*

Higgs q_T spectrum

gluon fusion

Higgs recoils against Jets

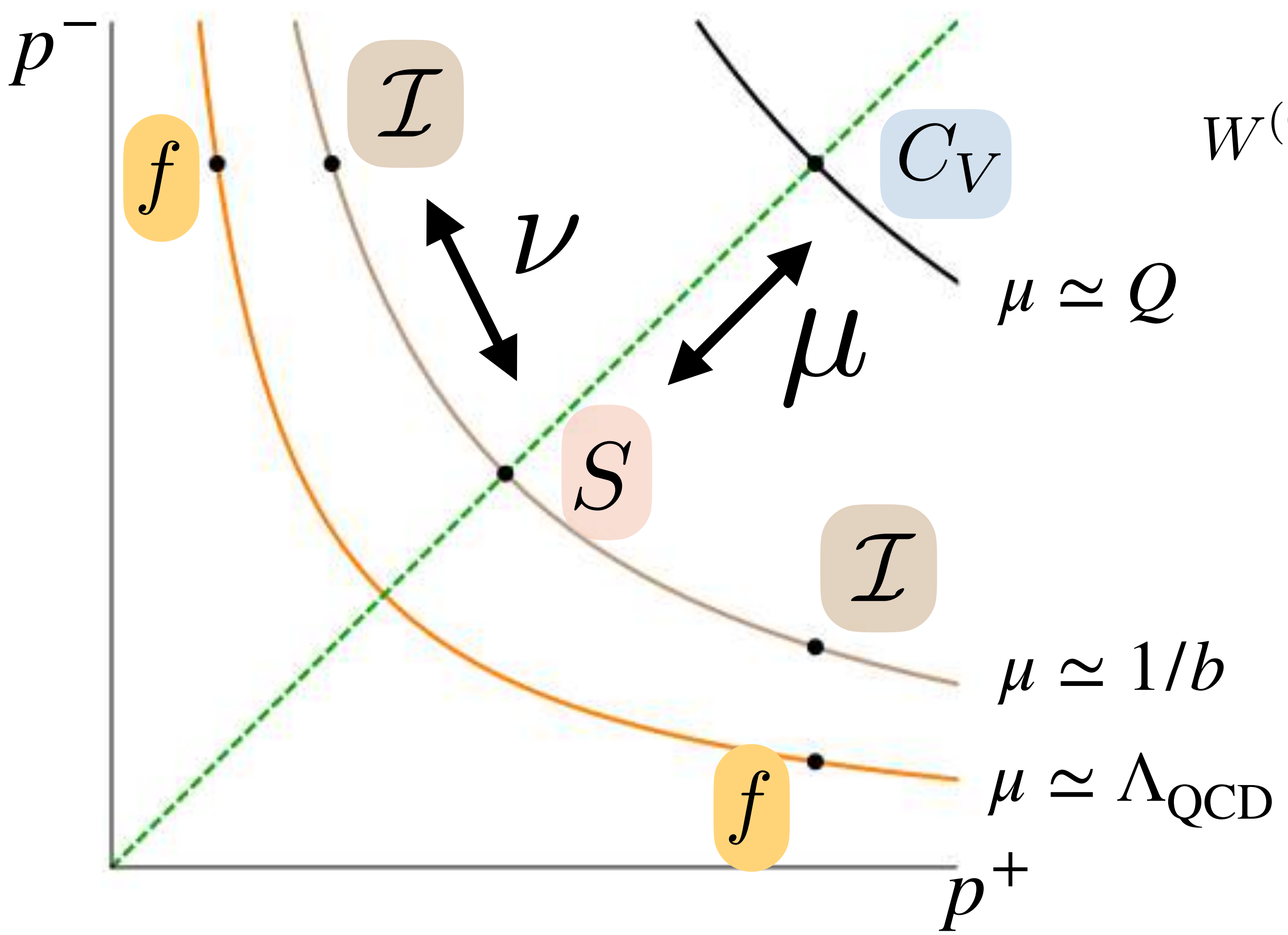




Small q_T factorization

$$\frac{d^2\sigma}{dq_T\,dY} = W^{(0)}(q_T,Y) + W^{\text{non.sing.}}(q_T,Y)$$

Collins, Soper, Sterman
SCET



$$W^{(0)}(q_T,Y) = \int \frac{d^2\vec{b}}{(2\pi)^2}\, e^{i\vec{b}\cdot\vec{q}_T}\, W(x_a, x_b, m_H, \vec{b})$$

μ = invariant mass scale

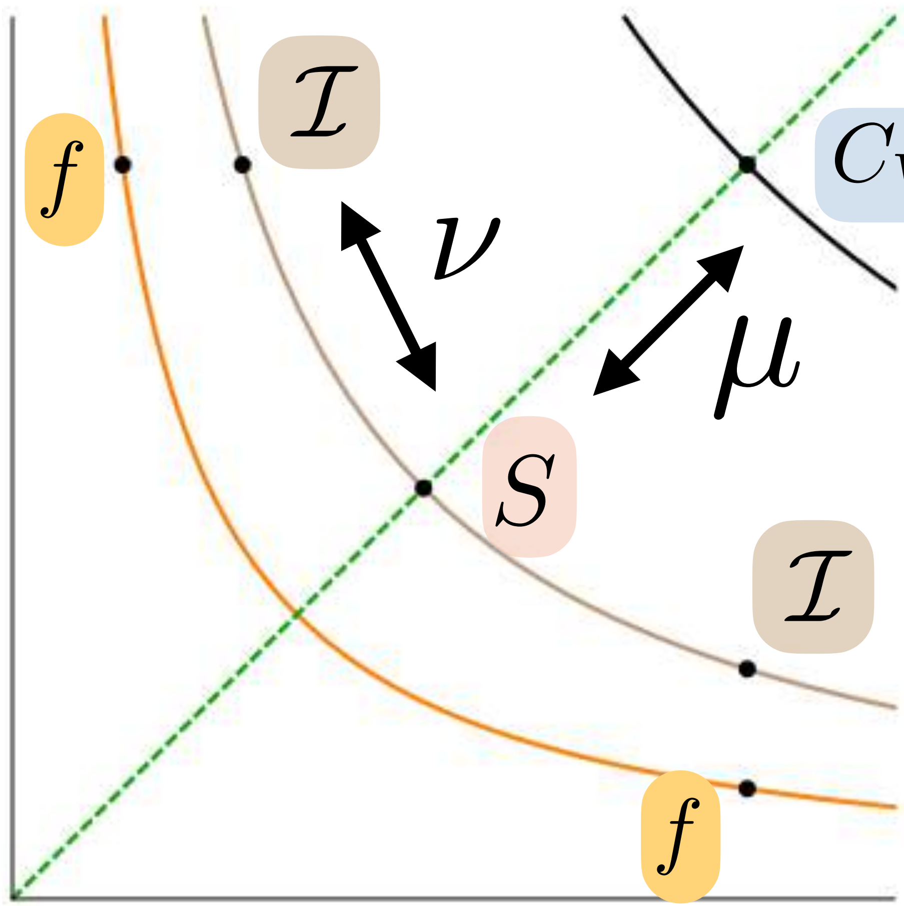
ν = "rapidity" RGE scale

$$W(x_a, x_b, m_H, \vec{b}) = \left| C_V\left(m_t, m_H, \mu\right)\right|^2 S(\vec{b},\mu,\nu)\, B^{\alpha\beta}_{g/N_1}(x_a, Q, \vec{b}, \mu, \nu)\, B^{\alpha\beta}_{g/N_2}(x_b, Q, \vec{b}, \mu, \nu)$$

$$B^{\alpha\beta}_{g/N}(x, Q, \vec{b}, \mu, \nu) = \sum_k \int \frac{d\xi}{\xi}\, \mathcal{I}^{\alpha\beta}_{gk}\left(\frac{x}{\xi}, \vec{b}, \mu, \nu\right) f_{k/N}(\xi,\mu) \quad + \mathcal{O}\left(\Lambda^2_{\rm QCD}\vec{b}^{\,2}\right)$$

Small q_T factorization

$$\frac{d^2\sigma}{dq_T dY} = W^{(0)}(q_T, Y) + W^{\text{non.sing.}}(q_T, Y)$$



Single Scale Functions:

hard function $\ln \frac{Q^2}{\mu^2}$

beam function $\ln(b^2\mu^2)$ $\ln \frac{Q^2}{\nu^2}$

soft function $\ln(b^2\mu^2)$ $\ln(b^2\nu^2)$

$$W(x_a, x_b, m_H, \vec{b}) = \left| C_V\left(m_t, m_H, \mu\right)\right|^2 S(\vec{b}, \mu, \nu) B^{\alpha\beta}_{g/N_1}(x_a, Q, \vec{b}, \mu, \nu) B^{\alpha\beta}_{g/N_2}(x_b, Q, \vec{b}, \mu, \nu)$$

Resummation:

$$\ln W = L\sum_k (\alpha_s L)^k + \sum_k (\alpha_s L)^k + \alpha_s \sum_k (\alpha_s L)^k + \alpha_s^2 \sum_k (\alpha_s L)^k$$

LL NLL NNLL N3LL

$$L = \ln(m_H b)$$

The Higgs p_T Spectrum and Total Cross Section with Fiducial Cuts at N^3LL'+N^3LO

Billis, Dehnadi, Ebert, Michel, Tackmann (2021)

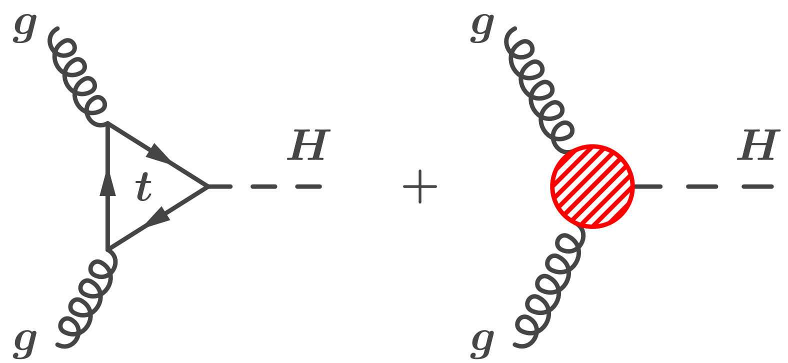
Consider $gg \to H \to \gamma\gamma$ with ATLAS fiducial cuts:

$$p_T^{\gamma 1} \geq 0.35\, m_H\,, \quad p_T^{\gamma 2} \geq 0.25\, m_H\,, \quad |\eta^\gamma| \leq 2.37\,, \quad |\eta^\gamma| \notin [1.37, 1.52]$$

$$\sigma^{\rm fid} = \int dq_T\, dY\, A(q_T, Y; \Theta)\, W(q_T, Y)$$

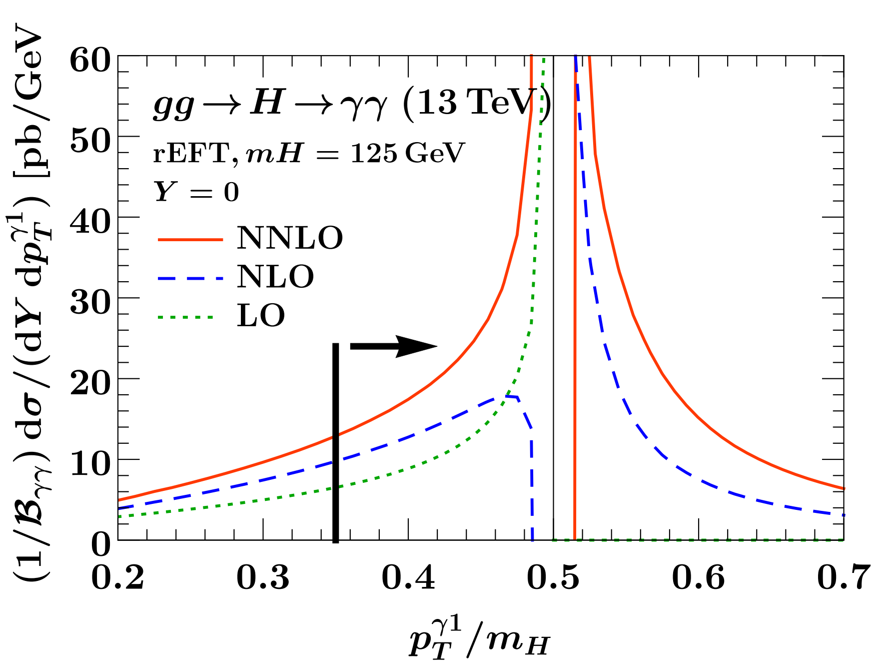
A=acceptance

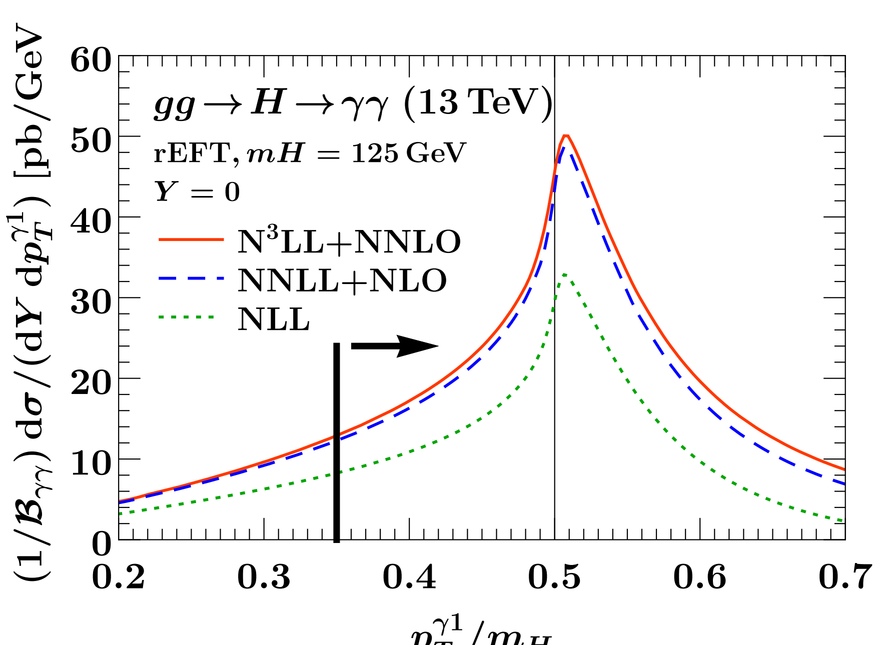
Fiducial cross section measures deviation from SM gluon-fusion:



$$= \left(\frac{\alpha_s}{12\pi v} C_t + \frac{v}{\Lambda^2} C_{HG}\right) H\, G^a_{\mu\nu} G^{a,\mu\nu}$$

Acceptance causes a need for resummation to obtain Fiducial cross section





cutting on photon p_T induces large logs

Billis, Dehnadi, Ebert, Michel, Tackmann (2021)

Resummation Inputs

- Three-loop soft and hard function …includes in particular the three-loop virtual form factor
  [Li, Zhu, '16] [Baikov et al. '09; Lee et al. '10; Gehrmann et al. '10]
- Three-loop unpolarized and two-loop polarized beam functions
  [Ebert, Mistlberger, Vita '20; Luo, Yang, Zhu, Zhu '20]
  [Luo, Yang, Zhu, Zhu '19; Gutierrez-Reyes, Leal-Gomez, Scimemi, Vladimirov '19]
- Four-loop cusp, three-loop noncusp anomalous dimensions
  [Brüser, Grozin, Henn, Stahlhofen '19; Henn, Korchemsky, Mistlberger '20; v. Manteuffel, Panzer, Schabinger '20] [Li, Zhu, '16; Moch, Vermaseren, Vogt '05; Idilbi, Ma, Yuan '06; Vladimirov '16]
- Four-loop CS kernel, from conformal relation between UV & rapidity anom. dims
  [Vladimirov, 1610.05791 ⟶ Duhr, Mistlberger, Vita, 2205.02242; Moult, Zhu, Zhu, 2205.02249]

Fixed Order Inputs

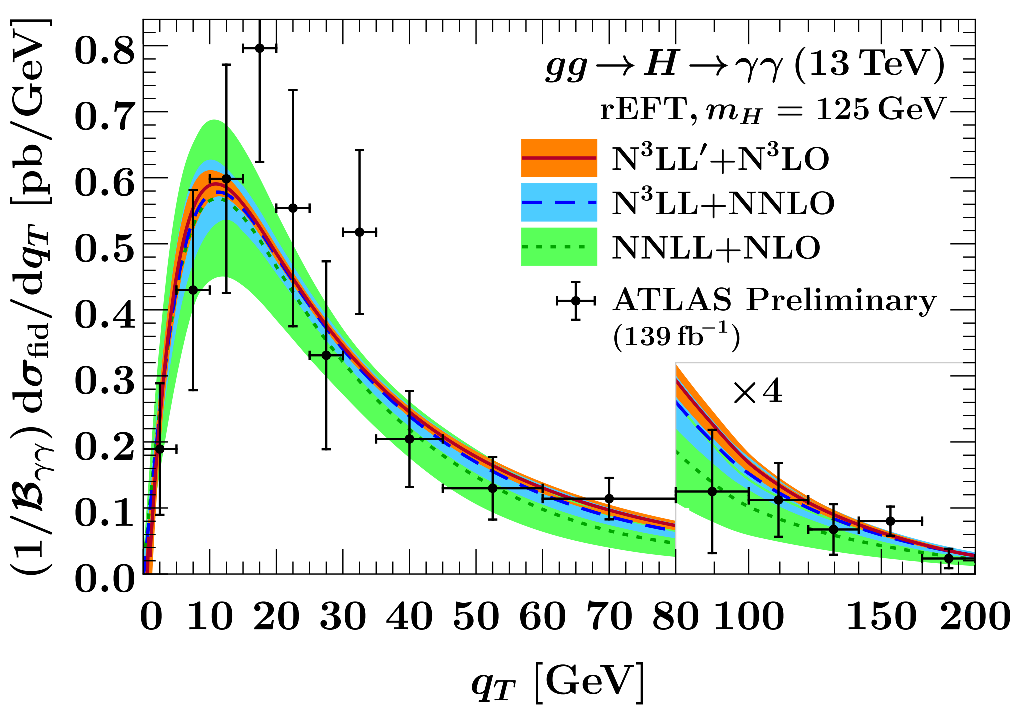
- At NNLO, renormalize & implement bare analytic results for $W(q_T, Y)$
  [Dulat, Lionetti, Mistlberger, Pelloni, Specchia '17]
- At N^{3}LO, use existing binned NNLO$_1$ results from `NNLOjet`
  [Chen, Cruz-Martinez, Gehrmann, Glover, Jaquier '15-16; as used in Chen et al. '18; Bizoń et al. '18]
- Use N^{3}LO total inclusive cross section as additional fit constraint on underflow
  [Mistlberger '18]

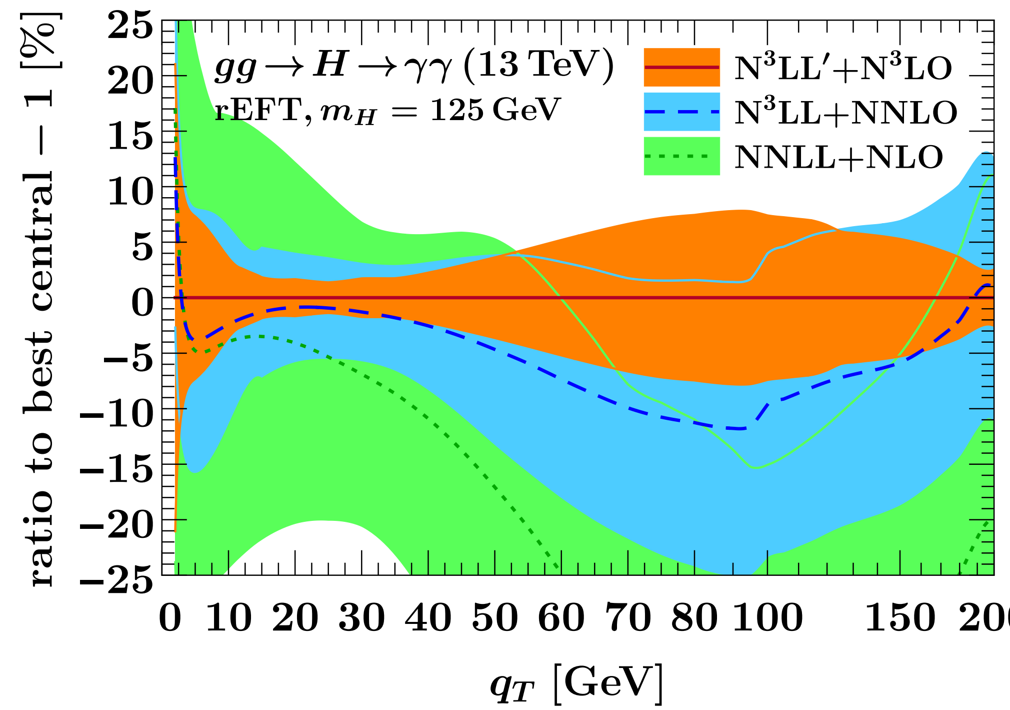
Implemented in C++ Library "SCETlib"

Higgs Results

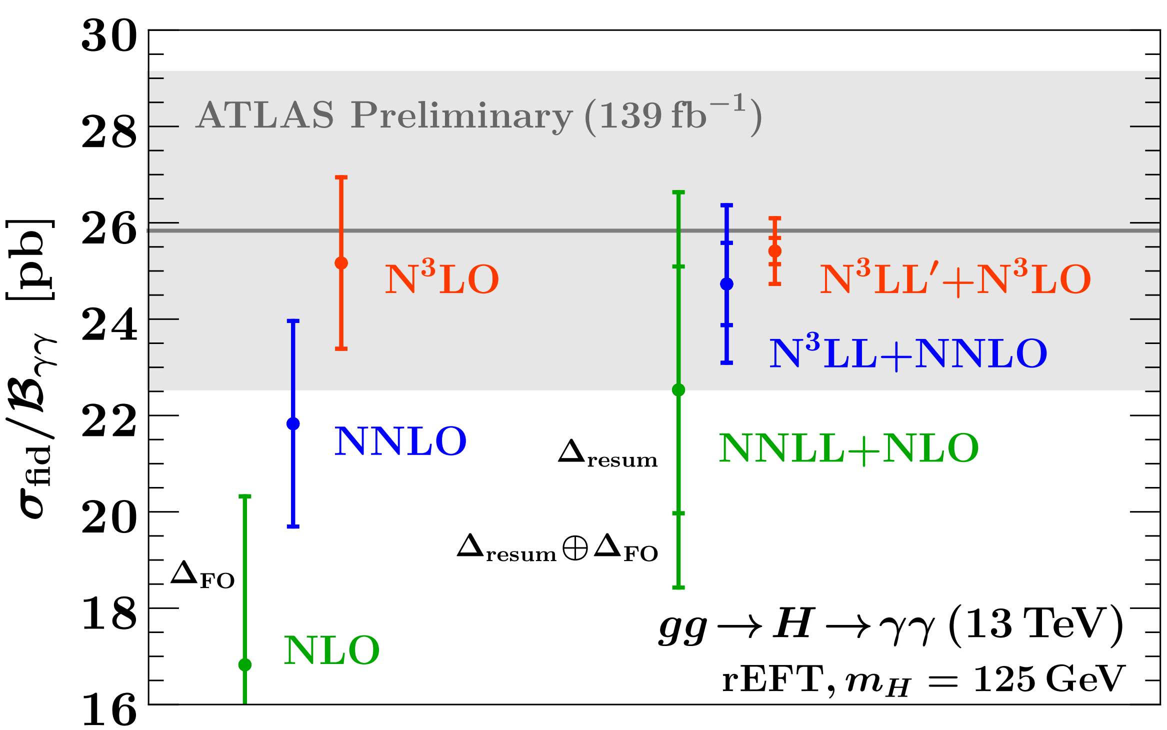
Billis, Dehnadi, Ebert, Michel, Tackmann (2021)

The fiducial q_T spectrum at N^{3}LL$'$+N^{3}LO





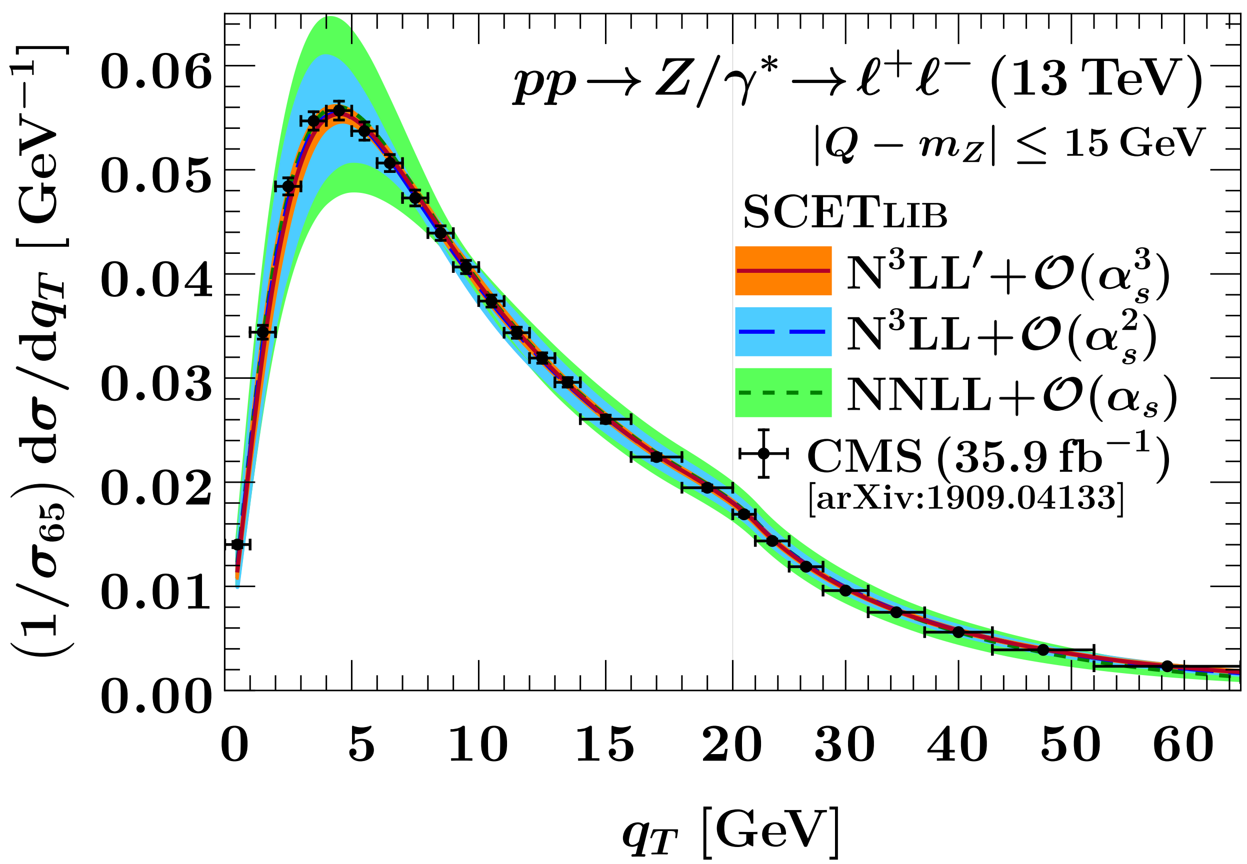
The total fiducial cross section at N^{3}LO and N^{3}LL$'$+N^{3}LO **(SM)**

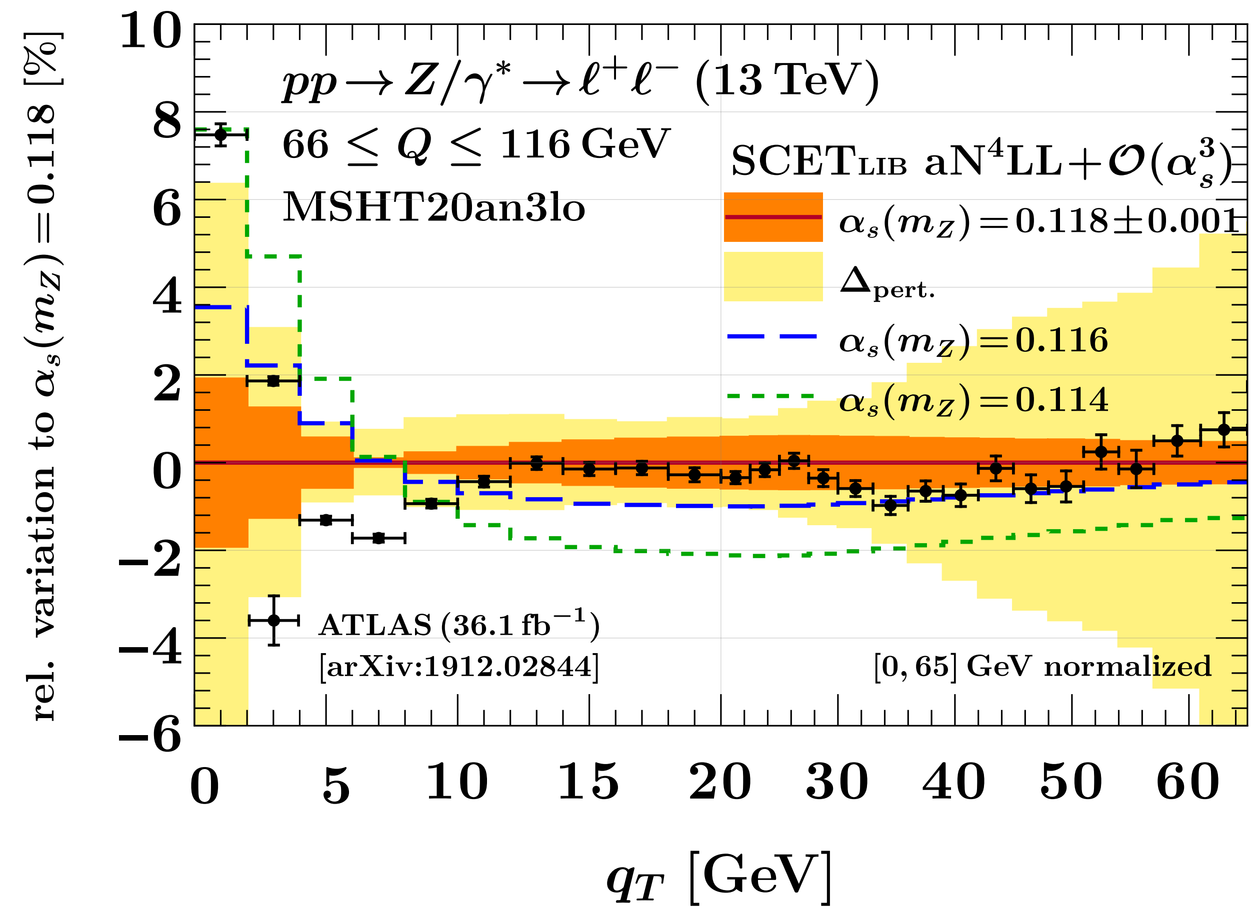


Precision and convergence improved

Drell-Yan Results

Billis, Michel, Tackmann
(in progress)





Fixed Order Inputs

- Fiducial Z+jet MC data at $\mathcal{O}(\alpha_s^2)$ from `MCFM`
  [Campbell, Ellis, et al. '99, '15]
- Very recently: Precise fiducial Z+jet MC data at $\mathcal{O}(\alpha_s^3)$ from `NNLOjet`
  [Chen et al., 2203.01565 – many thanks to the NNLOjet collaboration for providing the raw data.]

Power Corrections

SCET beyond leading power

$\mathcal{O}(\lambda^p)$

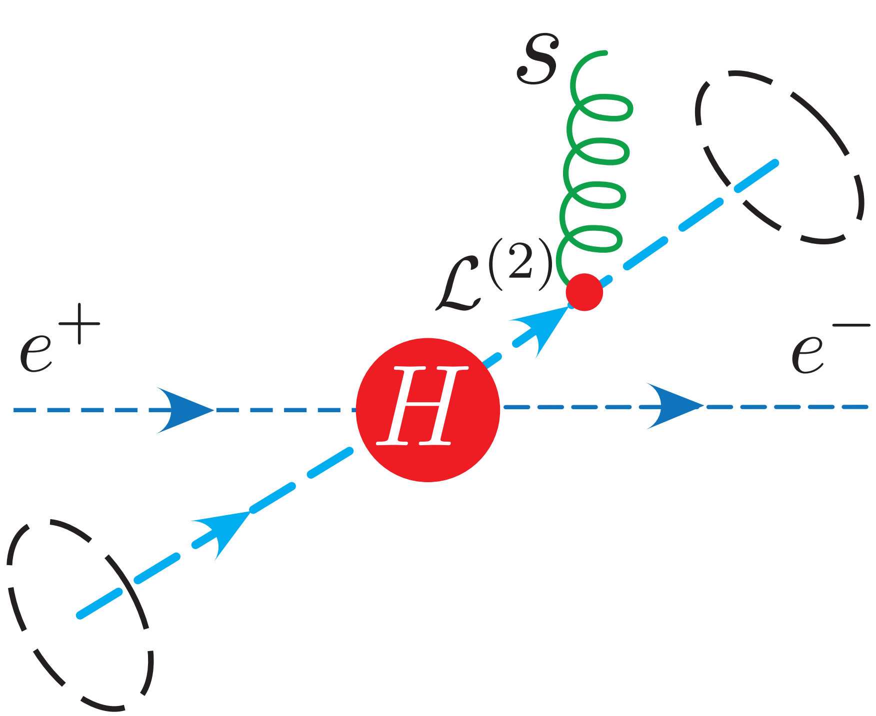
$$\mathcal{L} = \sum_{p\geq 0} \mathcal{L}^{(p)}_{\rm dyn} + \sum_{p} \mathcal{L}^{(p)}_{\rm hard} + \mathcal{L}^{(0)}_{\rm G}$$

Dynamics of infrared modes

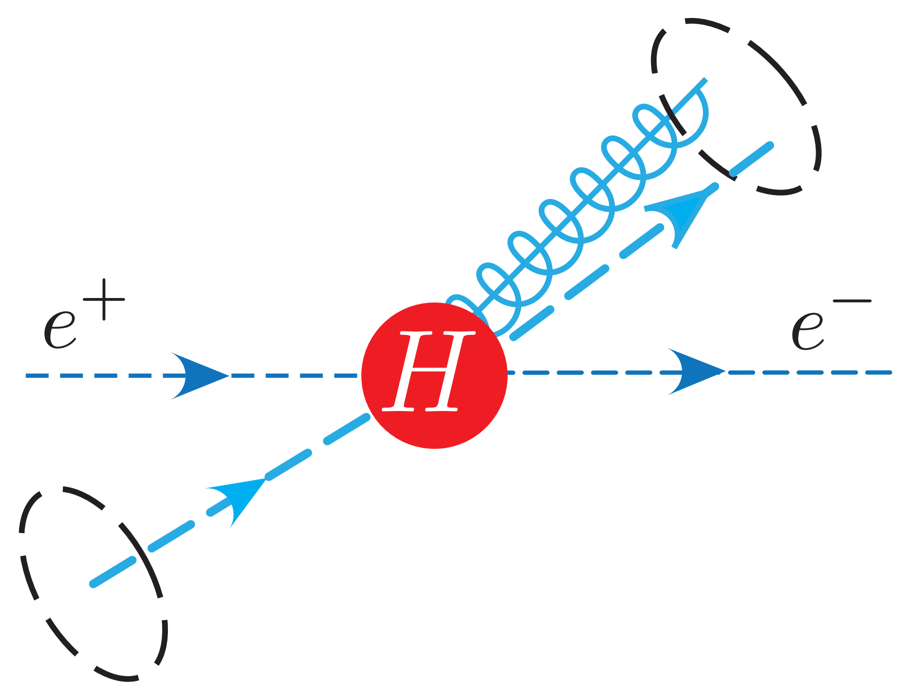
Hard Scattering operators (typically once)

Only leading term can spoil factorization

Subleading Lagrangians

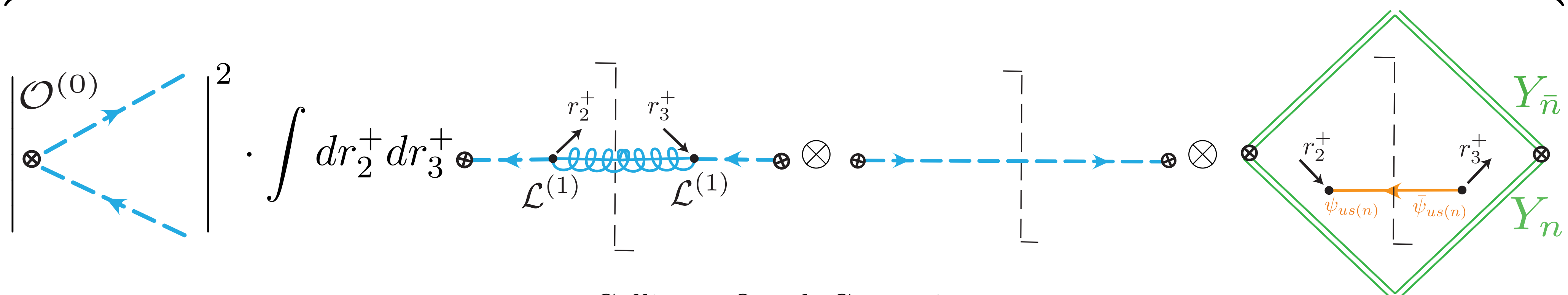


Subleading Hard Scattering Operators

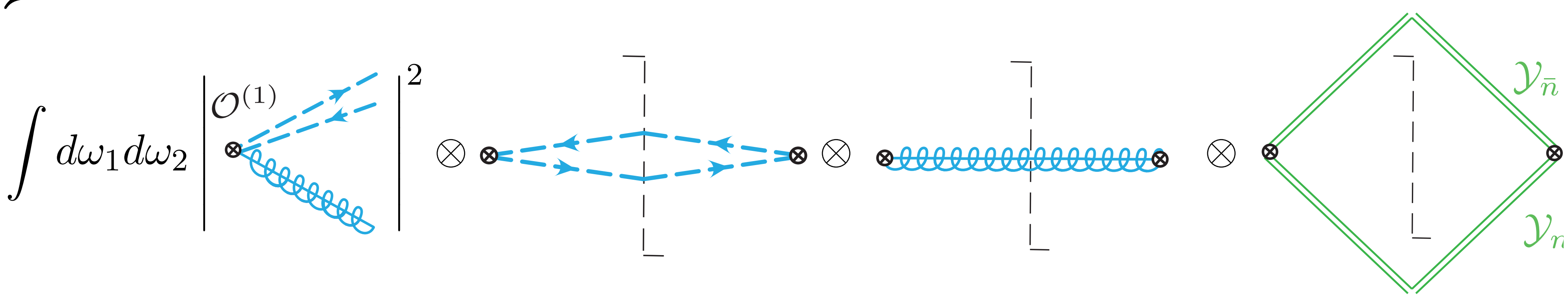


Sudakov suppression at subleading power? Eg. in Thrust

Soft Quark Correction



Collinear Quark Correction



$$= \int d\omega_1 \omega_2 \left| C^{(1)}_{\chi\chi n}(\omega_1) C^{(1)}_{\chi\chi n}(\omega_2) \right| J^{(2)}_{\chi\chi}(\omega_1,\omega_2) \otimes J^{(0)}_{g,\bar{n}} \otimes S^{(0)}_q + n \leftrightarrow \bar{n} \,.$$

- Proof (refactorization)

Beneke, Garny, Jaskiewicz, Strohm, Szafron, Vernazza, Wang `22

$$\frac{1}{\sigma_0}\frac{d\sigma^{(2),e^+e^-}_{\rm LL}}{d\tau} = \left(\frac{\alpha_s}{4\pi}\right) 8 C_F \log(\tau) e^{-4C_F\left(\frac{\alpha_s}{4\pi}\right)\log^2(\tau)} + \underbrace{\frac{C_F}{(C_F - C_A)\log(\tau)}\left(e^{-4C_F\left(\frac{\alpha_s}{4\pi}\right)\log^2(\tau)} - e^{-4C_A\left(\frac{\alpha_s}{4\pi}\right)\log^2(\tau)}\right)}_{\text{Soft Quark Sudakov}}$$

- Conjecture

Moult, IS, Vita, Zhu `19

Endpoint singularities! $\int_0 \frac{dx}{x}$

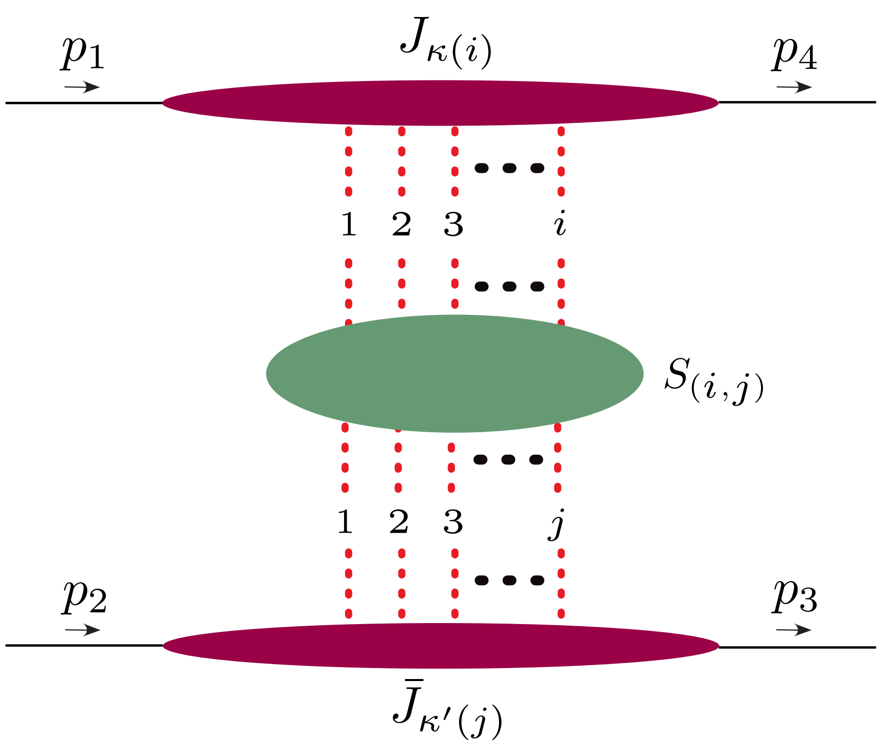
Regge Amplitudes

Regge Amplitudes in SCET

Rothstein, IS (2016)
Moult, Raman, Ridgway, IS (2022)
Gao, Moult, Raman, Ridgway, IS (2024)

$$\mathcal{O}_n^A \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_S^{AB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^B \quad , \quad \mathcal{O}_n^A \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^A$$

- $\mathscr{L}_G^{(0)}$ Lagrangian gives forward scattering amplitudes $s \gg |t|$



$\mathscr{A}(i,j)$

any loop order

both planar and non-planar graphs

any color channel

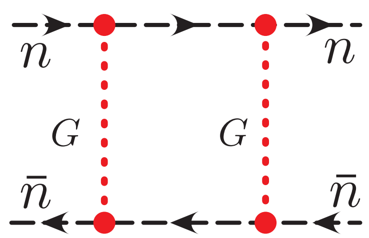
large (Regge) logs from rapidity RGE

$$\ln\left(\frac{s}{-t}\right) = \ln\left(\frac{s}{\nu^2}\right) + \ln\left(\frac{\nu^2}{-t}\right)$$

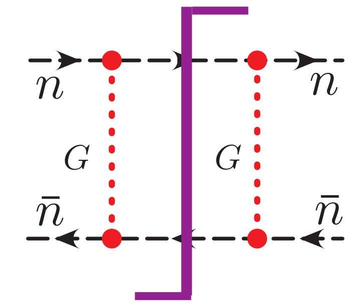
collinear loop   soft loop

- Glauber loops (simple)

require regulator
answer = pure cut

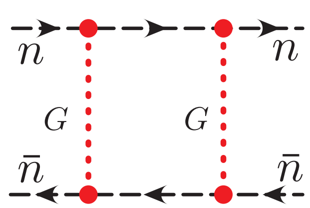


$$\propto (i\pi) \int \frac{d\!\!{}^{-}{}^{d-2}k_\perp}{(\vec{k}_\perp^{\,2})(\vec{k}_\perp+\vec{q}_\perp)^2}$$

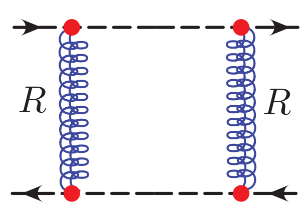


- Same color as QCD box graph (includes 8_A)

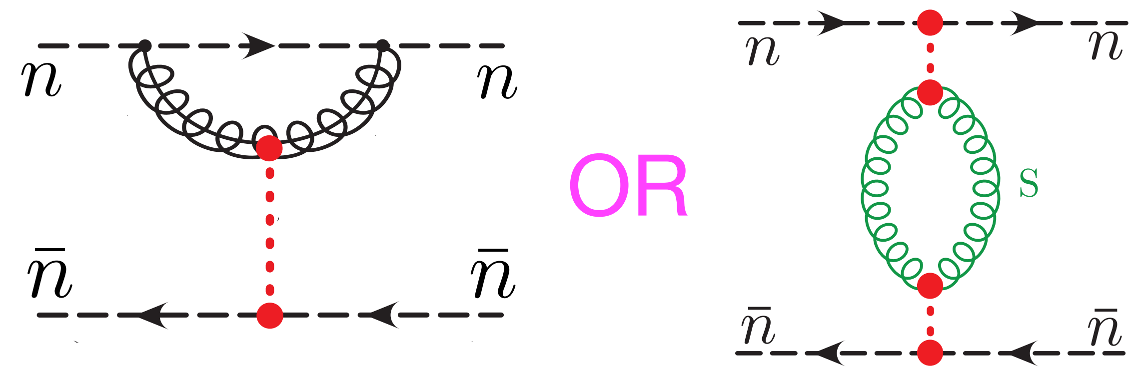
Glauber ≠ Reggeon



$\neq$



- eg. Gluon Reggeization $\mathscr{A}(1,1)$



single Glauber exchange (8_A)

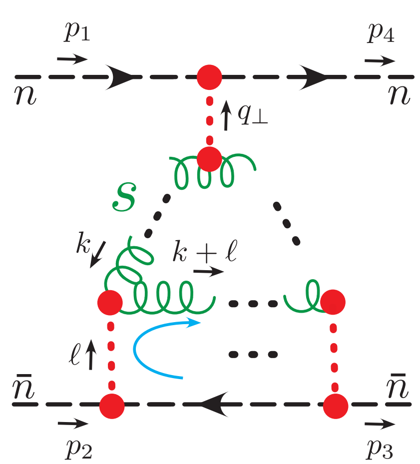
rapidity divergent due to Wilson lines in collinear (soft) operators

$$\nu \frac{d}{d\nu} J_{(1)} = -\alpha(t) J_{(1)}$$

evolve: $\nu^2 = s \ \rightarrow \ \nu^2 = -t$

gives: $\left(\frac{s}{-t}\right)^{\alpha(t)}$

- Turns out that there are no $1 \rightarrow (j \geq 2)$ transitions: $J_{(1)}^{\text{bare}} = Z_{(1,1)} J_{(1)}^{\text{ren}}$



$= 0$

provides natural definition for Gluon Regge trajectory at any loop order

- General rapidity renormalization

transverse momentum integrals

Glaubers

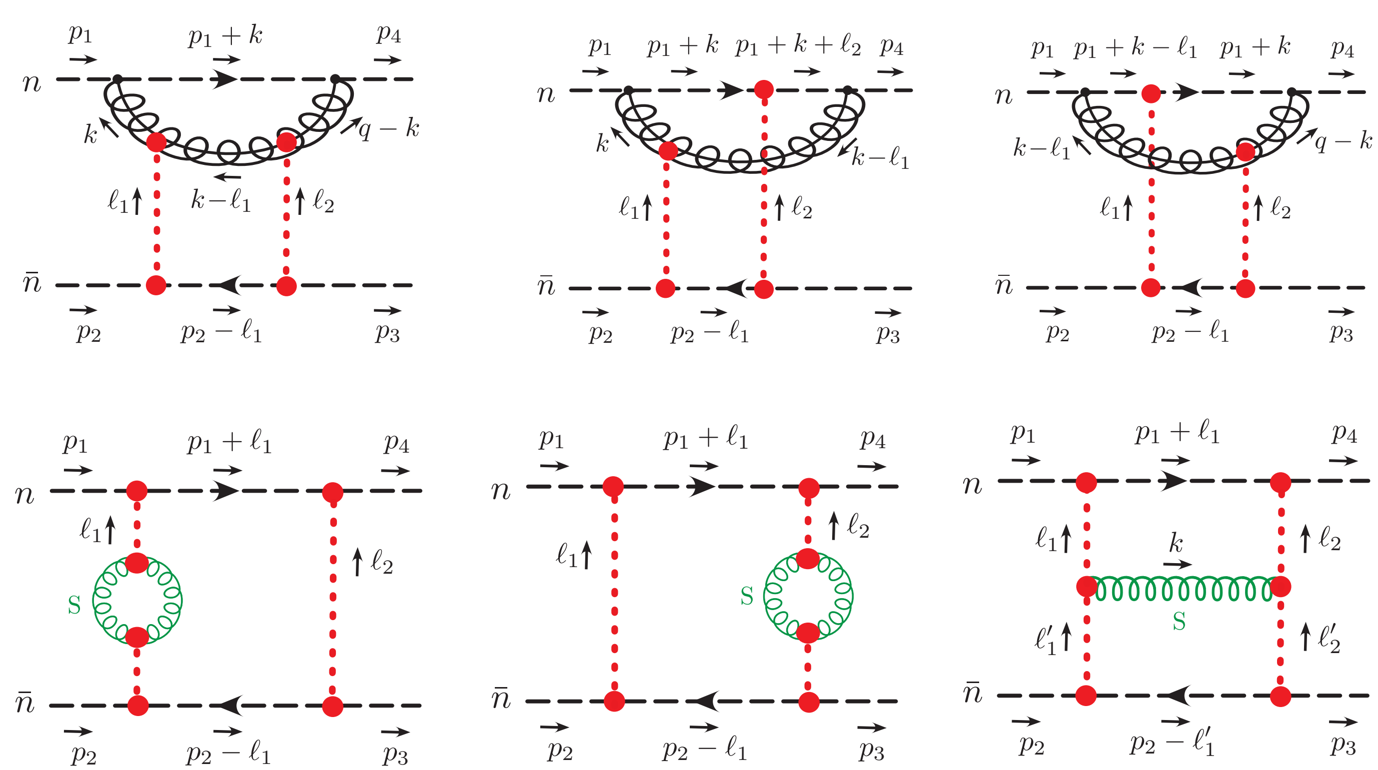
color

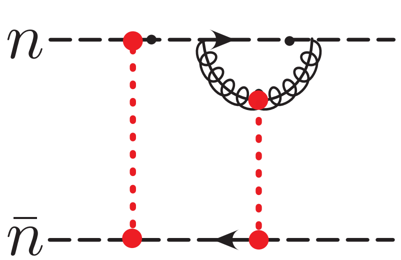
$$\mathcal{A} = \sum_{ij} \mathcal{A}_{(i,j)} = \sum_{ij} J^{\alpha}_{\kappa(i)} \otimes_i S^{\alpha\beta}_{(i,j)} \otimes_j J^{\beta}_{\kappa'(j)} = \mathbf{J}_\kappa \cdot \mathbf{S} \cdot \mathbf{J}_{\kappa'}$$

$$\mathbf{J}^{\rm bare} = \mathbf{J}^{\rm ren} \cdot \mathbf{Z}_J \qquad \mathbf{S}^{\rm bare} = \mathbf{Z}_S \cdot \mathbf{S}^{\rm ren} \cdot \mathbf{Z}_S \qquad \mathbf{Z}_S = \mathbf{Z}_J{}^{-1}$$

- 2 Glauber exchange reproduces 1_S (pomeron), 8_S, 27 BFKL equations

again 2 possible ways
to do calculation





= 0

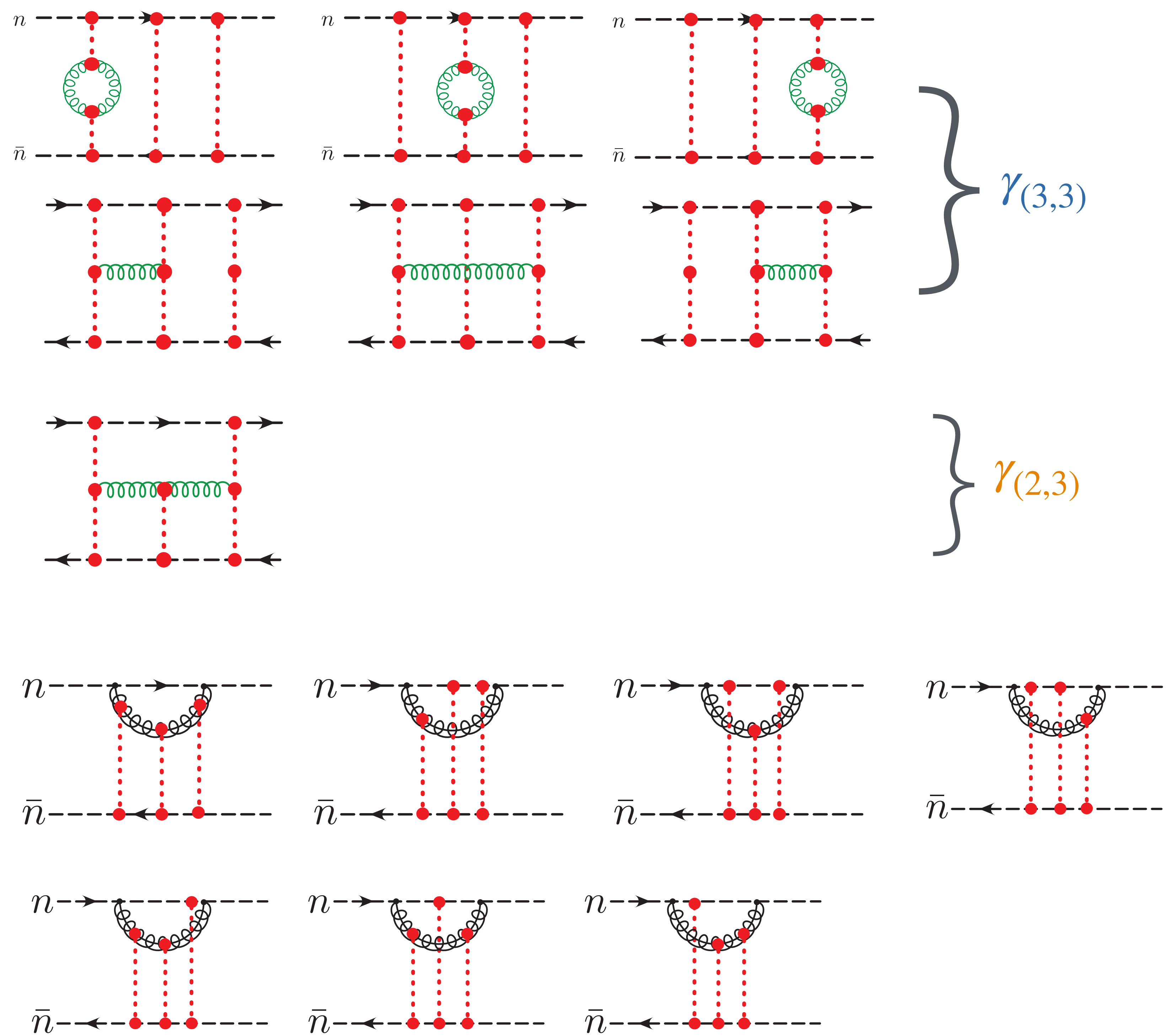
collapse rule

- 3 Glauber exchange

Meaning: $27 \oplus \cdots \oplus 27$, 6 copies of 27's

$$8 \otimes 8 \otimes 8 = 1^2 \oplus 8^8 \oplus 10^4 \oplus \overline{10}^4 \oplus 27^6 \oplus 35^2 \oplus \overline{35}^2 \oplus 64$$

Regge cuts
at this order



- Interesting complementarity to Reggeon EFTs

QCD ⟶ SCET ⇢ Wilson Line Regge EFT

$\mathscr{L}_G^{(0)}$

Caron-Huot, Gardi, Vernazza, …

★ operator definition for impact factors $\langle p | O_n^{A_1} \cdots O_n^{A_N} | p' \rangle$

★ collinear loop calculations for rapidity logs

★ different structure for vanishing transitions $1 \to j$ vs. eg. $(j-1) \to j$

★ signature and crossing symmetry not manifest from start

- Glauber operators can also be used to study factorization violation in hard scattering

Other Areas (no time to discuss)

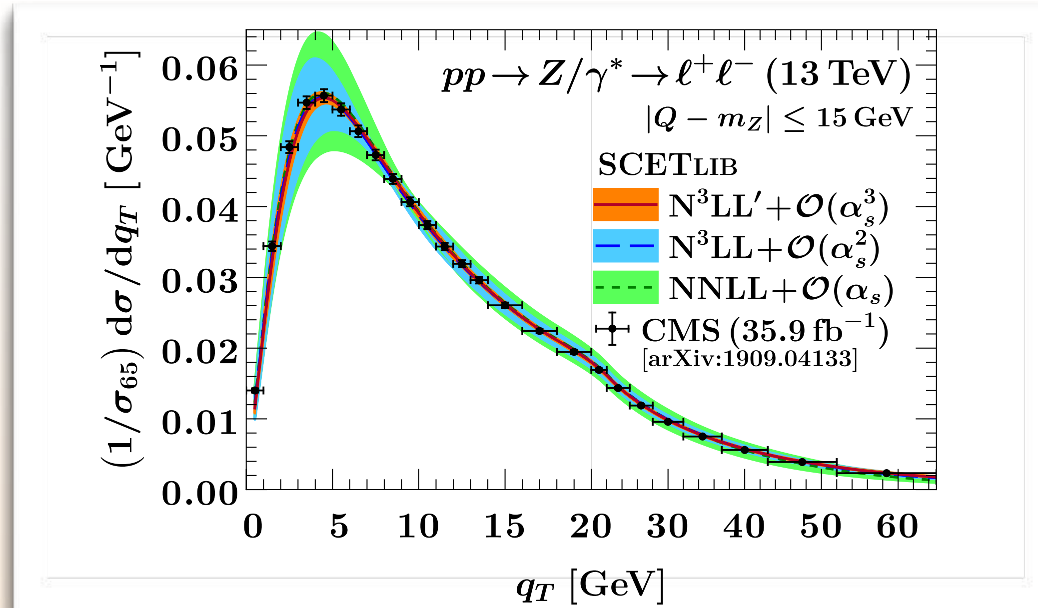
- SCET for B-physics (SCET+HQET)
- SCET for quarkonia (SCET+NRQCD)
- SCET for jet substructure, often called SCET_+
- SCET for heavy-ions (SCET coupled to medium)
- SCET for electroweak logarithms
- SCET for Dark Matter annihilation
- SCET for gravitational scattering amplitudes

⋮

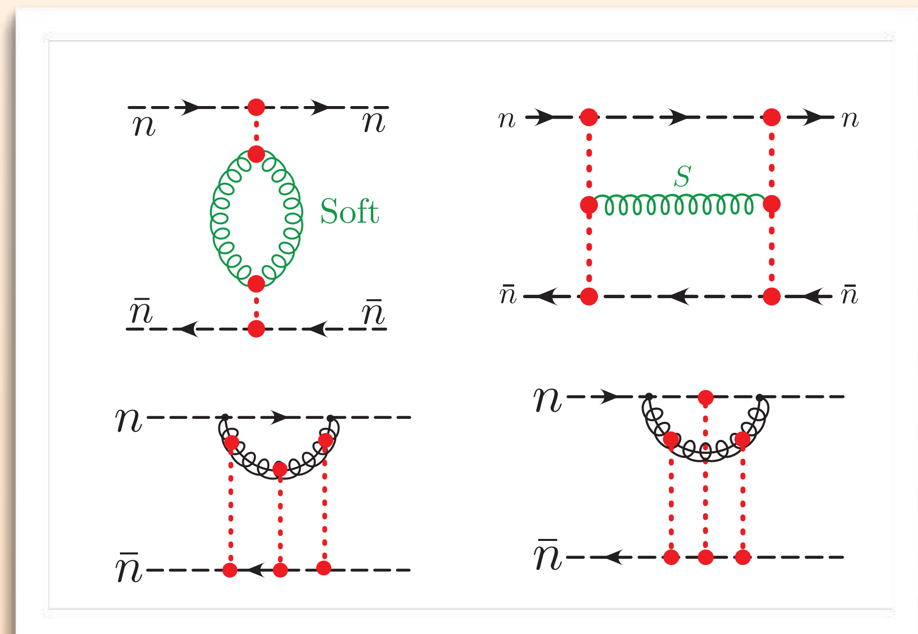
For further references see my SCET review in 50 yrs of QCD, 2212.11107

Summary:

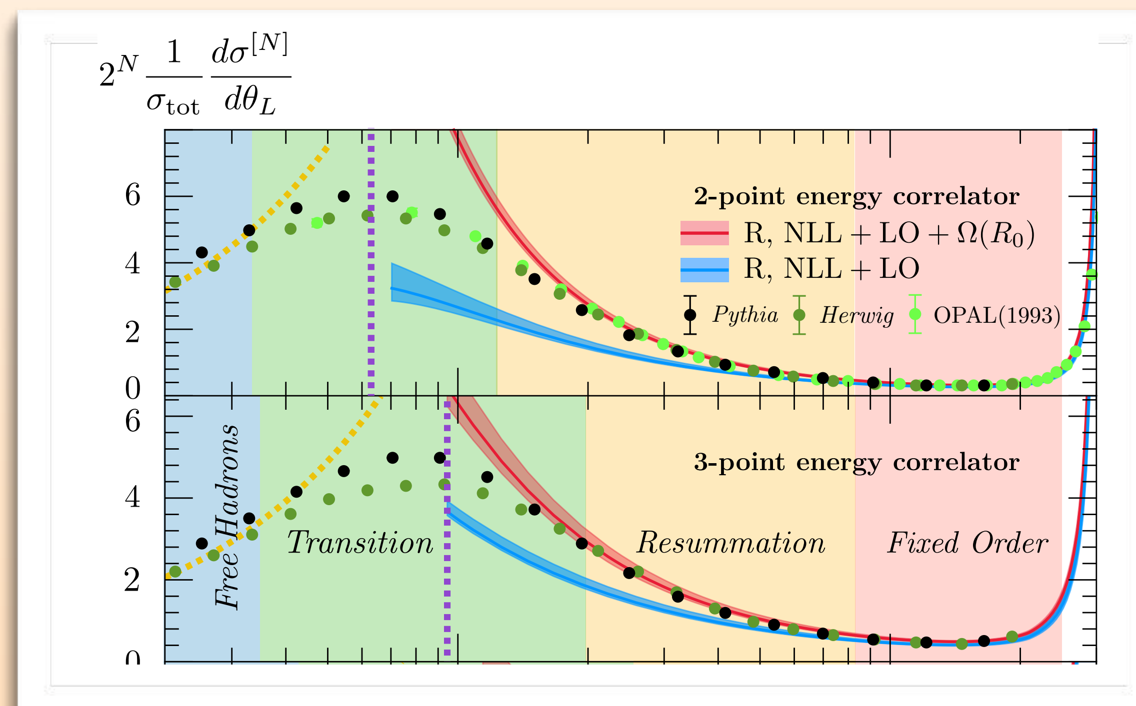
- Precision Resummation



- Regge Amplitudes



- Nonperturbative corrections



- Power Corrections

$$J_{\mathcal{P}}^{(1)\mu} \sim \frac{C_f^{(0)}}{2\omega_a} \bar{\chi}_{\bar{n},\omega_b} [S_{\bar{n}}^\dagger S_n] \gamma^\mu \mathcal{P}\!\!\!/_\perp \bar{n}\!\!\!/ \chi_{n,\omega_a}$$

$$J_{\mathcal{B}}^{(1)\mu} \sim (n^\mu + \bar{n}^\mu) \int d\omega_c\, C_f^{(1)}(Q,\omega_c)\ \bar{\chi}_{\bar{n},\omega_b} [S_{\bar{n}}^\dagger S_n] \mathcal{B}\!\!\!/_{\perp n,-\omega_c} \chi_{n,\omega_a}$$

SCET is a powerful tool for Collider Physics