

# Categorical Symmetries & $1 + 1d$ Scattering Amplitudes

Christian Copetti  
(Oxford)

Based on 2403.04835 and 2406.XXXXX with L. Cordova and S. Komatsu

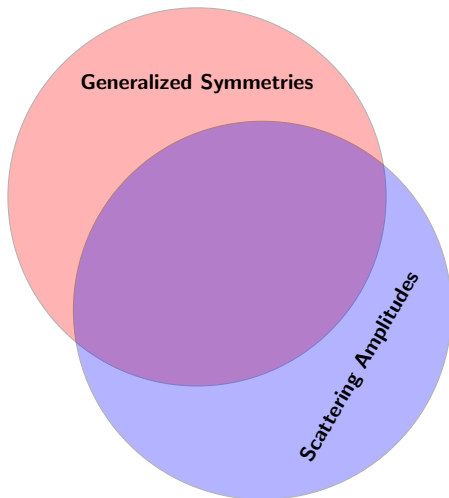


# What is this talk about?

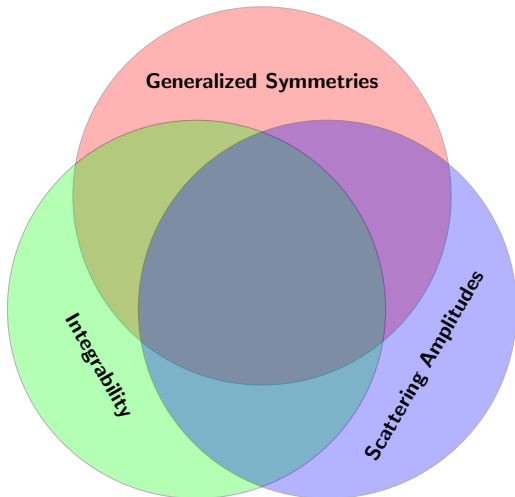
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**Generalized Symmetries**

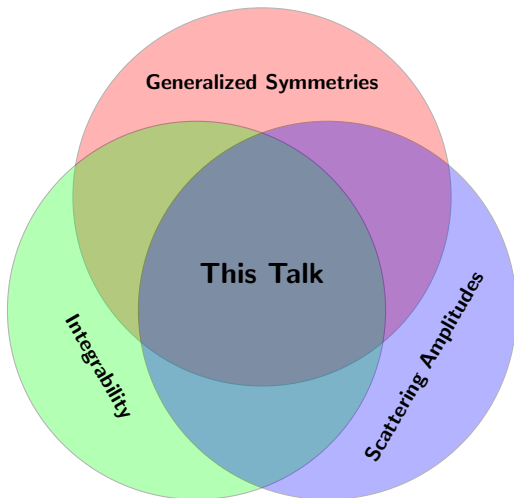
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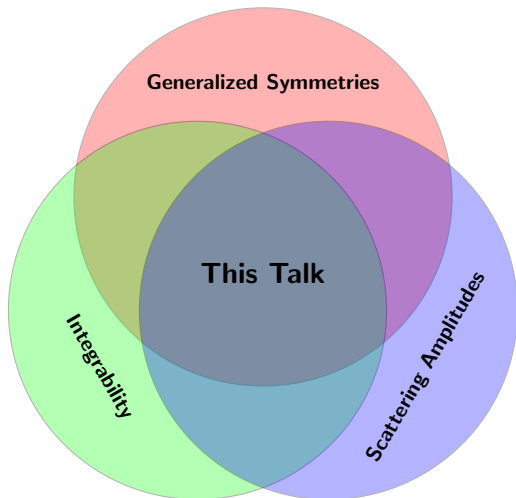
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**Aim:** Discuss action of **Generalized Symmetries** on **S-matrix** and derive physical consequences (**Integrable** examples, but conclusions more general!).

# Summary

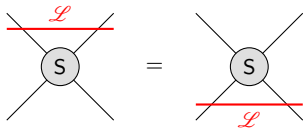


# Summary

- Categorical Symmetries act on (massive) kinks and lead to Ward identities for the  $2 \rightarrow 2$  S-Matrix:

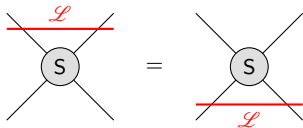
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$$\begin{array}{c} \text{L} \\ \hline \text{S} \end{array} = \begin{array}{c} \text{S} \\ \hline \text{L} \end{array} \quad (*)$$

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Diagrammatic equation (\*): A central grey circle labeled 'S' is crossed by two black lines. A red horizontal line labeled  $\mathcal{L}$  is drawn above the circle. This is equal to the same central grey circle labeled 'S' crossed by two black lines, but with the red horizontal line labeled  $\mathcal{L}$  drawn below the circle. The equation is labeled with an asterisk (\*) on the right.

Diagrammatic equation (\*\*): A loop with two vertices (small grey circles) on the left, where two black lines enter from the top and two exit from the bottom. This is equal to two parallel vertical black lines on the right. The equation is labeled with two asterisks (\*\*).

- Imposing **Symmetry** (\*), **Unitarity** (\*\*)

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- Categorical Symmetries act on (massive) kinks and lead to Ward identities for the  $2 \rightarrow 2$  S-Matrix:

Diagrammatic equation (\*): A kink (a circle labeled 'S') with two incoming lines from the top and two outgoing lines to the bottom. A red horizontal line labeled  $\mathcal{L}$  is drawn above the kink. This is equal to the same kink with the red horizontal line labeled  $\mathcal{L}$  drawn below it.

Diagrammatic equation (\*\*): A loop diagram with two vertices (circles) on the left and two vertices on the right. This is equal to two parallel vertical lines, one on the left and one on the right.

Diagrammatic equation (\*\*\*) shows a kink with two vertices on each side (left and right) equal to another kink with two vertices on each side, representing a different configuration of the same structure.

- Imposing **Symmetry** (\*), **Unitarity** (\*\*), and **YBE** (\*\*\*)

# Summary

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A diagrammatic equation labeled (\*). On the left, a gray circle labeled 'S' has four lines crossing at its center. A red horizontal line labeled 'L' is drawn above the circle. This is set equal to the same diagram but with the red line 'L' drawn below the circle.

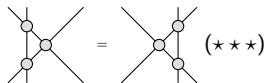
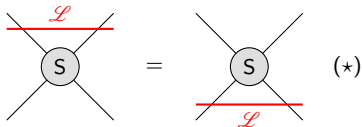
A diagrammatic equation labeled (\*\*). On the left, a circle with two vertices on its top and bottom edges, each connected to an external line. This is set equal to two parallel vertical lines.

A diagrammatic equation labeled (\*\*\*) showing two diagrams of a kink with three vertices. The left diagram has three vertices on a single vertical line. The right diagram has three vertices on a vertical line that is part of a larger structure. The two diagrams are set equal to each other.

- Imposing **Symmetry** (\*), **Unitarity** (\*\*), and **YBE** (\*\*\*) is incompatible with standard **Crossing**.

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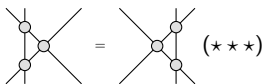
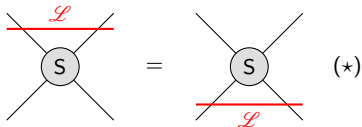
- Imposing **Symmetry** (\*), **Unitarity** (\*\*), and **YBE** (\*\*\*) is incompatible with standard **Crossing**. Instead:

$$S_{dc}^{ab}(\theta) = \sqrt{\frac{d_a d_c}{d_b d_d}} S_{ad}^{bc}(i\pi - \theta)$$



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$$S_{dc}^{ab}(\theta) = \sqrt{\frac{d_a d_c}{d_b d_d}} S_{ad}^{bc}(i\pi - \theta)$$

- Categorical symmetries can be used efficiently in the **Bootstrap** program. (See [Lucia's Lectures!](#))

# Philosophy





IR → UV



IR  $\rightarrow$  UV



IR Vacua

Symmetric TQFT  $\mathcal{M}$



IR  $\rightarrow$  UV



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Massive Kinks

$K_{ab}$



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Preserved by  $\phi$

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# Categorical symmetries (Review)

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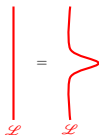
Implemented by topological lines:

[Petkova, Zuber '02; Gaiotto, Kapustin, Seiberg, Willett '14; Chang, Lin, Shao, Wang, Yin '18; ...]

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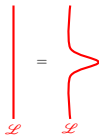
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Fusion structure

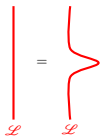
A diagram illustrating the fusion structure. On the left, two vertical red lines are shown, labeled with red  $\mathcal{L}$  and  $\mathcal{L}'$  at their bases. To the right of these lines is an equals sign. To the right of the equals sign is a mathematical expression: a sum over  $\mathcal{L}_3$  of  $N_{\mathcal{L} \mathcal{L}'}^{\mathcal{L}''}$  multiplied by a single vertical red line labeled with a red  $\mathcal{L}'''$  at its base.

$$N_{\mathcal{L} \mathcal{L}'}^{\mathcal{L}''} \in \mathbb{N}$$

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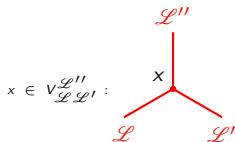


Fusion structure

A diagram illustrating the fusion structure. On the left, two vertical red lines are labeled  $\mathcal{L}$  and  $\mathcal{L}'$ . This is followed by an equals sign. On the right, there is a sum over  $\mathcal{L}_3$  of  $N_{\mathcal{L} \mathcal{L}'}^{\mathcal{L}''}$  multiplied by a single vertical red line labeled  $\mathcal{L}'''$ .

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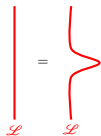
Topological junctions (vector space):



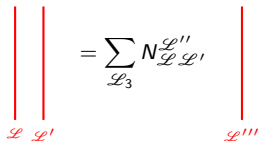
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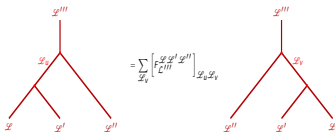


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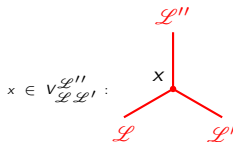


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Associativity (F-symbols):



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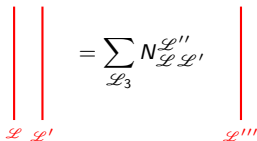
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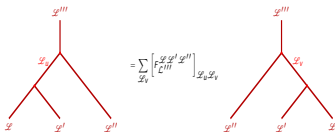


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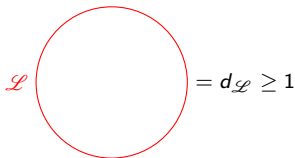


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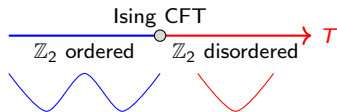
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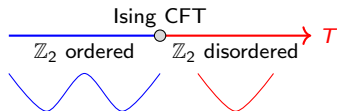
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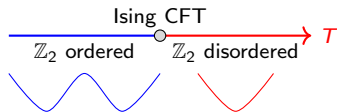
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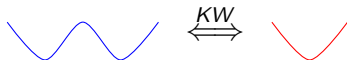
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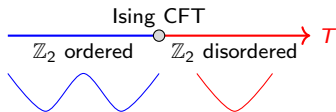


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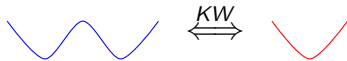


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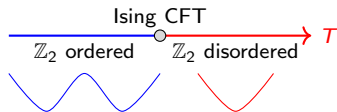
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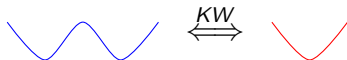
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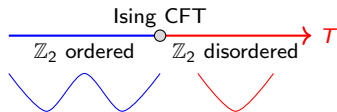


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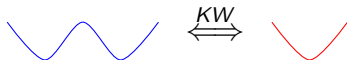
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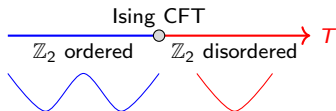
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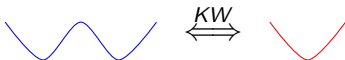
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Fusion algebra:

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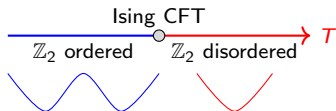
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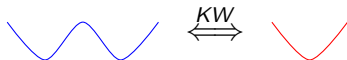
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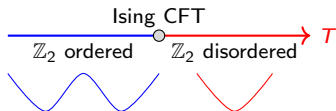
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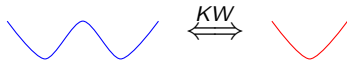
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The KW defect line  $\mathcal{N}$  is **non-invertible!**

# $\mathcal{C}$ -symmetric TQFTs

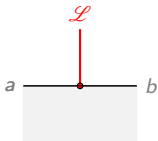
# $\mathcal{C}$ -symmetric TQFTs

We describe 1+1d TQFT  $\mathcal{M}$  via a collection of boundary conditions (states)  
 $a, b, c, \dots$  [Huang, Lin, Seifnashri '21, ...]



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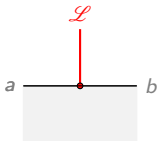
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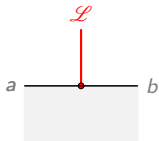
$$a \begin{array}{c} \mathcal{L} \\ | \\ \bullet \\ | \\ b \\ | \\ \bullet \\ | \\ \mathcal{L}' \\ | \\ c \end{array} = \sum_{\mathcal{L}''} \varphi_{\mathcal{L} \mathcal{L}' \mathcal{L}''}^{a b c} \begin{array}{c} \mathcal{L} \quad \mathcal{L}' \\ \diagdown \quad / \\ \bullet \\ | \\ \mathcal{L}'' \\ | \\ a \quad c \end{array}$$



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Parallel fusion is described by an integer-valued matrix:



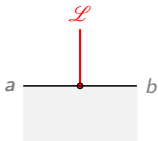
$$\begin{array}{c} \mathcal{L} \\ \hline \end{array} = (n_{\mathcal{L}})_a^b \begin{array}{c} \hline \end{array}$$

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$$\begin{array}{c} \mathcal{L} \quad \mathcal{L}' \\ | \quad | \\ a \quad b \quad c \end{array} = \sum_{\mathcal{L}''} \varphi_{\mathcal{L}''}^{a b c} \begin{array}{c} \mathcal{L} \quad \mathcal{L}' \\ \diagdown \quad / \\ \mathcal{L}'' \\ | \\ a \quad c \end{array}$$

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$$\overline{\mathcal{L}} = (n_{\mathcal{L}})_{ab}$$

The diagrammatic equation shows a red line labeled 'L' above a gray box with boundary 'a' on the left and 'b' on the right. This is equal to the matrix element  $(n_{\mathcal{L}})_{ab}$  multiplied by a gray box with boundary 'b' on the left and 'a' on the right.

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$$a \begin{array}{c} \mathcal{L} \\ | \\ \bullet \\ | \\ \mathcal{L}' \\ | \\ \bullet \\ | \\ c \end{array} b = \sum_{\mathcal{L}''} \varphi_{\mathcal{L} \mathcal{L}' \mathcal{L}''}^{abc} \begin{array}{c} \mathcal{L} \quad \mathcal{L}' \\ \diagdown \quad / \\ \bullet \\ / \quad \backslash \\ \mathcal{L}'' \end{array} c$$

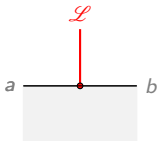
The diagrammatic equation shows a junction of two red lines labeled 'L' and 'L'' meeting at a point on a boundary labeled 'a' and 'c'. A third red line labeled 'L'' extends downwards from this junction to a point on the boundary labeled 'b'. This is equal to a sum over 'L''' of a coefficient  $\varphi_{\mathcal{L} \mathcal{L}' \mathcal{L}''}^{abc}$  times a junction where 'L' and 'L'' meet at a point on boundary 'a' and 'c', and a red line labeled 'L''' extends downwards from this junction to a point on the boundary labeled 'b'.

Satisfying the algebra:

$$\sum_b (n_{\mathcal{L}})_a^b (n_{\mathcal{L}'})_b^c = \sum_{\mathcal{L}''} N_{\mathcal{L} \mathcal{L}' \mathcal{L}''}^{\mathcal{L}''} (n_{\mathcal{L}''})_a^c,$$

# $\mathcal{C}$ -symmetric TQFTs

We describe 1+1d TQFT  $\mathcal{M}$  via a collection of boundary conditions (states)  $a, b, c, \dots$  [Huang, Lin, Seifnashri '21, ...]



Parallel fusion is described by an integer-valued matrix:

$$\overline{\mathcal{L}} = (n_{\mathcal{L}})^b_a \quad b$$

The equation shows a red line labeled  $\mathcal{L}$  above a gray box, followed by an equals sign, then a gray box above a red line labeled  $\mathcal{L}$ . The matrix element  $(n_{\mathcal{L}})^b_a$  is placed between the two gray boxes.

The symmetry action is described by topological junctions. Which satisfy associativity conditions

$$a \begin{array}{c} \mathcal{L} \\ | \\ \bullet \\ | \\ b \\ | \\ \bullet \\ | \\ \mathcal{L}' \\ | \\ c \end{array} = \sum_{\mathcal{L}''} \varphi_{\mathcal{L} \mathcal{L}' \mathcal{L}''}^{abc} \begin{array}{c} \mathcal{L} \quad \mathcal{L}' \\ \diagdown \quad \diagup \\ \bullet \\ \diagup \quad \diagdown \\ \mathcal{L}'' \\ | \\ a \end{array} c$$

The equation shows two diagrams. The left diagram has a horizontal line from 'a' to 'c'. Two vertical red lines, labeled  $\mathcal{L}$  and  $\mathcal{L}'$ , extend upwards from points on the line. The space between the lines is labeled 'b'. The right diagram has a horizontal line from 'a' to 'c'. A red line labeled  $\mathcal{L}''$  extends upwards from a point on the line. Two other red lines, labeled  $\mathcal{L}$  and  $\mathcal{L}'$ , branch off from the top of the  $\mathcal{L}''$  line. Below the horizontal line is a light gray shaded area.

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$$\sum_b (n_{\mathcal{L}})^b_a (n_{\mathcal{L}'})^c_b = \sum_{\mathcal{L}''} N_{\mathcal{L} \mathcal{L}' \mathcal{L}''}^{\mathcal{L}''} (n_{\mathcal{L}''})^c_a,$$

This endows  $\mathcal{M}$  with the mathematical structure of a module category over  $\mathcal{C}$ .

# Example: Ising TQFT

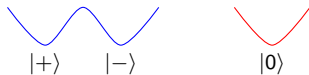
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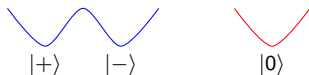
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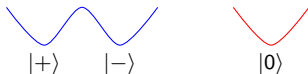
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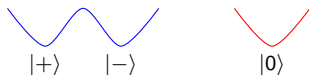
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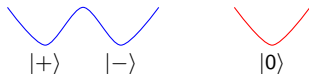
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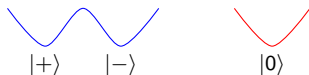
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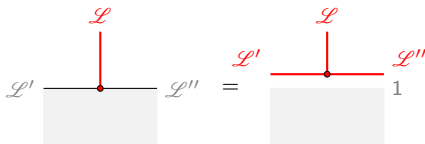


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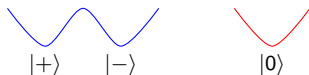
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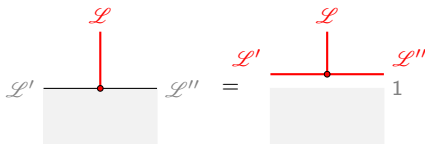


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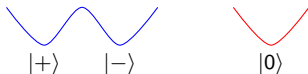
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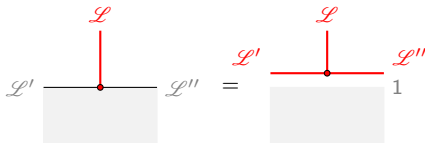


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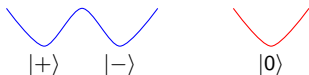
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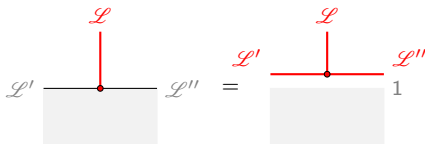
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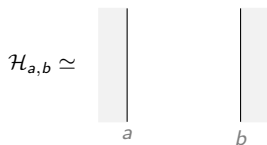
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To understand symmetry action on kinks we describe their Hilbert space  $\mathcal{H}_{ab}$  as the strip Hilbert space with  $L \gg 1/M_{\text{kink}}$  and TQFT b.c. [Cordova, Garcia-Sepulveda, Holfester '24] :



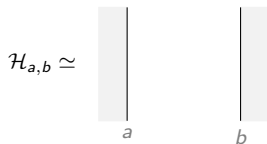
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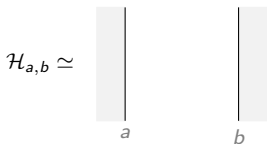
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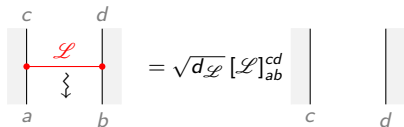
$$\begin{array}{|c|} \hline c \\ \hline \bullet \\ \hline a \end{array} \begin{array}{|c|} \hline d \\ \hline \bullet \\ \hline b \end{array} \xrightarrow{\mathcal{L}} \begin{array}{|c|} \hline \\ \hline c \end{array} \begin{array}{|c|} \hline \\ \hline d \end{array} = \sqrt{d_{\mathcal{L}}} [\mathcal{L}]_{ab}^{cd}$$

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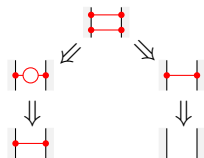
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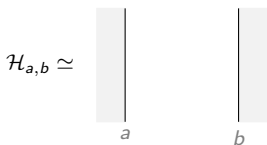


Composition of two lines  $\mathcal{L}, \mathcal{L}'$

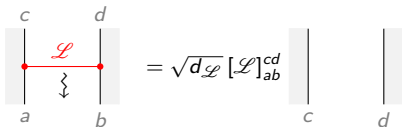


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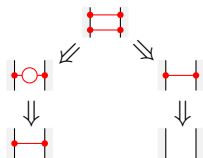
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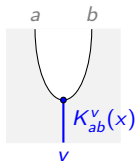
Gives the algebra:

$$[\mathcal{L}']_{cd}^{ef} \cdot [\mathcal{L}]_{ab}^{cd} = \sum_{\mathcal{L}''} \varphi_{\mathcal{L}\mathcal{L}'\mathcal{L}''}^{ace} \varphi_{\mathcal{L}\mathcal{L}'\mathcal{L}''}^{cdf} [\mathcal{L}'']_{ab}^{ef}.$$

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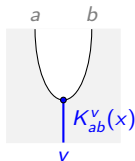
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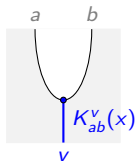


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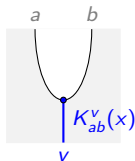
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The fusion algebra  $v \times v' = \sum_{v''} \tilde{N}_{vv'}^{v''} v''$  encodes the tensor product decomposition of irreps  $\rightarrow$  **kink bound states!**

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**Including the green piece turns out to be incompatible with the Ising symmetry.**

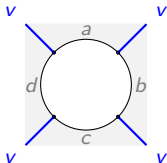
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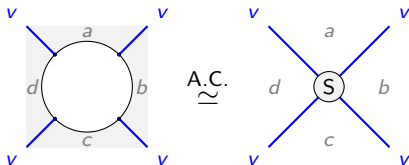
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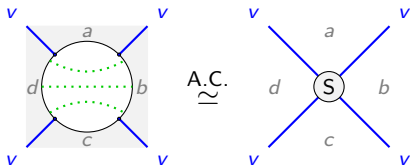
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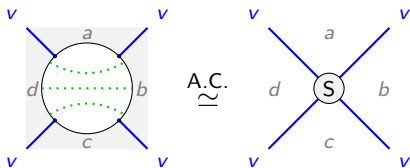
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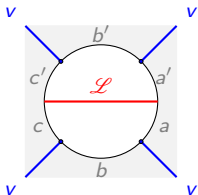


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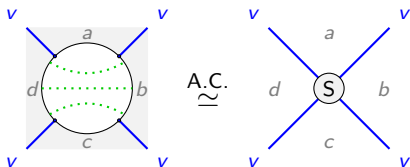


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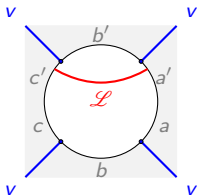


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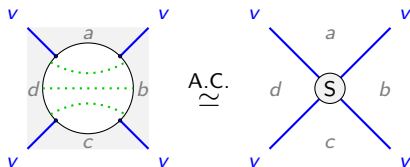


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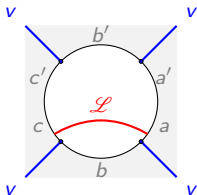


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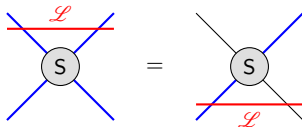
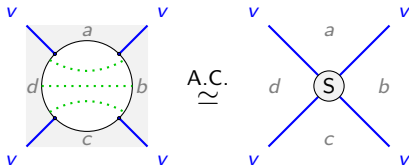


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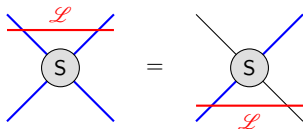
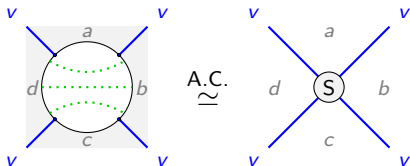
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$$\sum_e [\mathcal{L}; v]_{cb}^{c'e} [\mathcal{L}; v]_{ba}^{ea'} \sqrt{\frac{d_a}{d_c}} S_{a'e}^{c'b'}(\theta) = \sum_{e'} [\mathcal{L}; v]_{b'c'}^{e'c'} [\mathcal{L}; v]_{a'b'}^{ae'} \sqrt{\frac{d_{a'}}{d_{c'}}} S_{ab}^{ce'}(\theta).$$

# Ward identities and the S-matrix

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$\mathcal{M}_{4,3} \rightarrow$  Ising:

$$\mathcal{N}: S_{+0}^{0+}(\theta) = S_{0+}^{+0}(\theta) + S_{0-}^{+0}(\theta).$$

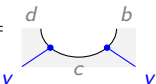
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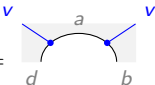
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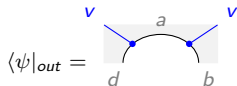
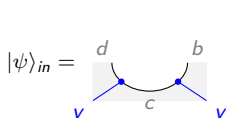
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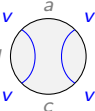
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Diagram 2: A shaded region with a concave-down boundary. Two blue dots are on the boundary, labeled 'd' (left) and 'b' (right). A blue line labeled 'a' connects the two dots. Four blue arrows labeled 'v' point outwards from the corners of the shaded region.

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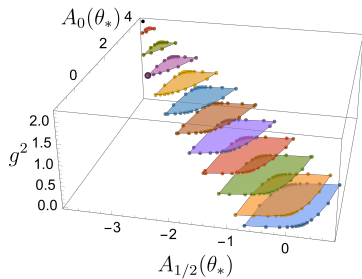
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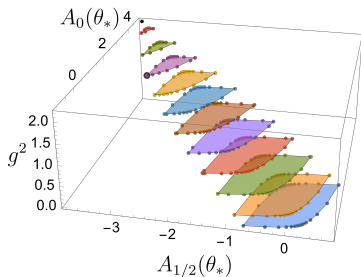




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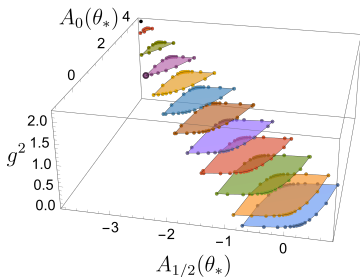
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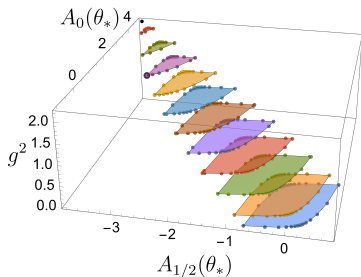


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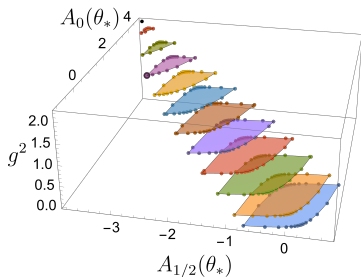


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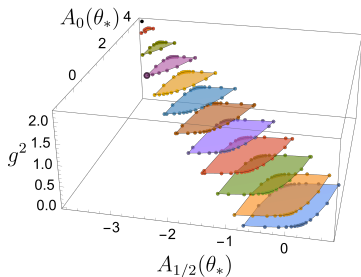


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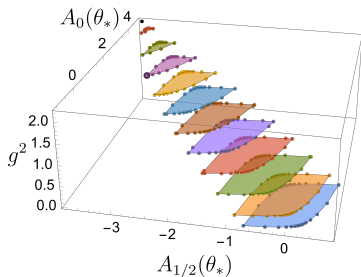
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