SELF-DUAL COSMOLOGY AND THE COLOR-KINEMATICS DUALITY

Based on 2406.XXXXX with A. Lipstein and S. Nagy

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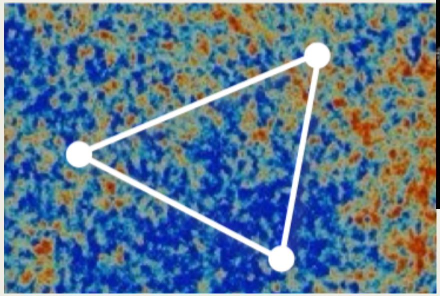
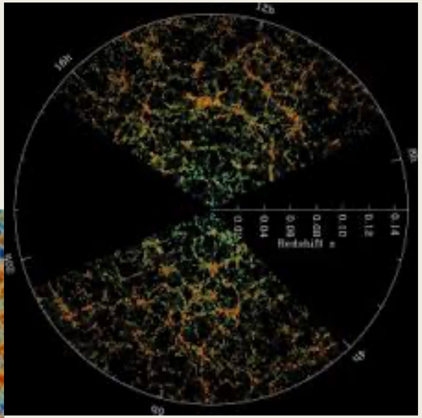
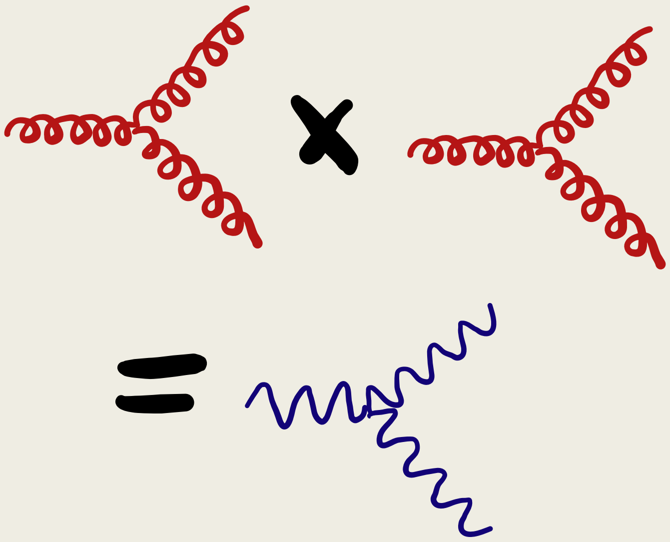
IMPERIAL

Amplitudes 2024







Calculation of correlators in curved space can be highly involved

Double copy can simplify calculations

?

Cleanest Double Copy Example

Self-dual solutions (SD)

$$\boxed{F_{\mu\nu} = \frac{i}{2}\varepsilon_{\mu\nu\rho\lambda}F^{\rho\lambda}} \qquad \boxed{R_{\mu\nu\lambda\rho} = \frac{i}{2}\varepsilon_{\mu\nu\eta\sigma}R^{\eta\sigma}{}_{\lambda\rho}}$$

On-shell $A^{\text{tree}} = 0$

Classically integrable

Exact at 1-loop

Full theory = SD + perturbations

SD Yang-Mills

Bardeen 95 Cangemi 97

$$ds^2 = du\,dv - dw\,d\bar{w}$$

$u = t+z, \; v = t-z, \; w = x+iy$

lightcone gauge $A_u = 0$

$$\boxed{F_{\mu\nu} = \frac{i}{2}\,\varepsilon_{\mu\nu\rho\lambda}\,F^{\rho\lambda}}$$

$$\boxed{A_\alpha = \Pi_\alpha \Phi} \qquad \Pi_\alpha = (\partial_w, \partial_u) \quad \alpha = v, \bar{w} \qquad \Phi = \Phi^a T^a$$

e.o.m $\;\Box\Phi - i g\,\varepsilon^{\alpha\beta}[\Pi_\alpha\Phi, \Pi_\beta\Phi] = 0$

Lie Bracket

$$[T^a, T^b] = i f^{abc} T^c$$

Scale invariant

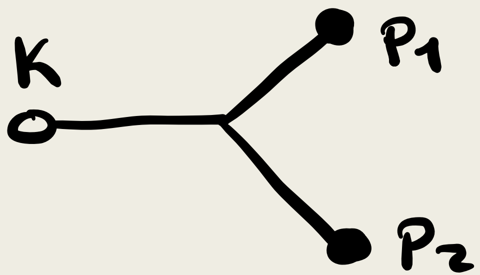
$\rightarrow$ holds for $g_{\mu\nu} = \Omega^2 \eta_{\mu\nu}$

Solve eom for $g \ll 1$

$$\Phi^{(0)a}(k) = j^a(k) \equiv \frac{J^a(k)}{k^2}$$

$$\Phi^{(1)a}(k) \sim \frac{g}{2}\int_{p_1 p_2} \frac{1}{k^2}\, F_{p_1 p_2}{}^{k}\, f^{abc}\, j^b(p_1)\, j^c(p_2)$$

$$F_{p_1 p_2}{}^{k} = \delta(p_1+p_2-k)\, \overbrace{p_{1w}\, p_{2u} - p_{1u}\, p_{2w}}^{\equiv X(p_1,p_2)}$$



$$\boxed{A_3^{abc} = -i \lim_{p_3^2 \to 0} p_3^2 \frac{\delta^2 \Phi^{(1)a}(-p_3)}{\delta j^b(p_1)\, \delta j^c(p_2)}} \longrightarrow A_n^{a_1 \cdots a_n}$$

SD Gravity

$$\boxed{R_{\mu\nu\lambda\rho} = \frac{i}{2}\varepsilon_{\mu\nu\eta\sigma} R^{\eta\sigma}{}_{\lambda\rho}}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

lightcone gauge $h_{u\mu} = 0$

$$\boxed{h_{\alpha\beta} = \frac{1}{4}\Pi_\alpha \Pi_\beta \phi,} \qquad \Pi_\alpha = (\partial_w, \partial_u) \qquad \alpha = v, \bar{w}$$

e.o.m. $$\Box\phi - \kappa\, \varepsilon^{\alpha\beta}\{\Pi_\alpha\phi, \Pi_\beta\phi\} = 0$$

Plebanski 75

Poisson Bracket

$$\{f, g\} = \epsilon^{\alpha\beta}\Pi_\alpha f\, \Pi_\beta g = \partial_w f \partial_u g - \partial_u f \partial_w g$$

SD Double Copy

Monteiro, O'Connell 2011

SDYM $\quad \Box \Phi - ig\left[\{\Phi, \Phi\}\right] = 0$

$\Phi \to \phi \qquad i[\;] \to \{\;\} \qquad f^{abc} \to F_{p_1 p_2}{}^{k}$

SDG $\quad \Box \phi - \kappa \left\{\{\phi, \phi\}\right\} = 0$

Kinematic Algebra

Area preserving diffeomorphisms: $L_k = e^{ik\cdot x}(k_u \partial_w - k_w \partial_u)$

SD solutions beyond flat space

MCG, Lipstein, Nagy

$$\boxed{W_{\mu\nu\rho\sigma} = \frac{i}{2}\sqrt{-g}\,\varepsilon_{\mu\nu\eta\lambda}W^{\eta\lambda}{}_{\rho\sigma}}$$

Scale invariant off-shell

on-shell Weyl $\quad W_{\mu\nu}{}^{\rho\sigma} \equiv R_{\mu\nu}{}^{\rho\sigma} - 2T_{[\mu}{}^{[\rho}g_{\nu]}{}^{\sigma]} + \frac{2}{3}T g_{[\mu}{}^{[\rho}g_{\nu]}{}^{\sigma]}$

SD eq $\Longrightarrow$ Trace reverse Einst eq $=$ Bianchi Id. $= 0$

SD de Sitter

$g_{\mu\nu} = g^{dS}_{\mu\nu} + h_{\mu\nu}$ τ - conformal time

$T^{\mu\nu} = -\Lambda g^{\mu\nu}$

$$h_{\alpha\beta} = \frac{L^2}{4\tau^2}\pi_{(\alpha}\tilde{\pi}_{\beta)}\phi$$

$\pi_\alpha = (\partial_w, \partial_u)$ $\alpha = v, \bar{w}$

$\tilde{\pi}_\alpha = (\partial_w, \partial_u - \frac{2}{\tau})$

$$\sqrt{|g|}\left(\Box_{dS} - \frac{2}{L^2}\right)\phi - \left\{\frac{L}{\tau}\phi, \frac{L}{\tau}\phi\right\}_\star = 0$$

Lipstein, Nagy 23

deformed Poisson Bracket = Jacobi Bracket

$$\{f, g\}_\star = \frac{1}{2}\epsilon^{\alpha\beta}\left(\pi_\alpha f\,\tilde{\pi}_\beta g - \pi_\alpha g\,\tilde{\pi}_\beta f\right)$$

Self-dual double copy in de Sitter

SDYM $\quad \sqrt{-g}\left(\Box_{dS} - \frac{2}{L^2}\right)\Phi - i\frac{L}{\tau}\left[\left\{\frac{L}{\tau}\Phi, \frac{L}{\tau}\Phi\right\}\right] = 0$

$$i[\;\;] \longrightarrow \frac{\tau}{L}\{\;\;\}_\star$$

$$f^{abc} \longrightarrow F^{\star}_{p_1 p_2}{}^{k}(\tau)$$

SDG^{dS}

$$\sqrt{-g}\left(\Box_{dS} - \frac{2}{L^2}\right)\phi - \left\{\left\{\frac{L}{\tau}\phi, \frac{L}{\tau}\phi\right\}\right\}_\star = 0$$

Cosmological spacetimes

Conformally flat!

$$ds^2 = a^2(u+v)\,(du\,dv - dw\,d\bar{w})$$
$$= a^2(\tau)\,(-d\tau^2 + d\bar{x}^2)$$

$$g^{SD}_{\mu\nu} = g^{FRW}_{\mu\nu} + h_{\mu\nu}$$

Ansatz:

$$h_{\alpha\beta} = \frac{a^2(\tau)}{4} D_{(\alpha} \tilde{D}_{\beta)} \phi$$

dif. operator

$$D \sim \partial + g(\tau)$$

source $T^{\mu\nu} \neq 0$

not required to be homogeneous, isotropic

5D off-shell Weyl tensor

$$h_{\alpha\beta} = \frac{a^2(\tau)}{4} \Pi_{(\alpha} \Pi^{\rho}_{\beta)} \phi \qquad \Pi_\alpha = (\partial_w, \partial_u)$$

$$\Pi^{\rho}_{\alpha} = (\partial_w, \partial_u + 2\rho(\tau))$$

$\alpha = 2\rho'/\rho^2 - 1$
$= \text{const.}$

3 conformal classes of 5D metrics

$\rho = -\frac{1}{\tau}, c, 0$

$$\sqrt{|g|}\left(\Box_{a=\rho} - \frac{R_{a=\rho}}{6}\right)\phi - \{\{\rho\phi, \rho\phi\}\}_\rho = 0$$

Jacobi bracket

New kinematic algebras

$$\{f, g\}_\rho = \{f, g\} + \alpha\, \rho(\tau)(f\partial_w g - g\partial_w f)$$

Cosmological self-dual double copy

SDYM $\sqrt{-g}\left(\Box_a - \frac{R}{6}\right)\Phi - i a\left[\{a\Phi, a\Phi\}\right] = 0$

$$i\,[\;] \rightarrow \frac{1}{S}\{\;\}$$

SDG

$$\sqrt{-g}\left(\Box_a - \frac{R}{6}\right)\phi - \frac{a}{g}\{\{a\phi, a\phi\}\} = 0$$

SD cosmological spacetimes!

Require $\underline{\underline{R^{SD} = R^{FRW}}}$ $\rightarrow$ $\mathcal{G} = \dfrac{a''(\tau)}{2a'(\tau)}$

SD : $a(\tau) = 1$ Flat $a(\tau) = \frac{L}{\tau}$ de Sitter

$a(\tau) = \frac{\tau}{L}$ Radiation

$R = T_\mu{}^\mu = 0, \; T^{\mu\nu} \neq 0$

$\mathcal{L}^{int} \sim a\{\{\partial\phi, \partial\phi\}\}$

$a(\tau) = e^{\mathcal{H}\tau}$ Non-accelerating FRW

$R \propto \frac{\mathcal{H}^2}{a^2}, \; T^{\mu\nu} \neq 0, \; \mathcal{L}^{int} = a\{\{\partial\phi, \partial\phi\}\}_{\frac{\mathcal{H}}{2}}$

Radiation SD spacetimes

$R = T_\mu{}^\mu = 0, \qquad g_{\mu\nu} = a^2 \left(\eta_{\mu\nu} + \frac{1}{4} \Pi_{(\alpha} \Pi^{\,g}_{\beta)} \phi \right)$

Conformally flat SD

$$\rho \sim 1/a^4 \sim 1/\tau^4 \,, \quad a \sim \tau$$

Conformally dS SD

$$\rho \sim \frac{1}{\tau^2} \,, \quad a = 1$$

Conformally non accel. FRN SD

$$\rho \sim \text{cnst} \,, \quad a = 1$$

Conformally SD spacetimes

Other FRW spacetimes?

$$g_{\mu\nu} = a^2\left(\eta_{\mu\nu} + \tfrac{1}{4}\Pi_{(\alpha}\Pi^{\beta}_{\beta)}\,\phi\right)$$

Any of the 3 conformal classes

Power law cosmologies $a(\tau) = \tau^p$

Sourced by viscous fluid with:

$$\rho \sim a(\tau)^{-2(1+p)/p}\left(\mathrm{cnst} + \hat{\Theta}(\tau)\,\partial_w^2\phi\right)$$

FRW

$$\omega = \frac{P}{\rho} = \frac{2-p}{3p} + \hat{\Omega}(\tau)\,\partial_w^2\phi$$

FRW

~FRW like for $\partial_w^2\phi \ll 1$

Cosmological SD Double Copy

$$\Box_{\mathbb{R}^4}\Phi - i[\{\Phi,\Phi\}] = 0$$

$$\sqrt{g}\left(\Box_{\text{FLRW}} - \frac{R}{6}\right)\Phi - i\,a[\{a\,\Phi, a\,\Phi\}] = 0$$

SD solution	color-kinematics replacements
Flat/Radiation	$\Phi \to \phi,\quad i[\{\ ,\ \}] \to \quad \{\{\ ,\ \}\}$
(A)dS	$\Phi \to \phi,\quad i[\{\ ,\ \}] \to \frac{1}{a}\{\{\ ,\ \}\}_*$
Non-acc. FLRW	$\Phi \to \phi,\quad i[\{\ ,\ \}] \to \quad \{\{\ ,\ \}\}_{\varsigma=\mathcal{H}/2}$

$\varsigma = 0$	flat	$\Box_{\mathbb{R}^4}\phi - \{\{\phi,\phi\}\} = 0$
$\varsigma = -1/\tau$	(A)dS	$\sqrt{g}\left(\Box_{\text{AdS}} + 2/L^2\right)\phi - \{\{a\phi, a\phi\}\}_* = 0$
$\varsigma = 0$	radiation	$\sqrt{g}\Box_{\text{rad.}}\phi - a\{\{a\phi, a\phi\}\} = 0$
$\varsigma = \mathcal{H}/2$	non-acc.	$\sqrt{g}(\Box_{\text{non-a.}} + (\mathcal{H}/a)^2)\phi - a\{\{a\phi, a\phi\}\}_{\varsigma=\mathcal{H}/2} = 0$

Cosmological SD Double Copy

$$\Box_{\mathbb{R}^4}\Phi - i[\{\Phi, \Phi\}] = 0$$

$$\sqrt{g}\left(\Box_{\mathrm{FLRW}} - \frac{R}{6}\right)\Phi - i\,a[\{a\,\Phi, a\,\Phi\}] = 0$$

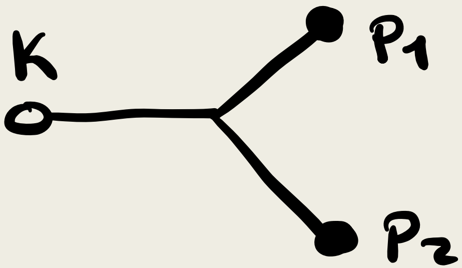
$$F_{P_1P_2}^{\varsigma\;\;K} = a(\tau)\, X^{\varsigma}(P_1, P_2)\, \delta(P_1 + P_2 - K)$$

$$X^{\varsigma}(P_1, P_2) = X(P_1, P_2) - i\alpha g(\tau)(P_1^{\omega} - P_2^{\omega})$$

$$\alpha = 2(\dot{\varsigma}/\varsigma^2 - 1) = \text{cnst.}$$

$$f^{abc} \longrightarrow F_{P_1P_2}^{\varsigma\;\;K}(\tau)$$

- $X^{\varsigma=-1/\tau} \longrightarrow X^{\varsigma=-1/\tau}/a$



flat	$\Box_{\mathbb{R}^4}\phi - \{\{\phi, \phi\}\} = 0$
(A)dS	$\sqrt{g}\left(\Box_{\mathrm{AdS}} + 2/L^2\right)\phi - \{\{a\phi, a\phi\}\}_* = 0$
radiation	$\sqrt{g}\Box_{\mathrm{rad.}}\phi - a\{\{a\phi, a\phi\}\} = 0$
non-acc.	$\sqrt{g}(\Box_{\text{non-a.}} + (\mathcal{H}/a)^2)\phi - a\{\{a\phi, a\phi\}\}_{\varsigma=\mathcal{H}/2} = 0$

Future Directions

→ Understand kinematic algebras
    + ~ Moyal deformations (chiral higher-spins)

→ Correlators

→ Perturbations around SD sol = full sol.

→ Integrability?

→ Other SD solutions?