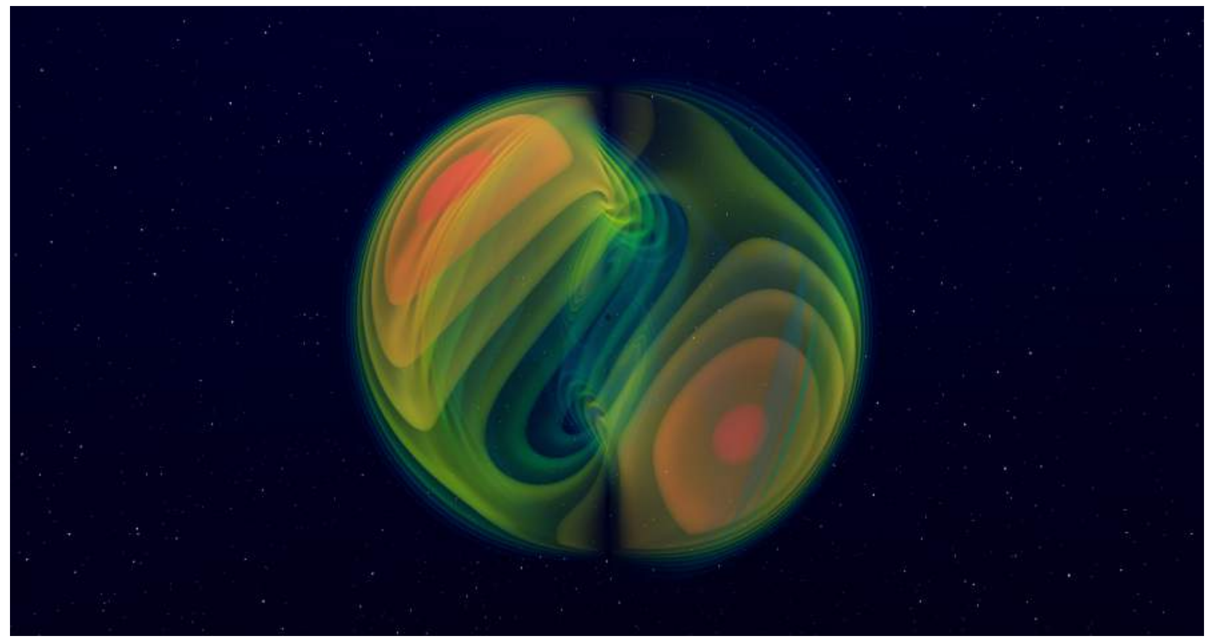
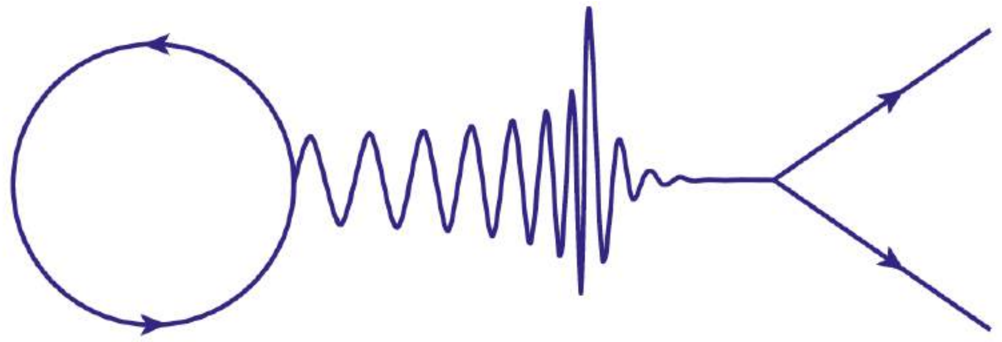
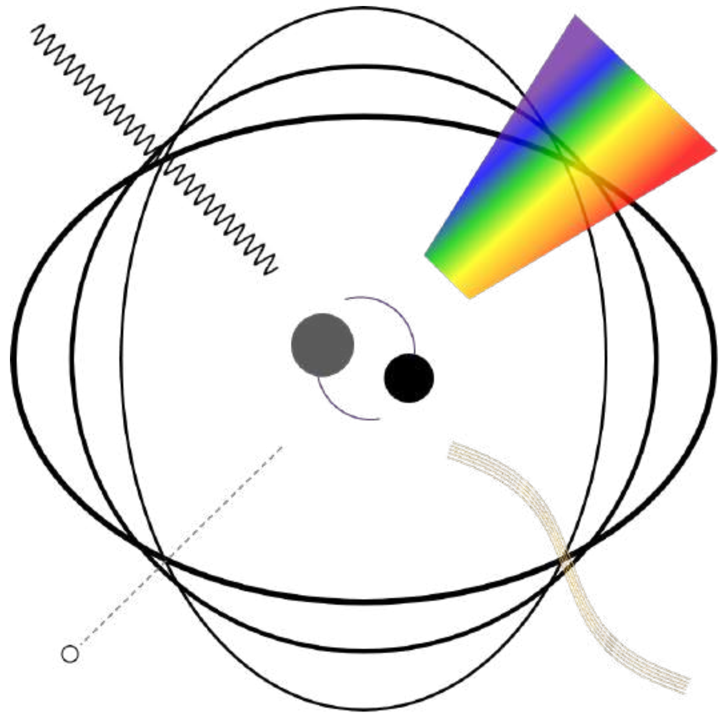
GW190412



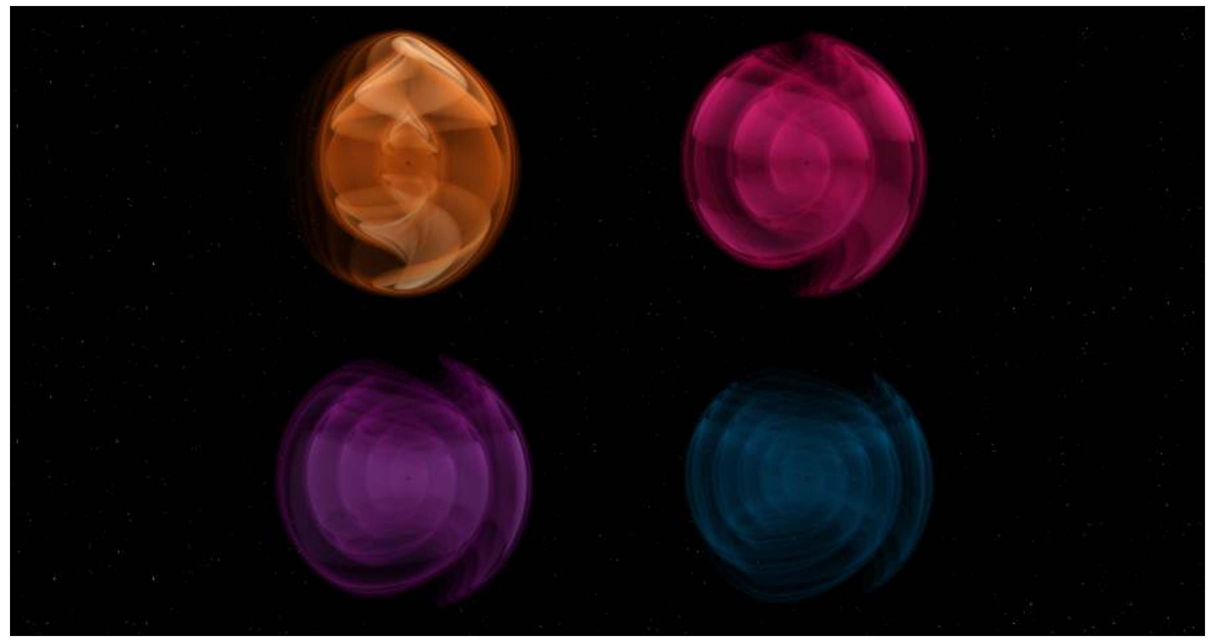


(credit: Vitor Cardoso & Paolo Pani)



(credit: Peter Shawhan)

GW190814



Gravitational-Wave Astronomy: Post-Minkowskian Theory Meets the Effective-One-Body Approach

Alessandra Buonanno

Max Planck Institute for Gravitational Physics

(Albert Einstein Institute), Potsdam

Deutsche Forschungsgemeinschaft
German Research Foundation

International Balzan Foundation

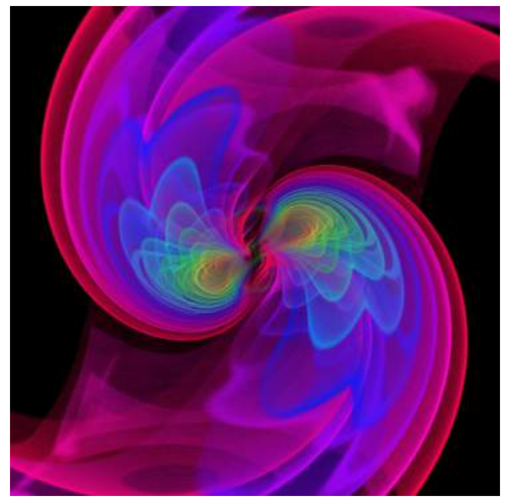
“Amplitudes 2024”, Institute for Advanced Study, Princeton

June 14, 2024

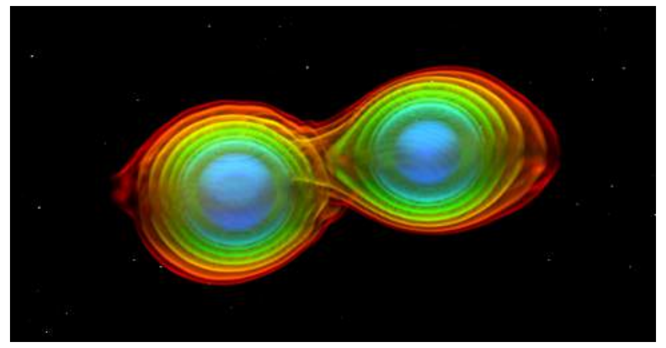
Discovering/Characterizing Black Holes & Neutron Stars in the Universe

- As today, GWs were observed by **LIGO-Virgo detectors** from **90 coalescences, plus tens of events** pulled out from public data **with independent analysis.** *(Abbott+ PRX 13 (2023) 4, 041039)* *(Nitz+23, Mehta+23, Wadekar+23)*

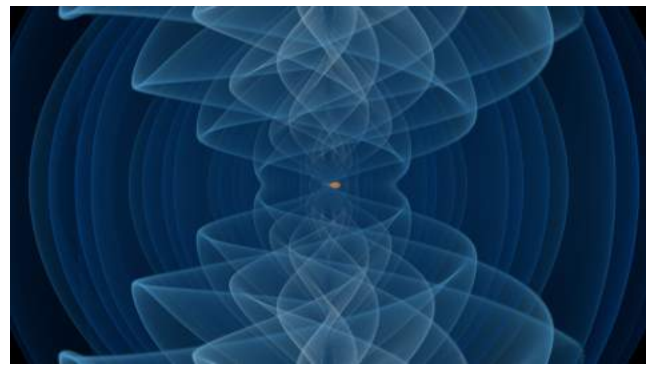
GW150914

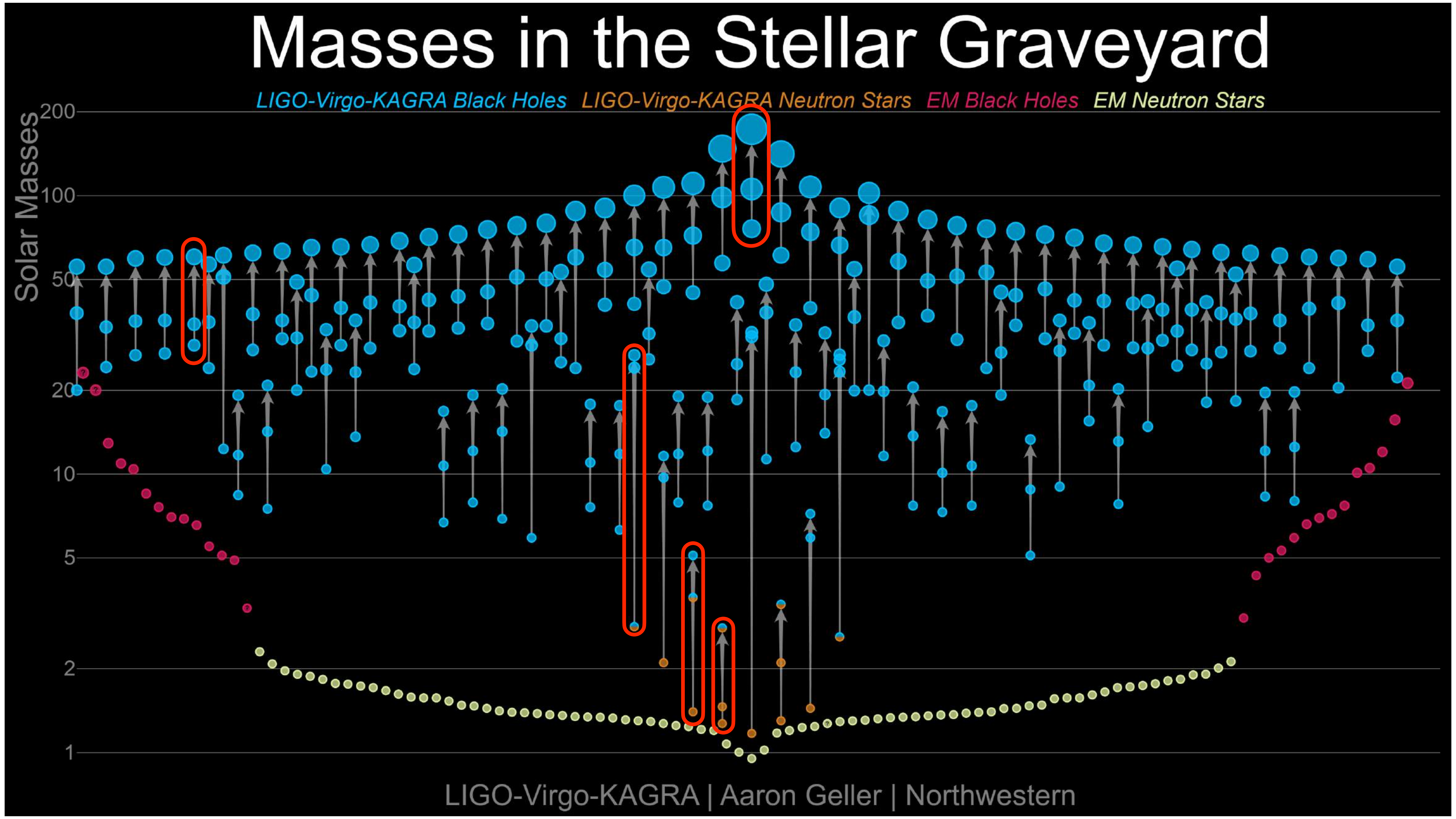


GW170817

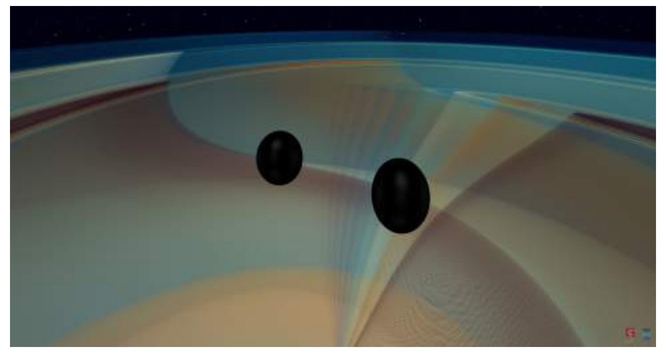


GW230529

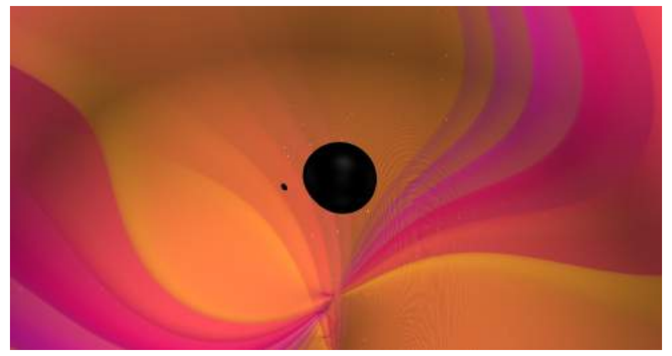


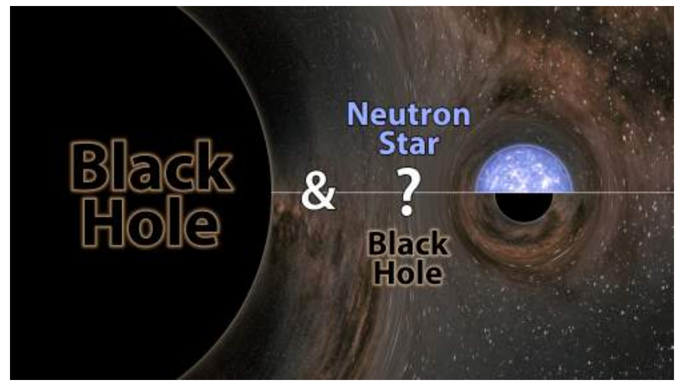


GW190521



GW190814





- Ongoing LIGO-Virgo-KAGRA observing run **O4 has already announced 105 signal candidates**.

Motivations/Outline/Collaborators

- What **role do waveform models play in detecting and interpreting** **LIGO-Virgo-KAGRA** **signals?**

- With ever more sensitive observational runs (O5, A#) and future detectors (LISA, Einstein Telescope, Cosmic Explorer), **precision GW astronomy will require ever more accurate waveforms**, with all physical effects (generic orbits, beyond-GR, matter/environment). Can **scattering amplitudes and worldline methods** help **to address the accuracy challenge?**

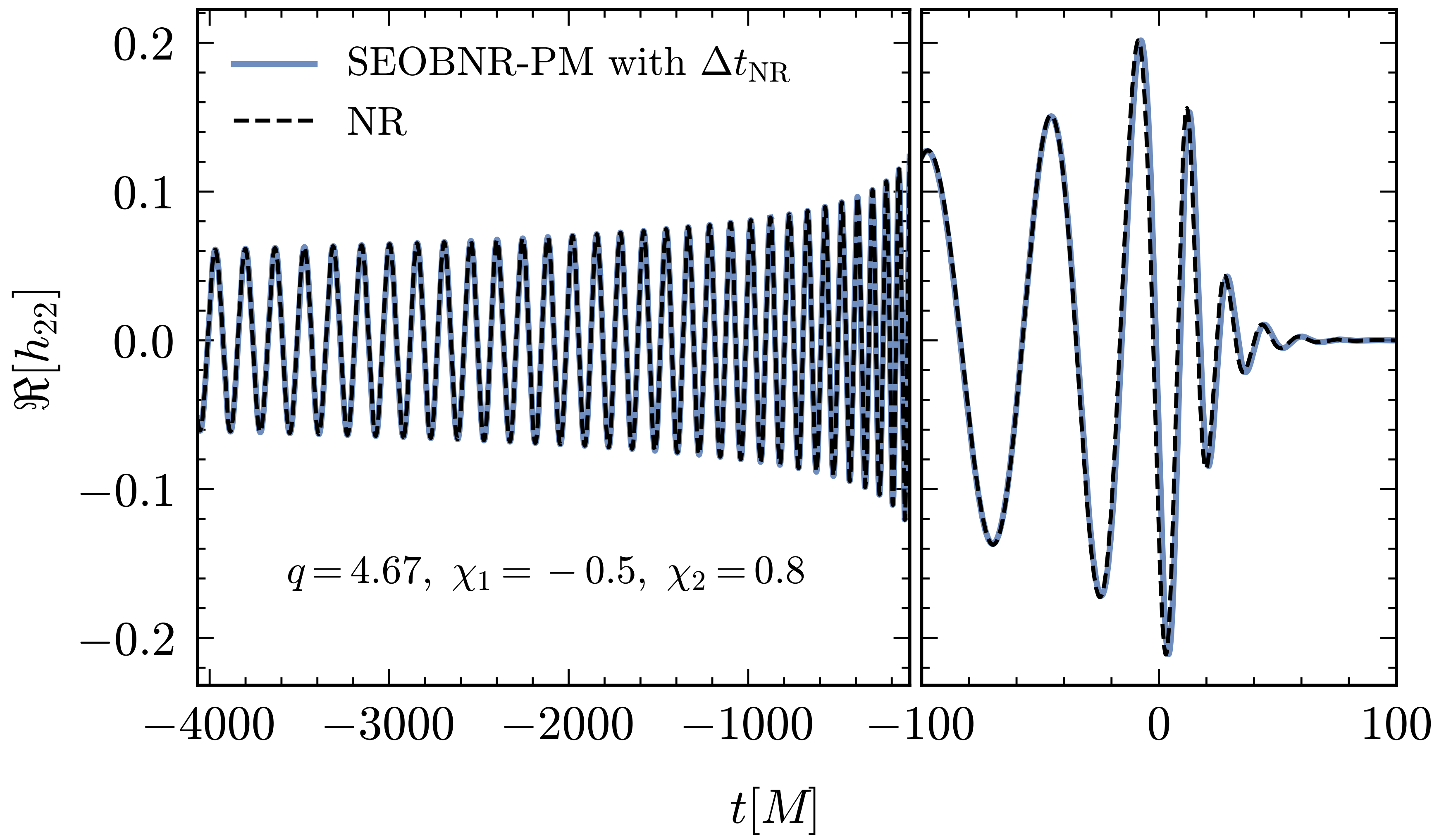
- **Very encouraging results for bound-orbit waveforms and scattering** by informing the **effective-one-body approach with** current **post-Minkowskian results.**

Gustav Jakobsen

Gustav Mogull

Raj Patil

Lorenzo Pompili

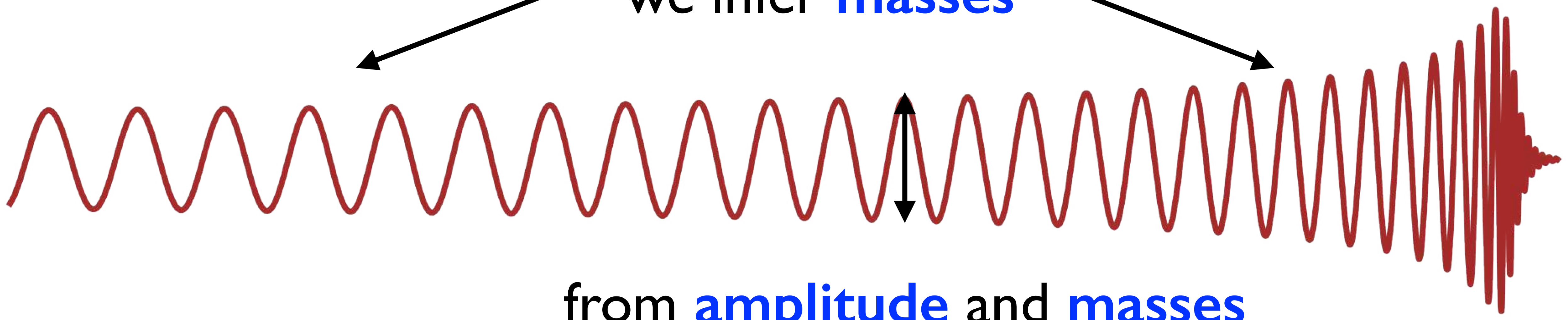


(AB, Jakobsen & Mogull arXiv: 2402.12342)

(AB, Mogull, Patil & Pompili arXiv: 2405.19181)

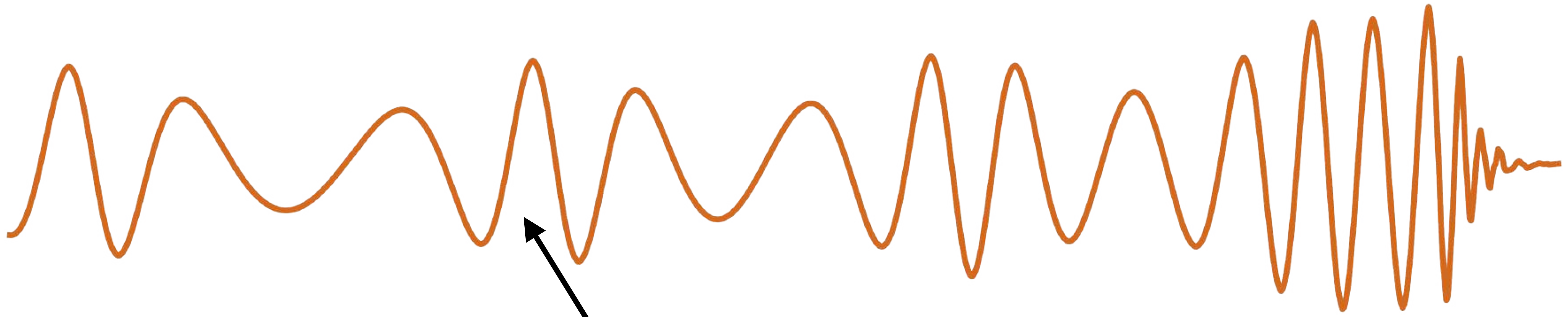
Properties of Astrophysical Sources via Gravitational Waves

from **frequency evolution**
we infer **masses**

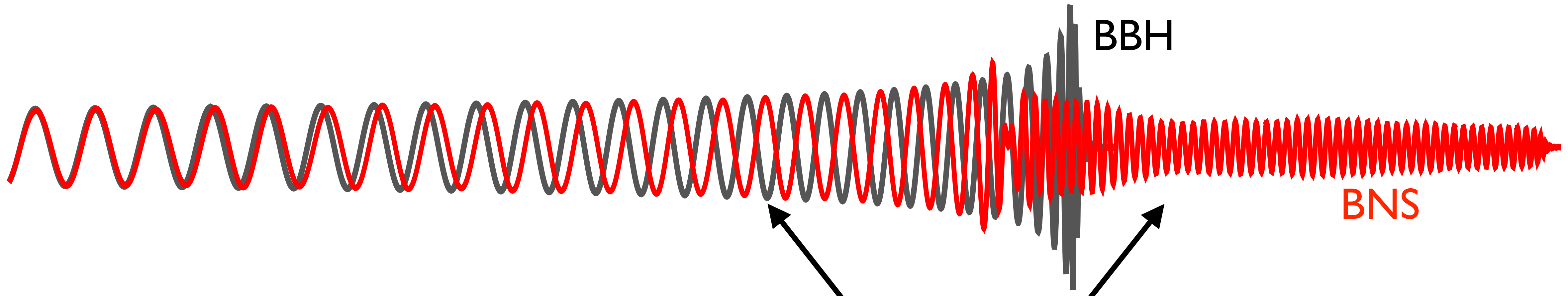


from **amplitude** and **masses**
we infer **distance**

from **time of arrival**, **amplitude and phase** at detectors we infer **sky location**



from **modulations** of **amplitude and phase**
we infer **spins and eccentricity**



from **differences in** late **inspiral and merger of BBHs**
we infer **tidal deformation**, and **NS composition**

By **comparing to waveforms with deviations from GR,** we can **probe** the theory of **gravity**

Solving Two-Body Problem in General Relativity

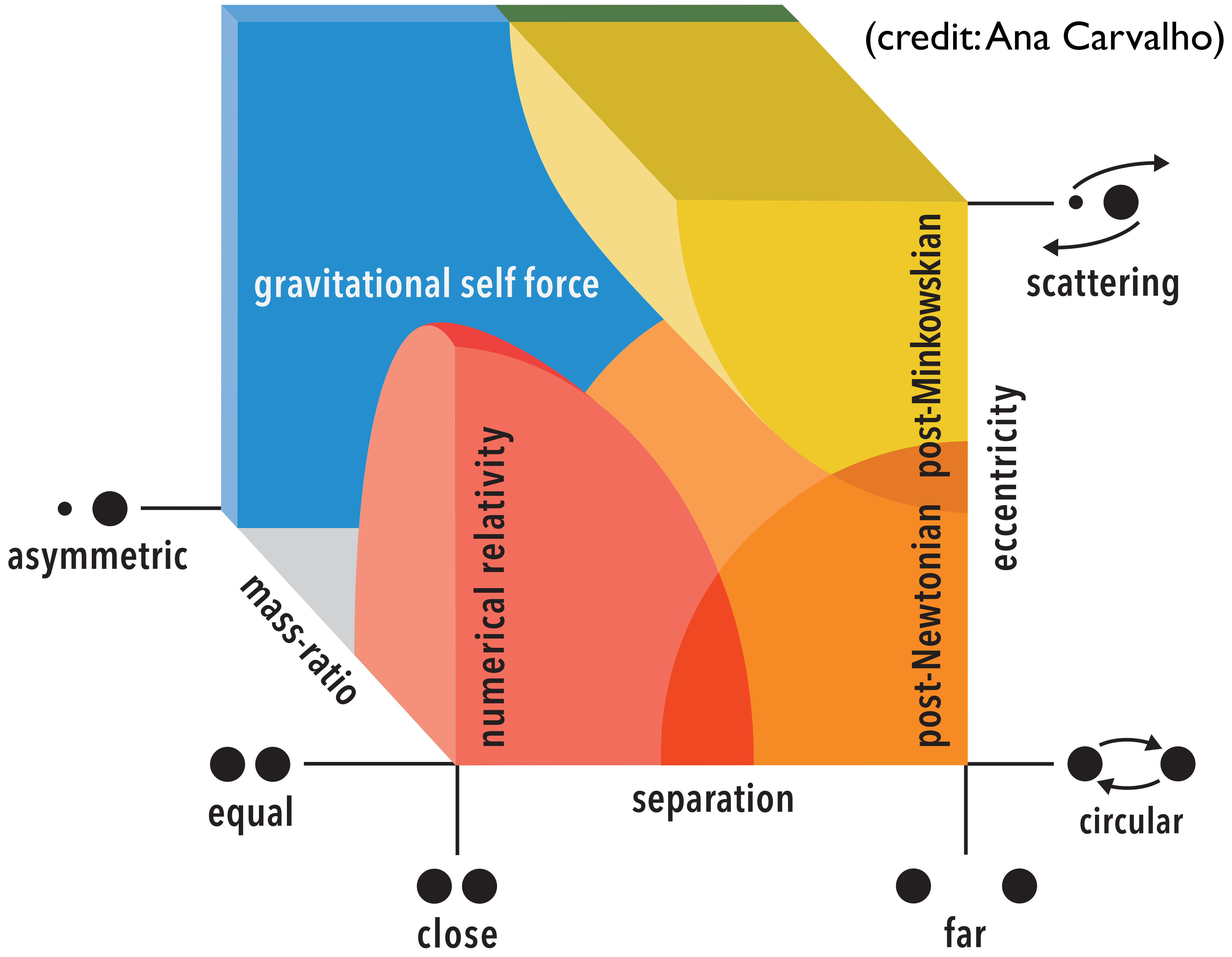
- **GR** is **non-linear theory**. $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$

- Einstein's field equations can be solved:

-**approximately,** but **analytically** (**fast** way)

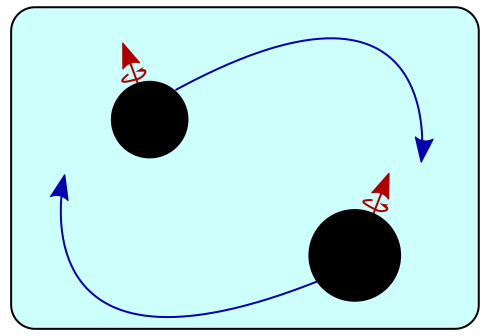
-**accurately**, but **numerically** on supercomputers (**slow** way)

- **Synergy** between **analytical** and **numerical relativity** is **crucial** to **provide GW detectors with templates** to use for **searches** and **inference analyses**.



- **Post-Newtonian** (large separation, and slow motion)

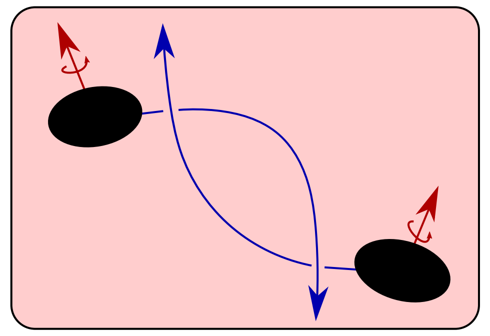
expansion in
$v^2/c^2 \sim GM/rc^2$



(Droste, Lorentz, Einstein, Infeld, Hoffmann, … Blanchet, Damour, Iyer, Jaranowski, Schäfer, Will, … Goldberger, Porto, Rothstein, …)

- **Post-Minkowskian** (large separation, and fast motion)

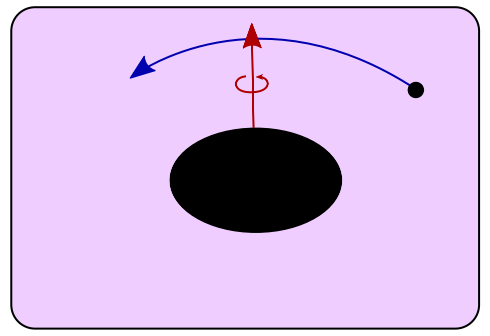
expansion in G



(Westpfahl, … Bern, Cheung, Hermmann, Parra-Martinez, Roiban, Rothstein, Solon, Shen, Zeng … Khälin, Porto, … Mogull, Jakobsen, Plefka, Steinhoff … Damgaard, Vanhove … Brandhuber, Travaglini …)

- **Gravitational self-force** (strong field)

expansion in m_2/m_1

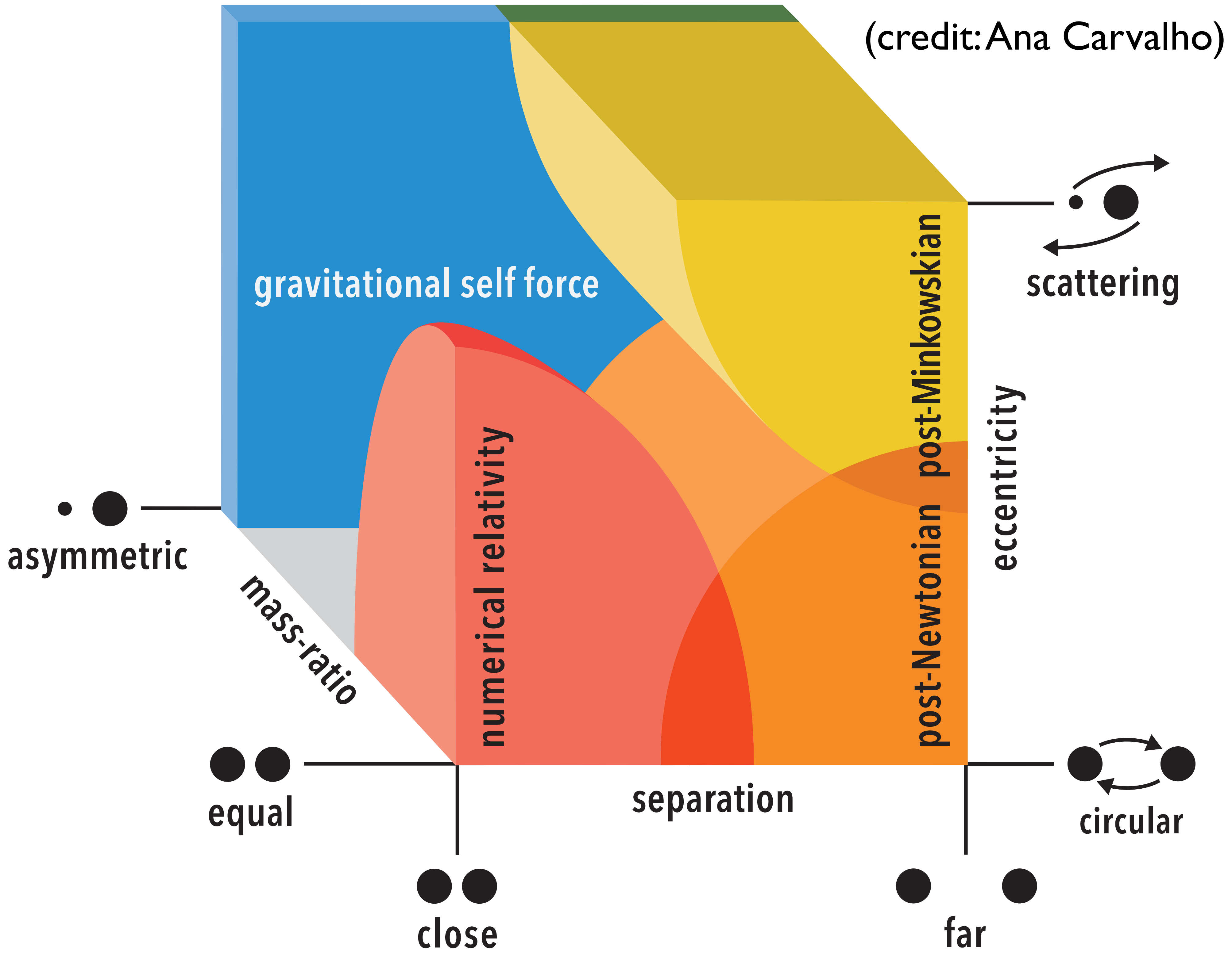


(Barack, Deitweiler, Mino, Poisson, Pound, Quinn, Sasaki, Tanaka, van de Meent, Wald, Warburton, Wardell, Whiting, …)

Solving Two-Body Problem in General Relativity

- **GR** is **non-linear theory**.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$



- Einstein's field equations can be solved:

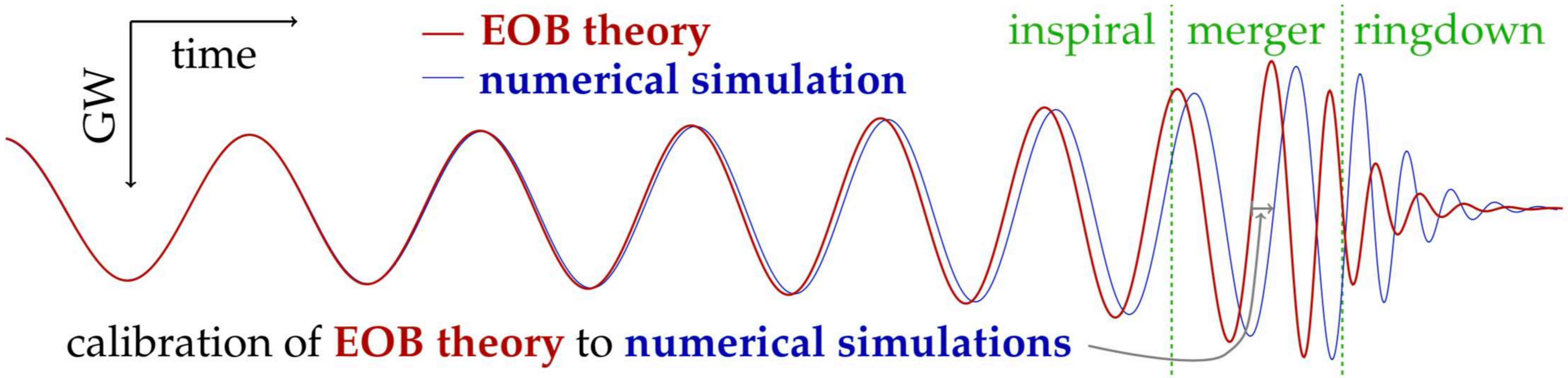
-**approximately,** but **analytically** (**fast** way)

-**accurately**, but **numerically** on supercomputers (**slow** way)

- **Synergy** between **analytical** and **numerical relativity** is **crucial** to **provide GW detectors with templates** to use for **searches** and **inference analyses**.

- **Effective-one-body** (EOB) **theory** (combines results from all methods, i.e., for **entire coalescence**)

(AB, Damour, … Barausse, Bohé, Cotesta, Estellés, Khalil, Mihaylov, Ossokine, Pan, Pompili, Pürrer, Ramos-Buades, Shao, Taracchini, … Nagar, Bernuzzi, Agathos, Albanese, Gamba, Messina, Rettegno, Riemenschneider, … Iyer, Jaranowski, Schäfer)



Solving Two-Body Problem in General Relativity

- **GR** is **non-linear theory**.

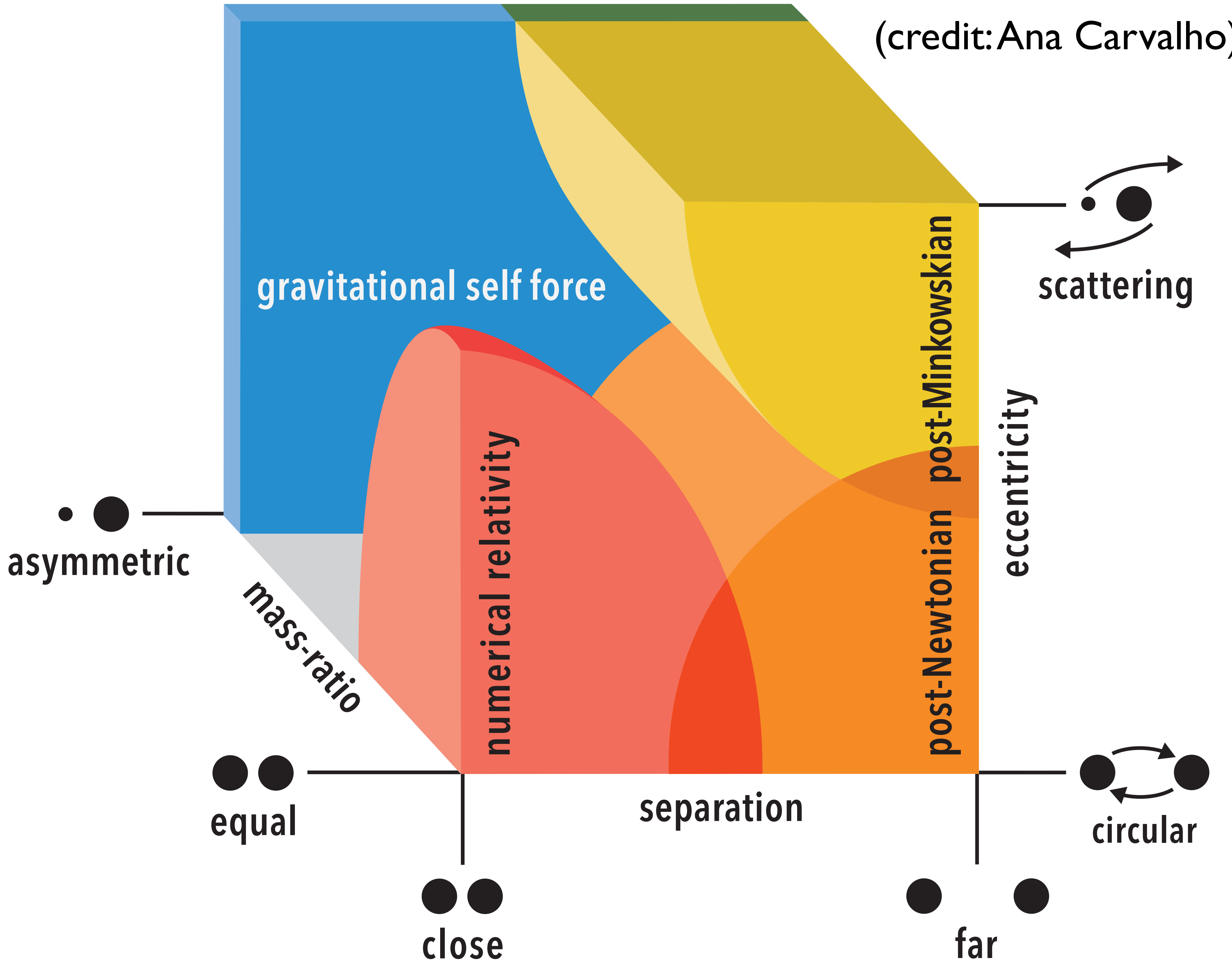
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- Einstein's field equations can be solved:

-**approximately,** but **analytically** (**fast** way)

-**accurately**, but **numerically** on supercomputers (**slow** way)

(credit: Ana Carvalho)

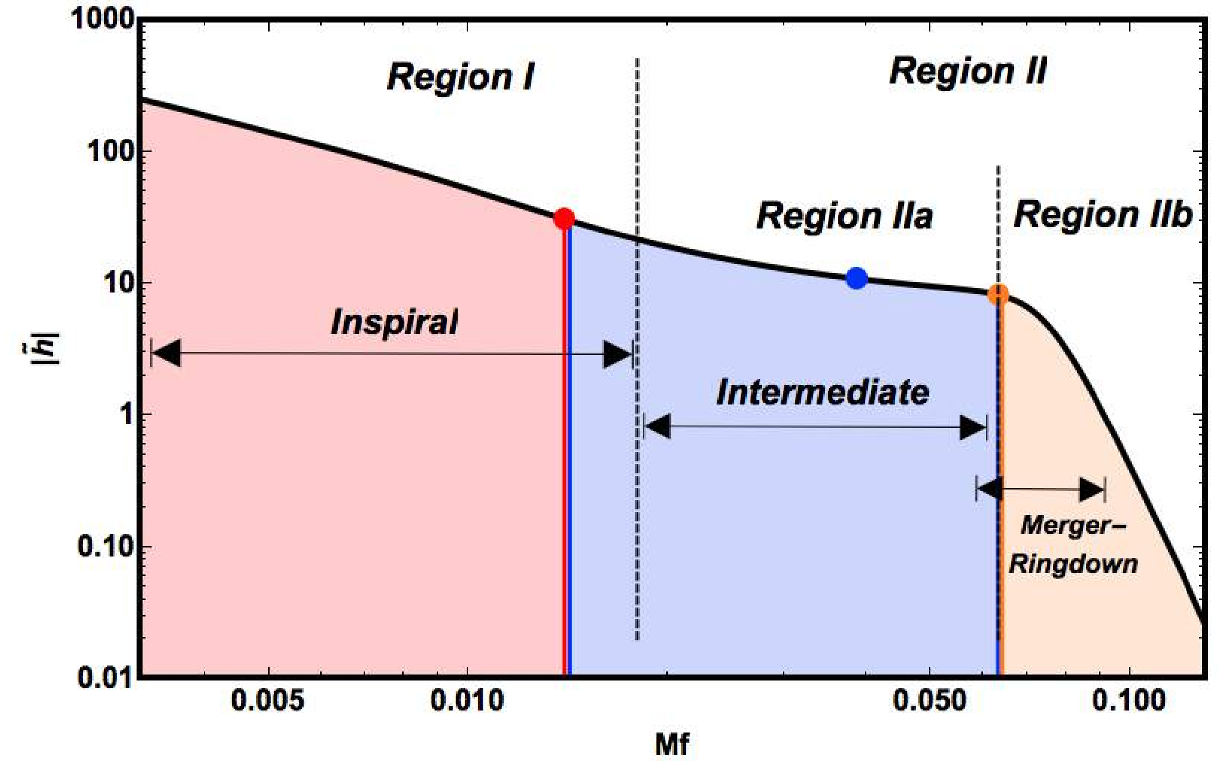


- **Synergy** between **analytical** and **numerical relativity** is **crucial** to **provide GW detectors with templates** to use for **searches** and **inference analyses**.

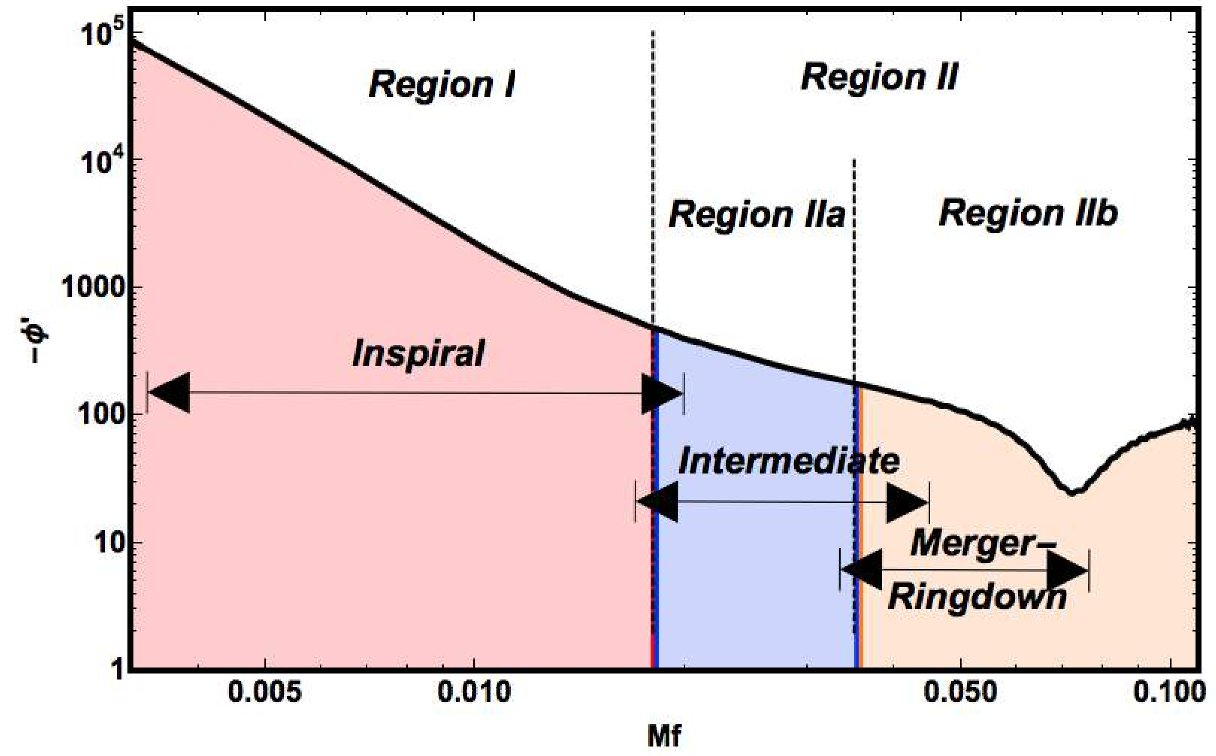
- **Phenomenological frequency-domain** waveforms (Phenom) built fitting to EOB, PN and NR.

(Ajith, Hannam, Husa, Ohme, … Bohé, Colleoni, García, Hamilton, Khan, London, Estellés, Pratten, Pürrer, Ramos-Buades, Quirós, Santamaria, Schmidt, Shrobana, Thompson, …)

Frequency-domain GW amplitude



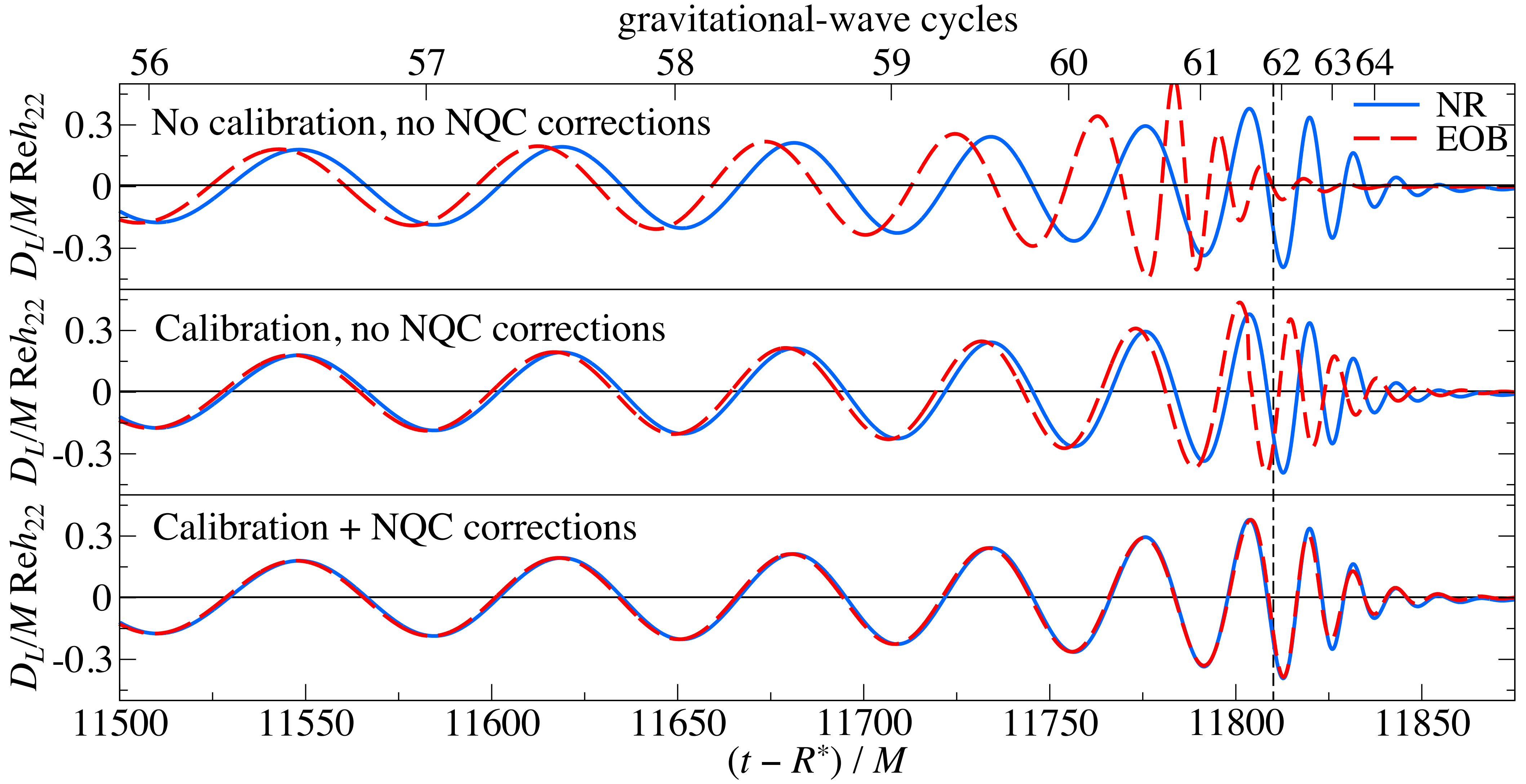
Frequency-domain GW phase derivative



(Khan+arXiv:1508.07253)

Completing Waveform Models with NR Information & Template Bank

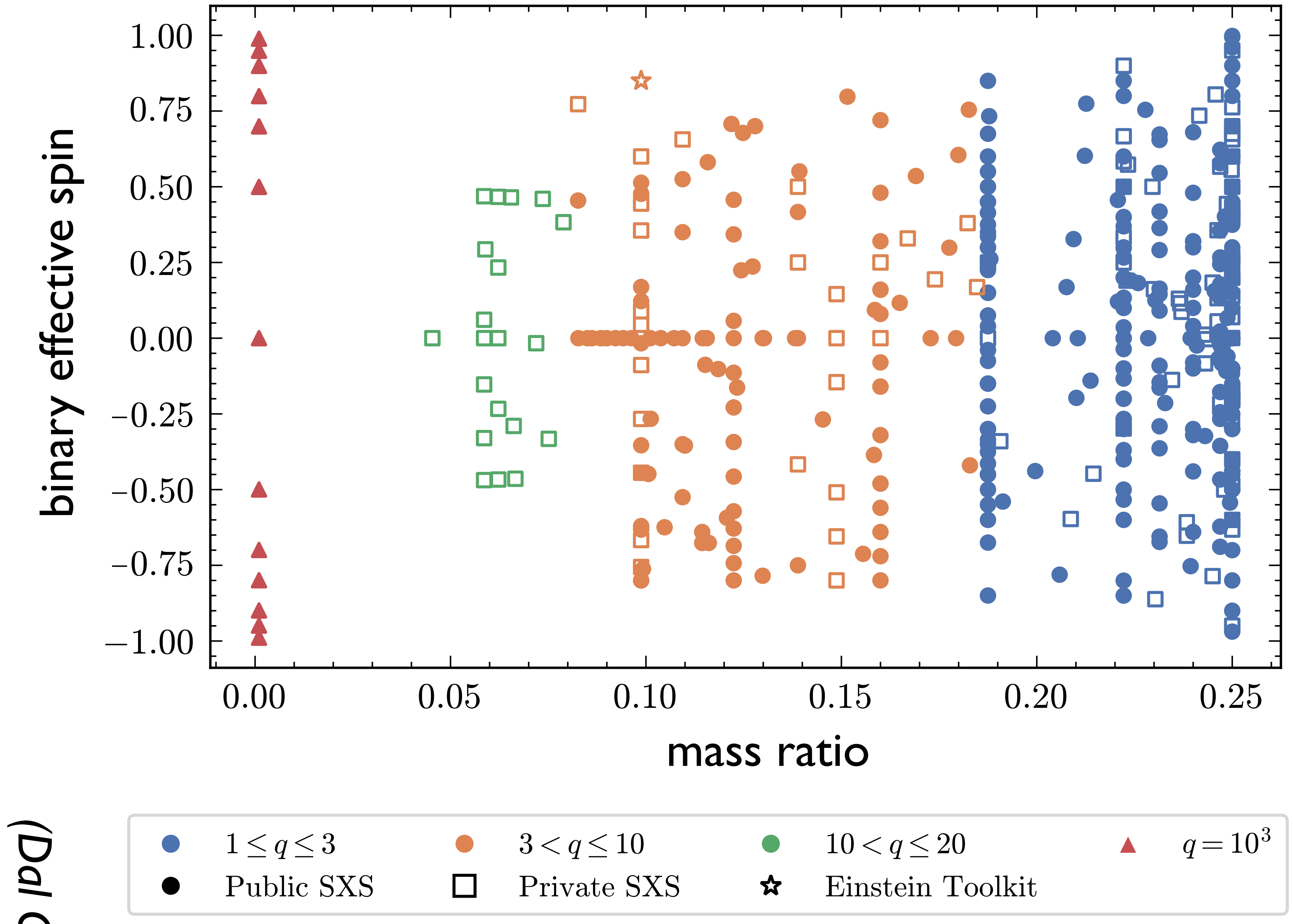
- We calibrate models to **inspiral-merger-ringdown NR** waveforms.



(credit: Andrea Taracchini)

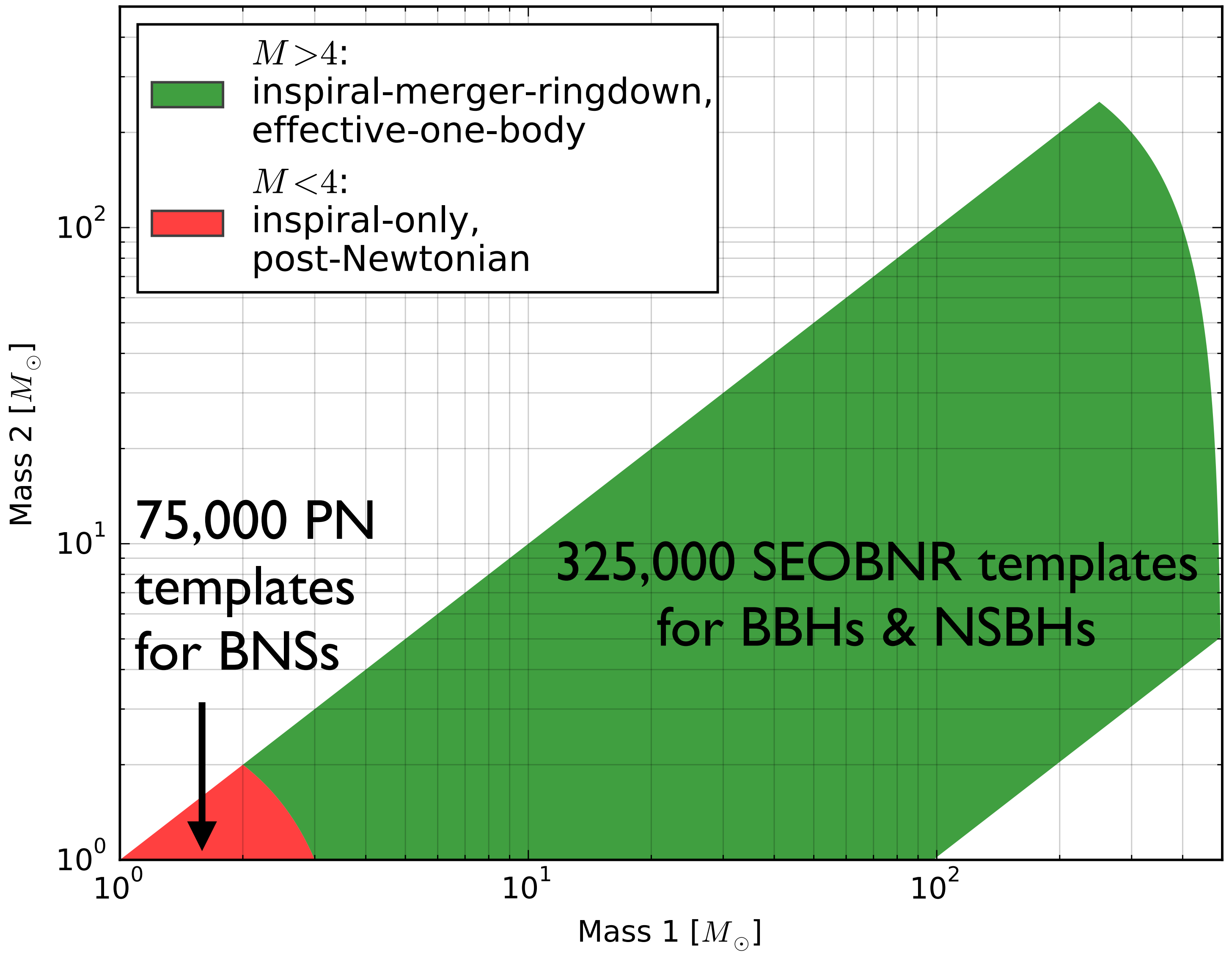
(NQC: non-quasi-circular corrections)

pySEOBNR calibration using **441 NR waveforms**



(Pompili+arXiv:2303.18039)

- **Matched filtering** employed in LIGO/Virgo searches.



(Dal Canton & Harry arXiv:1705.01845)

(**SXS**: Simulating eXtreme Spacetime)

*(**SEOBNR**: Pompili+23, van de Meent+23, Ramos-Buades+23, Mihaylov+23, Khalil+23)*

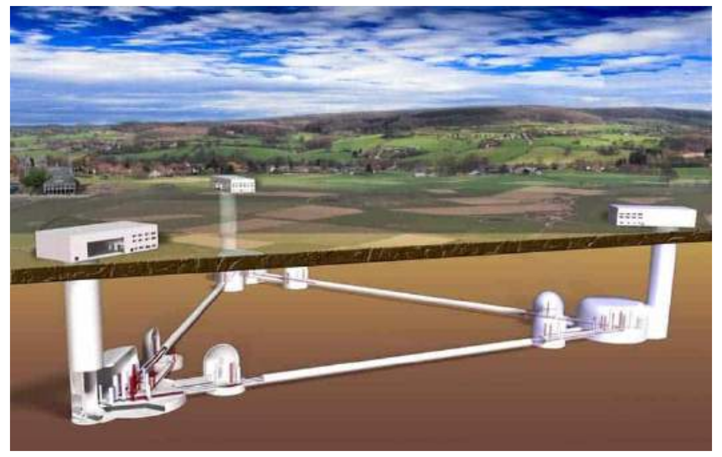
*(**IMRPhenom**: Pratten+20, García-Quíros+20, Estélles+21, Thompson+23)*

*(**TEOBResumS**: Akcay+21, Gamba+22, Nagar+23)*

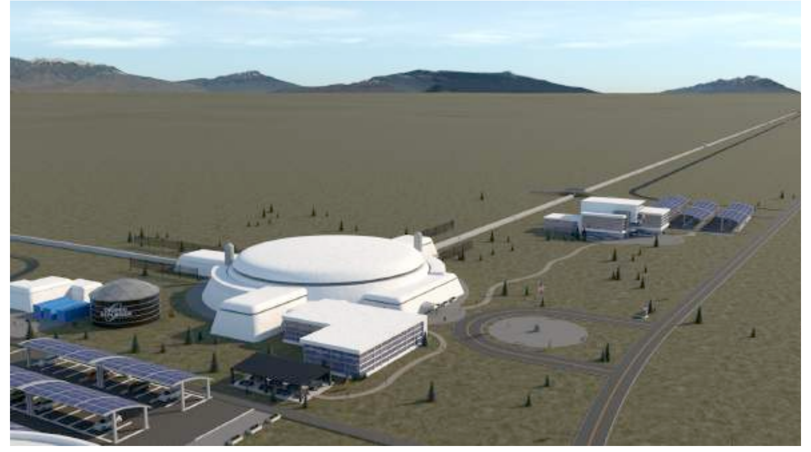
*(**NRSur**: Blackman+17, Varma+19, Yoo+23)*

GW Astronomy on the Ground & Space in 2030s: from hectoHz to milli Hz

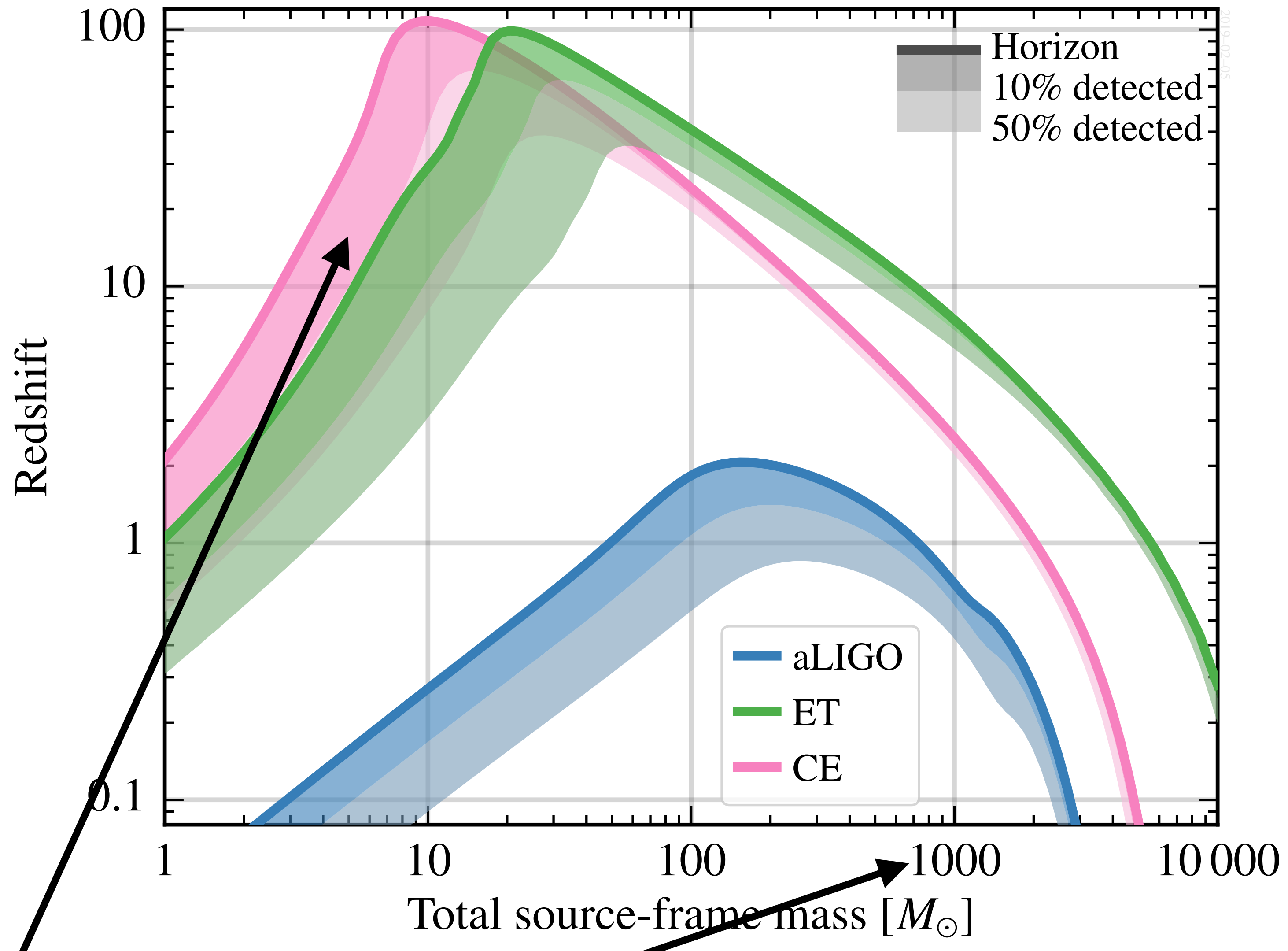
Einstein Telescope (ET)



Cosmic Explorer (CE)



(Kalogera+ arXiv:2111.0699)



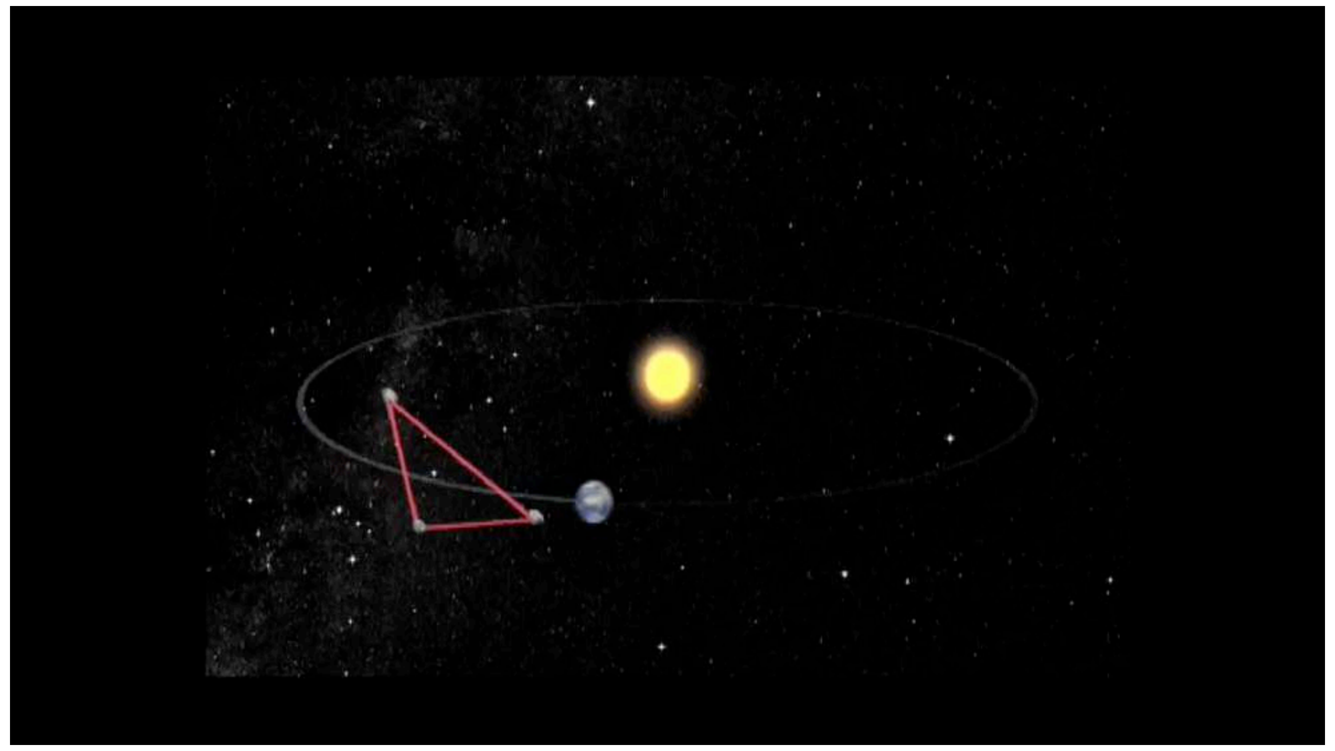
Observe BHs at much larger distance, when first stars formed, and more massive.

- Exquisite **characterization of binary BHs (NSs)**: the **number of events/yr** with **signal-to-noise ratio** > 100 will be $\sim 9{,}500\,(380)$.

(Borhanian & Sathyaprakash 22; Gupta et al. 23)

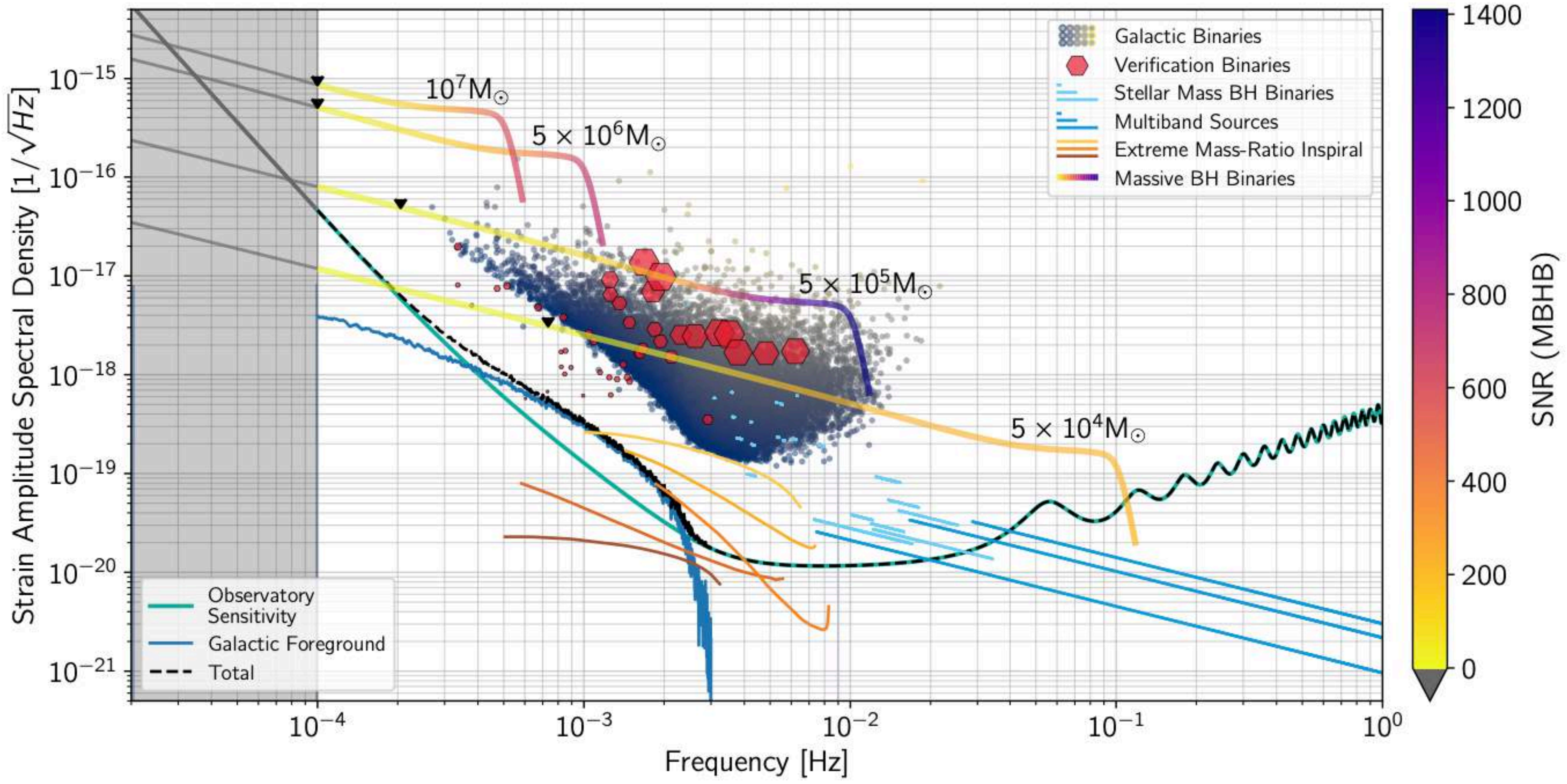
- **LISA adopted** as mission **by ESA** in Jan 2024; **launch** ~ 2035.

(credit: AEI/Milde Marketing/exozet)



- GW signals **will be loud and last for weeks/months.**

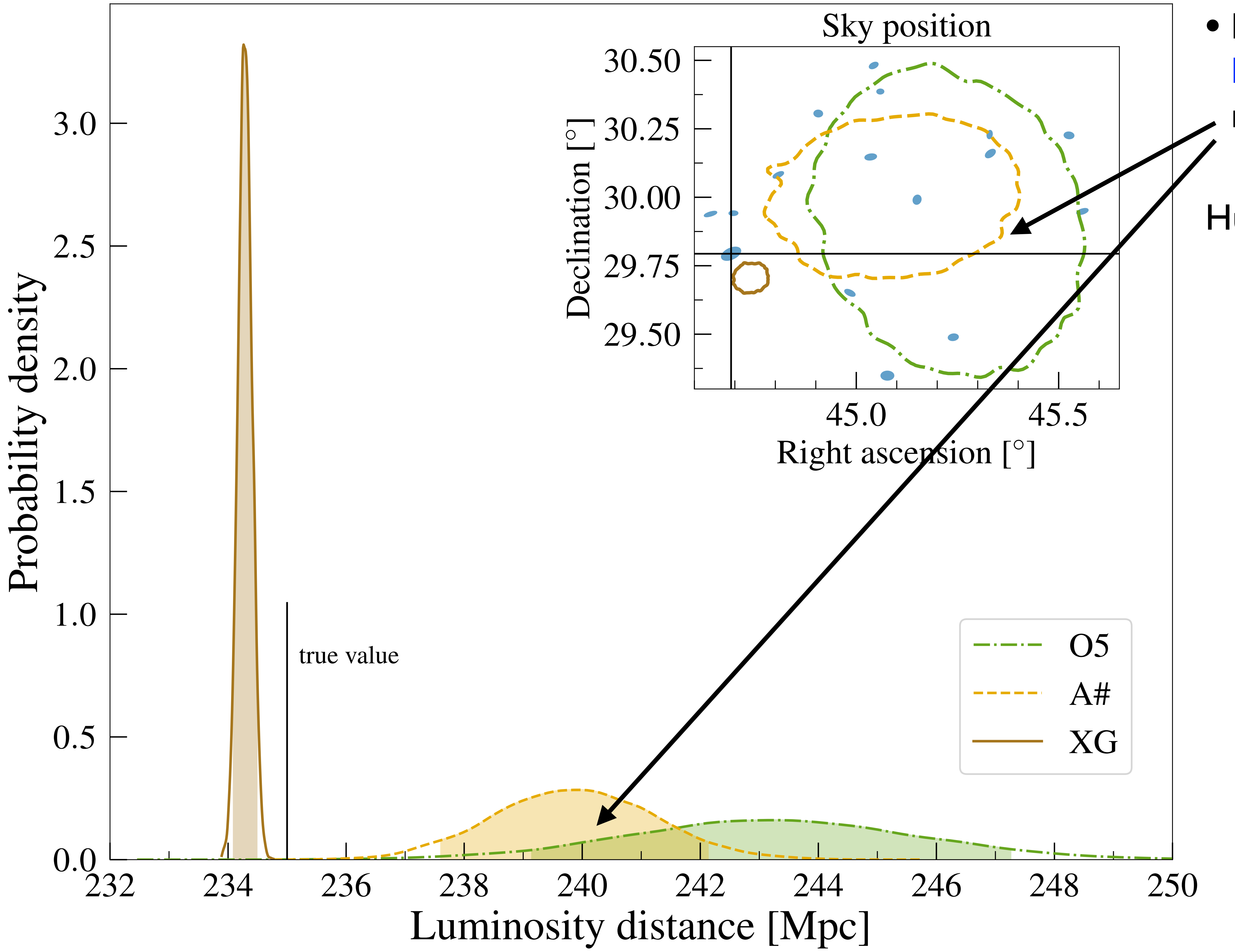
(LISA Red Book arXiv:2402.07571)



Precision GW Astronomy: The Accuracy Challenge

- BH binary **GW190814-like ($q \sim 10$), but highly precessing.**

$$\mathrm{SNR}_{\mathrm{O5}} = 119\,, \mathrm{SNR}_{\mathrm{A\#}} = 219\,, \mathrm{SNR}_{\mathrm{XG}} = 2490$$

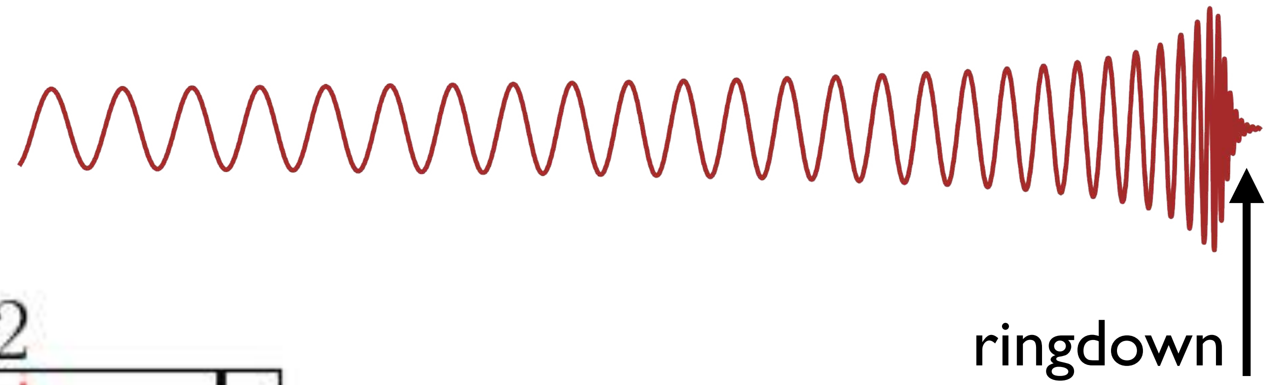


- Due to systematics, **wrong Hubble-Lemaitre parameter** (expansion rate of the Universe) **is measured.**

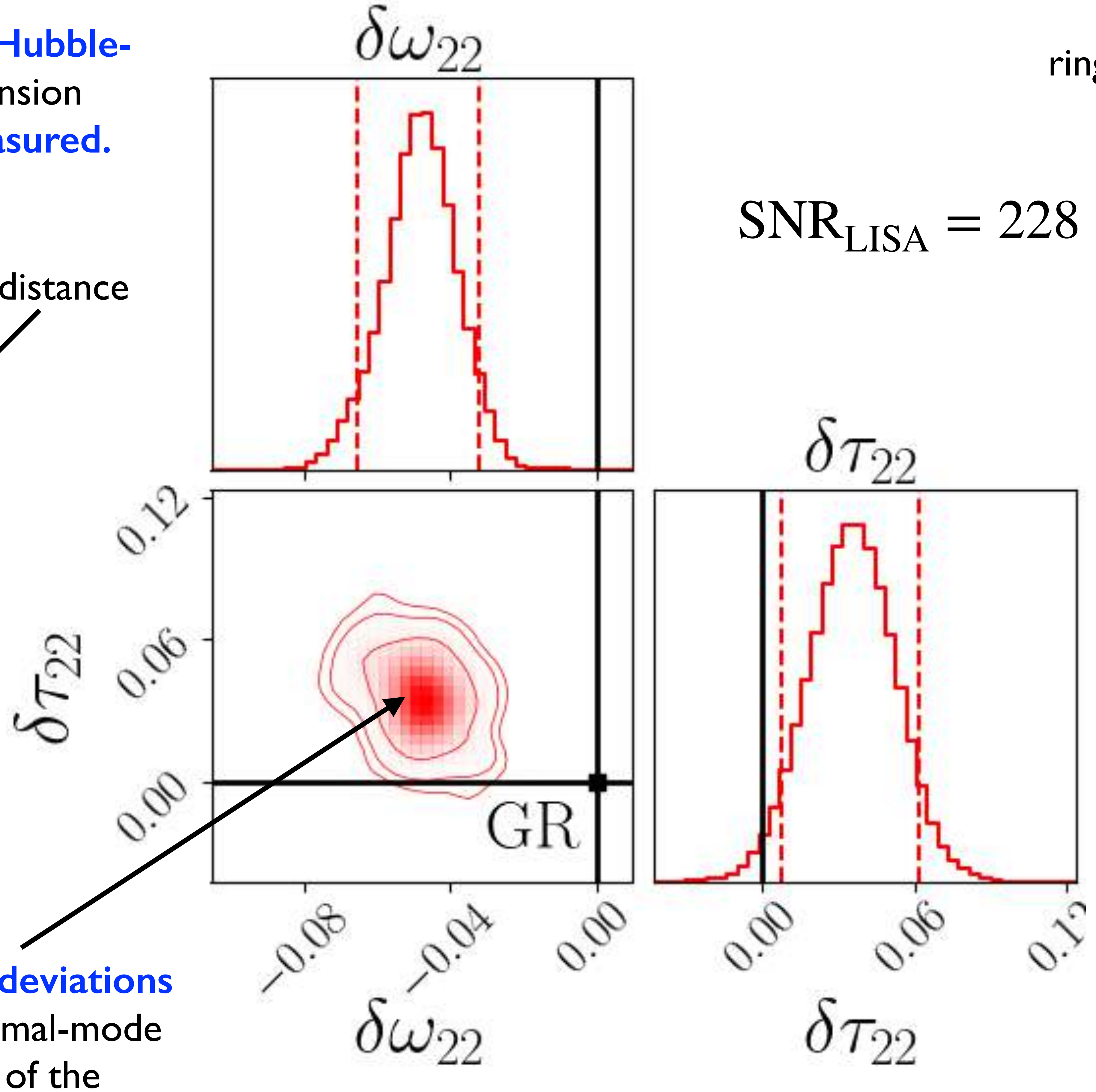
Hubble-Lemaitre flow velocity → $\mathrm{v_H} = H_0\, d$ ← luminosity distance

(Dhani, Völkel, AB, Estellés, Gair, Pfeiffer Pompili & Toubiana arXiv:2404.05811)

- Massive BH binary **with moderate mass ratio and spins.**



$$\mathrm{SNR}_{\mathrm{LISA}} = 228$$



- Due to systematics, **false deviations from GR** in the quasi-normal-mode frequency and decay time of the ringdown **are measured.**

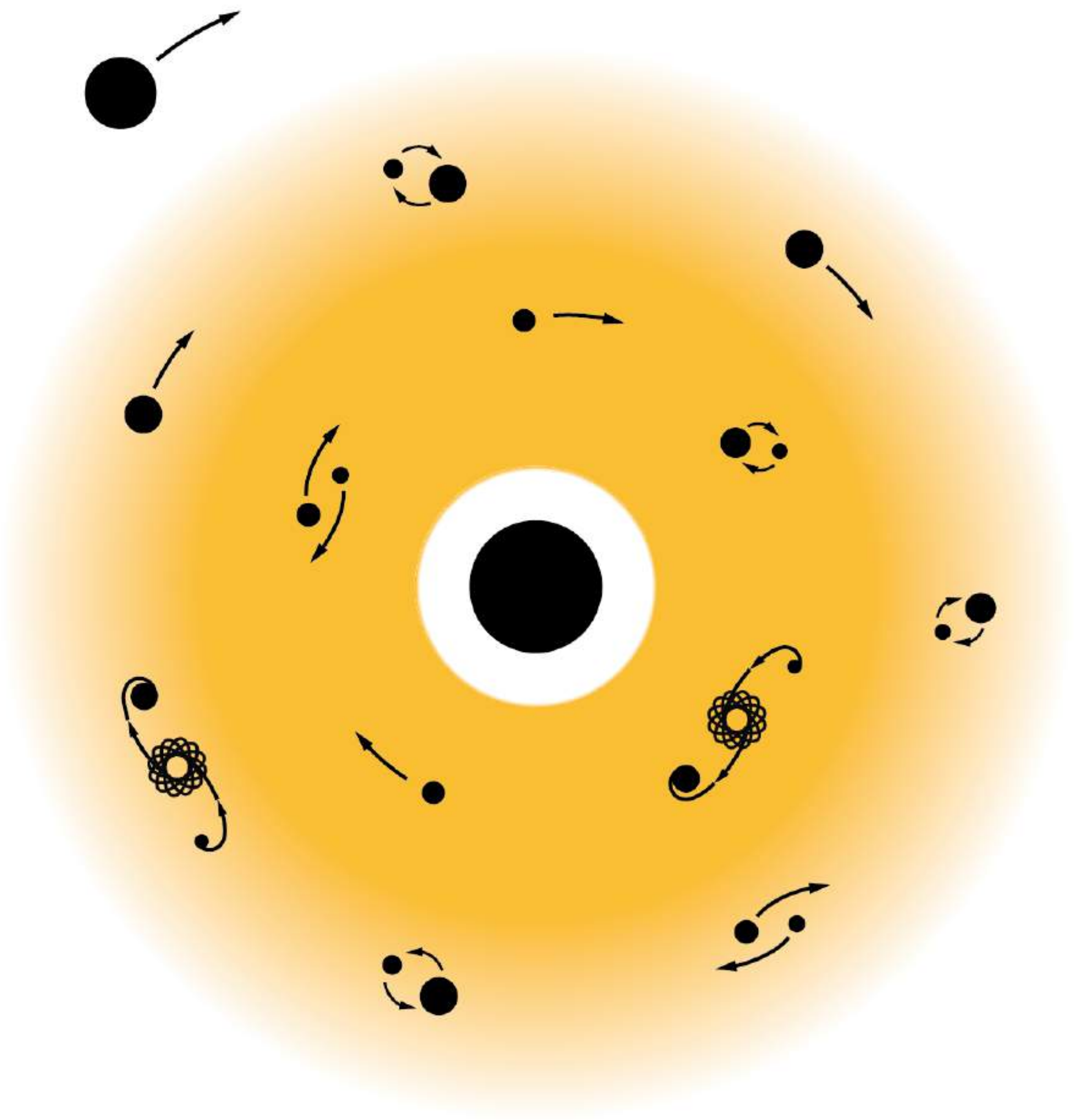
(Toubiana, Pompili, AB, Gair & Katz arXiv:2307.15086)

Theoretical Advances to Enable Precision GW Astronomy

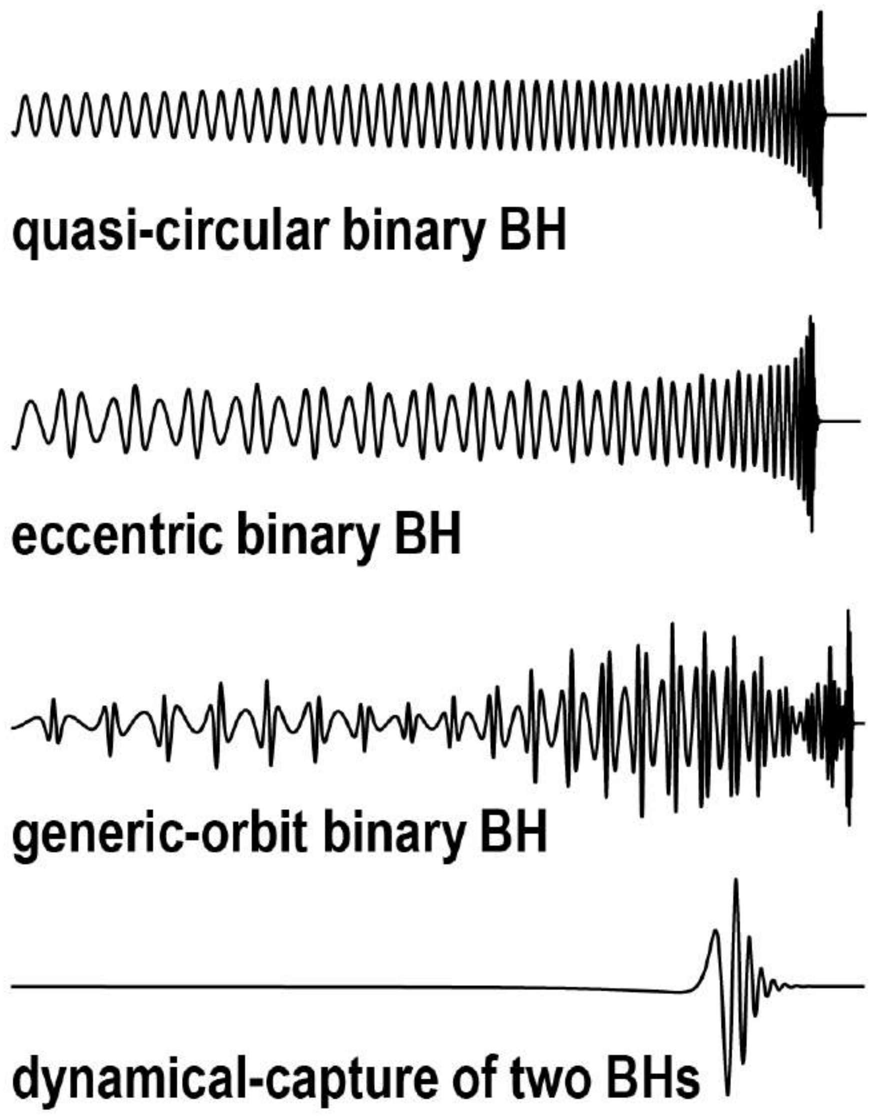
- **The accuracy of current waveform models** (for comparable mass binaries) would need **to be improved by 2 orders of magnitude**. **Numerical-relativity simulations** would also need to become **more accurate, for BBHs** and especially **BNS/NSBHs**.

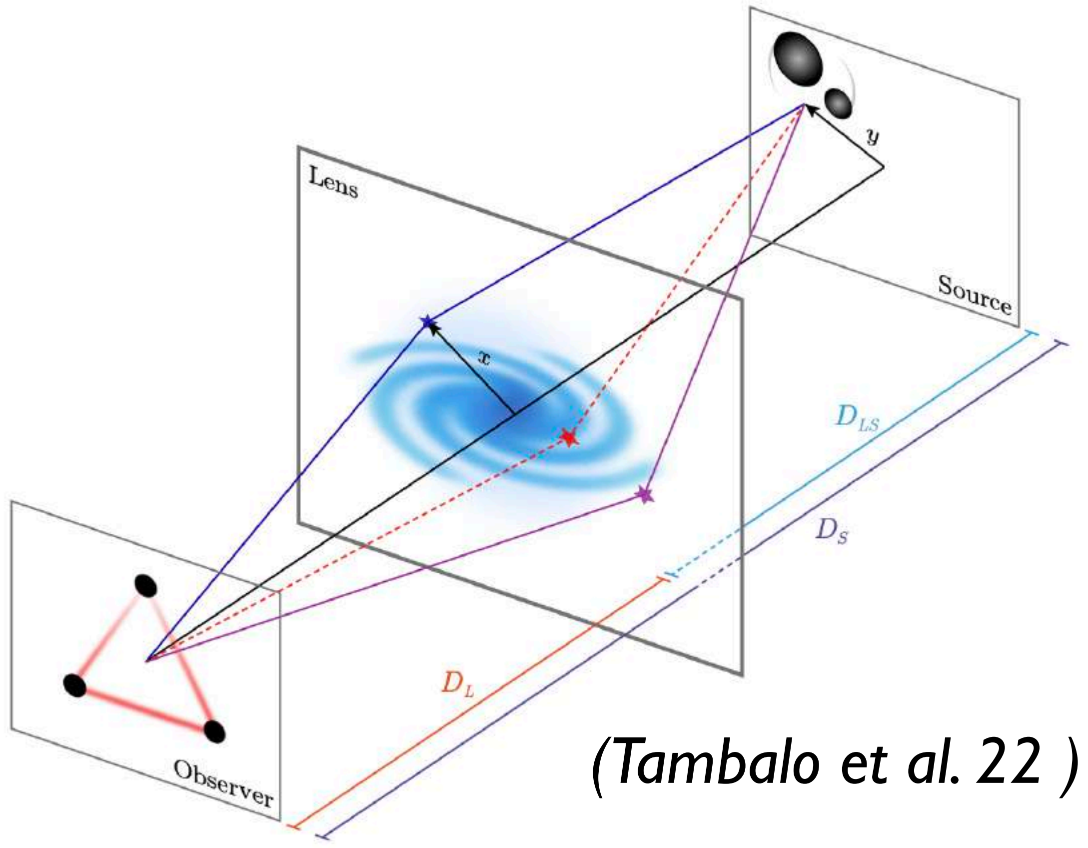
(Pürrer & Halster 19, Samajdar & Dietrich 18, Gamba et al 21, Dhani et al. 24)

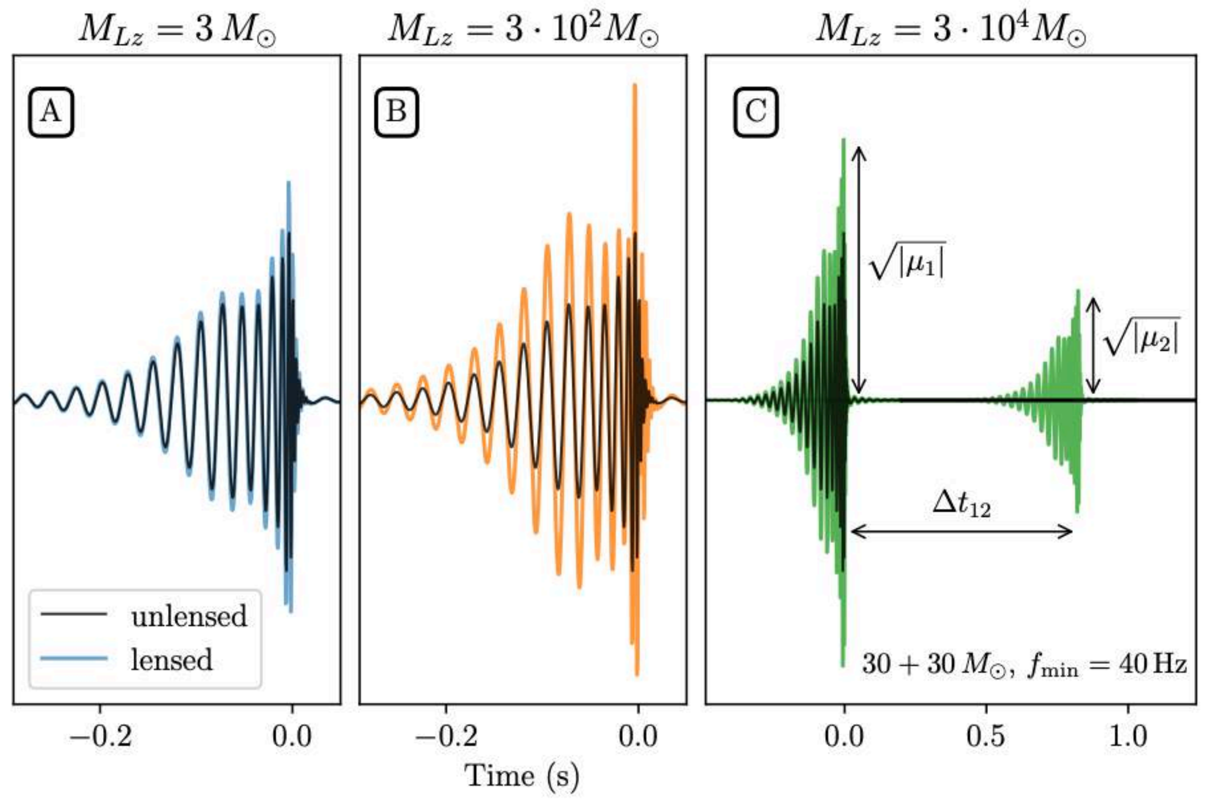
- **All physical effects** would **need to be included in waveform models** (generic orbits, astrophysical environmental effects, new physics beyond-GR, gravitational lensing, etc.) **to avoid wrong scientific** conclusions.



(credit: Ana Carvalho)







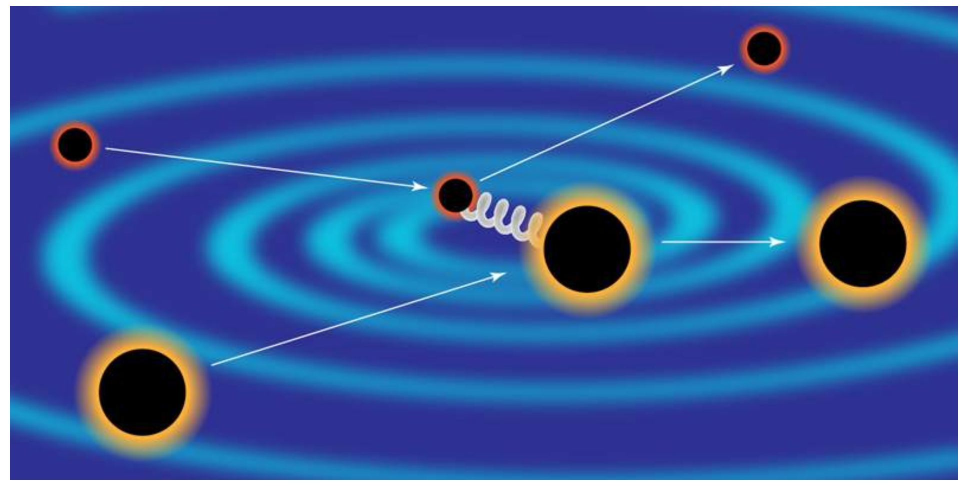
(Tambalo et al. 22)

Theoretical Advances to Enable Precision GW Astronomy (contd.)

- **PN, PM, GSF** should be **pushed at higher order** and **combined in EOB** approach **more effectively and in novel ways** to largely **improve analytical solutions** of two-body problem. **Calibration** to NR should be made **more effective**.

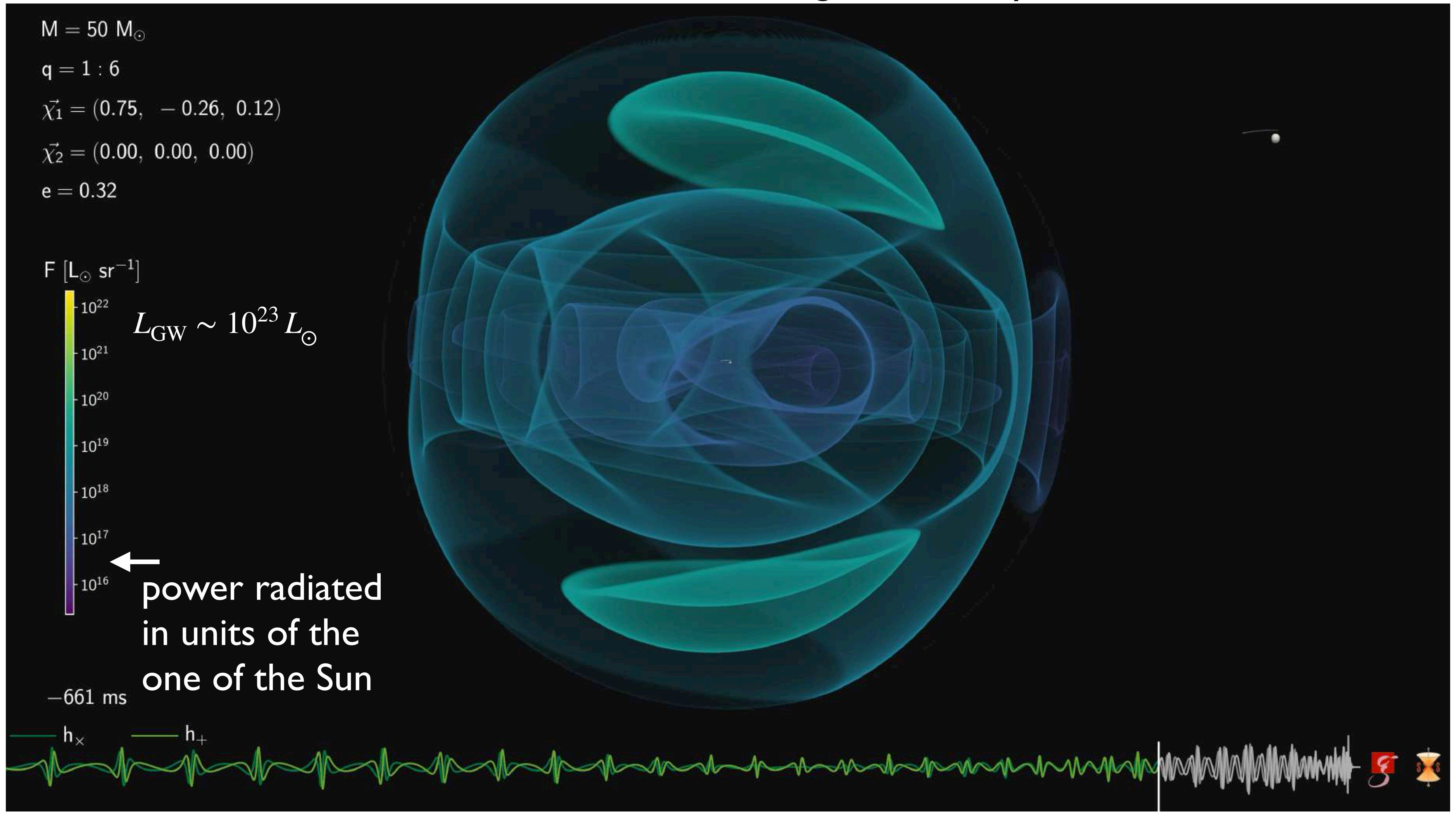
- **Scattering-amplitude/effective-field-theory/quantum-field-theory** methods from high-energy physics **have brought new tools to solve two-body problem in classical gravity**.

(Bjerrum-Bohr+18, Vines+18, Cheung+19; Bern+19, Kosower+19, Cristofoli+19, Damgaard+19, Blümlein+ 20, Bern+20, Kälin+ 20, Cheung & Solon 20, Parra-Martinez+20, Mogull+21, Brandhuber+21, Bern+21, Dlapa+21, Liu+21, Jakobsen+22, Bern+23, Jakobsen+23, Driesse+24, Dlapa+24, Bern+24, Bini+24)



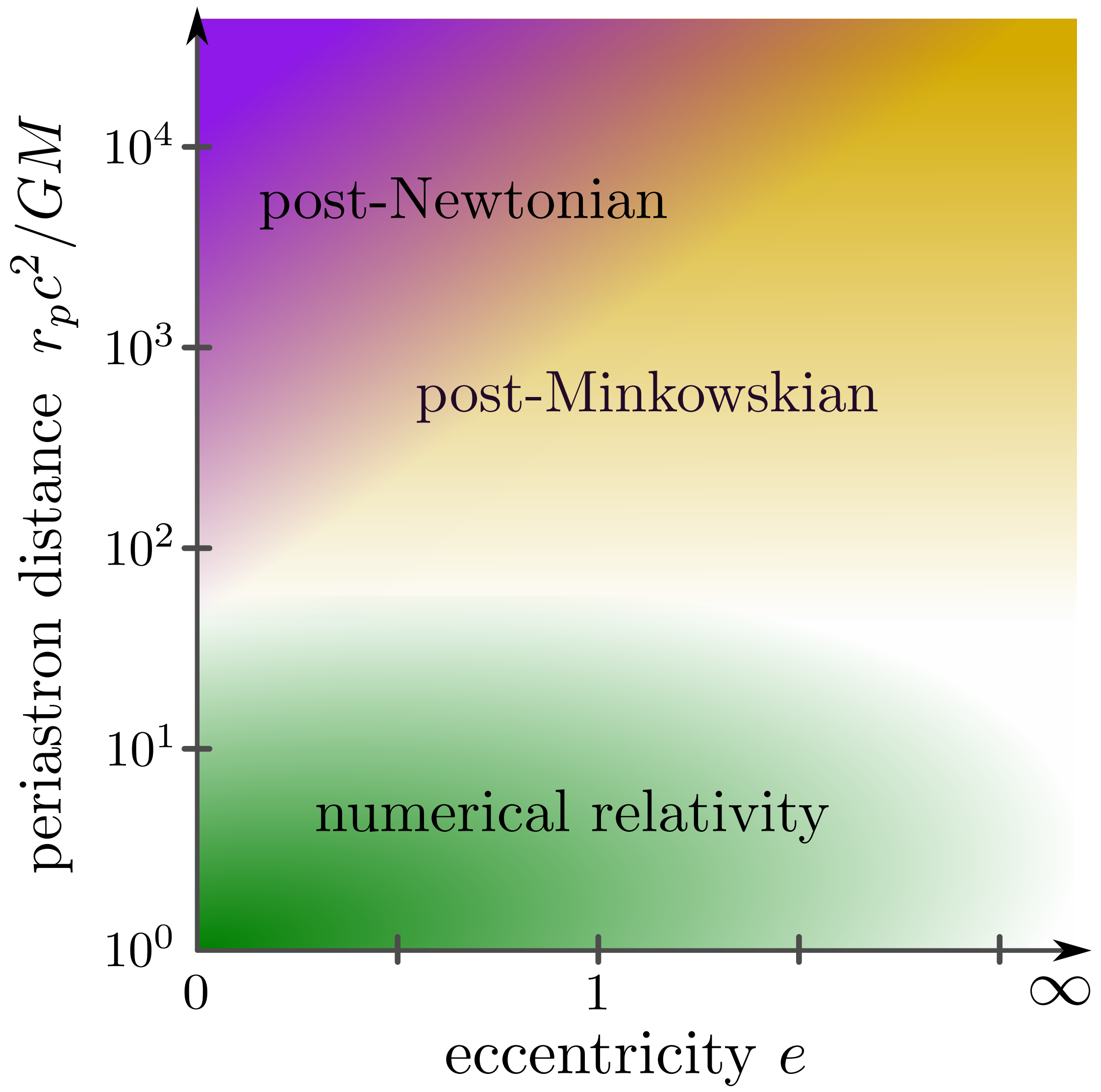
(APS/Stonebraker)

Frontier of GW modeling: eccentricity



(credit: Ramos-Buades, Markin & Pfeiffer)

- The **PM approximation is more accurate than PN** for scattering encounters **at large velocities,** or equivalently **large eccentricities at fixed periastron distance**.

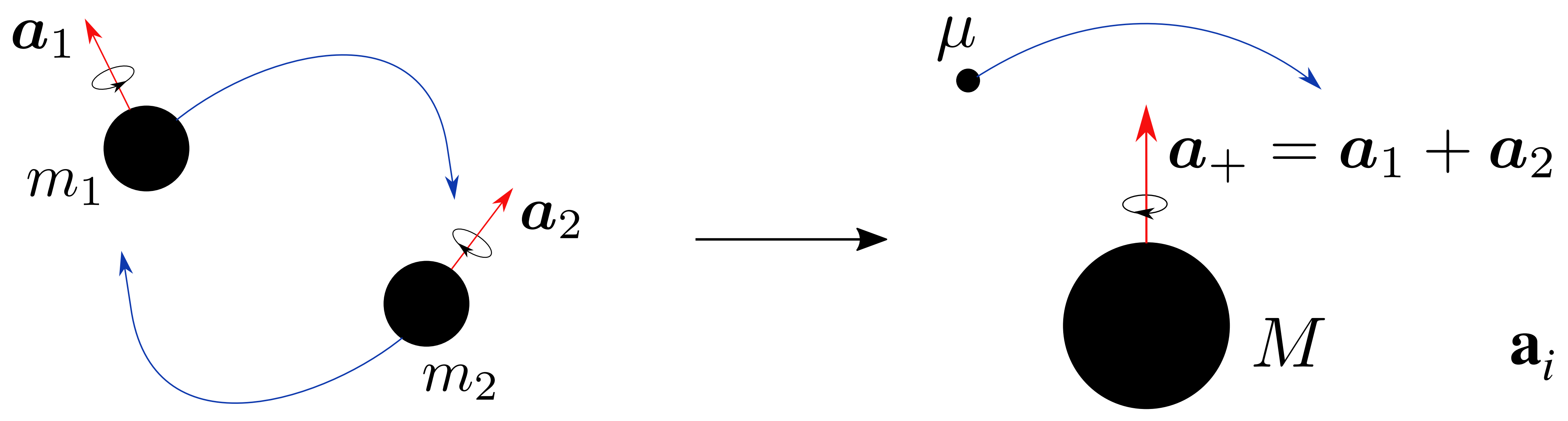


(Khalil, AB, Steinhoff & Vines 22)

EOB-PN Hamiltonian: Non-Spinning Bodies

- **Two-body dynamics is mapped** into dynamics of **one-effective body** moving **in deformed black-hole spacetime**, deformation being the mass ratio.

$\mu = m_1 m_2/M \qquad M = m_1 + m_2 \qquad \nu = \mu/M \qquad 0 \leq \nu \leq 1/4$



(credit: Khalil)

$\mathbf{a}_i = m_i \chi_i \quad i = 1,2$

$0 \leq \chi_i \leq 1$

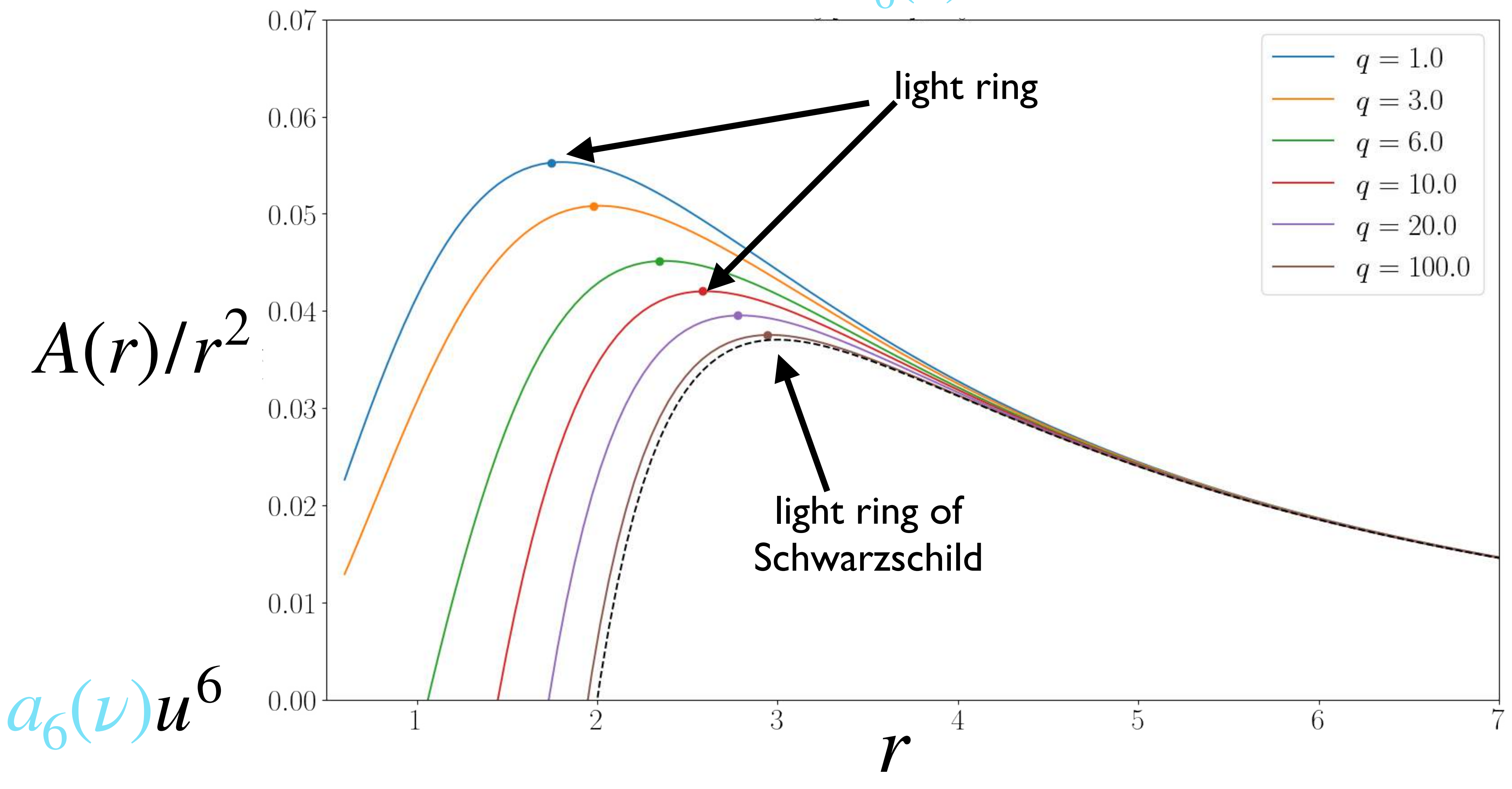
$$H_{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}}{\mu} - 1\right)}$$

(AB & Damour 99; Damour 00; AB, Chen & Damour 05; Damour, Jaranowski & Schafer 08; Barausse, Racine & AB 10; Barausse & AB 11; Damour & Nagar 14; Balmelli & Damour 15; Khalil, Steinhoff, Vines & AB 20; Khalil, AB, Estelles, Pompili, Ossokine & Ramos-Buades 23)

$\mathbf{a}_i = 0 \quad i = 1,2 \qquad g_{\rm eff}^{\mu\nu}\, p_\mu\, p_\nu + \mu^2 + \cdots = 0 \qquad G = 1 = c$

$$H_{\rm eff} = \sqrt{A(r,\nu;a_6)\left[\mu^2 + p_r^2\, B_{np}(r,\nu) + \frac{L^2}{r^2} + Q(r,p_r,\nu)\right]}$$

$a_6(\nu) = 0$



- Historically, **effective Hamiltonian based on PN results:**

$$A(u,\nu;a_6) = 1 - 2u + 2\nu u^3 + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\nu u^4 + [a_5(\nu) + a_5^{\log}(\nu)\,\log(u)]u^5 + a_6(\nu)u^6$$

$u = M/r \qquad$ 4PN $\qquad$ 5PN

with Padé resummation of $A(r)$ potential

PM Theory Meets the EOB Approach for Bound Orbits

- **The SEOB-PM Hamiltonian** is a **deformation of the Kerr Hamiltonian**, it is **informed by *available* PM results,** and it is **complemented by PN bound-orbit** corrections.

(Bini+17-18, Antonelli, AB+19, Khalil, AB+22, Khali, AB+23, AB, Jakobsen & Mogull 24)

(AB, Mogull, Patil & Pompili arXiv: 2405.19181)

$$H_{\rm eff} = \frac{M\, p_\phi\,(g_{a_+}\, a_+ + g_{a_-}\, \delta\, a_-)}{r^3 + a_+^2\,(r+2M)} + \sqrt{A\left(\mu^2 + \frac{p_\phi^2}{r^2} + (1+B_{\rm np}^{\rm Kerr})\, p_r^2 + B_{\rm npa}^{\rm Kerr}\,\frac{p_\phi^2\, a_+^2}{r^2}\right)}$$

$$H_{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}}{\mu} - 1\right)}$$

$$A = \frac{(1 - 2u + \chi_+^2\, u^2 + \Delta A)}{[1 + \chi_+^2\, u^2\,(2u+1)]} \qquad g_{a_\pm} = \frac{\Delta g_{a_\pm}}{u^2} \qquad u = M/r$$

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$$B_{\rm npa}^{\rm Kerr} = -\,(1+2u)/[r^2 + a_+^2(1+2u)] \qquad B_{\rm np}^{\rm Kerr} = \chi_+^2 u^2 - 2u$$

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(AB, Jakobsen & Mogull arXiv: 2402.12342)

	S^0 (Spin-0)	S^1 (Spin-1/2)	S^2 (Spin-1)	S^3 (Spin-3/2)	S^4 (Spin-2)	S^5 (Spin-5/2)
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5PM (4 loops)	G^5	G^6	G^7	G^8	G^9	G^{10}

tails

PM results for conservative dynamics in the last 5 years

(Guevara, Ochirov & Vines 19, Chen, Chung, Huang, & Kim 22, Bern, Kosmopoulos, Luna, Roiban & Teng 23, Aoude, Haddad & Helset 23, Bautista 23)

(Bern, Cheung, Roiban, Shen, Solon & Zeng 19, Kälin, Liu & Porto 20, Cheung & Solon 20, Di Vecchia, Heissenberg, Russo & Veneziano 20, Jakobsen & Mogull 22, 23, Febres Cordero, Kraus, Lin, Run & Zeng 23, Brandhuber+21)

(Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon et al. 22, Dlapa, Kälin, Liu & Porto 22, Jakobsen, Mogull, Plefka, Sauer & Xu 23, Jakobsen, Mogull, Plefka & Sauer 23, Dlapa, Kälin, Liu & Porto 24, Damour & Bini 24)

(Driesse, Jakobsen, Mogull, Plefka, Sauer & Usovitsch 24)

PM Theory Meets the EOB Approach for Bound Orbits

- **The SEOB-PM Hamiltonian** is a **deformation of the Kerr Hamiltonian**, it is **informed by *available* PM results,** and it is **complemented by PN bound-orbit** corrections.

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$\delta = (m_1 - m_2)/M$

we complement with 4PN corrections for bound orbits, including tails

$$\Delta A = \sum_{n=2}^{5} u^n\,\Delta A^{(n)} + \Delta A^{\rm 4PN} \qquad \Delta g_{a_\pm} = \sum_{n=2}^{5} u^n\,\Delta g_{a_\pm}^{(n)}$$

even-in-spin PM corrections (through 5PM)

odd-in-spin PM corrections (through 5PM)

(AB, Jakobsen & Mogull arXiv: 2402.12342)

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tails

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$\delta = (m_1 - m_2)/M$

- **Coefficients** of effective Hamiltonian **determined by computing the EOB scattering angle and matching to PM results** (only conservative sector):

$$p_r^2 = \frac{1}{(1+B_{\rm np}^{\rm Kerr})}\left\{\frac{1}{A}\left[E_{\rm eff} - \frac{ML\,(g_{a_+}\, a_+ + g_{a_-}\, \delta\, a_-)}{r^3 + a_+^2\,(r+2M)}\right]^2 - \left(\mu^2 + \frac{L^2}{r^2} + B_{\rm npa}^{\rm Kerr}\frac{L^2\, a_+^2}{r^2}\right)\right\}$$

$$\theta + \pi = -2\int_{r_{\rm min}}^{+\infty} dr\, \frac{\partial p_r}{\partial L} \qquad p_\infty = \mu\sqrt{\gamma^2 - 1}\,, \quad \gamma = \frac{E_{\rm eff}}{\mu} > 1$$

(AB, Jakobsen & Mogull arXiv: 2402.12342)

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$M = m_1 + m_2 \quad \nu = \mu/M \quad 0 \le \nu \le 1/4 \quad G = 1 = c$

$\delta = (m_1 - m_2)/M$

$$\Delta A^{(n)} = \sum_{s=0}^{\lfloor (n-1)/2\rfloor}\sum_{i=0}^{2s} \alpha^{(n)}_{(2s-i,i)}\,\delta^{\sigma(i)}\,\chi_+^{2s-i}\chi_-^{i}$$

$$\Delta g^{(n)}_{a_+} = \sum_{s=0}^{\lfloor (n-2)/2\rfloor}\sum_{i=0}^{s} \alpha^{(n)}_{(2(s-i)+1,2i)}\,\chi_+^{2(s-i)}\chi_-^{2i}$$

$$\Delta g^{(n)}_{a_-} = \sum_{s=0}^{\lfloor (n-2)/2\rfloor}\sum_{i=0}^{s} \alpha^{(n)}_{(2(s-i),2i+1)}\,\chi_+^{2(s-i)}\chi_-^{2i}$$

$\alpha_s^{(n)} \to$ are function of γ, ν

(logarithms, dilogarithms, and elliptic functions of the first and second kind)

$$\gamma = \frac{E_{\rm eff}}{\mu}$$

$$\gamma = \gamma_{\rm Kerr} + \sum_{n\ge 2}\sum_{s\ge 0}\Delta^{(n)}_{(s)}(\gamma_{\rm Kerr})$$

(AB, Jakobsen & Mogull arXiv: 2402.12342)

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tails

Inspiral-Plunge SEOB-PM Dynamics

- **EOB equations of motion**

  non-precessing spins

  $G = 1 = c$

$$\dot r = \frac{\partial H_{\rm SEOB-PM}}{\partial p_r} \qquad \dot p_r = -\frac{\partial H_{\rm SEOB-PM}}{\partial r} + \frac{p_r}{p_\phi}\mathscr{F}_\phi$$

$$M\Omega = \dot\phi = \frac{\partial H_{\rm SEOB-PM}}{\partial p_\phi} \qquad \dot p_\phi = \mathscr{F}_\phi$$

RR force from resummed PN/GSF results

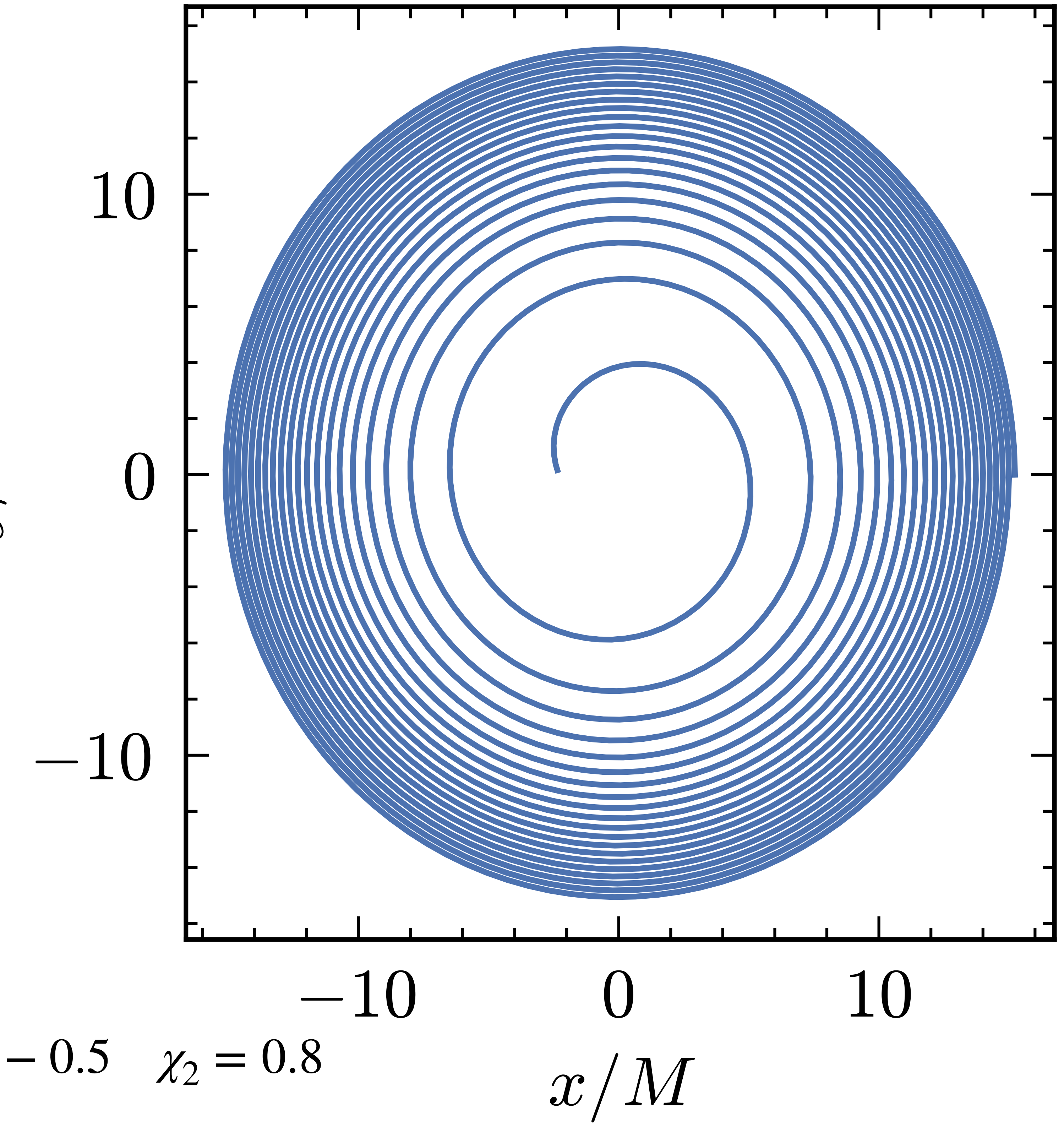
- **SEOB-PM Hamiltonian, dynamics and waveforms are developed using the** flexible and efficient **Python code (pySEOBNR).**

  (Mihaylov, … AB+23)

pySEOBNR

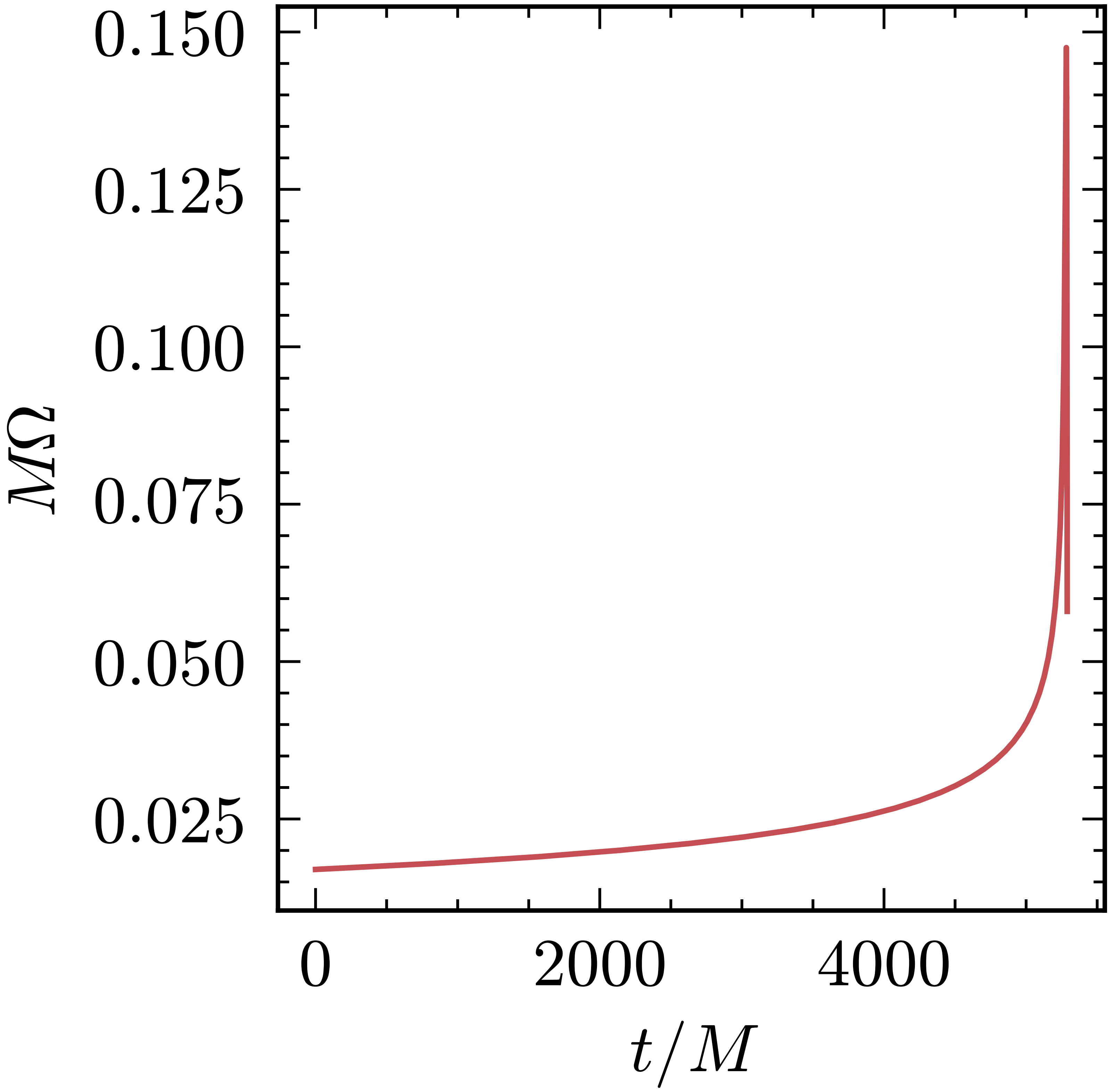
https://git.ligo.org/waveforms/software/pyseobnr

- Evolve **two-body dynamics toward merger**



(credit: Pompili)

$q = 4.67 \quad \chi_1 = -0.5 \quad \chi_2 = 0.8$



(credit: Pompili)

Comparing SEOB-PM Binding Energy with Numerical Relativity

$G = 1 = c$

- **Binding energy** is computed **along quasi-circular inspiral** (and a circular orbit):

$$\mathscr{E} = \frac{H_{\mathrm{EOB}} - M}{\mu} \qquad v = (M\dot{\phi})^{1/3}$$

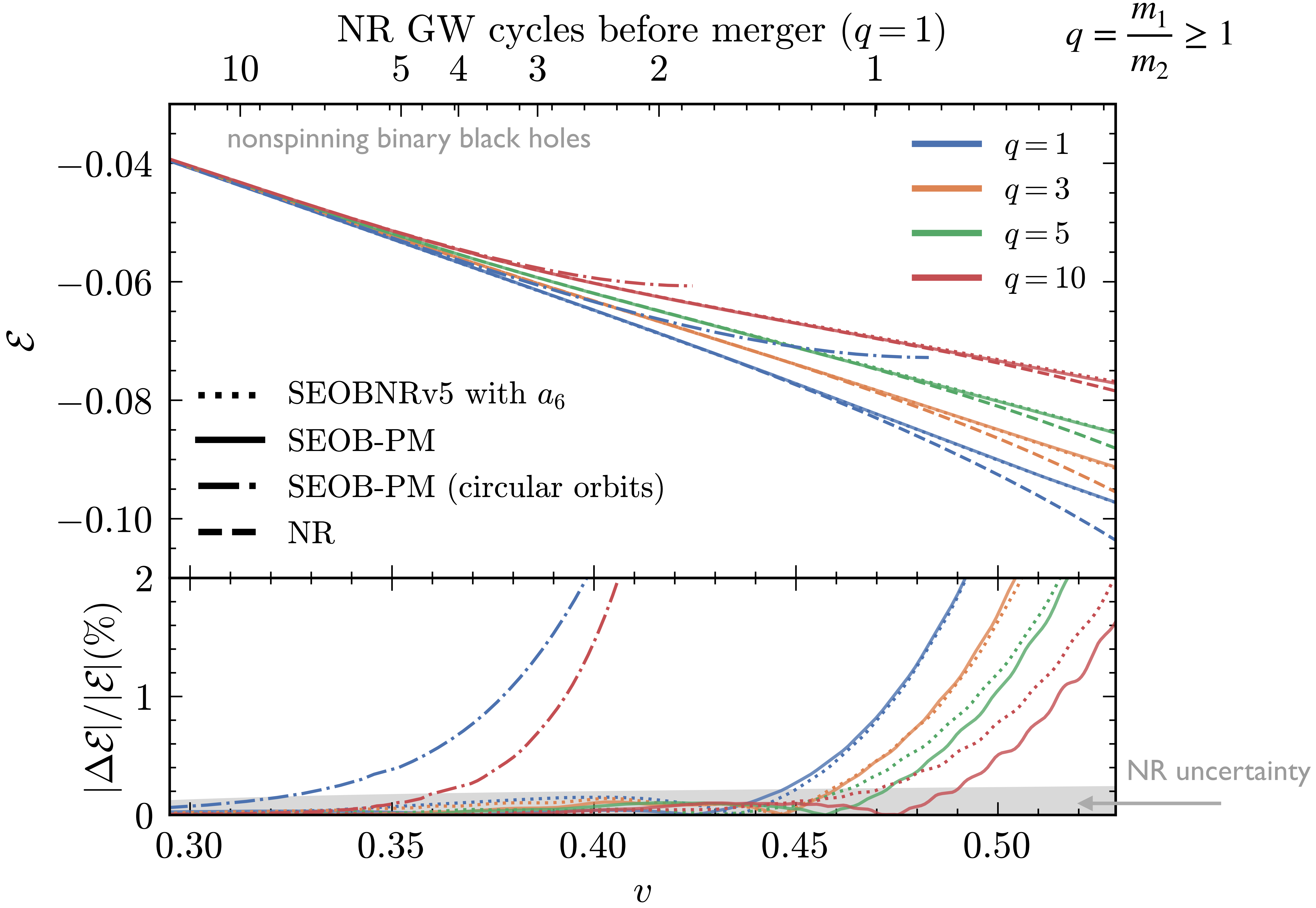
SEOBNRv5 → state-of-the-art waveform model (from SEOBNR family) based on PN/GSF, developed for the ongoing LVK run (O4)

(Pompili,AB+23, van de Meent,AB+23, Ramos-Buades,AB+23, Khalil,AB+23)

NR → waveform from the Simulating eXtreme Spacetimes (SXS) Collaboration

(Boyle+19, Ossokine+20)

$q = \frac{m_1}{m_2} \geq 1$

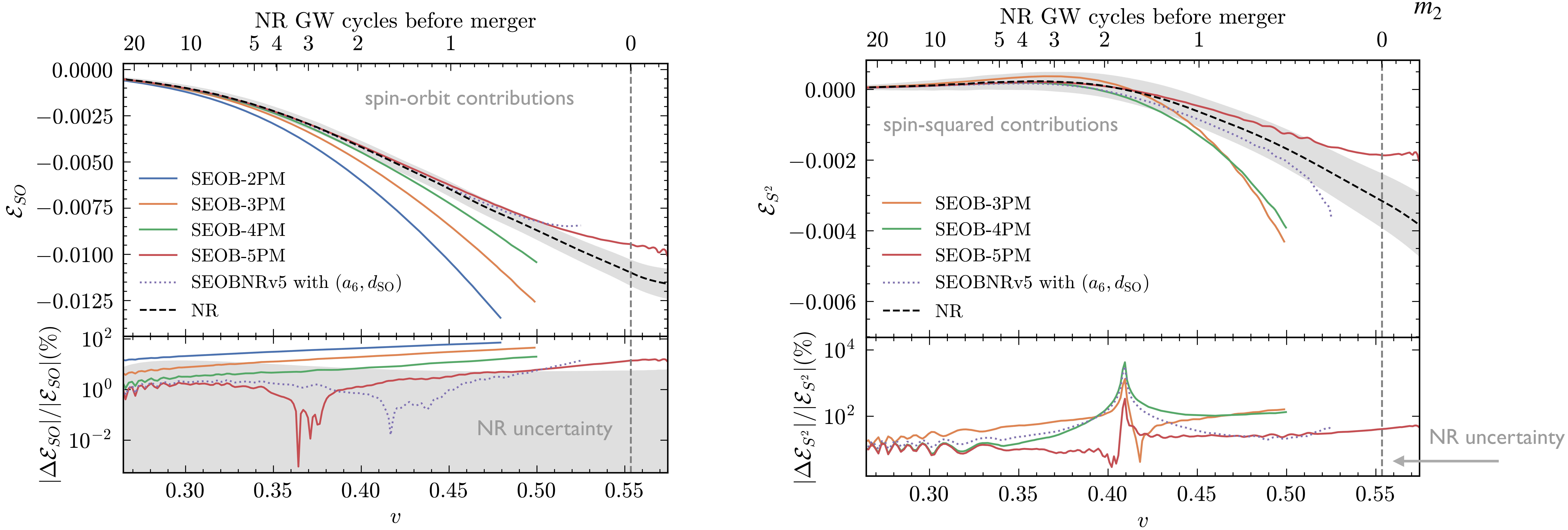


- **SEOB-PM binding energy has excellent agreement with NR** without resummation or calibration.

Comparing SEOB-PM Binding Energy with Numerical Relativity (contd.)

$G = 1 = c$

$q = \frac{m_1}{m_2} = 1$



- **Despite not being calibrated to NR, SEOB-PM shows excellent agreement with NR**, with a clear convergence. Its **accuracy is somewhat better than SEOBNRv5**, despite the latter being calibrated in the non-spinning (a_6) and spin-orbit coupling (d_{SO}) sectors.

Inspiral-Plunge SEOB-PM Waveform & Frequency

- **EOB equations of motion**

non-precessing spins

$$\dot{r} = \frac{\partial H_{\rm SEOB-PM}}{\partial p_r} \qquad \dot{p}_r = -\frac{\partial H_{\rm SEOB-PM}}{\partial r} + \frac{p_r}{p_\phi}\mathcal{F}_\phi$$

$$\dot{\phi} = \frac{\partial H_{\rm SEOB-PM}}{\partial p_\phi} \qquad \dot{p}_\phi = \mathcal{F}_\phi$$

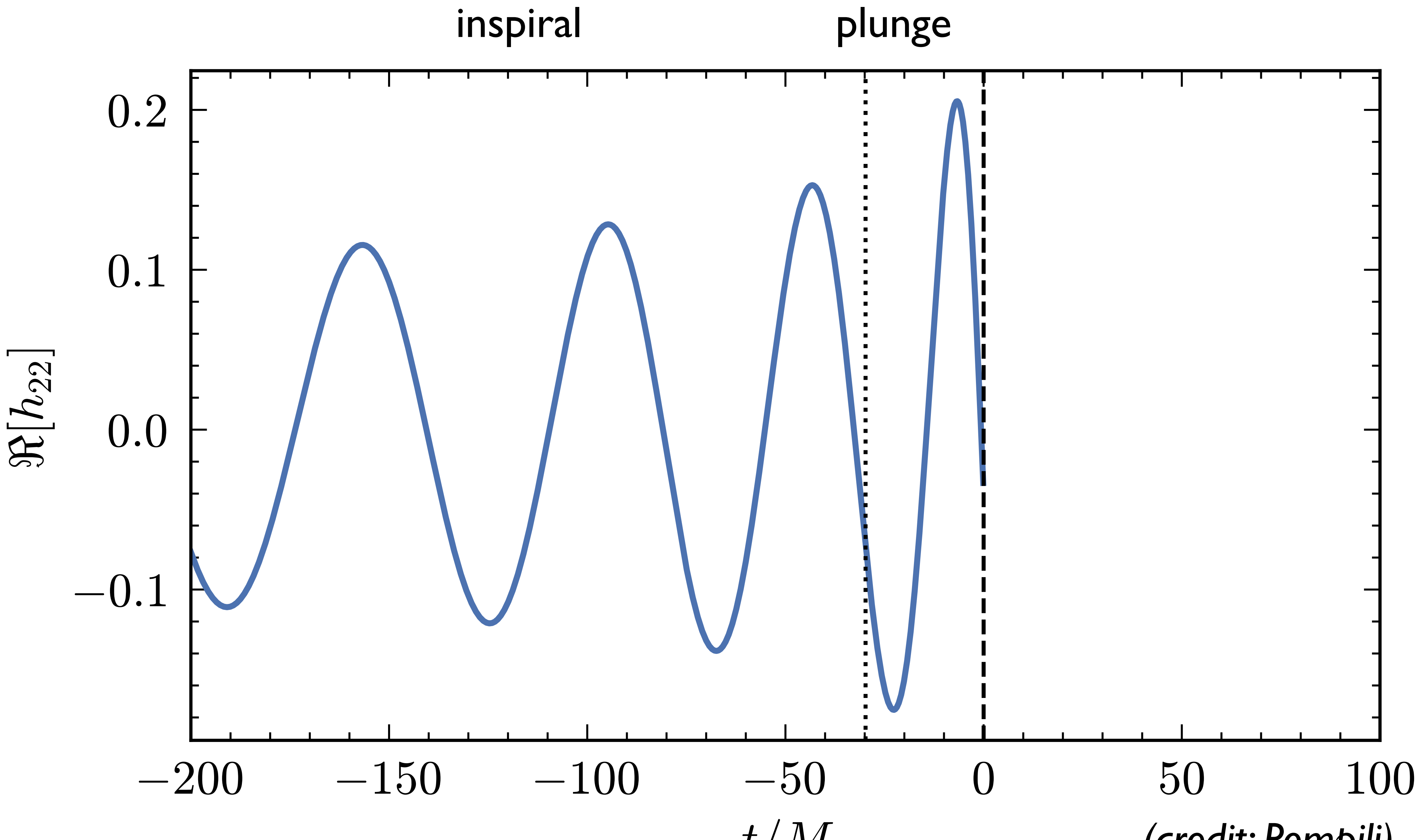
$$h_+ - i h_\times = \sum_{\ell,m} {}_{-2}Y_{\ell m}(\varphi, \iota)\, h_{\ell m}(t)$$

$$h_{22}^{\rm insp-plunge}(t) = h_{22}^{\rm Newt}\, S_{\rm eff}\, T_{22}\, f_{22}\, e^{i\delta_{22}}\, h_{22}^{\rm NQC}$$

(Damour+09, Pan, AB+11, Pompili, AB+23, van de Meent, AB+23)

GW modes from **resummed PN/GSF results**

- Evolve **two-body dynamics up to close to light ring** (or photon orbit) and then …



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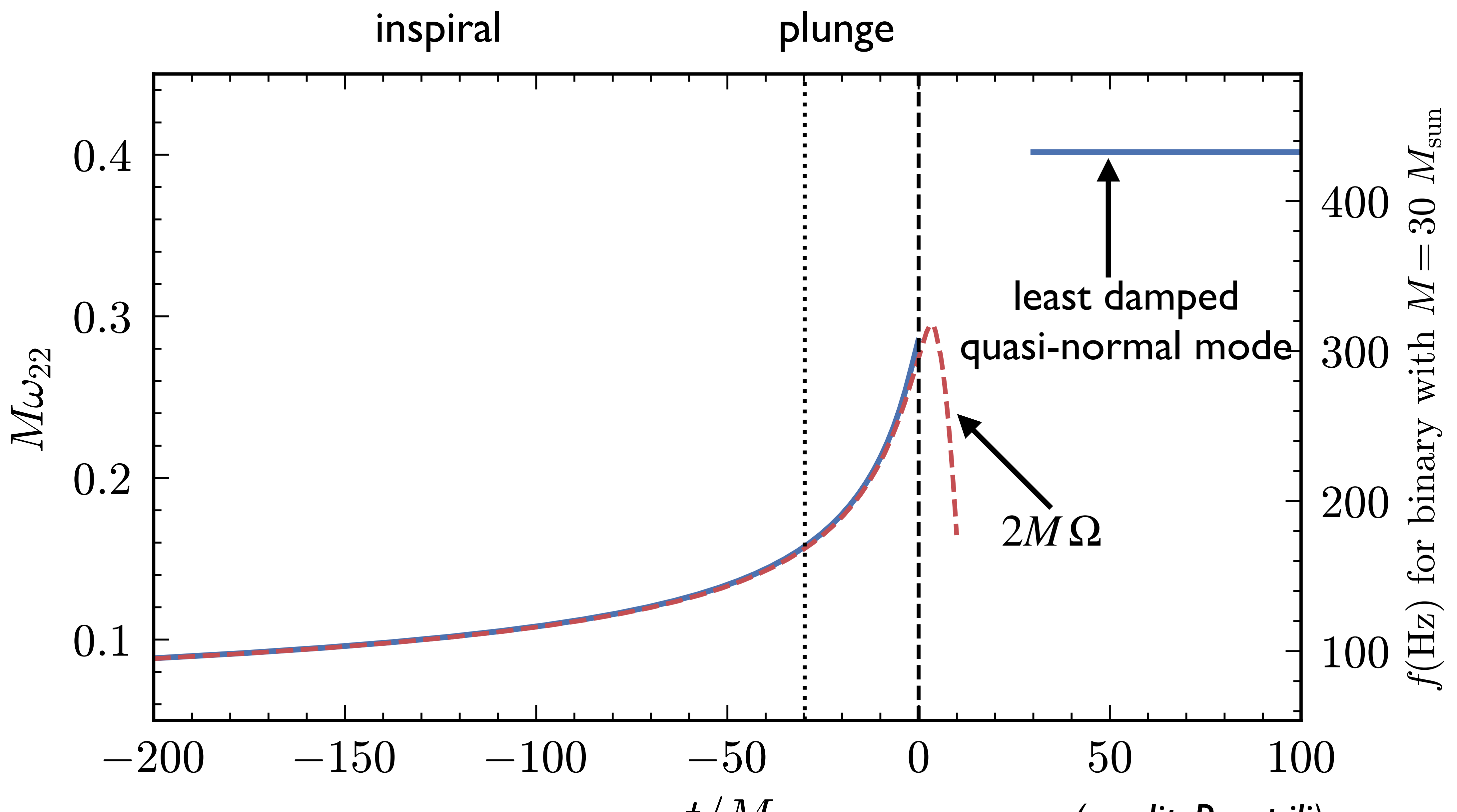
(credit: Pompili)



(credit: Pompili)

- **Quasi-normal modes** excited around **light-ring crossing.** *(Goebel 1972; Davis, Ruffini & Tiomno 1972; Ferrari et al. 1984; Price and Pullin 1994)*

Inspiral-Merger-Ringdown SEOB-PM Waveform & Frequency

- **EOB equations of motion**

non-precessing spins

$$\dot{r} = \frac{\partial H_{\mathrm{SEOB-PM}}}{\partial p_r} \qquad \dot{p}_r = -\frac{\partial H_{\mathrm{SEOB-PM}}}{\partial r} + \frac{p_r}{p_\phi}\mathcal{F}_\phi$$

$$\dot{\phi} = \frac{\partial H_{\mathrm{SEOB-PM}}}{\partial p_\phi} \qquad \dot{p}_\phi = \mathcal{F}_\phi$$

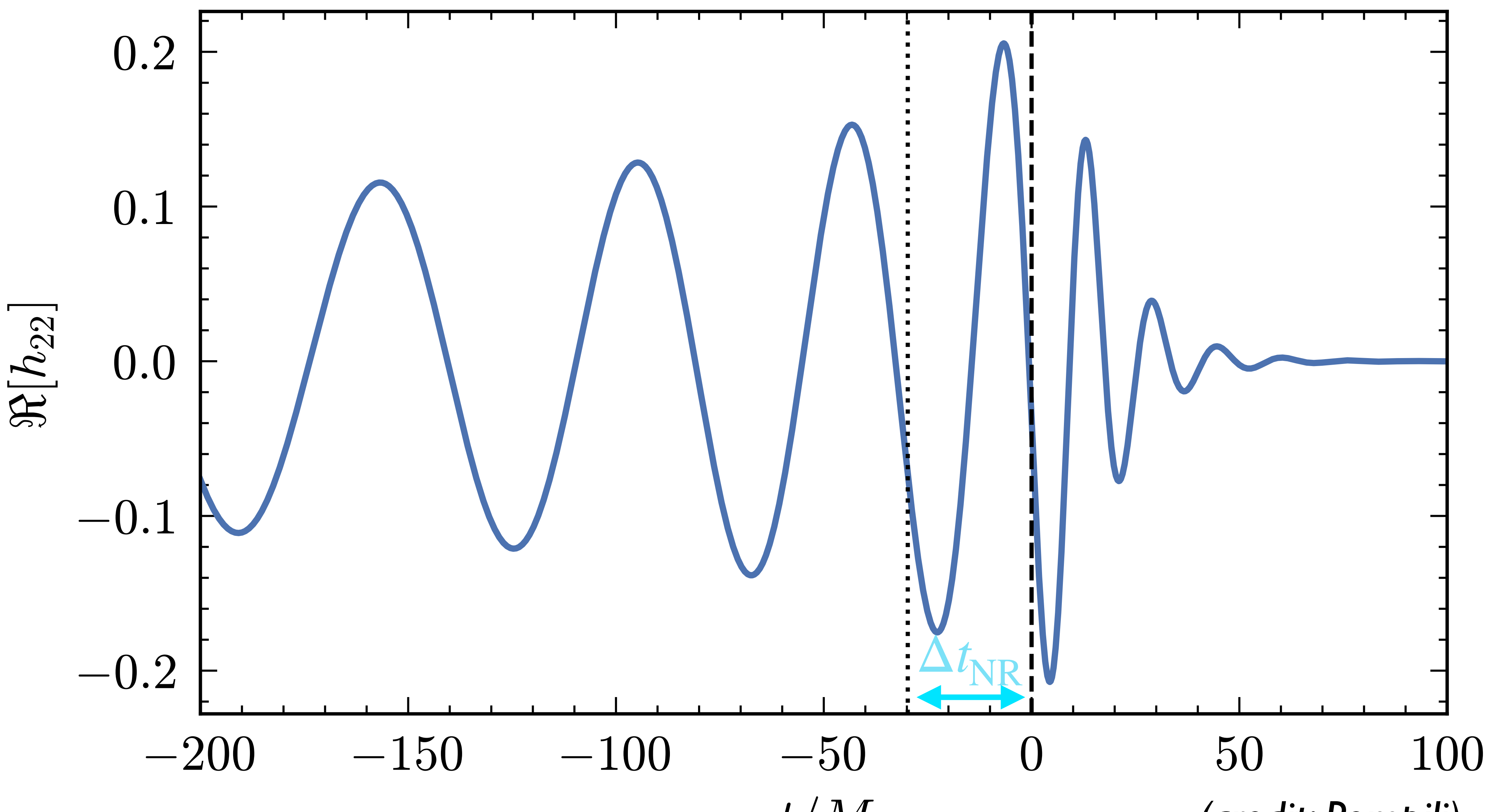
BH quasi-normal modes

$$h_{22}^{\mathrm{merger-RD}}(t) = \nu\, \tilde{A}_{22}(t)\, e^{i\,\tilde{\phi}_{22}(t)}\, e^{-i\sigma_{220}(t-t_{22}^{\mathrm{peak}})}$$

(Baker+08, Damour & Nagar14, London+14, Bohé, … AB+17, Cotesta, AB+19, Pompili, AB+23)

- … attach a function representing **quasi-normal mode ringing** of remnant BH.

$$t_{22}^{\mathrm{peak}} = t_{\mathrm{ISCO}} + \Delta t_{\mathrm{NR}}$$



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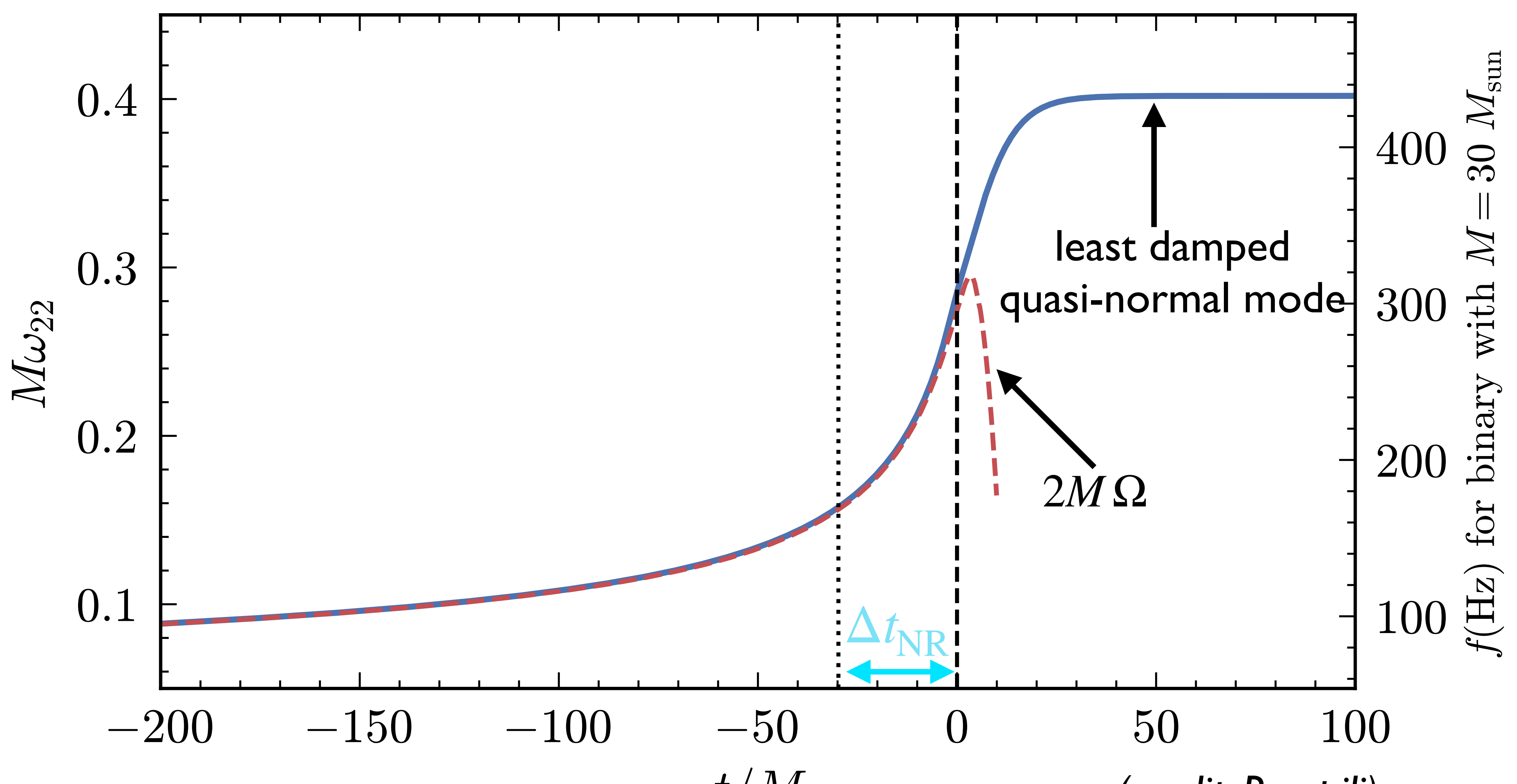
(credit: Pompili)



(credit: Pompili)

$$h_{22}(t) = h_{22}^{\mathrm{insp-plunge}}(t)\, \theta(t_{22}^{\mathrm{peak}} - t) + h_{22}^{\mathrm{merger-RD}}(t)\, \theta(t - t_{22}^{\mathrm{peak}})$$

Comparing SEOBNR-PM Waveforms with Numerical Relativity

$G = 1 = c$

pySEOBNR

- **Calibrating *only* the time to merger $\Delta t_{\rm NR}$** *(Pompili+23)*:

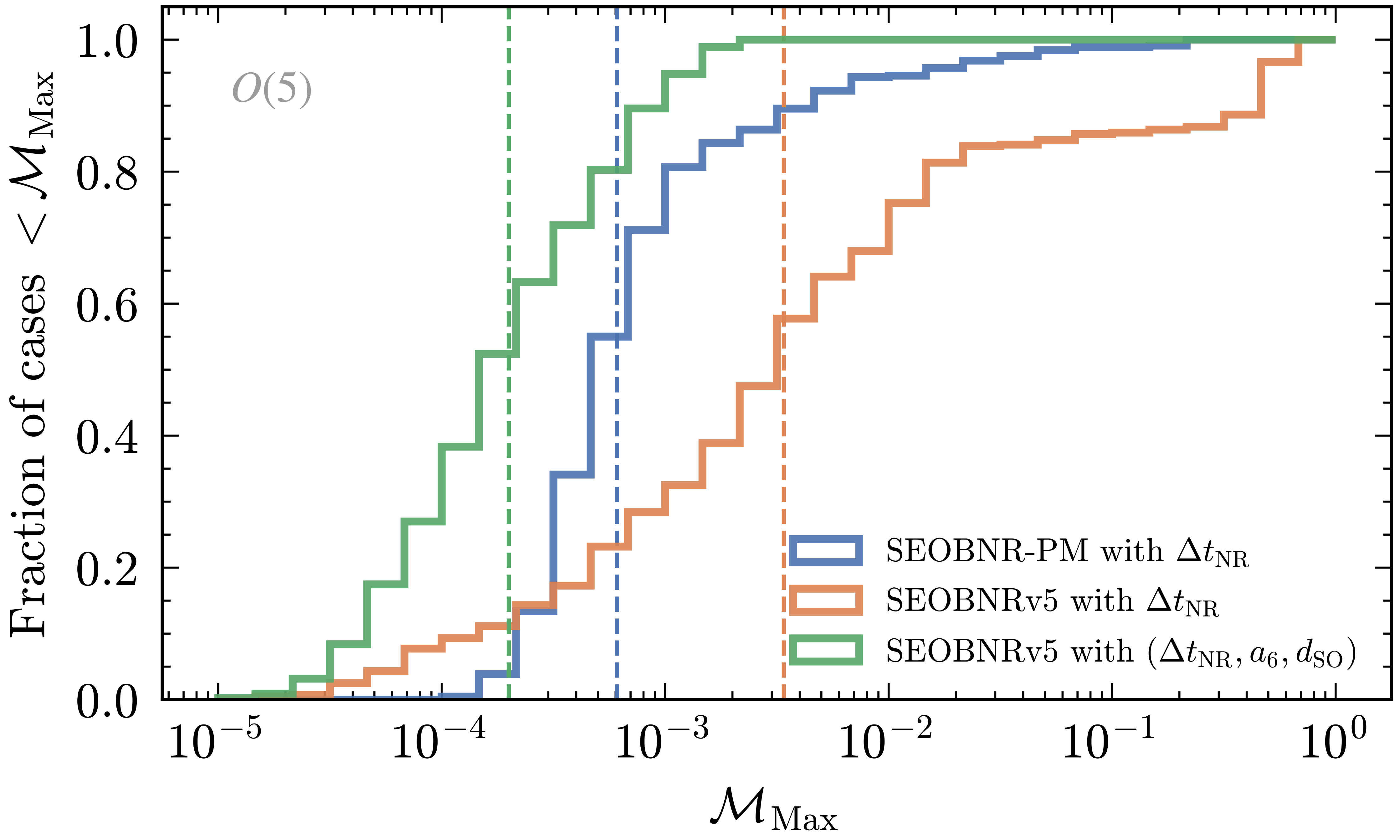
$$h_{22}(t) = h_{22}^{\rm insp-plunge}(t)\,\theta(t_{22}^{\rm peak} - t) + h_{22}^{\rm merger-RD}(t)\,\theta(t - t_{22}^{\rm peak})$$

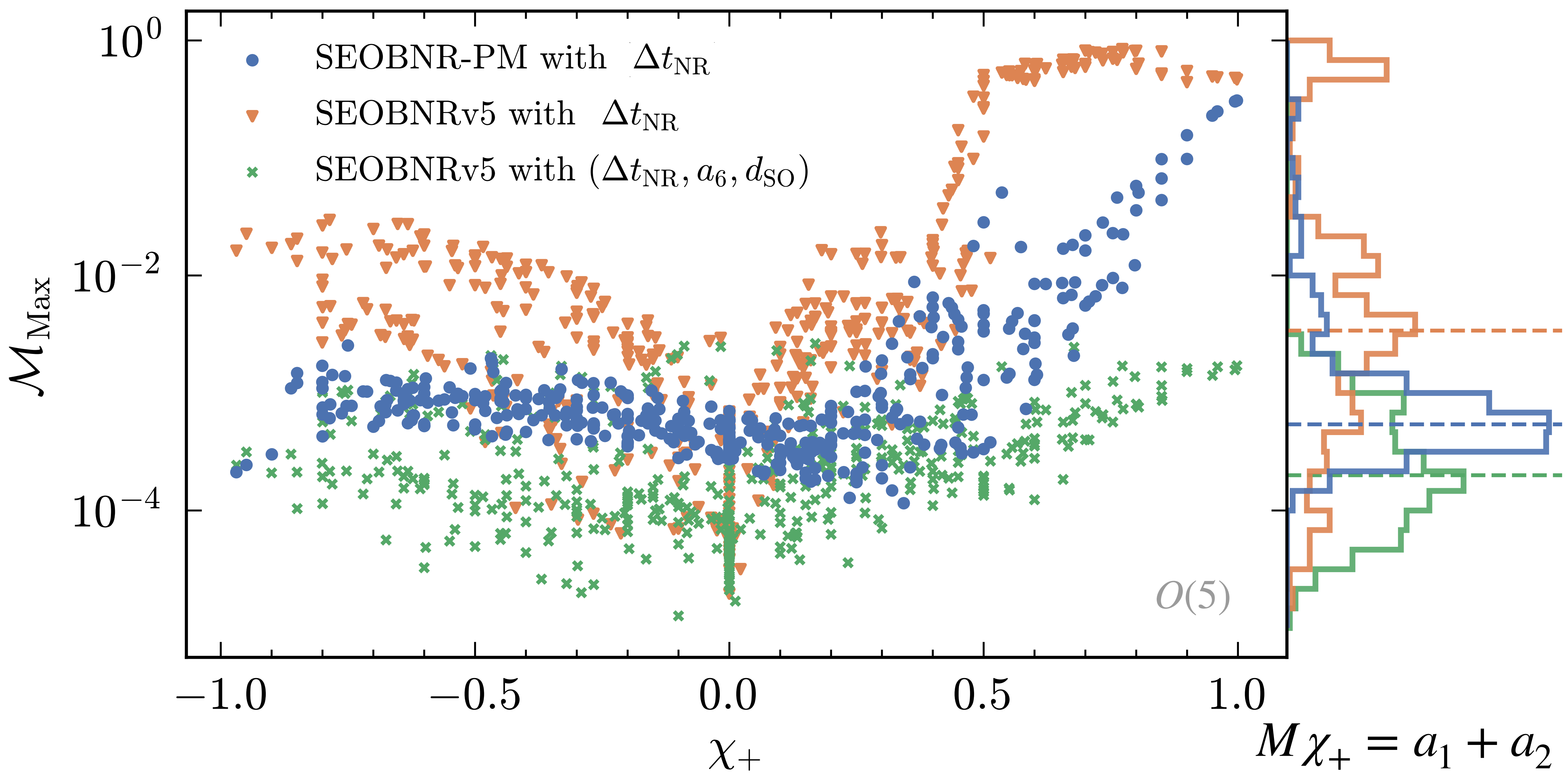
$$t_{22}^{\rm peak} = t_{\rm ISCO} + \Delta t_{\rm NR}$$

$$\mathcal{M} = 1 - \max_{t_0,\phi_0} \frac{(h_{\rm model}, h_{\rm NR})}{\sqrt{(h_{\rm model}, h_{\rm model})(h_{\rm NR}, h_{\rm NR})}} \qquad (h,g) = 4{\rm Re}\left[\int_{f_{\rm min}}^{f_{\rm max}} \frac{h(f)\,g^*(f)\,df}{S_n(f)}\right]$$

Mismatch $\mathcal{M} = 0$ implies models & NR match perfectly

- **Mismatch against 441 NR SXS waveforms**



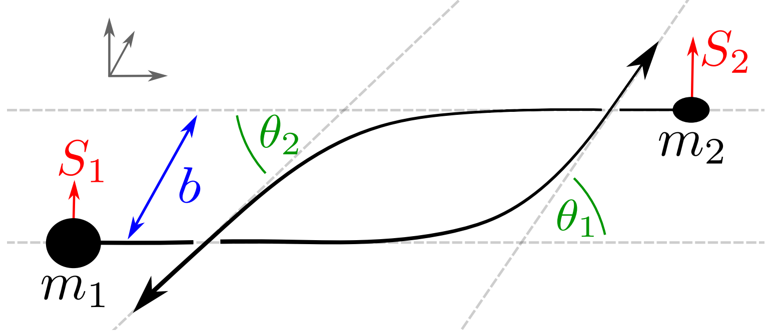


$M\chi_+ = a_1 + a_2$

- **SEOBNR-PM has remarkably good agreement with NR.** When calibrating only $\Delta t_{\rm NR}$, the **accuracy of both SEOBNR-PM and SEOBNRv5 tends to degrade for large positive spins**, but much more for the latter.

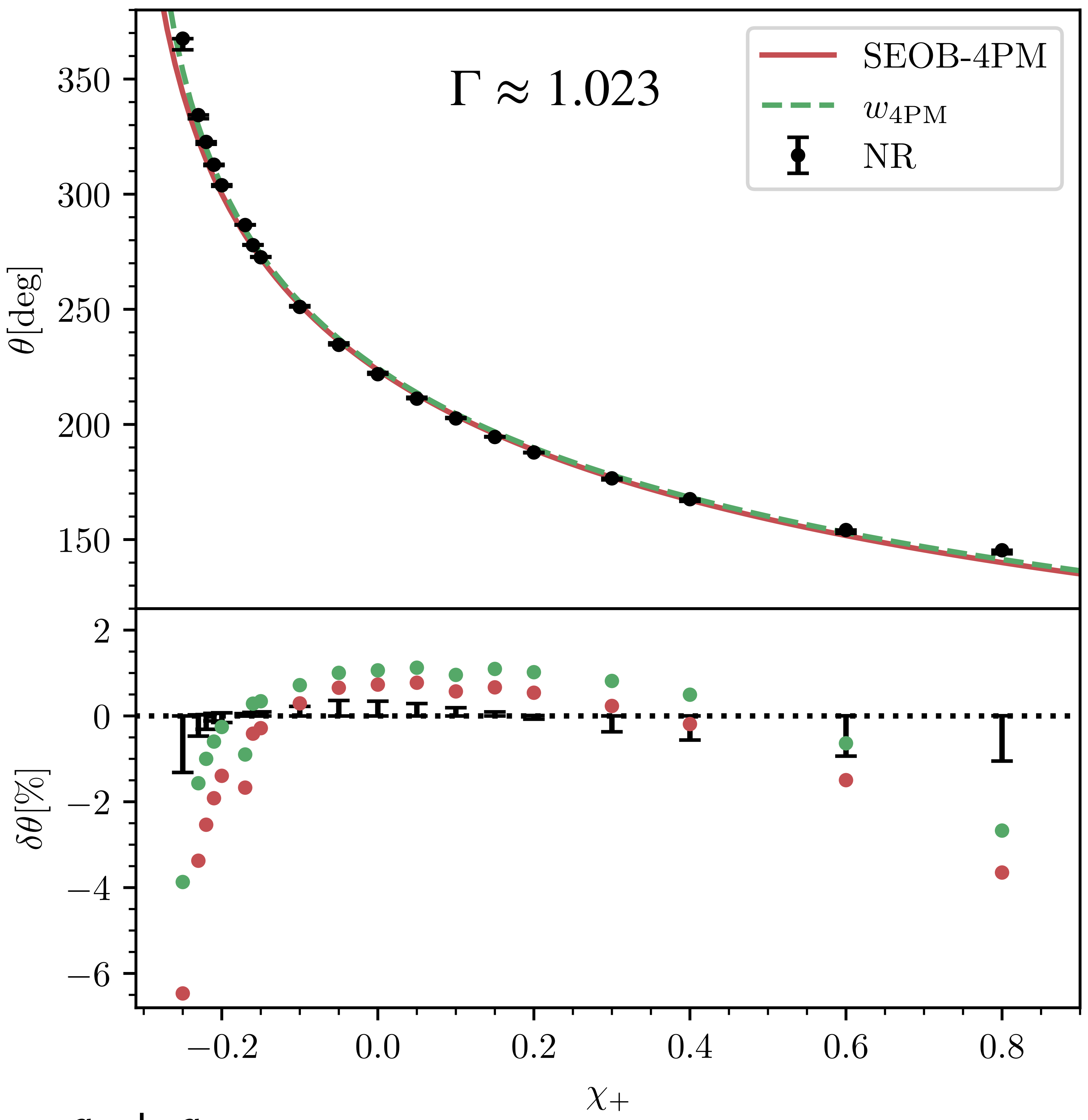
PM Theory Meets the EOB Approach for Scattering

$$\frac{E_{\rm eff}}{\mu} = \frac{E^2 - m_1^2 - m_2^2}{2 m_2 m_2} \qquad \Gamma = \frac{E}{M}$$



($w_{\rm PM}$ → Damour & Rettegno 23, Rettegno+23) (SEOB-PM → *AB, Jakobsen & Mogull 24)*

- Here, **deformation coefficients of SEOB-PM** Hamiltonian also **depend on dissipative terms** of PM scattering angle.



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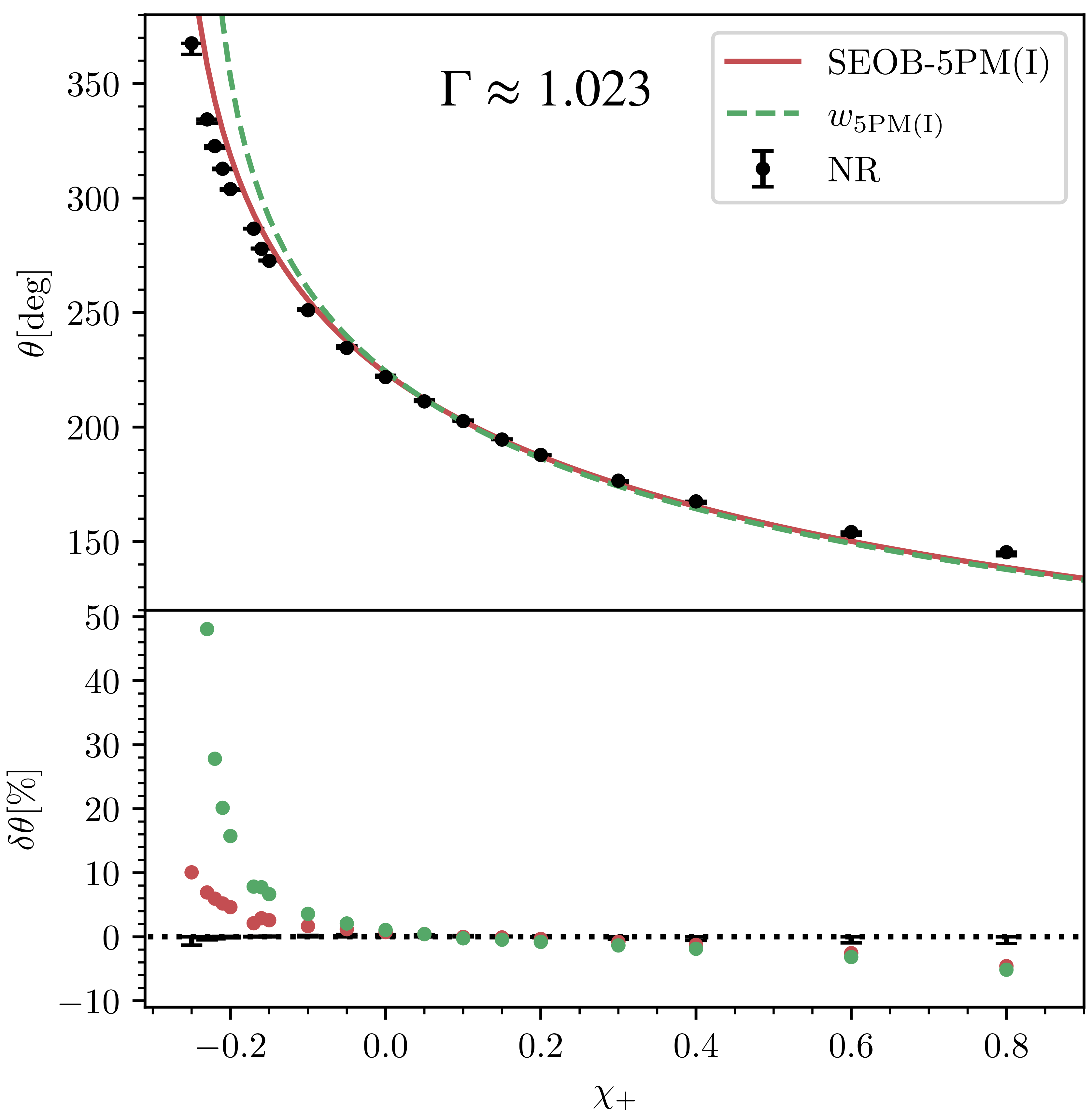

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$$q = \frac{m_1}{m_2} = 1$$

$$M\chi_+ = a_1 + a_2$$

- Important to **complete 5PM and push spin results at higher PM order.**

Summary & Outlook

- To **correctly interpret** future GW observations, and **avoid drawing wrong scientific conclusions**, the **precision** of theoretical GW predictions **in vacuum GR must improve by two orders of magnitude or more**, depending on the binary's parameter space, and **must include *all* physical effects** (generic orbits, beyond-GR, matter/environment).

- To **address the accuracy challenge, perturbative calculations** (PN, PM, GSF), should be **pushed at higher orders**, and **combined in** EOB approach **more effectively and in novel ways**.

- Built the **first inspiral-merger-ringdown EOB waveform model (SEOBNR-PM)** for aligned/antialigned-spin BHs that uses a PM-informed Hamiltonian (i.e., expanded in G, but at all orders in the velocity).

- **SEOB-PM non-spinning binding energy,** computed along an inspiraling trajectory, **at 4PM, and its spin-orbit and spin-spin contributions through 5PM**, **agree remarkably well with the NR data** up to about 1 GW cycle before merger.

- When calibrated to NR by **adjusting the time to merger, SEOBNR-PM performs better than the state-of-the-art SEOBNR** based on PN.

- **Future: resum PM-EOB potentials, fully calibrate SEOB-PM to NR, include PM results in EOB RR force and gravitational modes** when available at high PM order, **extend to eccentricity** … **use new PM results** in EOB potentials when available!