Loops in the Sky:
Recent Applications of Amplitude Methods in Galaxy Clustering and Gravitational Waves

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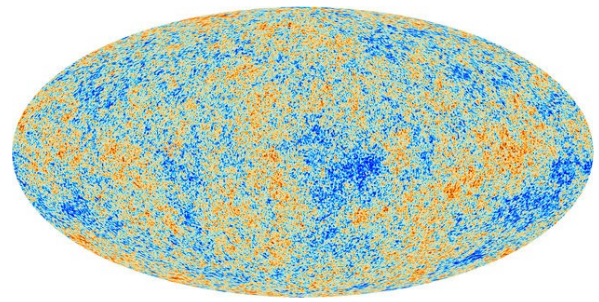
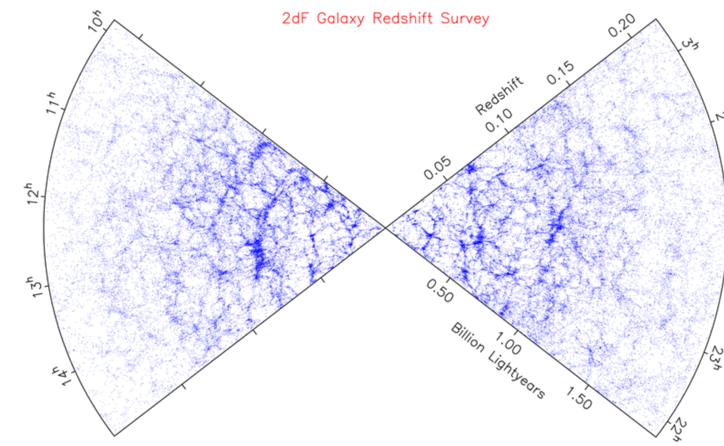
Amplitudes 2024, 10-14 June 2024, IAS

Cosmology

inflation



CMB



galaxies

$\Lambda\mathrm{CDM}$: Inflation, Cold Dark Matter, Lambda

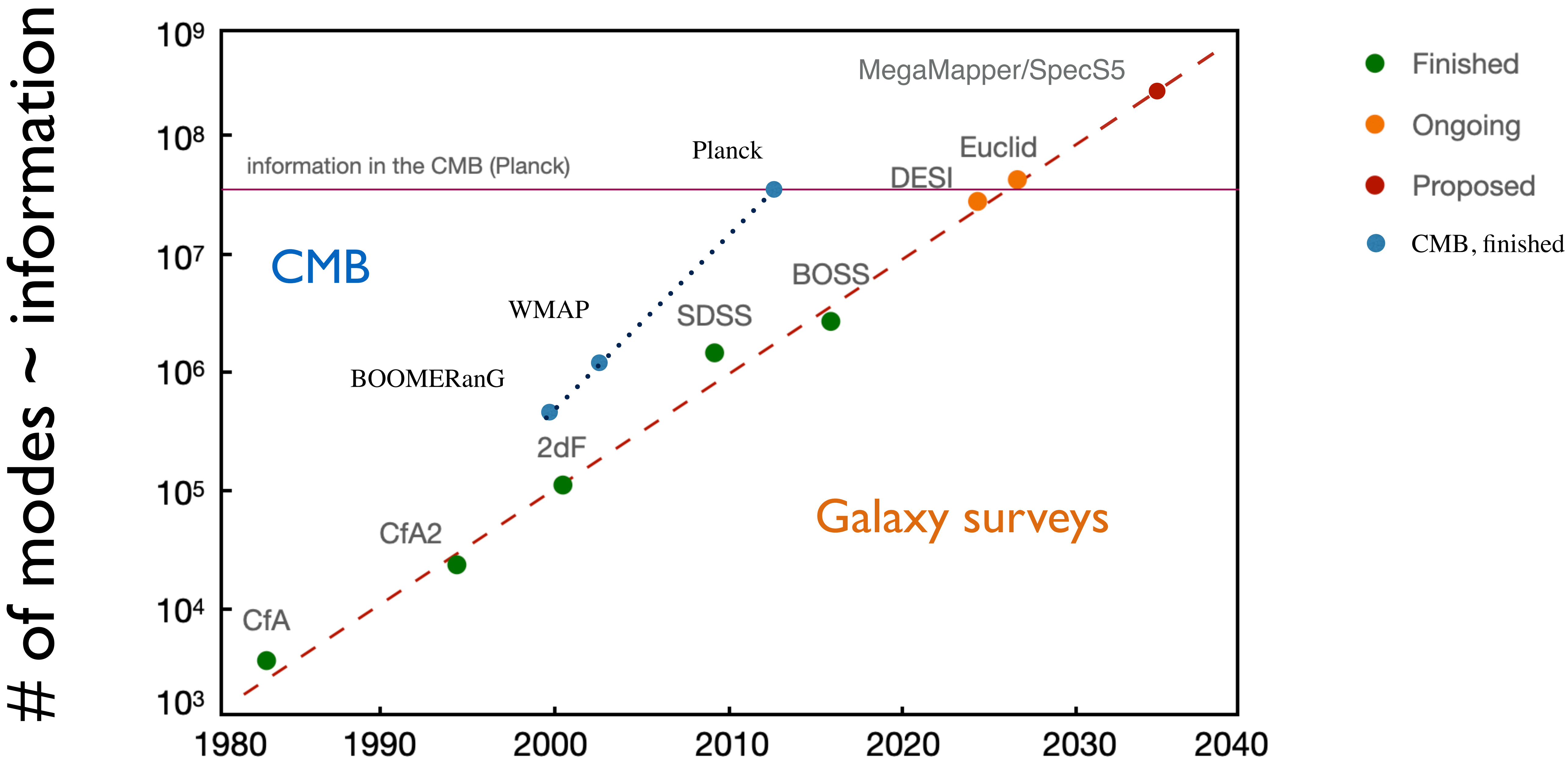
Known Unknowns:
What was inflation, exactly?
Is DM really cold?
many more …

Unknown Unknowns:
Surprises ?

- Need to continue measuring fluctuations: galaxies

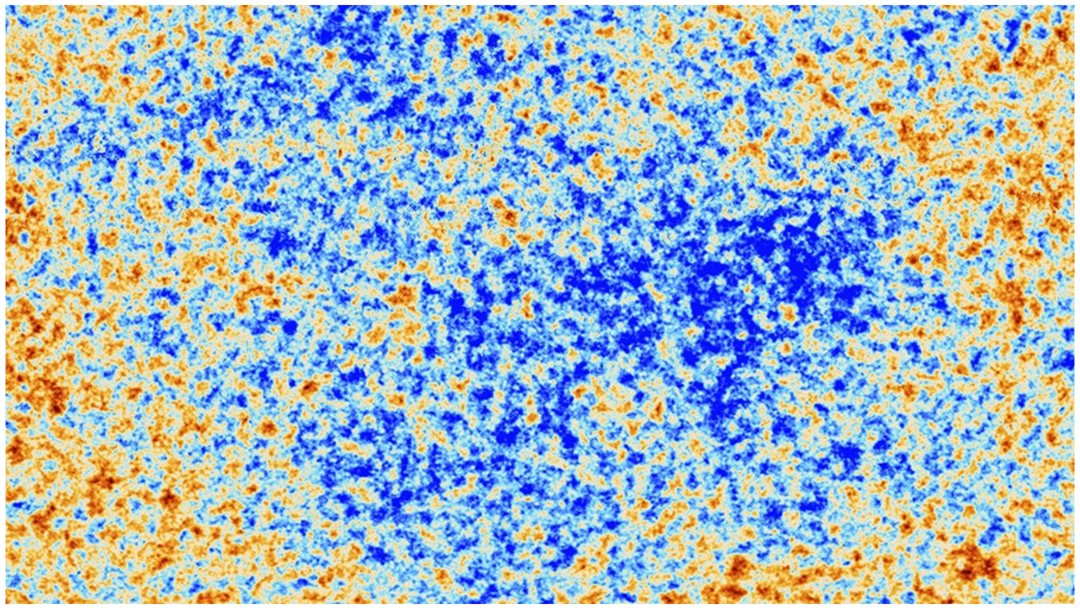
also see Hayden Lee's talk

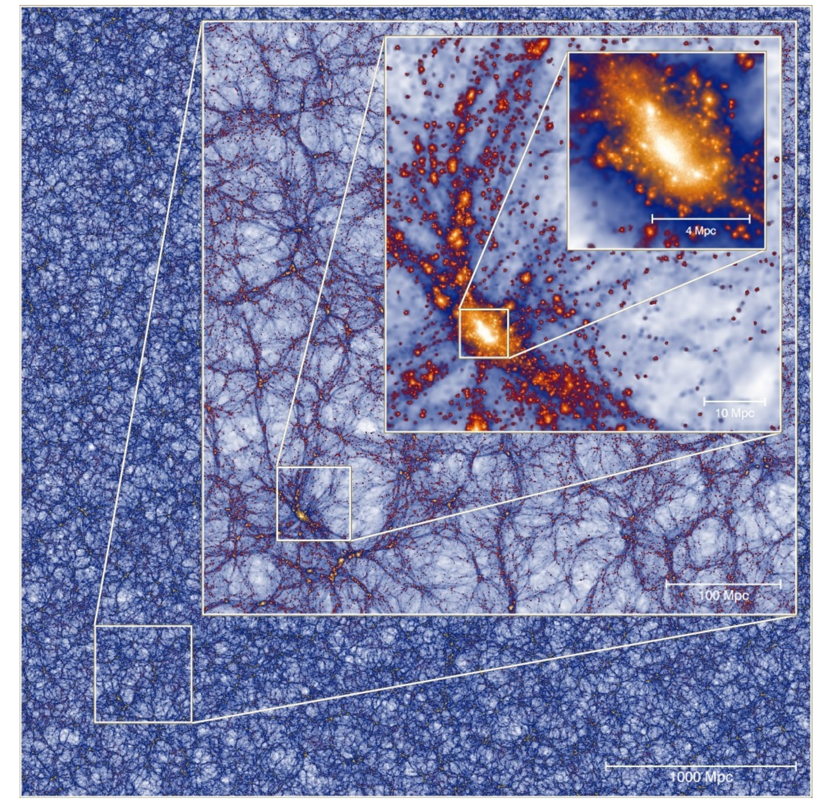
Information in Galaxy Surveys

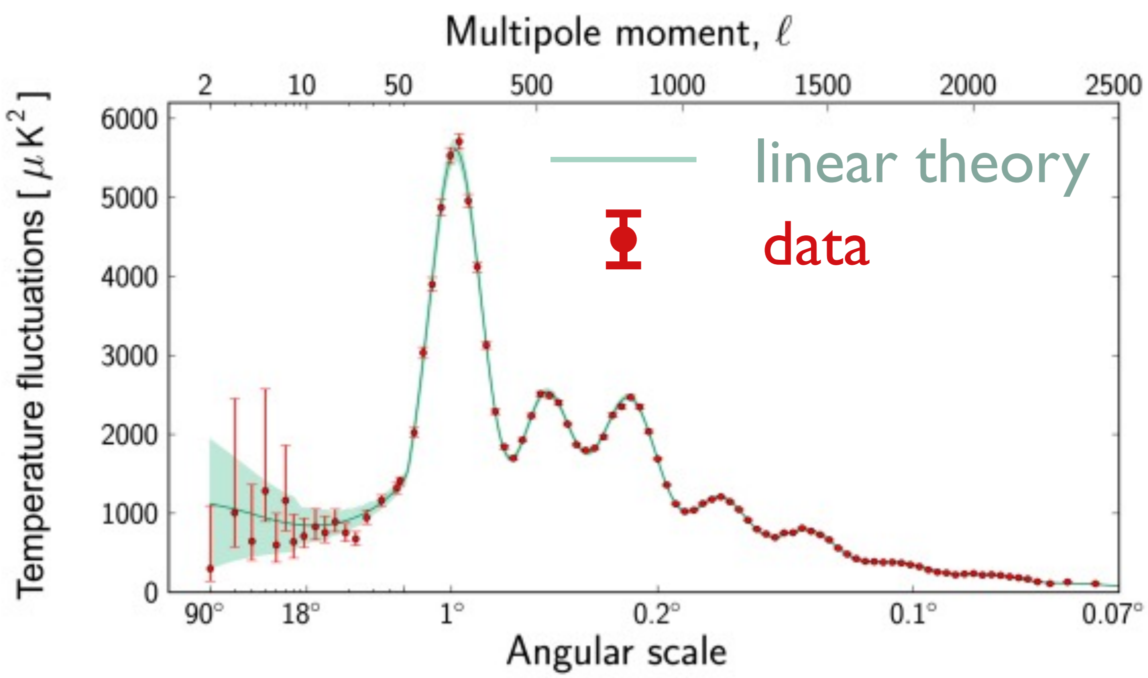


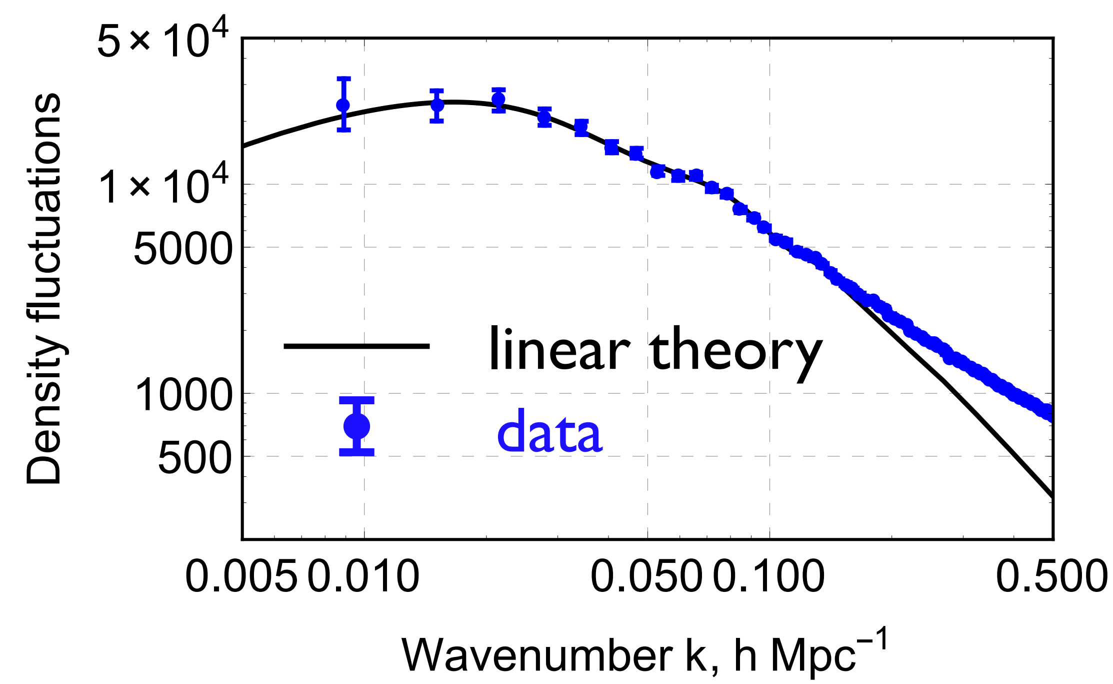
“No mode left behind” © Nima Arkani-Hamed

The big problem







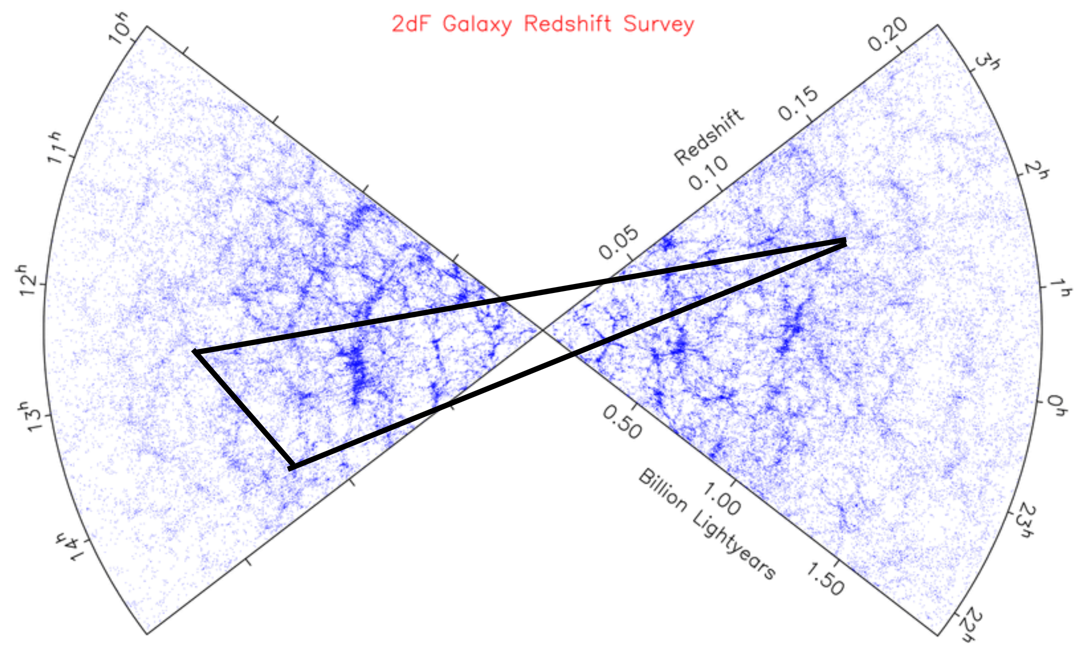


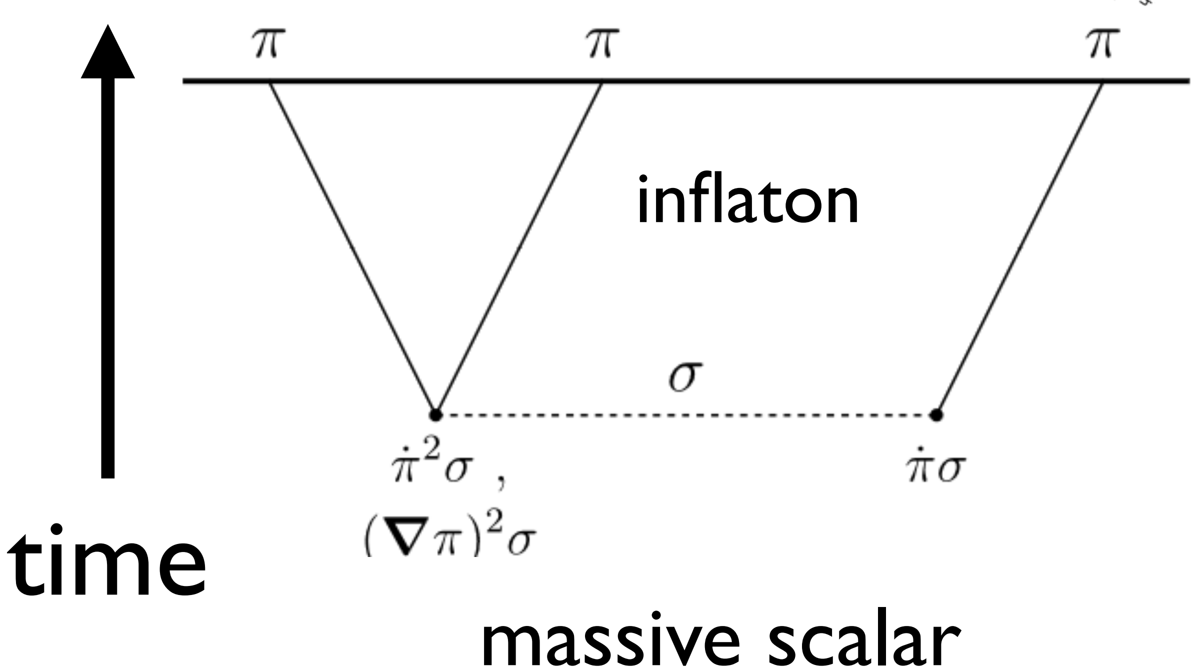
- non-linearity (non-Gaussianity) is important

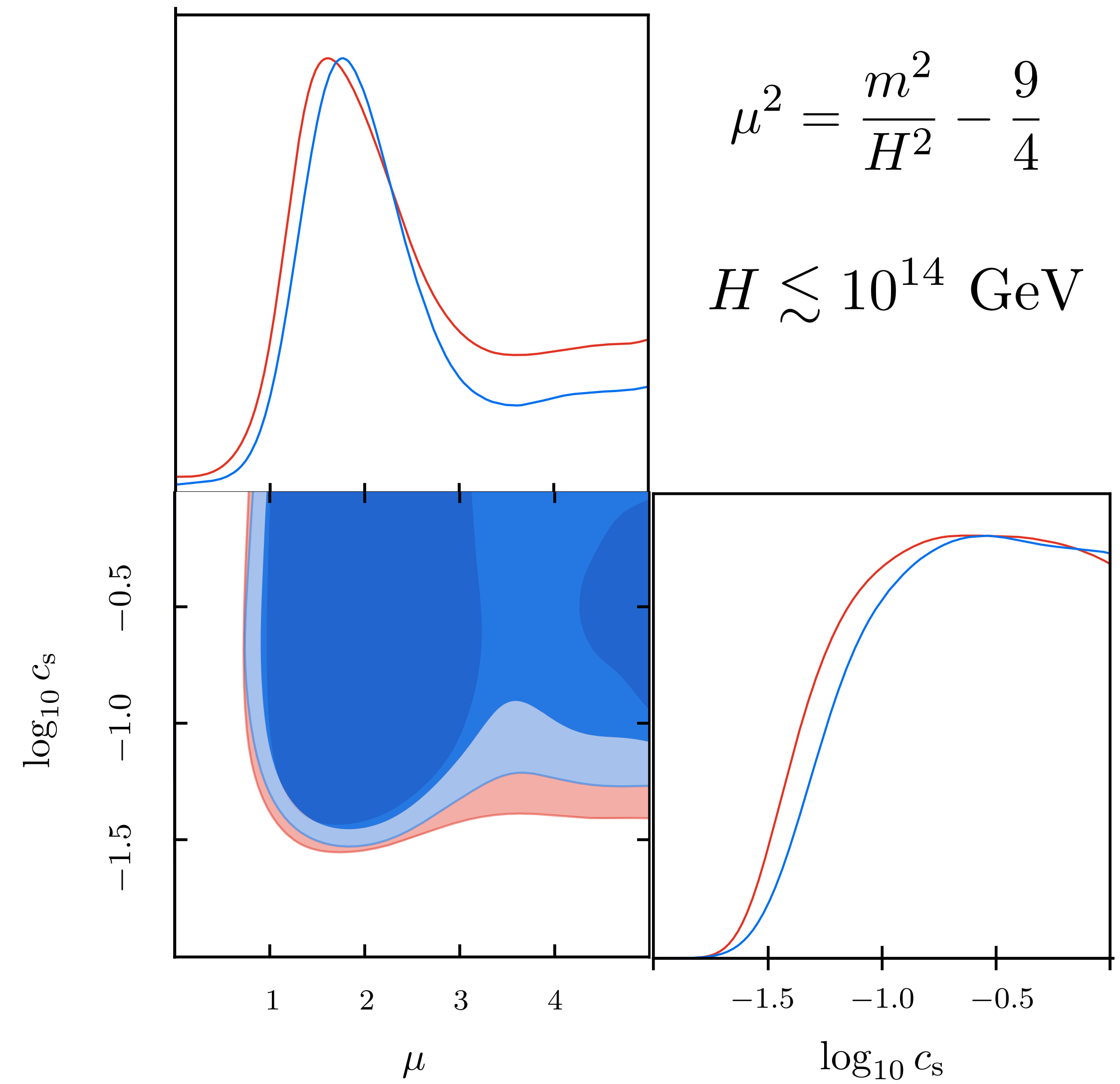
Example: Cosmological Collider in Action

- non-Gaussianity probes particle content, interactions, and the speed of propagation
- Example: decay of massive particles during inflation


Cabass, Philcox, MI ++ (2024)
Chan, Wang (2009)
Arkani-Hamed, Maldacena (2015)







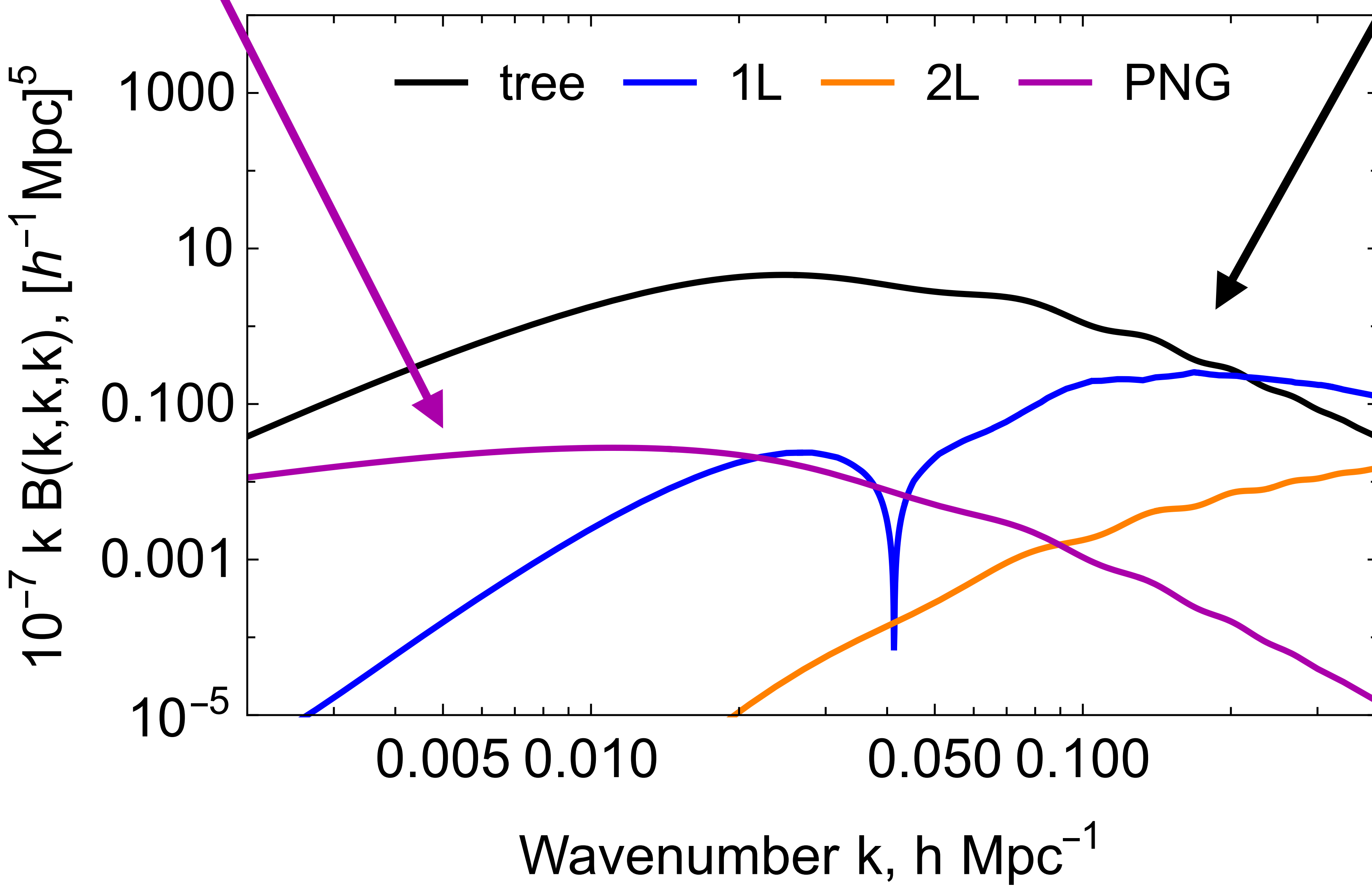
$$\mu^2 = \frac{m^2}{H^2} - \frac{9}{4}$$

$$H \lesssim 10^{14}\ \mathrm{GeV}$$

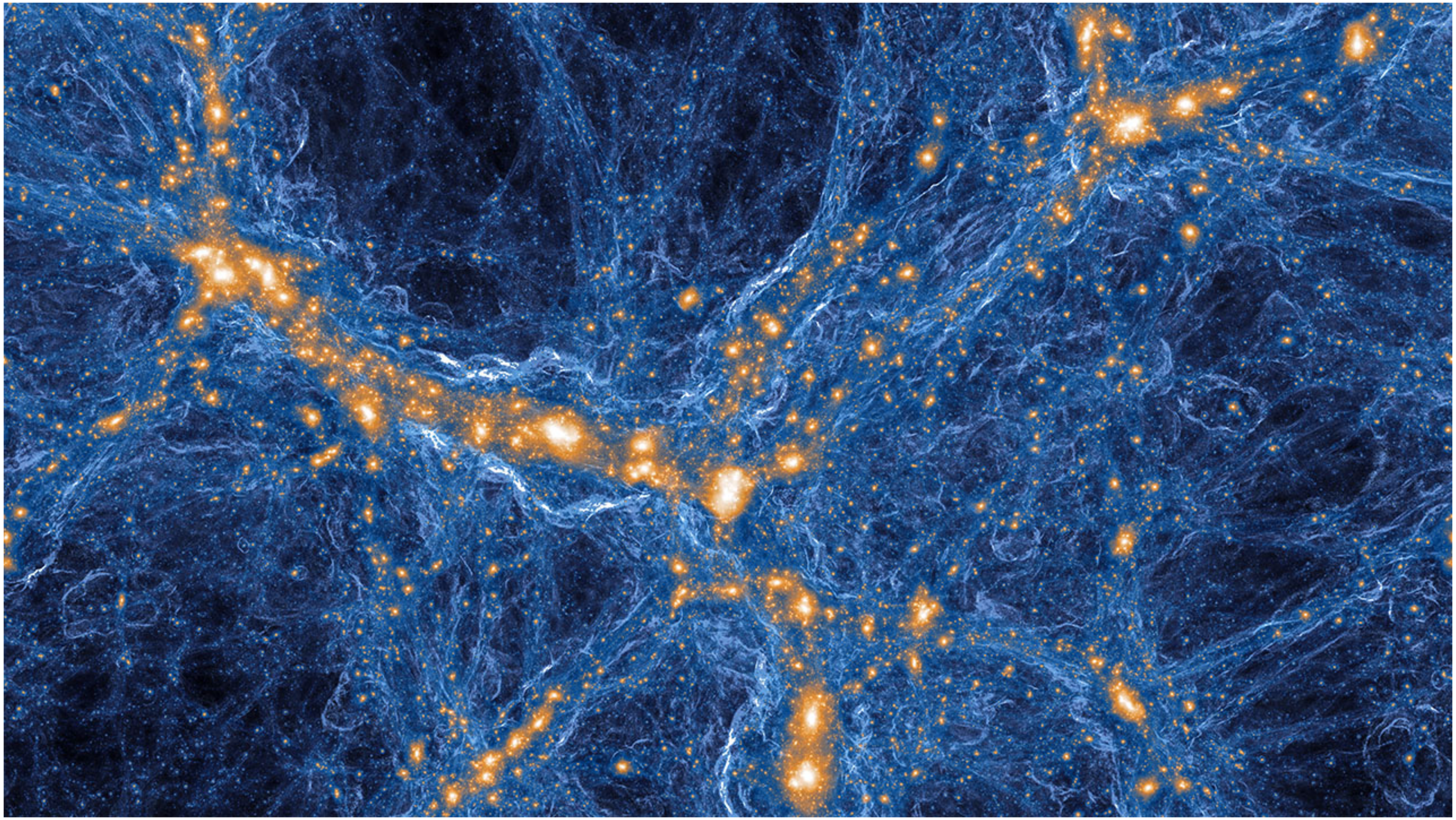
$$\delta = \frac{\rho}{\langle \rho \rangle} - 1 \qquad \langle \delta_g(\mathbf{k}_1)\delta_g(\mathbf{k}_2)\delta_g(\mathbf{k}_3)\rangle = B(k_1, k_2, k_3)\delta_D^{(3)}(\sum \mathbf{k}_i)$$

New physics
(signal)

galaxy formation loops
(LCDM background)



Simplest example: galaxy bias



IllustrisTNG

$$\delta = \frac{\rho}{\langle\rho\rangle} - 1$$

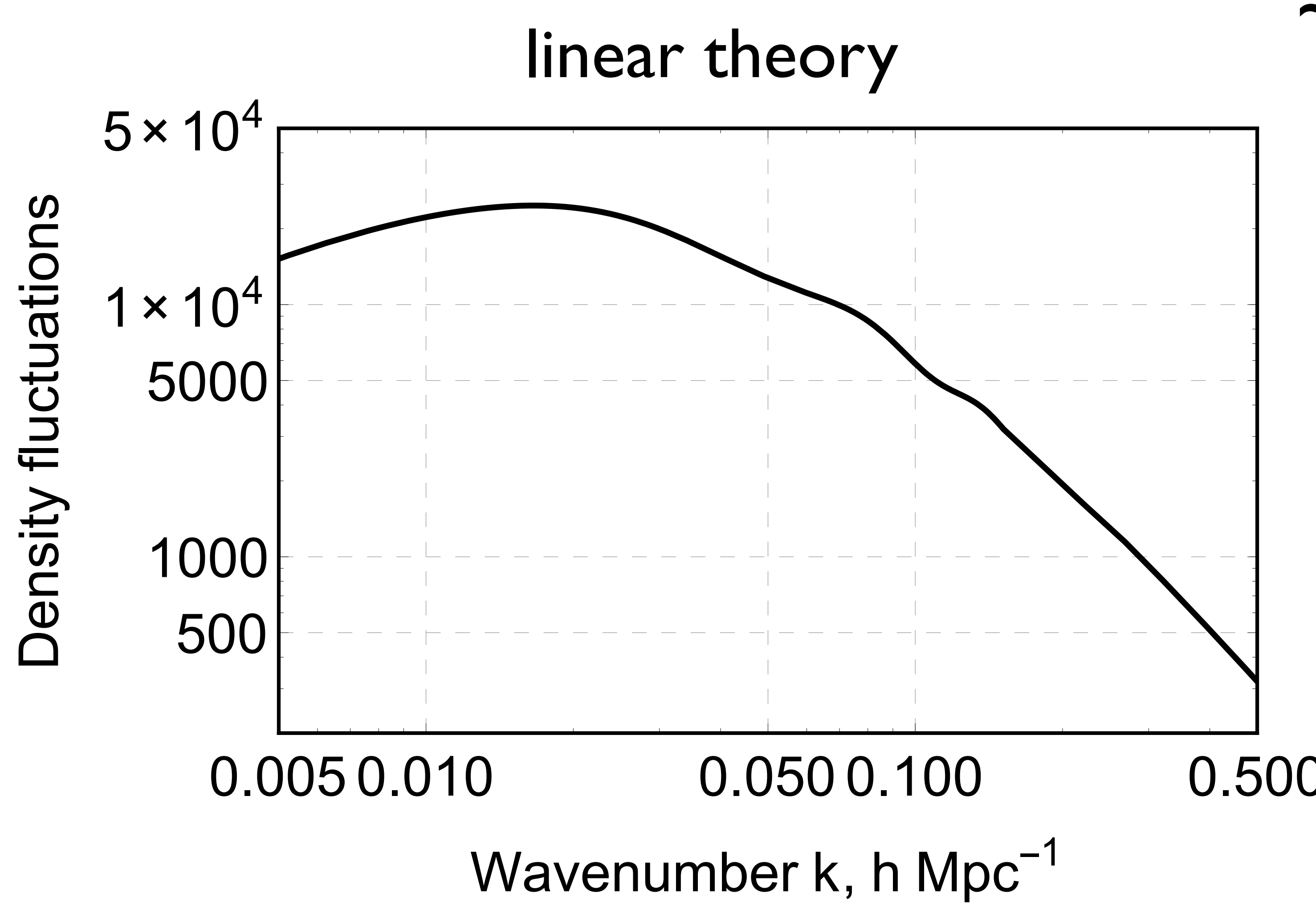
$$\delta_g = b_1\delta_m + b_2\delta_m^2 + \ldots$$

$$\delta_g = b_1 \delta_m + b_2 \delta_m^2 + \ldots$$

- **Statistics:** $\langle \delta_m(\mathbf{k}) \delta_m(\mathbf{k}') \rangle = \delta_D^{(3)}(\mathbf{k} + \mathbf{k}') P_{\rm lin}(k)$

Gaussian random field

~ propagator



- depends on radiation, baryons, DM, etc. from Boltzmann codes

$$\delta_g = b_1 \delta_m + b_2 \delta_m^2 + ...$$

Statistics: $\langle \delta_m(\mathbf{k}) \delta_m(\mathbf{k}') \rangle = \delta_D^{(3)}(\mathbf{k} + \mathbf{k}') P_{\rm lin}(k)$

Non - linear theory

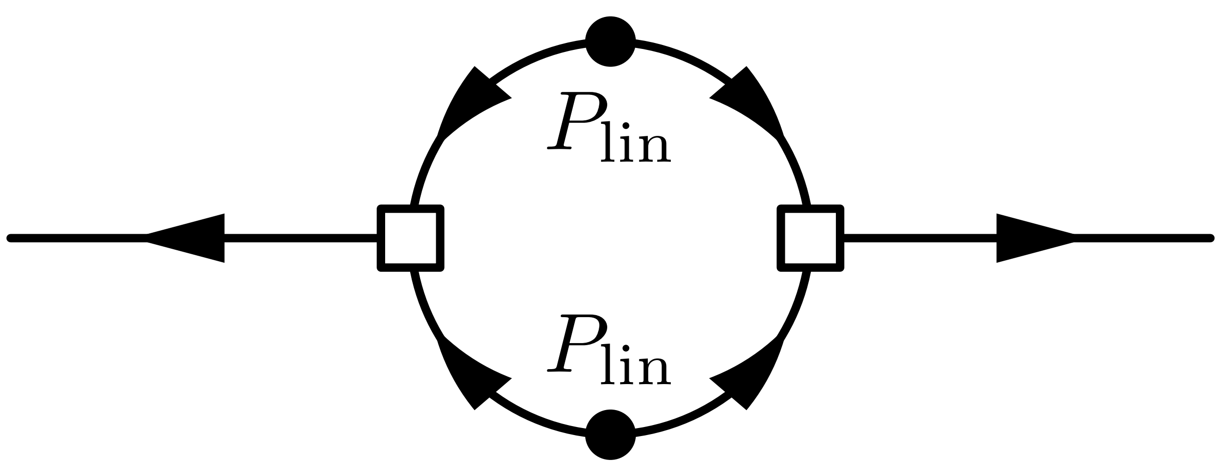
$$\langle \delta_g(\mathbf{k}) \delta_g(\mathbf{k}') \rangle = b_1^2 \langle \delta_m \delta_m \rangle + 2 b_1 b_2 \langle \delta_m \delta_m \delta_m \rangle + b_2^2 \langle \delta_m \delta_m \delta_m \delta_m \rangle$$

$\langle \delta_m \delta_m \delta_m \rangle \to 0$

$$\langle \delta_m \delta_m \delta_m \delta_m \rangle' = \int \frac{d^3 q}{(2\pi)^3} P_{\rm lin}(q) P_{\rm lin}(|\mathbf{k} - \mathbf{q}|)$$

Galaxy loops

$$\langle \delta_m \delta_m \delta_m \delta_m \rangle' = \int \frac{d^3q}{(2\pi)^3} P_{\rm lin}(q) P_{\rm lin}(|\mathbf{k} - \mathbf{q}|)$$



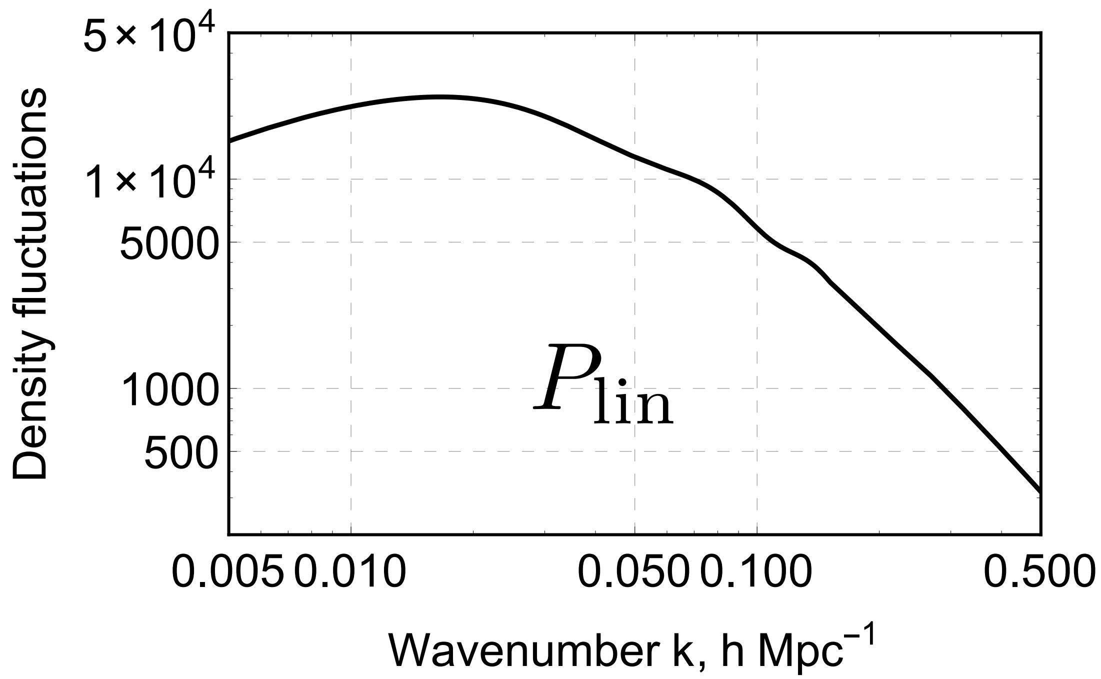
MI "EFT for Large Scale Structure" (2212.08488)

Baumann, Nicolis, Senatore, Zaldarriaga (2012)

Loop evaluation: FFTLog

Simonovic, Zaldarriaga+ (2017)

$$\langle \delta_m \delta_m \delta_m \delta_m \rangle' = \int \frac{d^3 q}{(2\pi)^3} P_{\rm lin}(q) P_{\rm lin}(|\mathbf{k} - \mathbf{q}|)$$



$$P_{\rm lin}(k) \approx \sum_m c_m k^{i\eta_m}$$
$$= \sum_m c_m e^{i\eta_m \ln k}$$

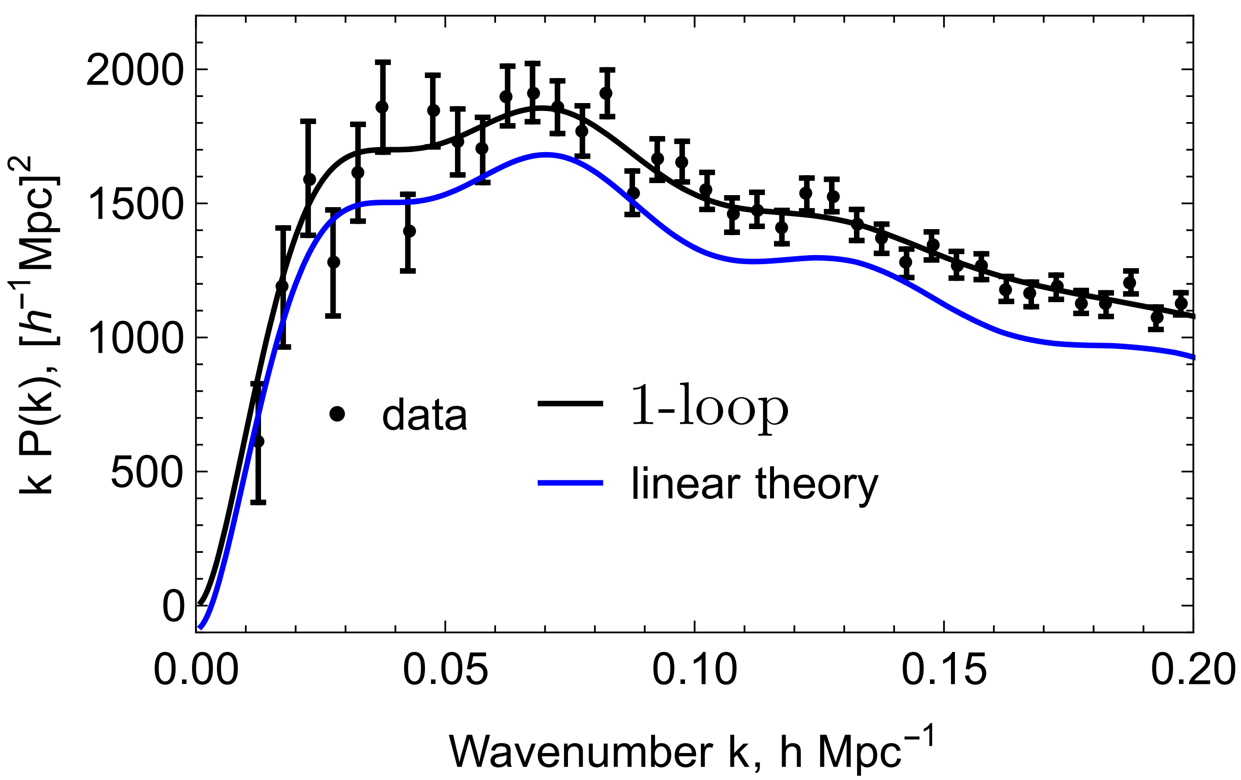
$$\int \frac{d^3 q}{(2\pi)^3} \frac{1}{q^{\nu_1} |\mathbf{k} - \mathbf{q}|^{\nu_2}} = \frac{k^{3-\nu_1-\nu_2}}{8\pi^{3/2}} \frac{\Gamma\left(\frac{3}{2} - \frac{\nu_1}{2}\right) \Gamma\left(\frac{3}{2} - \frac{\nu_2}{2}\right) \Gamma\left(\frac{\nu_1+\nu_2}{2} - \frac{3}{2}\right)}{\Gamma(\nu_1/2)\, \Gamma(\nu_2/2)\, \Gamma(3 - \nu_1/2 - \nu_2/2)}$$

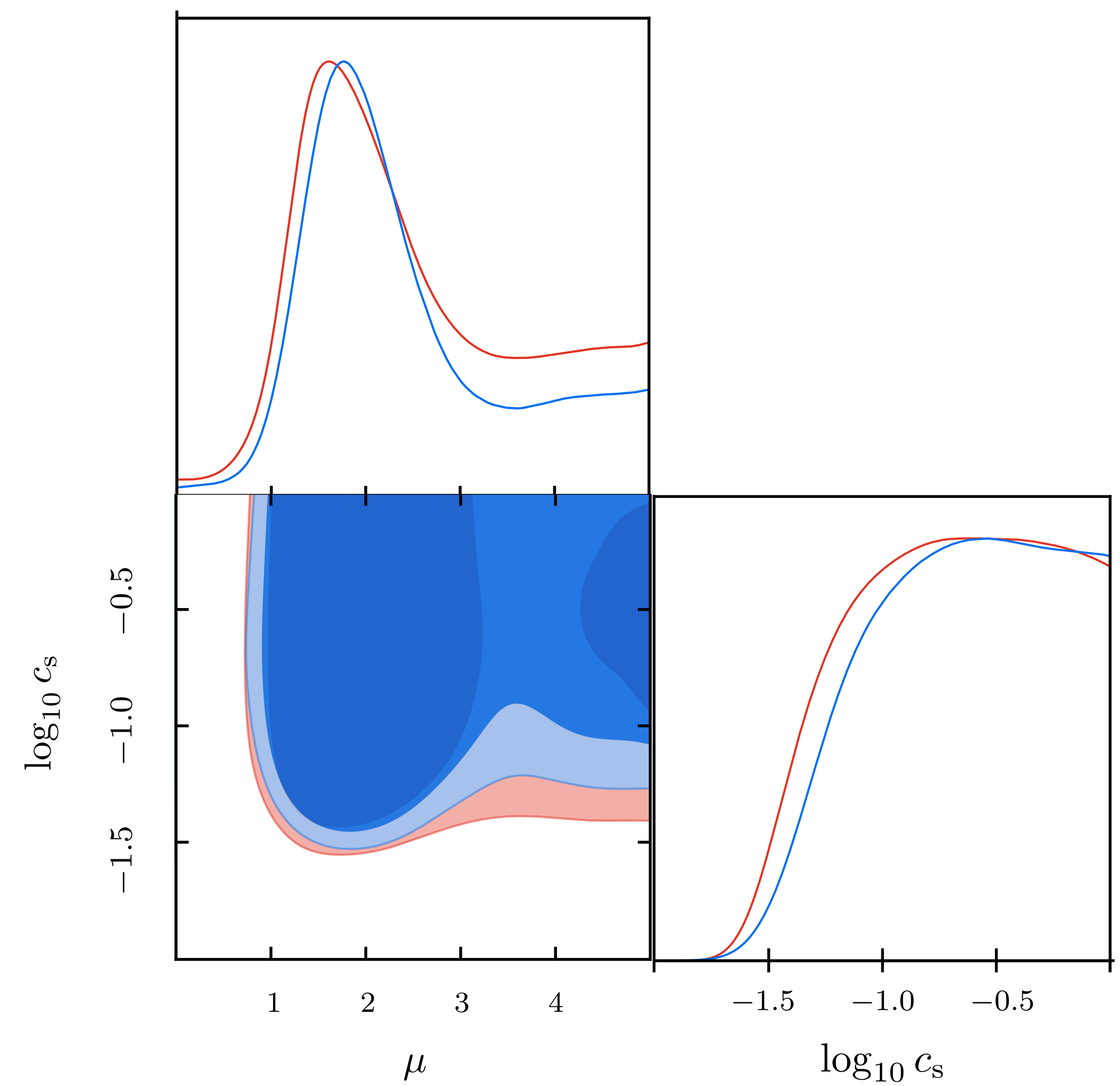
Smirnov (1991)

- 3D massless Euclidean QFT

$$\int \frac{d^3q}{(2\pi)^3} \frac{1}{q^{\nu_1}|\mathbf{k}-\mathbf{q}|^{\nu_2}} = \frac{k^{3-\nu_1-\nu_2}}{8\pi^{3/2}} \frac{\Gamma\left(\frac{3}{2}-\frac{\nu_1}{2}\right)\Gamma\left(\frac{3}{2}-\frac{\nu_2}{2}\right)\Gamma\left(\frac{\nu_1+\nu_2}{2}-\frac{3}{2}\right)}{\Gamma(\nu_1/2)\,\Gamma(\nu_2/2)\,\Gamma(3-\nu_1/2-\nu_2/2)}$$

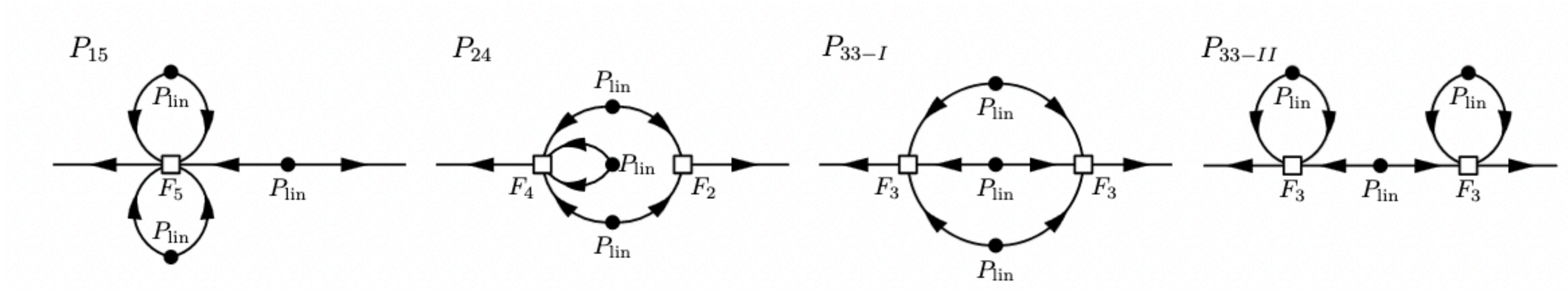
- CLASS-PT code, 150 loops in 1 sec. *Ivanov, Simonovic, Zaldarriaga (2019)*





Two Loops

Simonovic, Zaldarriaga+ (2017)



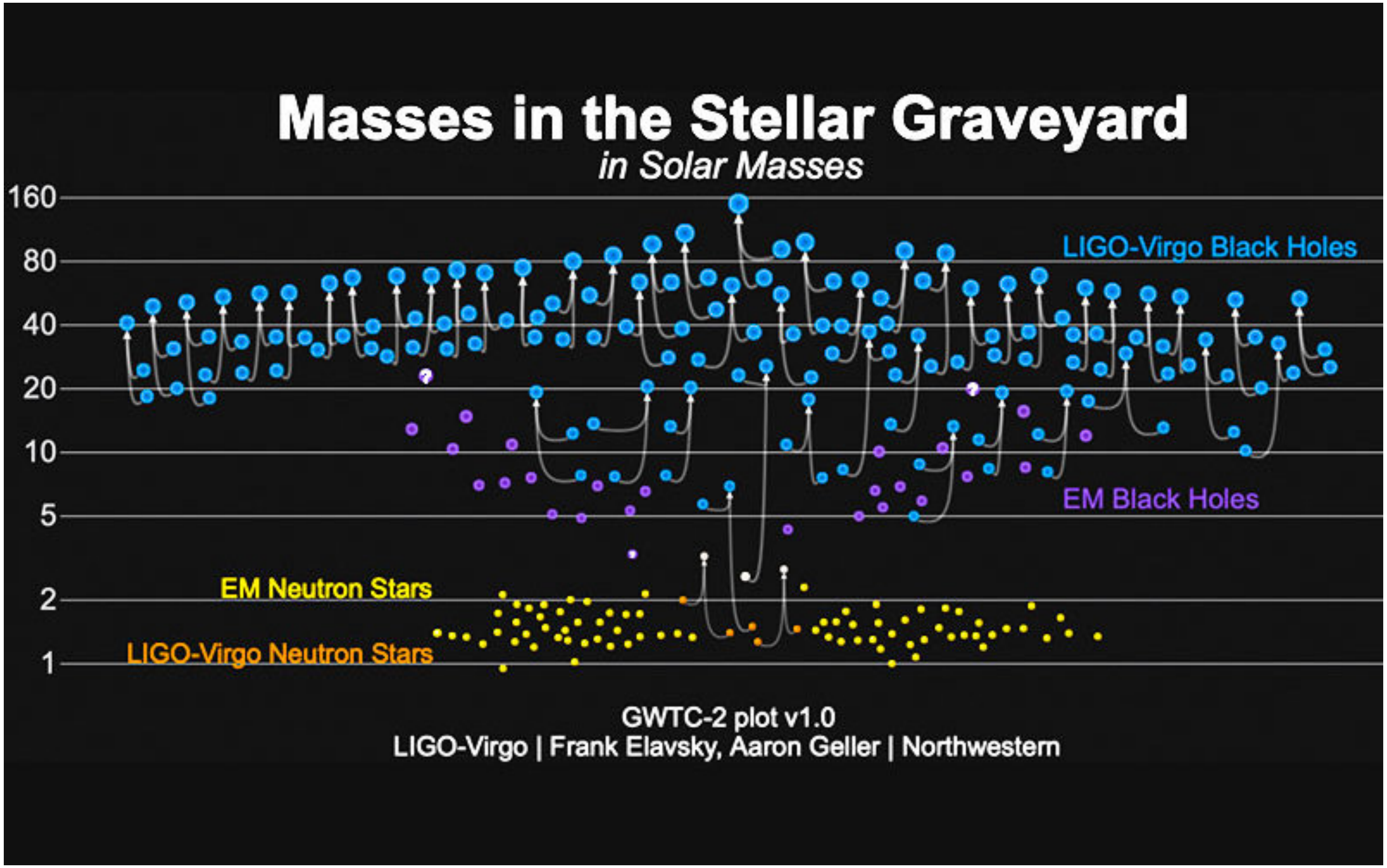
$$\int_{\boldsymbol{q}} \frac{1}{q^{2\nu_4}|\boldsymbol{k}-\boldsymbol{q}|^{2\nu_5}} \int_{\boldsymbol{p}} \frac{1}{p^{2\nu_1}|\boldsymbol{k}-\boldsymbol{p}|^{2\nu_2}|\boldsymbol{q}-\boldsymbol{p}|^{2\nu_3}} \equiv k^{6-2\nu_{12345}}\, \mathsf{K}(\nu_1,\ldots,\nu_5)$$

$$K(\nu_1,\nu_2,\nu_3,\nu_4,\nu_5) = \frac{\sec(\pi\nu_{23})}{8\sqrt{\pi}\Gamma(\nu_1)\Gamma(\nu_2)\Gamma(\nu_3)\Gamma(3-\nu_{123})}$$
$$\times\sum_{n=0}^{\infty}\Big[a_n(\nu_1,\nu_2,\nu_3)\,\kappa\left(\tfrac{3}{2}-\nu_{235}+n,-\nu_4,\nu_1+n,\tfrac{3}{2}-\nu_2+n,3-\nu_{23}+2n\right)$$
$$-a_n(\tilde\nu_1,\tilde\nu_2,\tilde\nu_3)\,\kappa\left(-\nu_5+n,\tfrac{3}{2}-\nu_{134},\tilde\nu_1+n,\tfrac{3}{2}-\tilde\nu_2+n,3-\tilde\nu_{23}+2n\right)\Big]$$

$$\kappa(\alpha,\beta,a,b,c) = \frac{1}{1+\alpha}\left[\frac{1}{1+\beta}\,{}_3F_2\left(\begin{matrix}1,a,b;\\ 2+\beta,c;\end{matrix}\,1\right) - \quad \ldots\right]$$

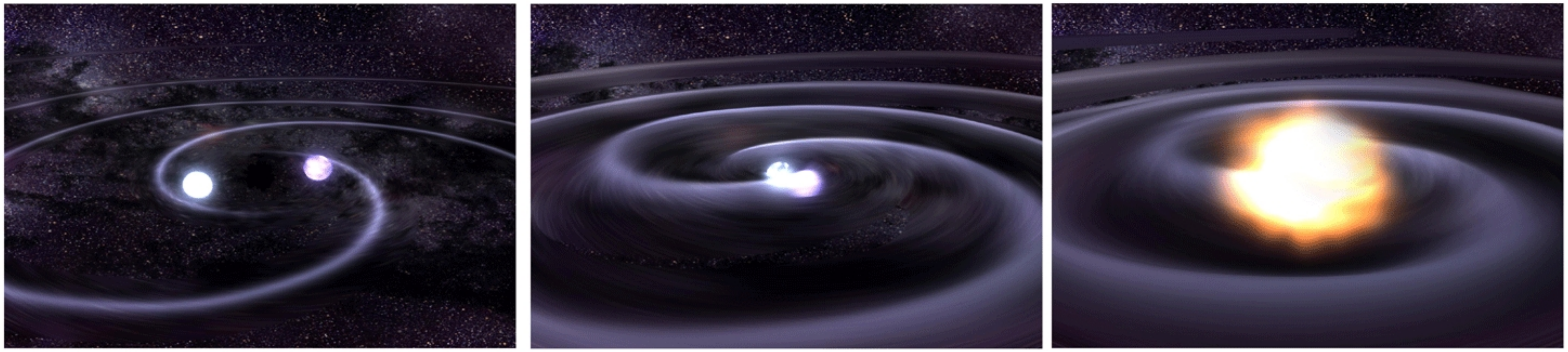
- The FFTLog formula converges too slow
- New ideas from you are welcome!

The dawn of precision GW science



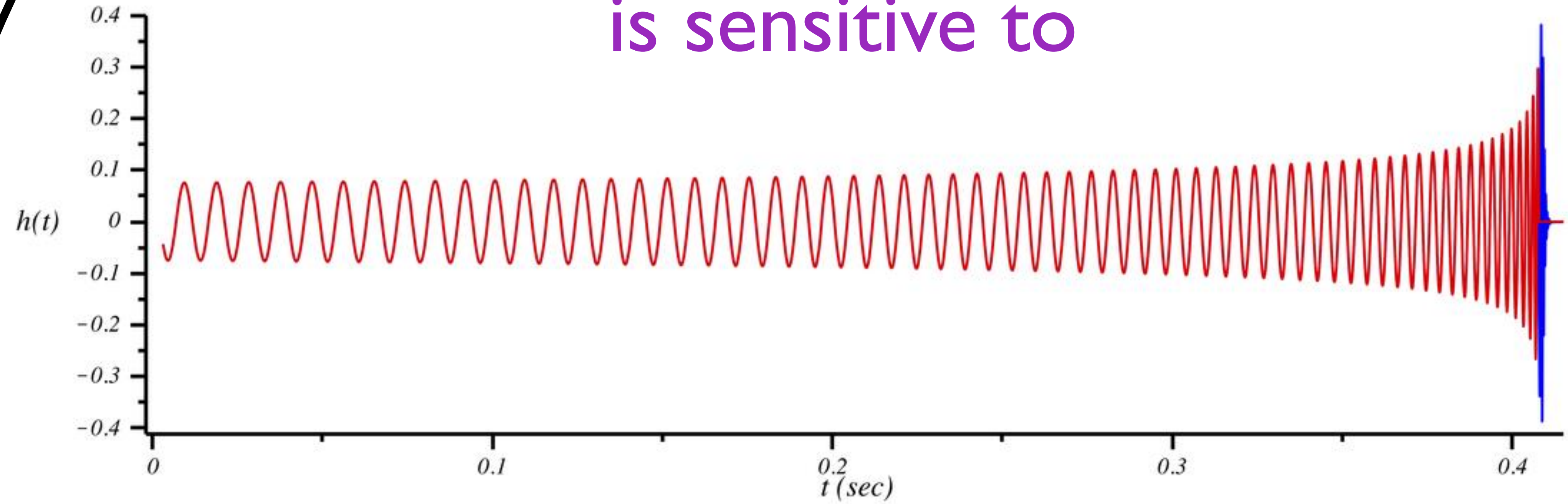
also see Zvi Bern's, Alessandra Buonanno's +++ talks

A typical binary merger



© NASA

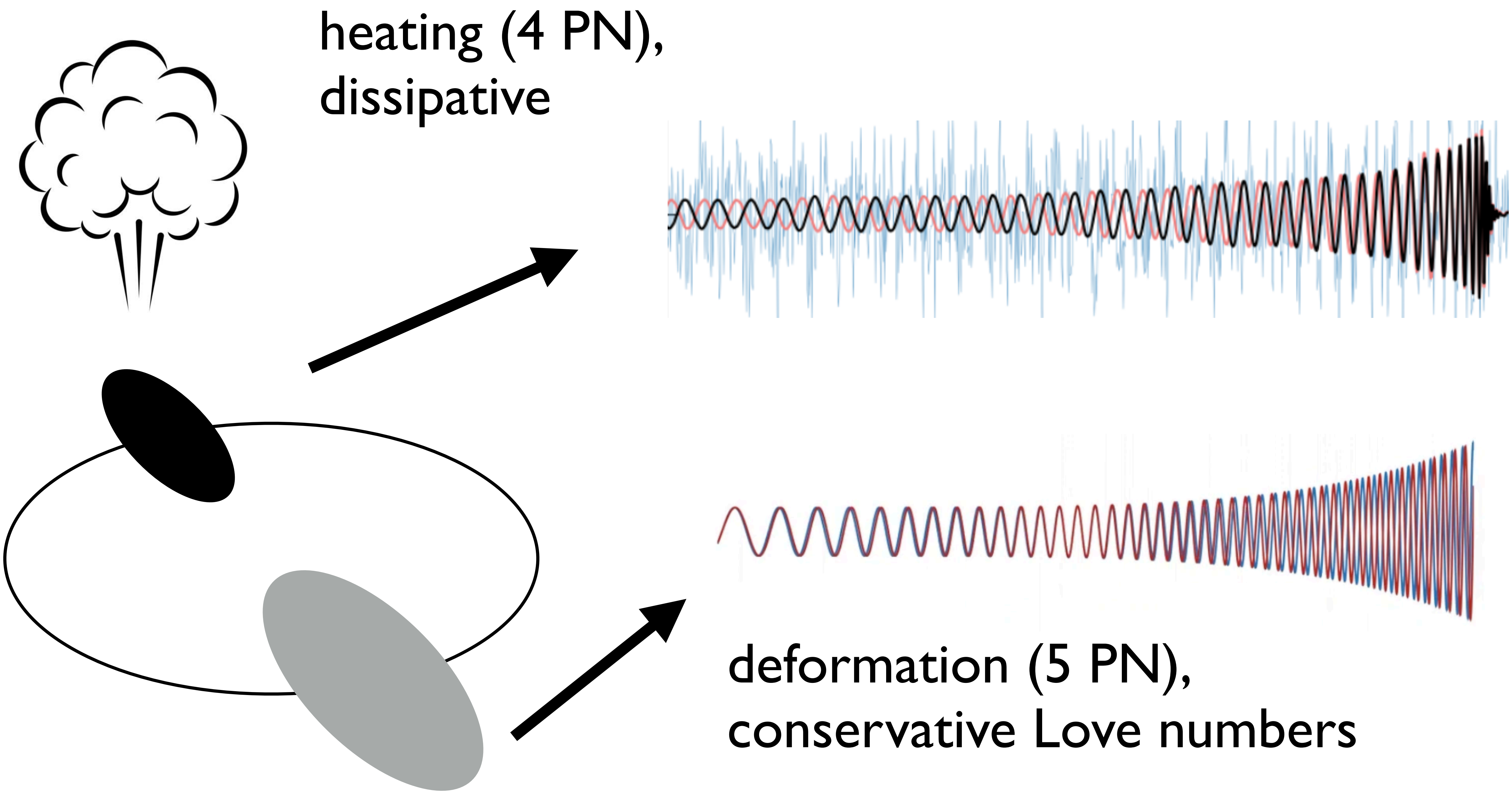
is sensitive to



Quasinormal modes (QNMs)

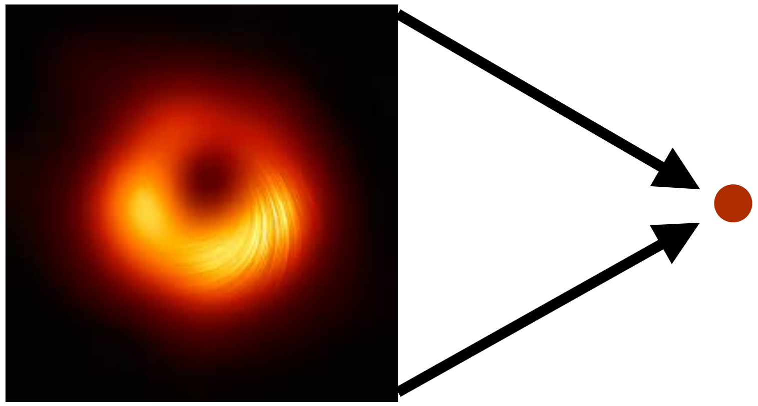
tidal deformation of the sources

Tides probe the nature of compact objects



- One can test EoS for neutron stars *Flanagan, Hinderer (2007) ++*
- Use LVK data to test exotic compact objects *Chia, Zhou, MI (2404.14641)*

Love numbers in Worldline EFT



at $r \gg r_s$ a black hole is described by the word line effective action

$$S_{\text{eff}} = -m \int ds \; + \lambda_2 R^5 \int ds \; E_{\mu\nu} E^{\mu\nu} + \int ds \; E_{\mu\nu} Q^{\mu\nu}(X)$$

$$E_{ij} = \partial_i \partial_j \Phi \quad \leftarrow \quad u^\mu u^\lambda R_{\mu\nu\lambda\rho}$$

- Local operators capture conservative dynamics
- Wilson coefficients = Love numbers
- 2pf of Q encodes dissipative dynamics

Goldberger, Rothstein+ (2004,2005,2020)

Matching of Tides in Worldline EFT

$$S_{\text{eff}} = -m \int ds \ + \lambda_2 R^5 \int ds \ E_{\mu\nu} E^{\mu\nu} + \int ds \ E_{\mu\nu} Q^{\mu\nu}(X)$$

- Wilson coeffs extracted from GR matching calculations, use for new predictions
- Normally done with off-shell quantities (~Newton potential)
- Works well* @LO (static LNs), but matching conditions are trivial

Damour, Poisson,+'09+

@NLO (dynamical LNs) some confusion: gauge, coordinate dependence

$$\int ds \dot{E}_{\mu\nu} \dot{E}^{\mu\nu}$$

e.g. Poisson (2021a,b) vs *Chakrabarti +'13, Charalambous, Dubovsky, MI'21*
Saketh, Zhou, MI'21

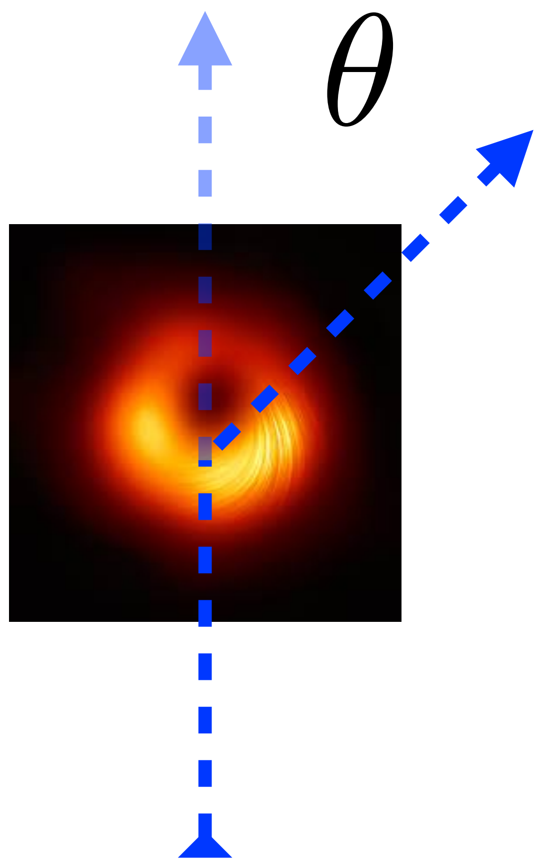
Alternative: on-shell scattering amplitudes

Free of gauge, coordinate, and field-redefinition

MI, Li, Parra-Martinez, Zhou'24

Gravitational Raman Scattering

MI, Li, Parra-Martinez, Zhou'24



$\phi, A^\mu, h_{\mu\nu}^{TT}$

frequency ω

$$S_{\text{eff}} = -m\int ds + \int ds[C_1 R^3(\partial_i\phi)^2 + C_{0,\omega^2}R^3\dot{\phi}^2 + C_{1,\omega^2}R^5(\partial_i\dot{\phi})^2] + \int ds\ [\phi Q(X) + \partial_i\phi Q^i(X) + \ldots]$$

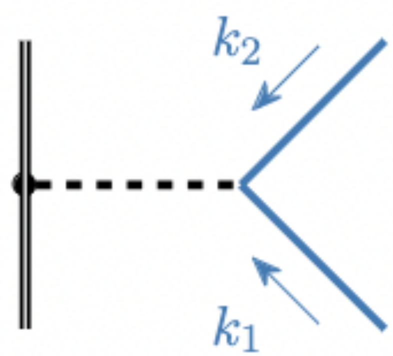
- Partial wave basis:

$$i\mathcal{M}(\omega,\theta) = \frac{2\pi}{\omega}\sum_{\ell=0}^{\infty}(2\ell+1)(\eta_\ell e^{2i\delta_\ell} - 1)P_\ell(\cos\theta)$$

- (Real part of) phase encode conservative tides (Love numbers)
- Inelasticity encodes tidal heating

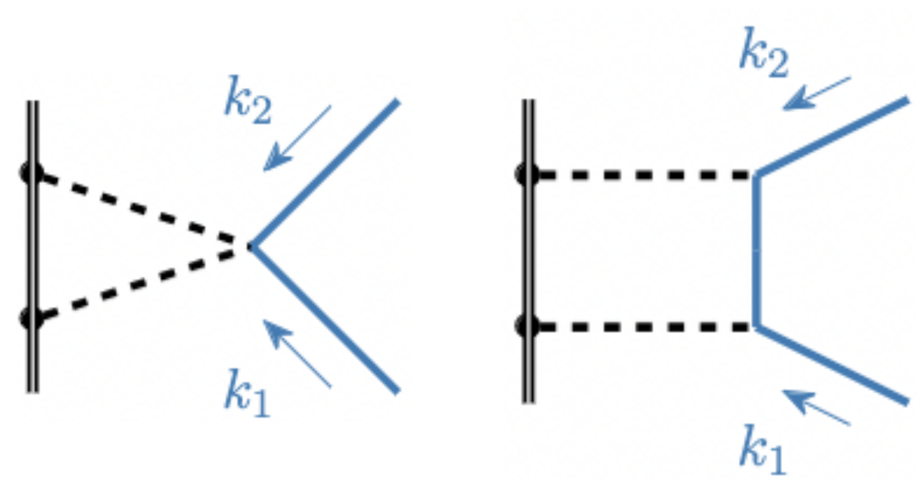
Non-linearity of Gravity

$$i\mathcal{M}(\omega) \sim \underbrace{(mG)}_{\text{“Rutherford”}} \underbrace{(1 + mG\omega + (mG\omega)^2 + \ldots)}_{\text{relativ.corrections}} + \underbrace{(mG)^5\omega^4}_{\text{tides}}$$

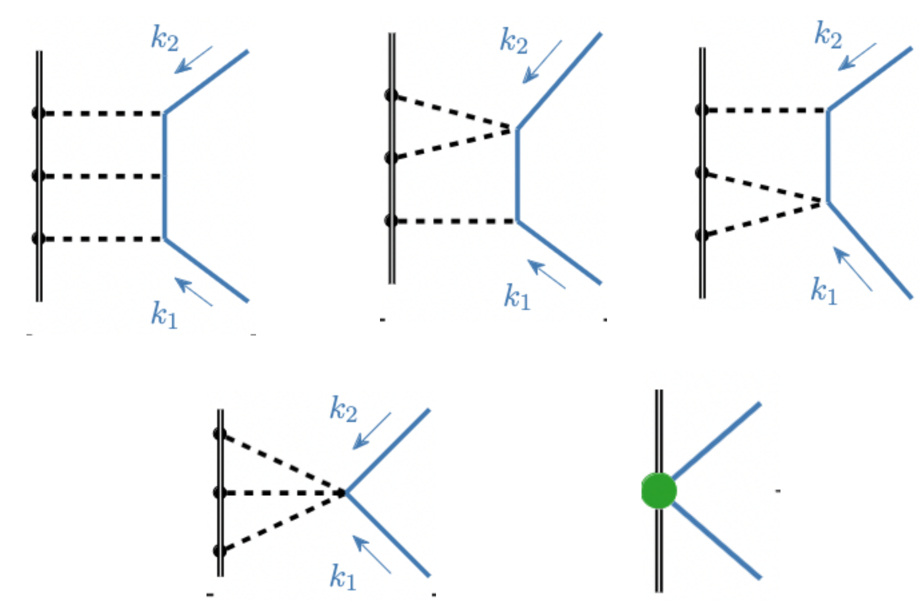
1PM: $i\mathcal{M}_{\text{tree}} =$ 

$$\left.\frac{d\sigma}{d\Omega}\right|_{1\text{PM}} = \frac{G^2 m^2}{\sin^2\left(\frac{\theta}{2}\right)} \cos^4\left(\frac{\theta}{2}\right)$$

2PM:



3PM:



- 3PM calculated for the first time thanks to IBP+diff. equations
- Background field method integrands *Cheung, Parra-Martinez, Rothstein+'23*

Results:

- 3PM EFT: $$\delta_\ell\Big|_{\rm EFT} = -\frac{\lambda}{2\epsilon_{\rm IR}} + \frac{\lambda}{2}\ln\left(\frac{4\omega^2}{\bar\mu_{\rm IR}^2}\right) + \sum_{n=1}^{3}\nu_n^\ell\lambda^n + \delta_\ell^{G^3}\,,$$

$$1-\eta_\ell\Big|_{\rm EFT} = \frac{\ell!\omega^{2\ell+1}{\rm Im}F_\ell(\omega)}{2\pi(2\ell+1)!!}\left(1+\pi\lambda+\lambda^2\eta_\ell^{G^2}\right)\,,\qquad \lambda\equiv 2Gm\omega\ll 1$$

$$\delta_0^{G^3}\Big|_{\rm EFT} = \lambda^3\left[\frac{1}{4\epsilon_{\rm UV}} + \frac{49}{24} - \frac{1}{2}\ln\left(\frac{4\omega^2}{\bar\mu^2}\right)\right] + \frac{C_{0,\omega^2}\omega^3}{4\pi}\,,$$

$$\delta_1^{G^3}\Big|_{\rm EFT} = \frac{C_1\omega^3}{12\pi}\left(1+\pi\lambda+\lambda^2\eta_1^{G^2}\right) + \frac{C_{1,\omega^2}\omega^5}{12\pi}\,,$$

$$\eta_0^{G^2}\Big|_{\rm EFT} = \frac{67}{12} - \frac{11}{6}\left(-\frac{1}{2\epsilon_{\rm UV}} + \ln\left(\frac{4\omega^2}{\bar\mu^2}\right)\right) + \frac{\pi^2}{3}$$

$$\eta_1^{G^2}\Big|_{\rm EFT} = \frac{413}{100} - \frac{19}{30}\left(-\frac{1}{2\epsilon_{\rm UV}} + \ln\left(\frac{4\omega^2}{\bar\mu^2}\right)\right) + \frac{\pi^2}{3}$$

- Comparison with known results for black holes in GR:

$$\delta_\ell\Big|_{\rm GR} = (r_s\omega)\ln(2\omega r_s) + \sum_{n=1}^{3}\nu_n^\ell(r_s\omega)^n + \delta_\ell^{G^3}$$

$$\delta_0^{G^3}\Big|_{\rm GR} = (r_s\omega)^3\left[\frac{7}{12} - \gamma_E - \ln(2r_s\omega)\right]$$

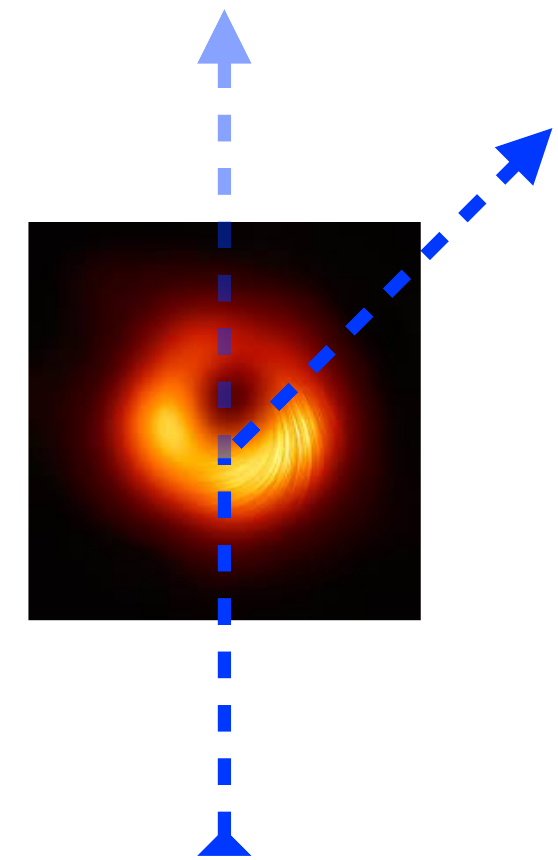
Static Love numbers are zero and do not run: $C_1 = 0$

Dynamical Love numbers non zero and do run:

$$C_{0,\omega^2}(\bar\mu)^{\overline{\rm MS}} = -4\pi r_s^3\left[\frac{1}{4\epsilon_{\rm UV}} + \ln(\bar\mu r_s) + \frac{35}{24} + \gamma_E\right]$$

Scalar Raman Scattering: Upshot

- Perfect match: consistency of the EFT and the renormalization program!
- IR/ UV logs neatly separated in EFT
- Photon/graviton results underway!
- Running part can be matched thanks to the factorization of GR amplitudes



Chakrabarti ++'13, Charalambous, Dubovsky, MI'21
Zhou MI'22, Saketh, Zhou, MI'23

- Dynamical Love numbers of BHs are non-zero & run, ~ natural from the EFT perspective

$$\int ds \dot{E}_{\mu\nu} \dot{E}^{\mu\nu}$$

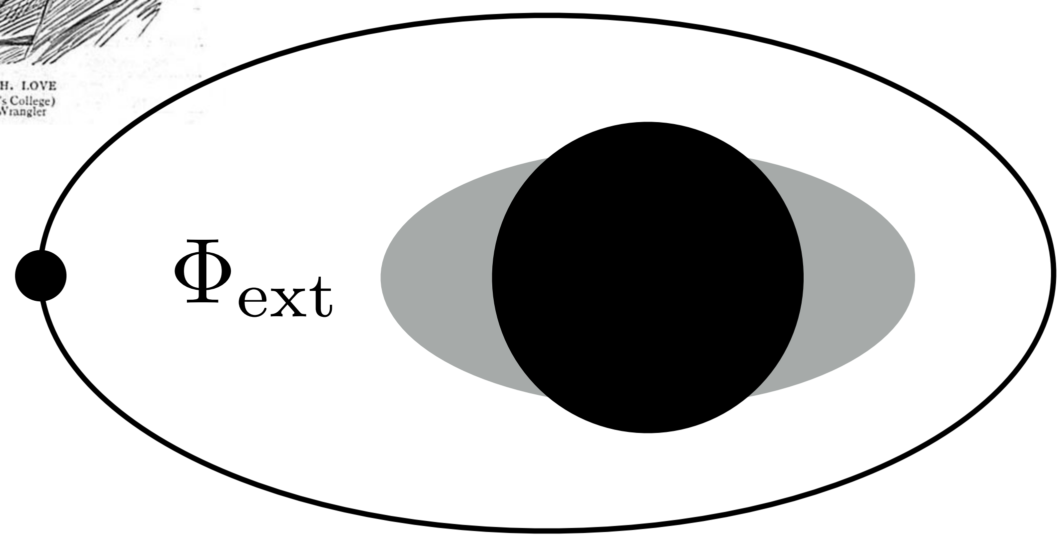
Universal running: same for BH & NS

Summary

- Large scale structure - a powerful probe of physics
- Our Universe is the largest collider
- Loops thanks to QFT, but need more :)
- Gravitational waves - probe nature of compact objects
- Tides understood thanks to scattering amplitudes

Thank you!

Love numbers in Newtonian gravity



$$\Phi = \Phi_0 + \Phi_2 + \Phi_3 + \ldots$$

$$\Phi_0 = -\frac{M}{r}$$

$$\Phi_2 = r^2 E_{ij} n^i n^j + \frac{Q_{ij} n^i n^j}{r^3}$$

tidal source   tidal response

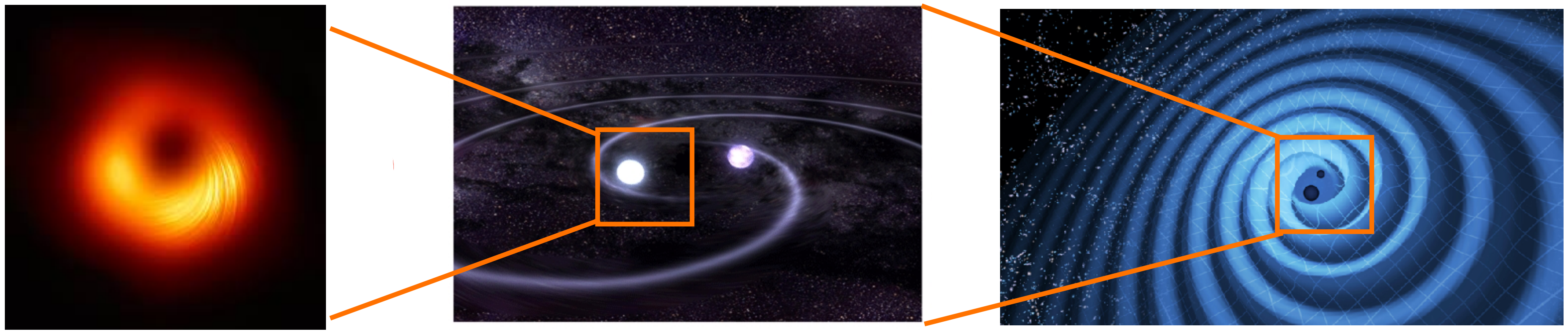
$$\Phi_\ell = r^\ell E_{i_1 \ldots i_\ell} n^{i_1} \ldots n^{i_\ell} + Q_{i_1 \ldots i_\ell} \frac{n^{i_1} \ldots n^{i_\ell}}{r^{\ell+1}}$$

Linear response theory: $Q_\ell = \Lambda_\ell r_s^{2\ell+1} E_\ell$

Love number

$(G = c = \hbar = 1)$

Worldline EFT

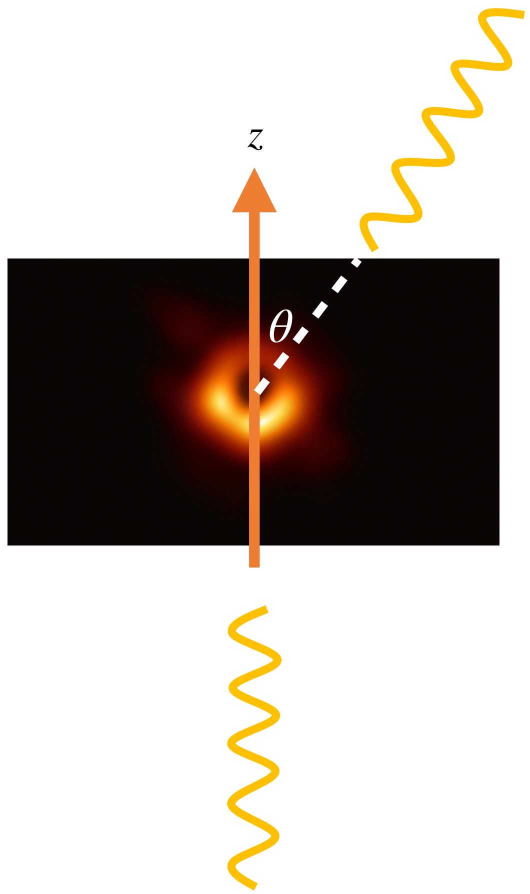


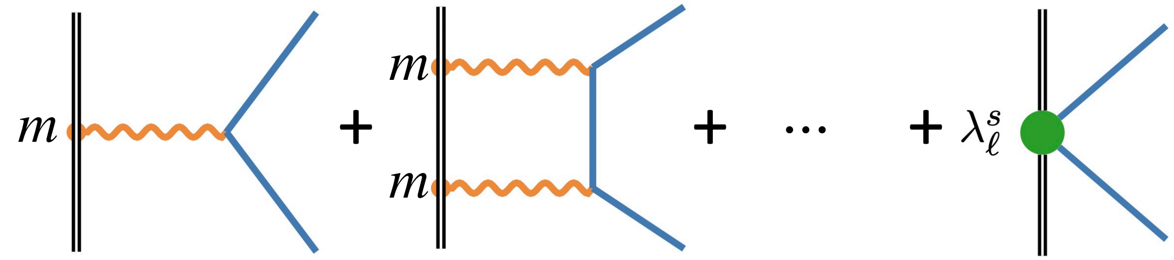
EFT framework: disentangle physics from different scales

at $r \gg r_s$ a black hole is described by the word line effective action

$$S = -m \int ds + \ldots$$

Matching LN via Raman scattering amplitudes





PM expansion: $G + G^2 + \dots$

$$\lambda_\ell^s \sim R^{2\ell+1} \propto G^{\frac{2\ell}{D-3}+1}$$

Analytic continuation in angular momentum (Gribov-Froissart) separates LN and PM

$$i\mathcal{A} \sim (Gm) + (Gm)^2\omega + \dots + (Gm)^n\omega^{n-1} + (Gm)^{2\ell+1}\omega^{2\ell} + \dots$$

N.B. the limit $\ell \to n \in \mathbb{N}$ leads to singularities if LNs run (there's an actual mixing between LN and PM corrections)

Zhou, MI'22

MI, Li, Parra-Martinez, Zhou'24

Dynamical Love numbers

- Problem: RG running of LNs (PN loops + local counterterms)
- EFT gives a consistent definition via matching (~coupling running)
- Dynamical LNs of BHs are not zero and run *Chakrabarti ++'13, MI ++'21*

$$C_{\dot{E}^2} \int d\tau \; \dot{E}^{ij} \dot{E}_{ij} \qquad\qquad C_{E\dot{E}} \int d\tau \; \varepsilon^{ijk} \hat{S}_i E_{jl} \dot{E}^l_k$$

- Running can be matched analytically with FG, e.g.

$$\frac{dC_{\dot{E}^2}}{d\log\mu} = \frac{32}{45} m^7 G^6 \qquad\qquad \frac{dC_{E\dot{E}}}{d\log\mu} = -\frac{32}{45}\frac{a}{m} m^6 G^5$$

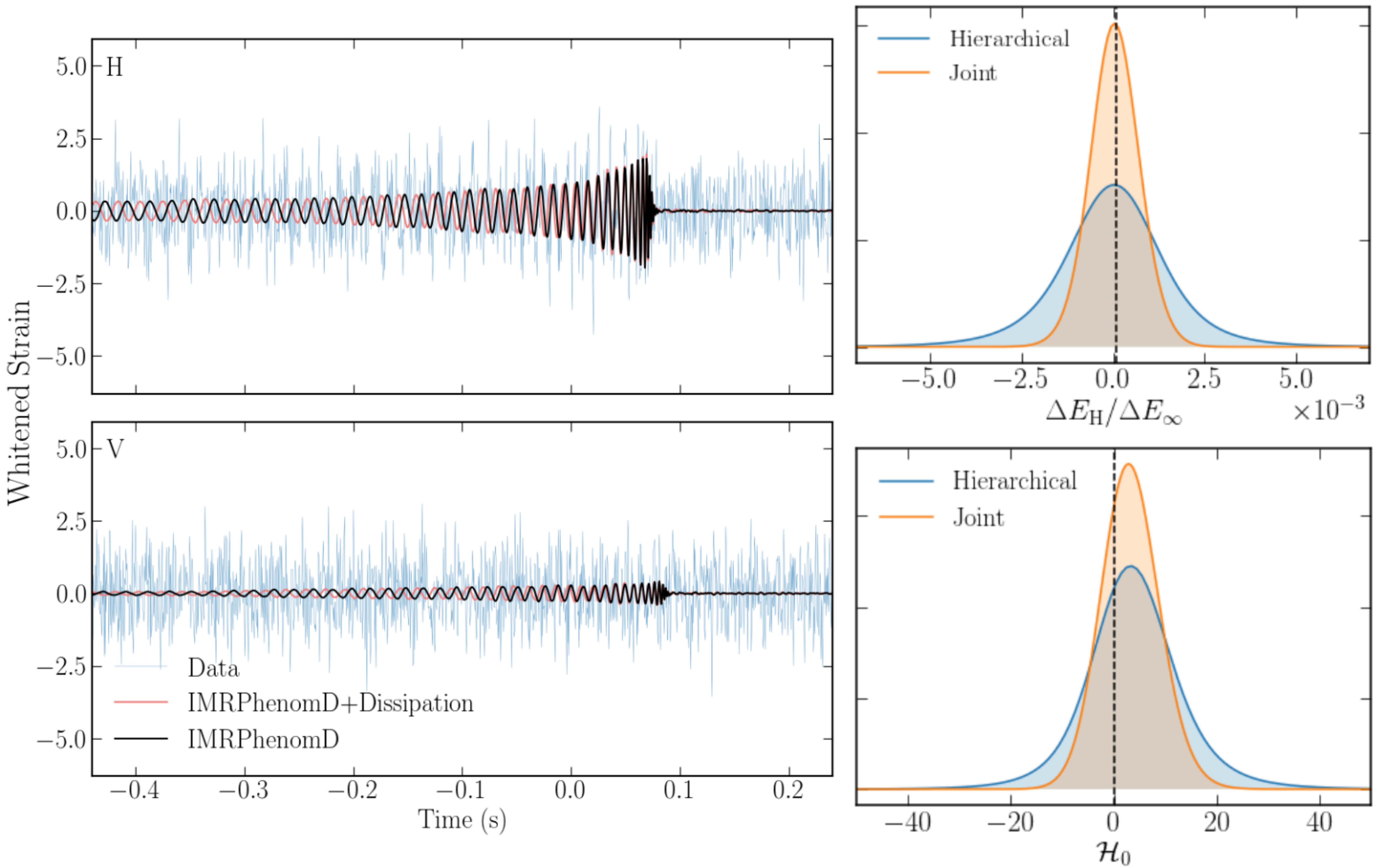
Saketh, Zhou, MI' 23

- Or extracted fully from Raman process amplitudes:

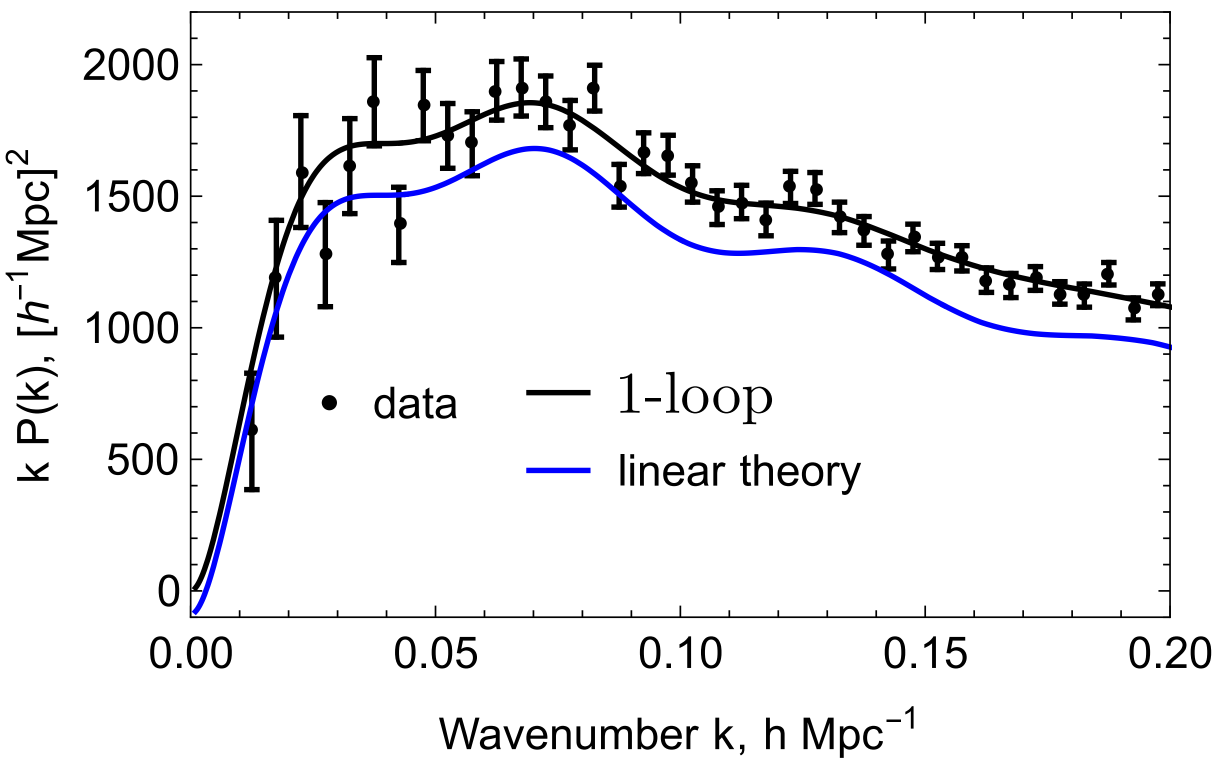
$$C_{0,\omega^2}(\bar{\mu})^{\overline{\rm MS}} = -4\pi r_s^3 \left[\frac{1}{4\epsilon_{\rm UV}} + \ln(\bar{\mu} r_s) + \frac{35}{24} + \gamma_E\right] \qquad \sim \int \dot{\phi}^2$$

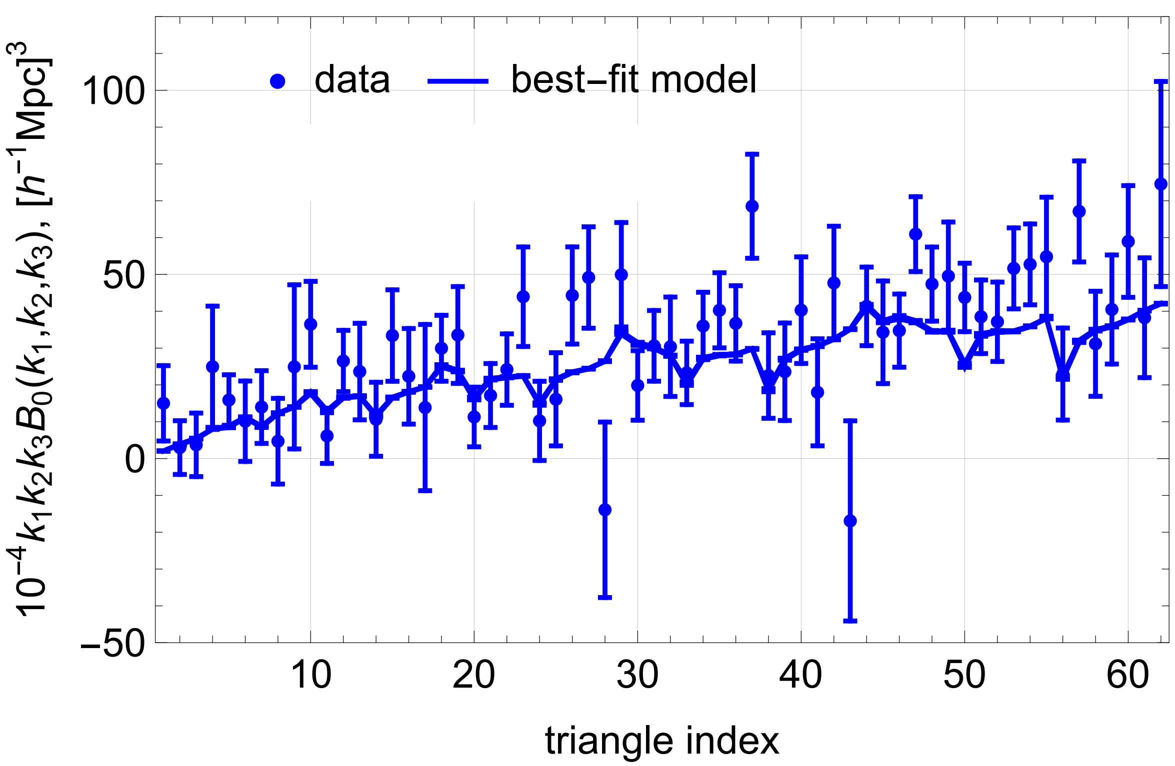
MI, Li, Parra-Martinez, Zhou'24

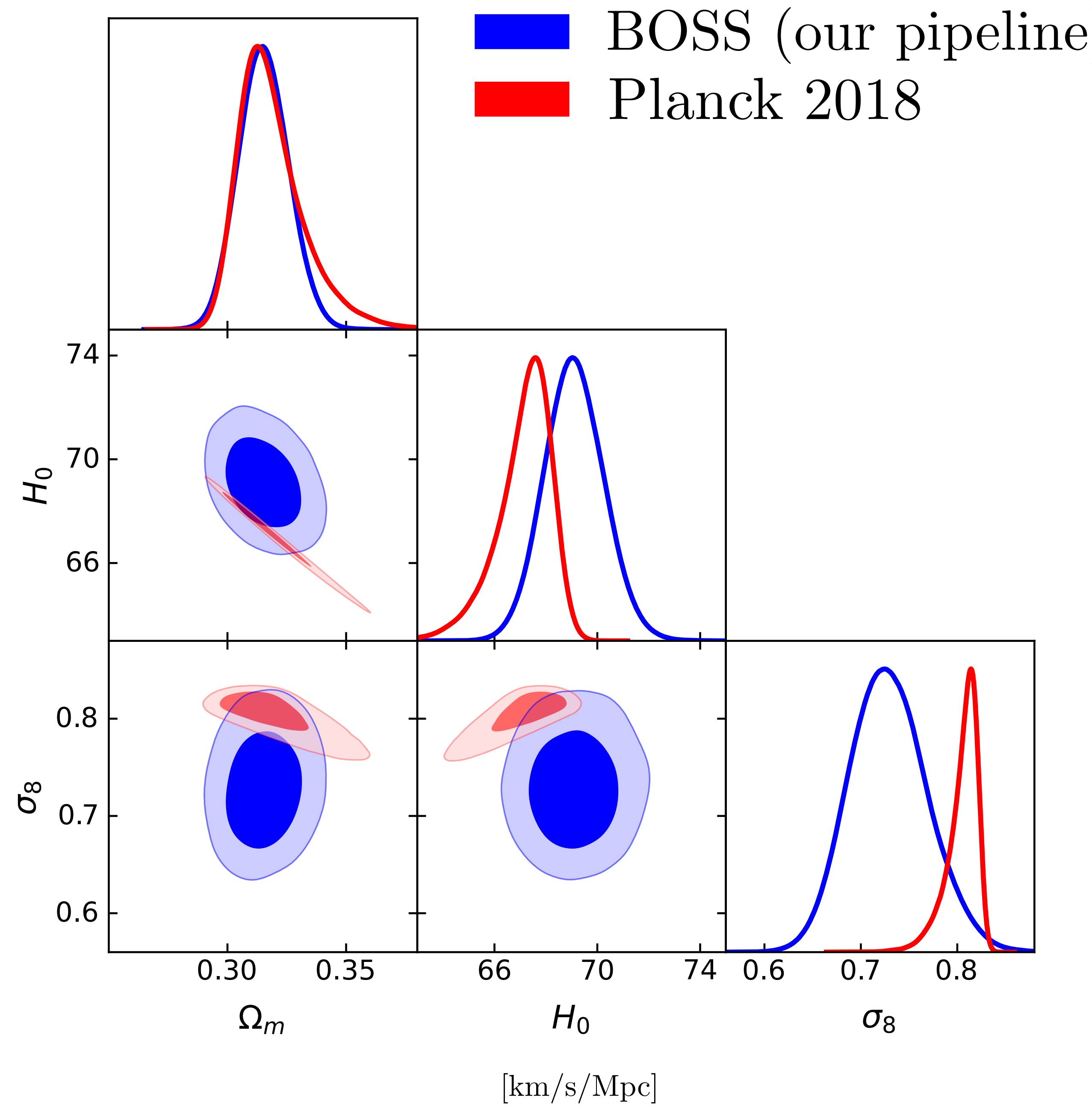
- Worldline EFT: consistent definition of tidal effects in GR
- First constrains on exotic compact objects with LIGO +VK



Large-scale structure: re-analysis of BOSS data







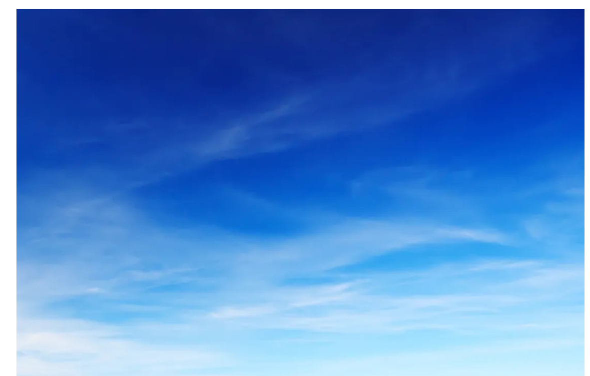
MI, Simonovic, Zaldarriaga (2019), Philcox, MI (2021) ++
D'Amico, Kokron++(2019), Chen, White, Vlah (2021)

Why is the sky blue ?

Scattering of light by neutral atoms (~ nitrogen in the air):

$$S_{\rm eff} = Q\int ds\, A_0 + \chi \int ds\ E_i E^i + \dots \qquad \chi \sim [{\rm cm}]^3$$

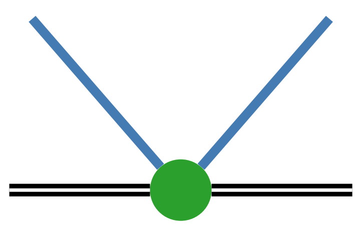
$$i\mathcal{M} \sim \chi\omega^2$$



Scattering of GWs by a compact object:

$$S_{\rm eff} = -m\int ds + \lambda_2 R^5 \int ds\ E_{\mu\nu}E^{\mu\nu}$$

$$i\mathcal{M} =$$



$$\sim \lambda_2 R^5 \omega^4$$

Goldberger, Rothstein (2004,2007)
Ivanov, Zhou (2022)

Universality of EFT dictates that the same param's appear in waveforms !

- Comparison with known results for black holes in GR:

$$\delta_\ell\Big|_{\rm GR} = (r_s\omega)\ln(2\omega r_s) + \sum_{n=1}^{3}\nu_n^\ell (r_s\omega)^n + \delta_\ell^{G^3} \qquad \delta_0^{G^3}\Big|_{\rm GR} = (r_s\omega)^3\left[\frac{7}{12} - \gamma_E - \ln(2r_s\omega)\right]$$

Renormalization flow of dynamical tides:

$$C_{0,\omega^2}(\bar\mu)^{\overline{\rm MS}} = -4\pi r_s^3\left[\frac{1}{4\epsilon_{\rm UV}} + \ln(\bar\mu r_s) + \frac{35}{24} + \gamma_E\right]$$

$$\begin{aligned}{\rm Im}F_0(\omega;\bar\mu)^{\overline{\rm MS}} = 4\pi r_s^2|\omega|\Bigg(1 + (r_s\omega)^2\Bigg[&\frac{\pi^2}{3} - \frac{5}{18} \\ &- \frac{11}{3}\left(\ln(\bar\mu r_s) + \gamma_E\right)\Bigg]\Bigg)\,.\end{aligned}$$

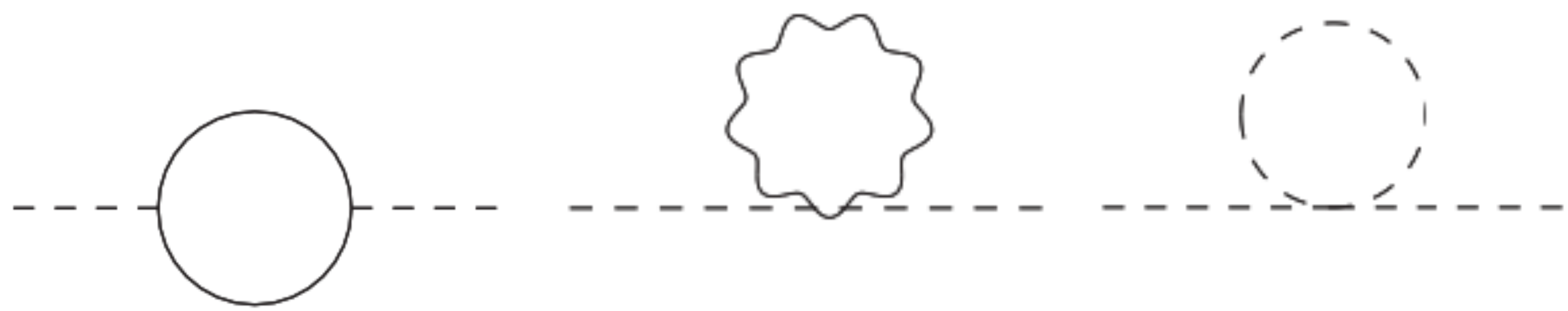
Static Love numbers are zero and do not run: $C_1 = 0$

- Perfect match: consistency of the EFT and the renormalization program!

Loops in Large Scale Structure

- Perturbation theory + Ensemble average = Loops
- Particle physics: ensemble of virtual particles

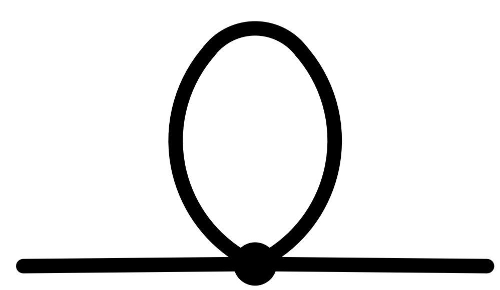
$$\delta m_H^2 =$$



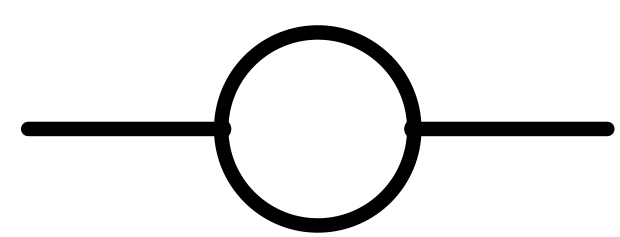
© Donaghue, Golowich, Holstein SM textbook

- Cosmology: ensemble of virtual Universes

$$\delta P(k) =$$



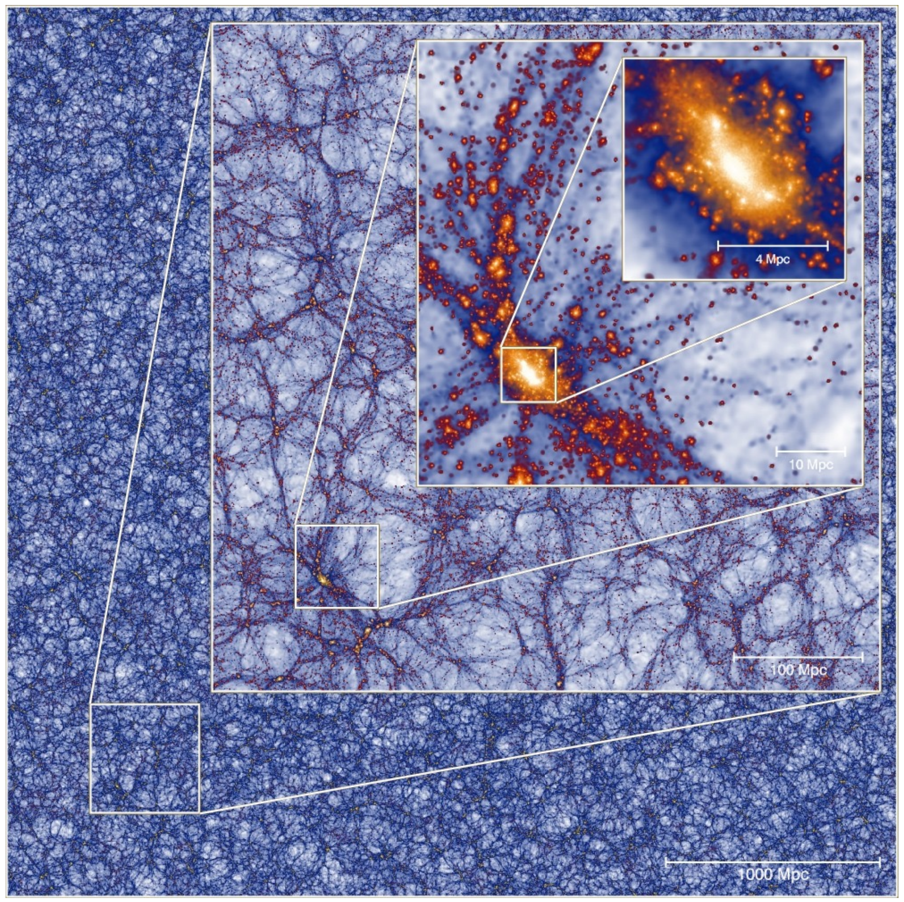
$+$



Large scale structure perturbation theory

difficulty

r_{halo}



L

simplicity

r_{halo} r_{Hubble}

Halo formation *effective fluid* *horizon*

[5 Mpc] [5 Gpc]

- Coarse-grained fields:

$$\frac{\partial u^i}{\partial t} + aH\partial u^i + u^j \nabla_j u^i = -\nabla^i \Phi - \frac{1}{\rho}\partial^j \sigma_{ij}$$

- Dim. analysis + symmetries:

$$\sigma_{ij} = c_s^2 \rho \delta_{ij} + \ldots$$

- Caveats: time evolution is slow ~ age of Universe
- Radiation effects can make R_{gal} very large

Mirbabayi, Schmidt ++ (2014), ++

Cabass, Schmidt (2018)