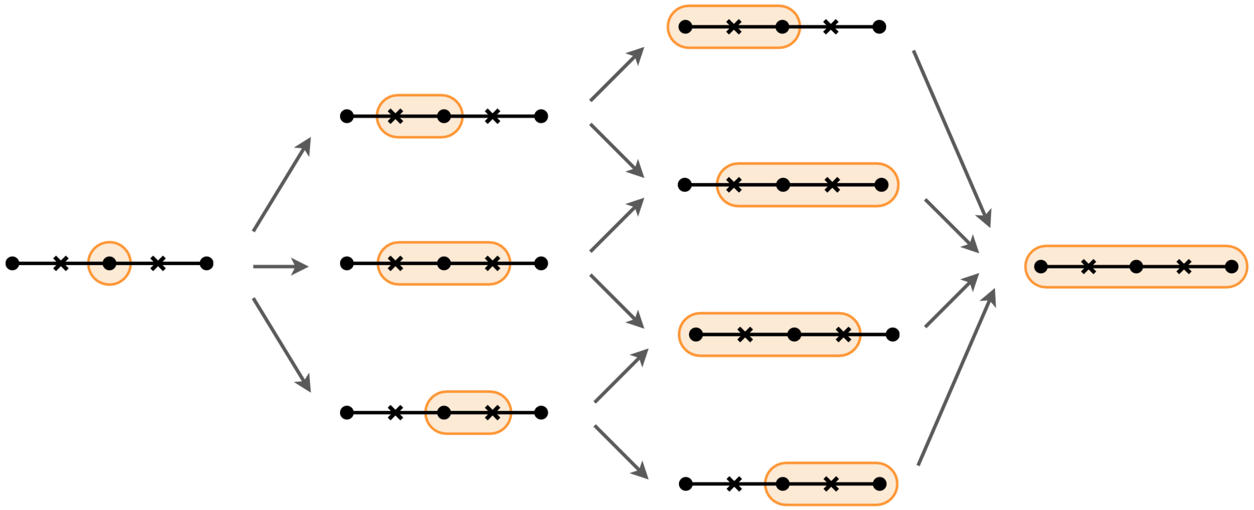
Kinematic Flow and the Emergence of Time



Hayden Lee

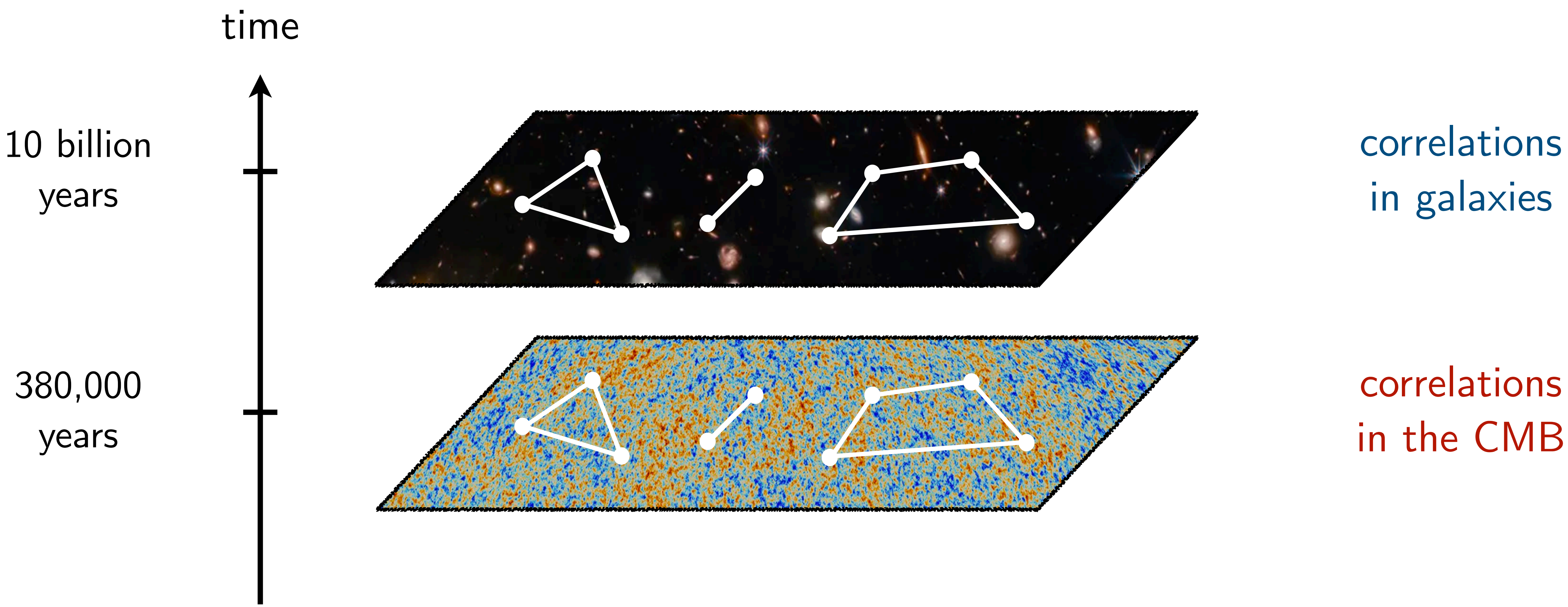
University of Chicago

w/ N. Arkani-Hamed, D. Baumann, A. Hillman, A. Joyce, G. Pimentel

[2312.05300, 2312.05303]

[Amplitudes 2024 @ IAS]

Cosmological Correlators



time

10 billion years

380,000 years

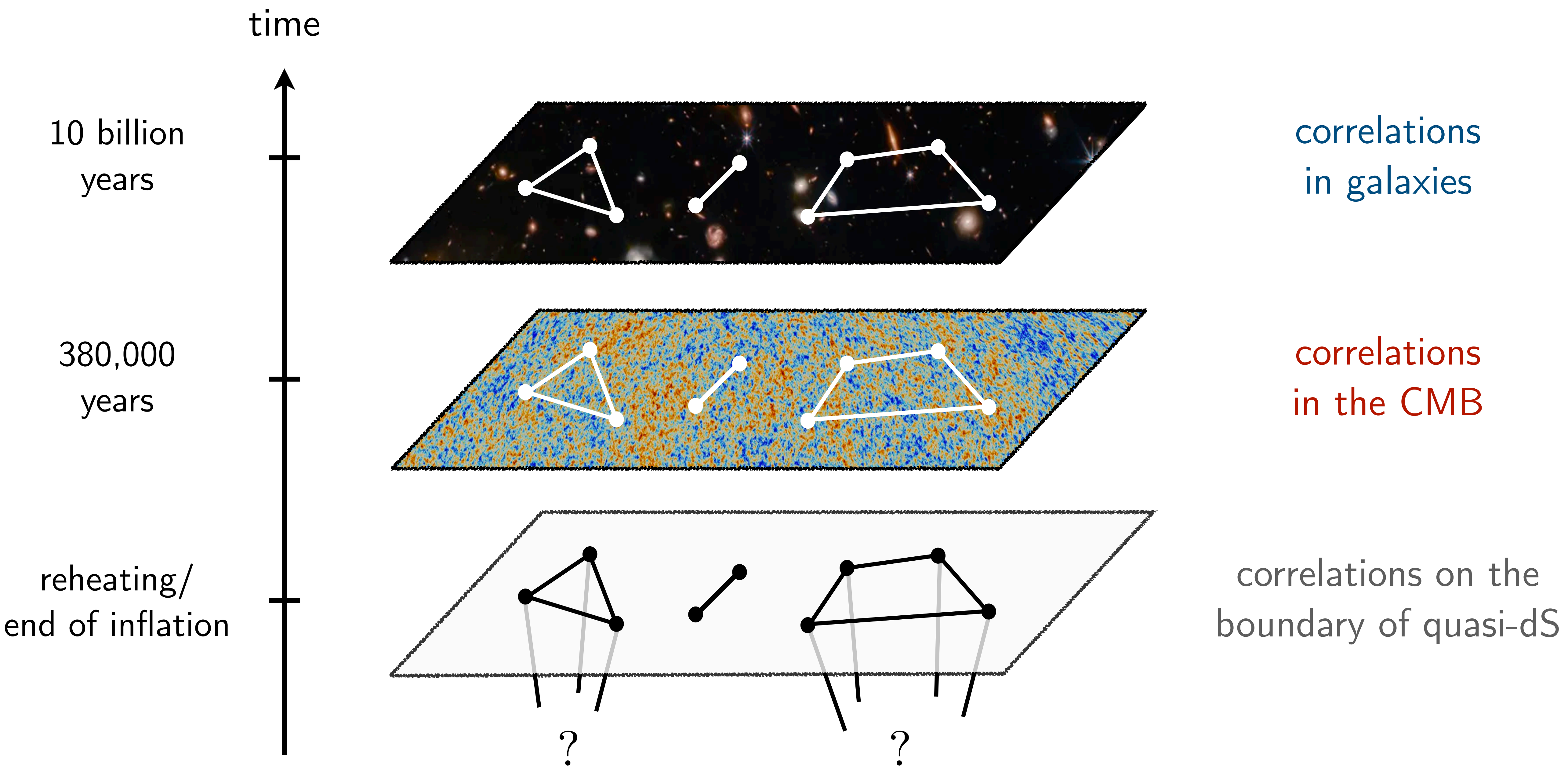
correlations in galaxies

correlations in the CMB

Structures in the universe are correlated over large distances.

Cosmological correlators encode the physics of the primordial universe.

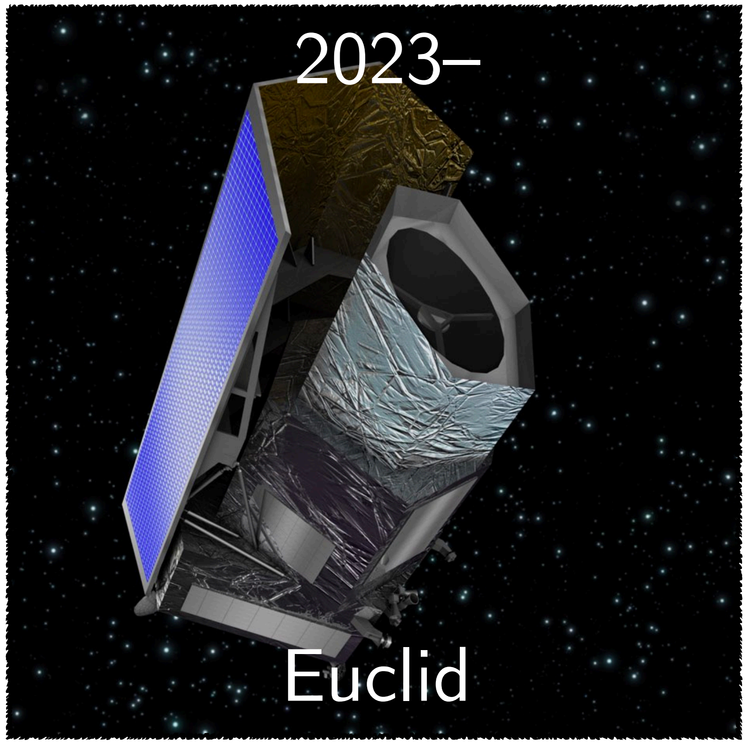
Cosmological Correlators



Dynamical information of inflation is encoded in **higher-point functions**.

Observational Frontier

An exciting era awaits for observational cosmology in the next decade.



10^{-2} 10 10^{5}

lower bound

LSS

~~CMB~~

~~upper bound~~

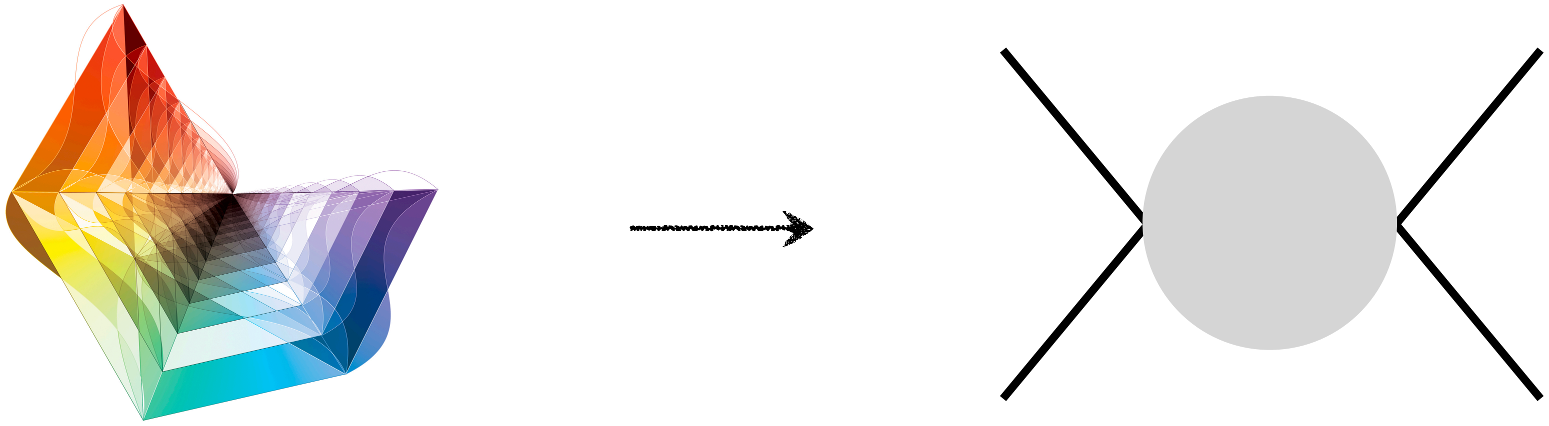
$$f_{\rm NL} = \frac{\langle \phi^3 \rangle}{\langle \phi^2 \rangle^2}$$

[See also Ivanov's talk]

What governs the patterns we observe in cosmological correlators?

Emergence of Spacetime

Over the past decade, we have seen scattering amplitudes emerge from new mathematical structures in boundary kinematic space.



positive geometries

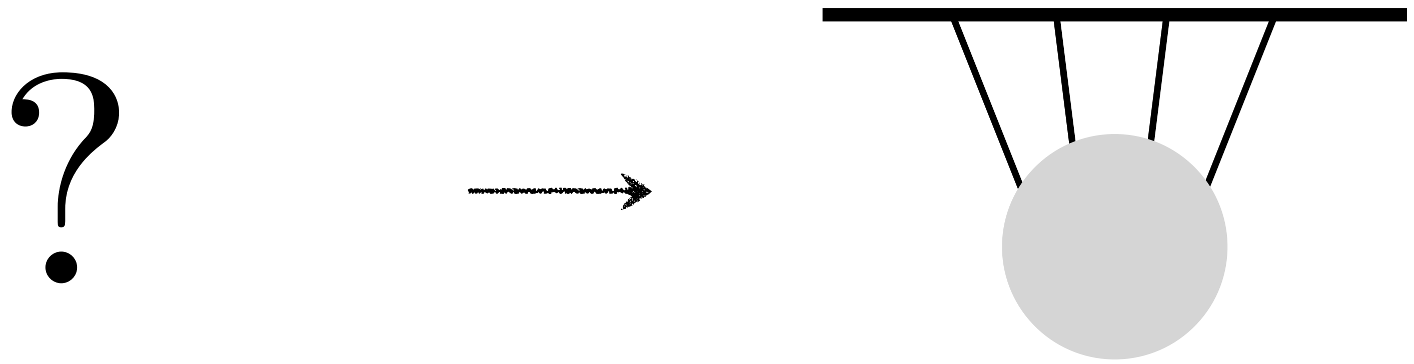
scattering amplitudes

Amplitudes can be obtained from the volumes of **positive geometries**.

Emergence of Time?

Are there similar structures for cosmological correlators?

Is it possible for bulk time evolution to arise from boundary phenomena?



cosmological correlators

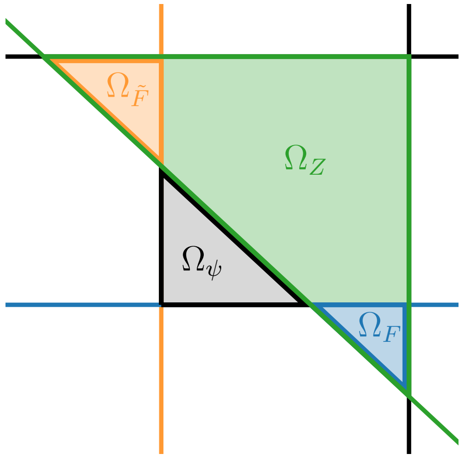
In this talk, I'll present (possibly) the first glimpses of such structures.

Arkani-Hamed, Baumann, Hillman, Joyce, HL, Pimentel [2023]

Outline

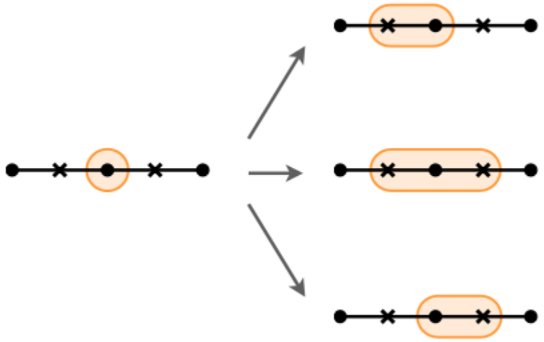
I.

Correlators as Twisted Integrals



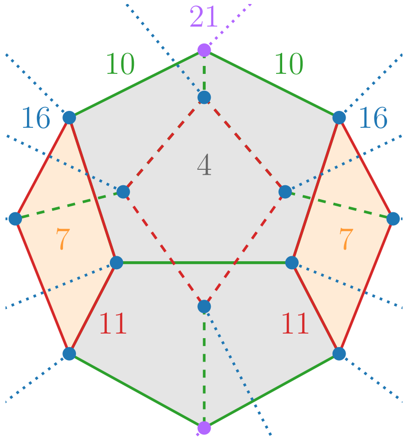
II.

A Hidden Pattern



III.

Conclusion & Outlook



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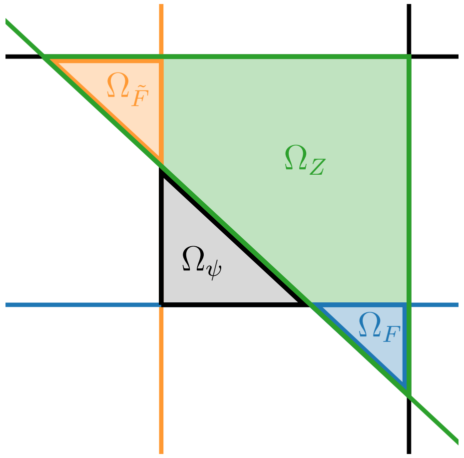

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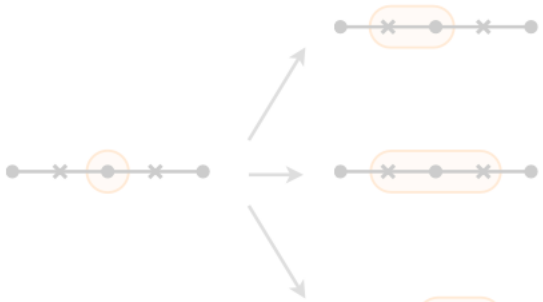

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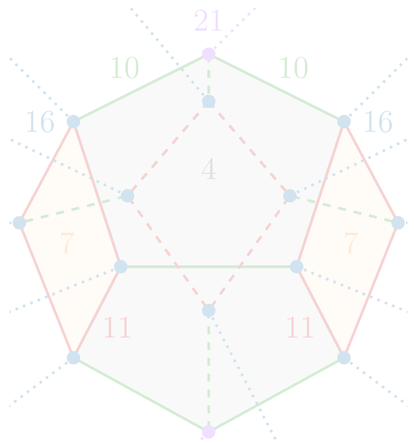


Toy Model

Consider a **conformally coupled scalar** with **polynomial interactions**:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}(\partial\phi)^2 - \frac{1}{12}R\phi^2 - \frac{\lambda}{3!}\phi^3\right]$$

conformal mass

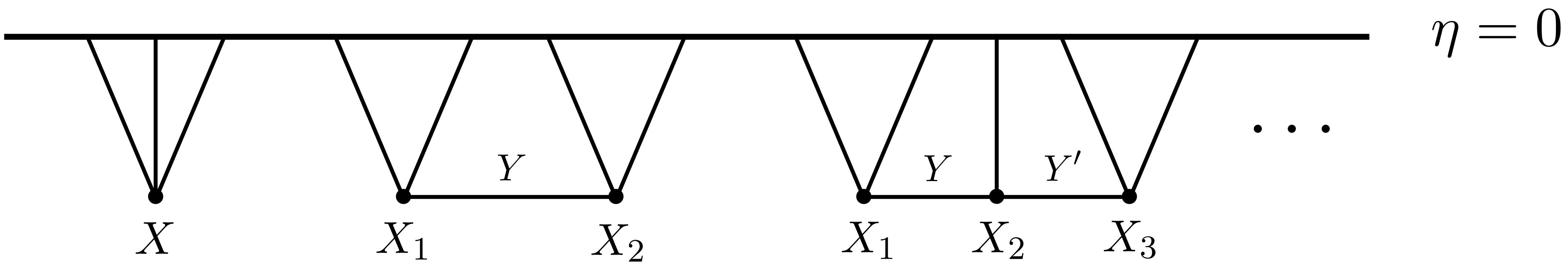
non-conformal interaction

in an **FRW** spacetime expanding as a **power law**:

$$\mathrm{d}s^2 = a^2(\eta)[-\mathrm{d}\eta^2 + \mathrm{d}\vec{x}^2] \qquad a(\eta) \propto \frac{1}{\eta^{1+\varepsilon}} \quad \begin{cases} \varepsilon = 0: & \mathrm{dS} \\ \varepsilon = -1: & \mathrm{flat} \\ \varepsilon = -2: & \mathrm{radiation} \\ \varepsilon = -3: & \mathrm{matter} \end{cases}$$

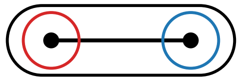
Wavefunction in Flat Space

We will study the **tree-level wavefunction** in this theory.



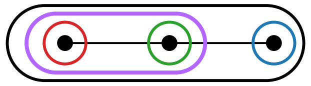
In **flat space**, the WF is given by **rational functions** with simple poles.

$$\psi^{(2)}_{\text{flat}} =$$

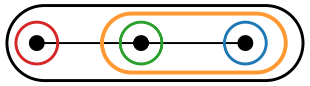


$$= \frac{1}{(X_1+X_2)(X_1+Y)(X_2+Y)}$$

$$\psi^{(3)}_{\text{flat}} =$$



$$+$$



$$= \frac{1}{(X_1+X_2+X_3)(X_1+Y)(X_2+Y+Y')(X_3+Y')}\left(\frac{1}{X_1+X_2+Y'}+\frac{1}{X_2+X_3+Y}\right)$$

Arkani-Hamed, Benincasa, Postnikov [2017]

Wavefunction in FRW

In **FRW**, the wavefunction can be represented as **twisted integrals**:

$$u = \prod_v x_v^{\varepsilon}$$

twist (multi-valued)

$$\boxed{\psi_{\mathrm{FRW}}(\mathbf{X}, \mathbf{Y}) = \int_0^{\infty} u\, \Omega_{\psi}}$$

rational form (single-valued)

$$\Omega_{\psi} = \psi_{\mathrm{flat}}(\mathbf{X} + \mathbf{x}, \mathbf{Y}) \bigwedge_v \mathrm{d}x_v$$

Modern amplitude approaches to compute integrals of this type include:

- twisted cohomology
- method of differential equations

Twisted Cohomology

Two integrands differing by a total differential give the same integral.

$$0 = \int \mathrm{d}(u\,\Omega) = \int u\,\underbrace{(\mathrm{d} + \mathrm{d}\log u\,\wedge)}_{\equiv \nabla_\omega}\,\Omega \qquad \Rightarrow \qquad \Omega \sim \Omega + \nabla_\omega \xi$$

The set of equivalence classes of integrands $=$ **twisted cohomology**

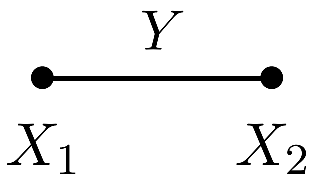
Basis size $=$ # **bounded regions** formed by the singular hyperplanes.

$\Rightarrow$ satisfies a closed system of **differential equations**.

Aomoto, Kita [2011]
Mastrolia, Mizera [2018]

Two-Site Chain (Four-Point Function)

The integral for the **two-site chain** takes the form



$$= \int_0^\infty (x_1 x_2)^\varepsilon \frac{\mathrm{d}x_1 \mathrm{d}x_2}{(x_1 + X_1 + Y)(x_2 + X_2 + Y)(x_1 + x_2 + X_1 + X_2)}$$

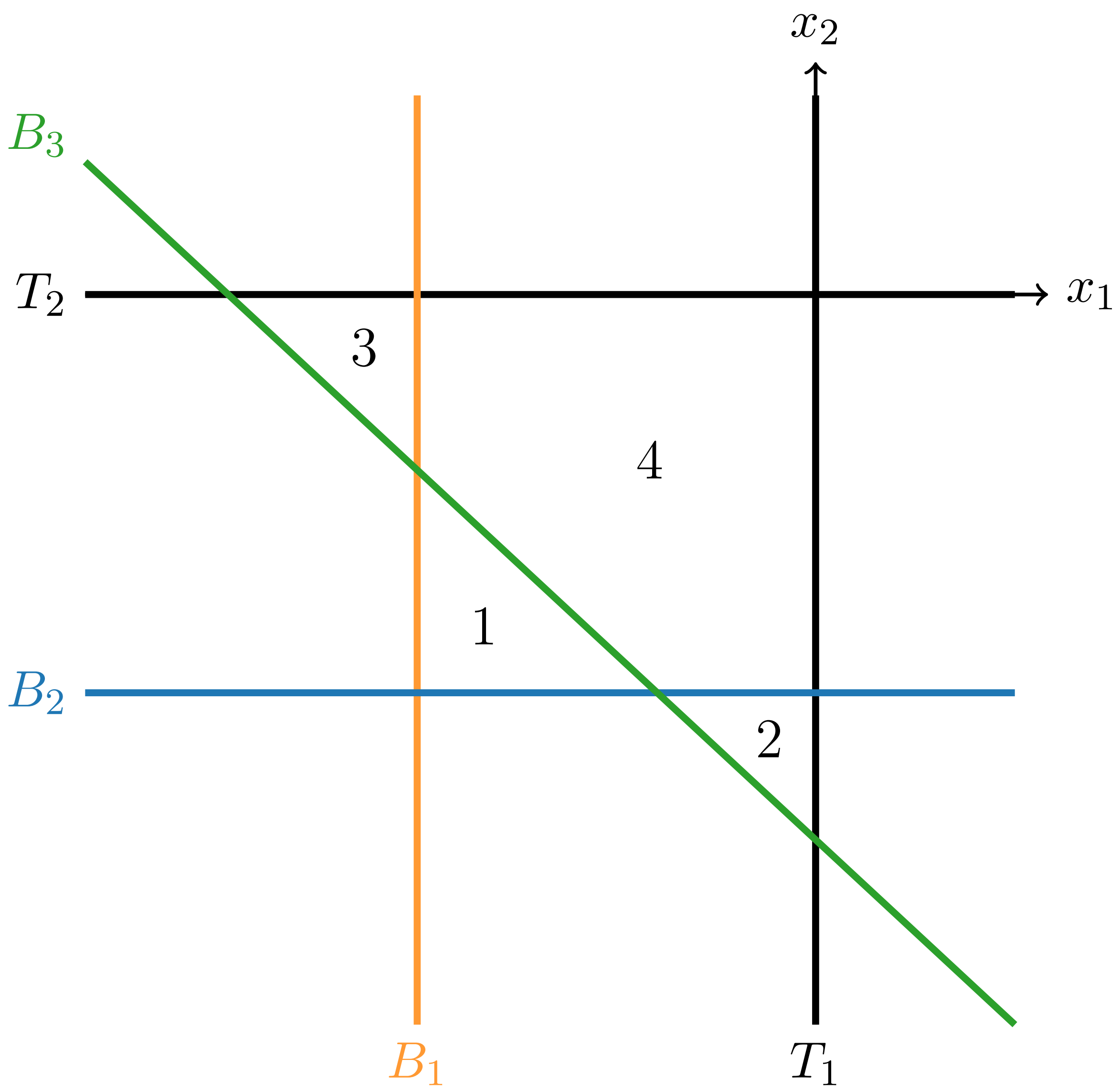
We consider a family of integrals with the same singularities:

$$\boxed{\int_0^\infty (x_1 x_2)^\varepsilon \, \Omega_{\mathbf{n}} \,, \qquad \Omega_{\mathbf{n}} = \frac{\mathrm{d}x_1 \mathrm{d}x_2}{T_1^{n_1} T_2^{n_2} B_1^{n_3} B_2^{n_4} B_3^{n_5}}} \qquad (n_i \in \mathbb{Z})$$

$$T_1 = x_1 \,, \qquad B_1 = x_1 + X_1 + Y \,,$$
$$T_2 = x_2 \,, \qquad B_2 = x_2 + X_2 + Y \,, \qquad B_3 = x_1 + x_2 + X_1 + X_2$$

Master Integrals

master integrals = # bounded regions formed by the singular hyperplanes.



$$T_1 = x_1$$

$$T_2 = x_2$$

$$B_1 = x_1 + X_1 + Y$$

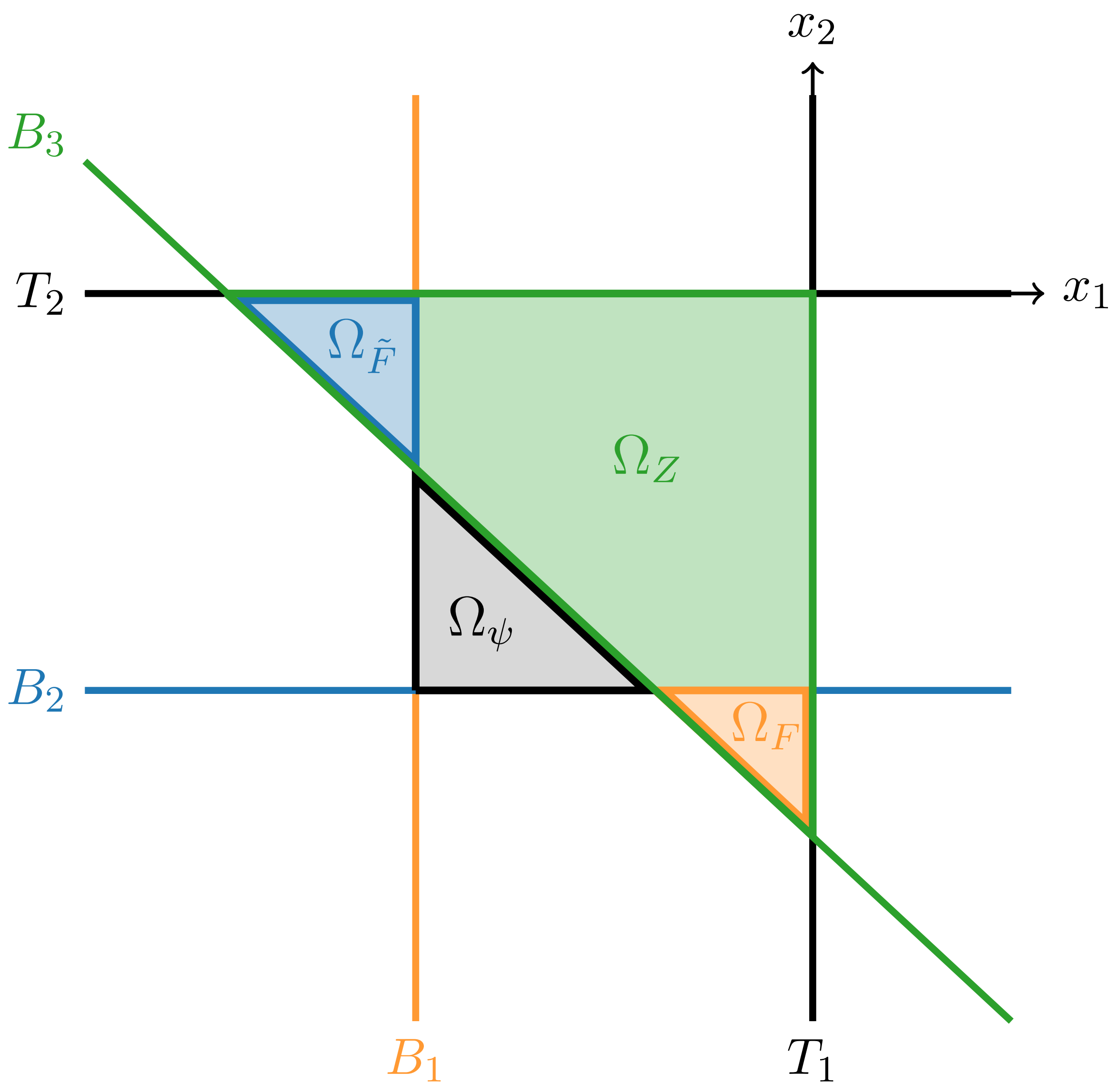
$$B_2 = x_2 + X_2 + Y$$

$$B_3 = x_1 + x_2 + X_1 + X_2$$

De, Pokraka [2023]
Arkani-Hamed, Baumann, Hillman, Joyce, HL, Pimentel [2023]

Master Integrals

A good basis choice is given by the **canonical forms** of the bounded regions.



$$\vec{I} = \begin{bmatrix} \psi \\ F \\ \tilde{F} \\ Z \end{bmatrix} = \int (x_1 x_2)^\varepsilon \begin{bmatrix} \Omega_\psi \\ \Omega_F \\ \Omega_{\tilde{F}} \\ \Omega_Z \end{bmatrix}$$

$$\Omega_{\text{can}}[\Delta_{L_1 L_2 L_3}] = \mathrm{d}\log\left(\frac{L_1}{L_3}\right) \wedge \mathrm{d}\log\left(\frac{L_2}{L_3}\right)$$

De, Pokraka [2023]
Arkani-Hamed, Baumann, Hillman, Joyce, HL, Pimentel [2023]

Differential Equations

Taking the differential of the basis vector and performing IBP gives

$$\boxed{\mathrm{d}\vec{I} = \varepsilon A \vec{I}}$$

$$\mathrm{d} = \sum_i \mathrm{d}X_i \frac{\partial}{\partial X_i} \qquad\qquad A = \sum_i \alpha_i \, \mathrm{d}\log R_i$$

letters

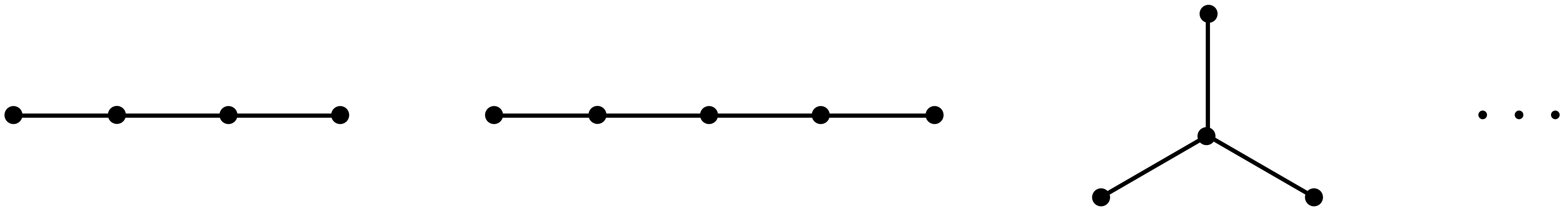
with

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix} \mathrm{d}\log(X_1 + X_2) + \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mathrm{d}\log(X_1 + Y) + \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mathrm{d}\log(X_1 - Y)$$

$$+ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mathrm{d}\log(X_2 + Y) + \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mathrm{d}\log(X_2 - Y)$$

General Tree Graphs

Unfortunately, this approach breaks down for more complicated graphs.

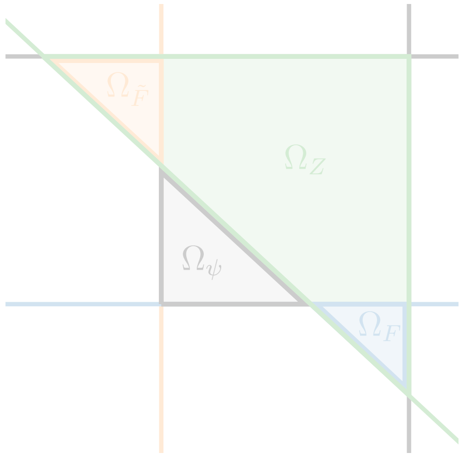


- **Twisted cohomology** gives an **unphysical, over-complete basis**.
- Deriving equations using **IBP relations** is **highly challenging**.

Remarkably, these are solved by simple graphical rules (kinematic flow).

I.

Correlators as Twisted Integrals



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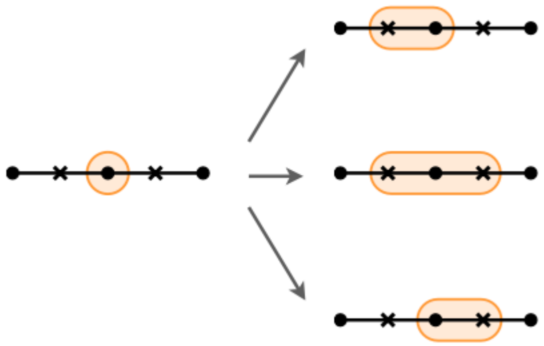

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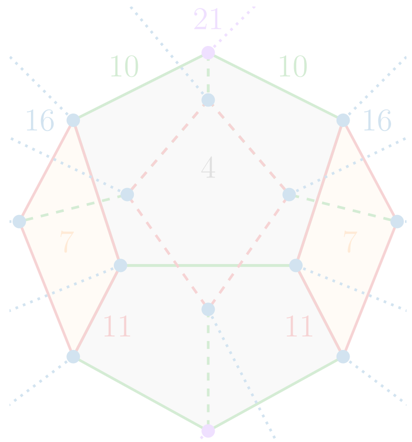
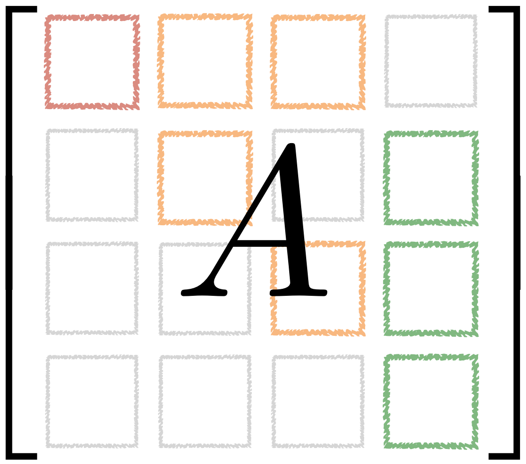
Conclusion & Outlook



Differential Equations

We derived the system of differential equations for the two-site chain.



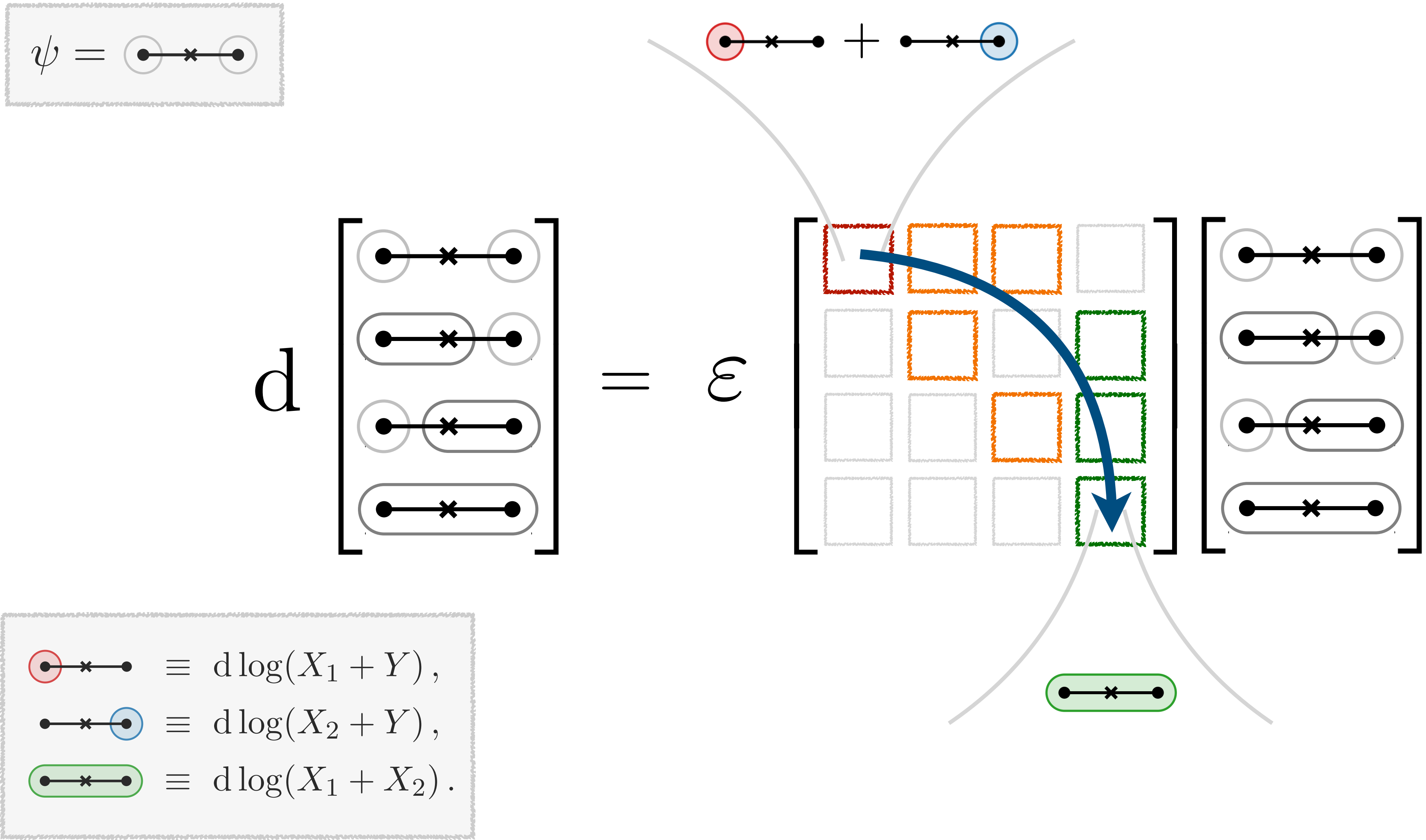
$$\mathrm{d}\begin{bmatrix}\psi\\ F\\ \tilde{F}\\ Z\end{bmatrix} = \varepsilon\begin{bmatrix} & & & \\ & & & \\ & & A & \\ & & & \end{bmatrix}\begin{bmatrix}\psi\\ F\\ \tilde{F}\\ Z\end{bmatrix}$$

$$A = \sum_i \alpha_i \,\mathrm{d}\log R_i$$

However, the explicit result isn't very illuminating and is hard to generalize.

A Hidden Pattern

A **hidden pattern** was revealed when we drew pictures of the results!



$\equiv \mathrm{d}\log(X_1+Y)\,,$

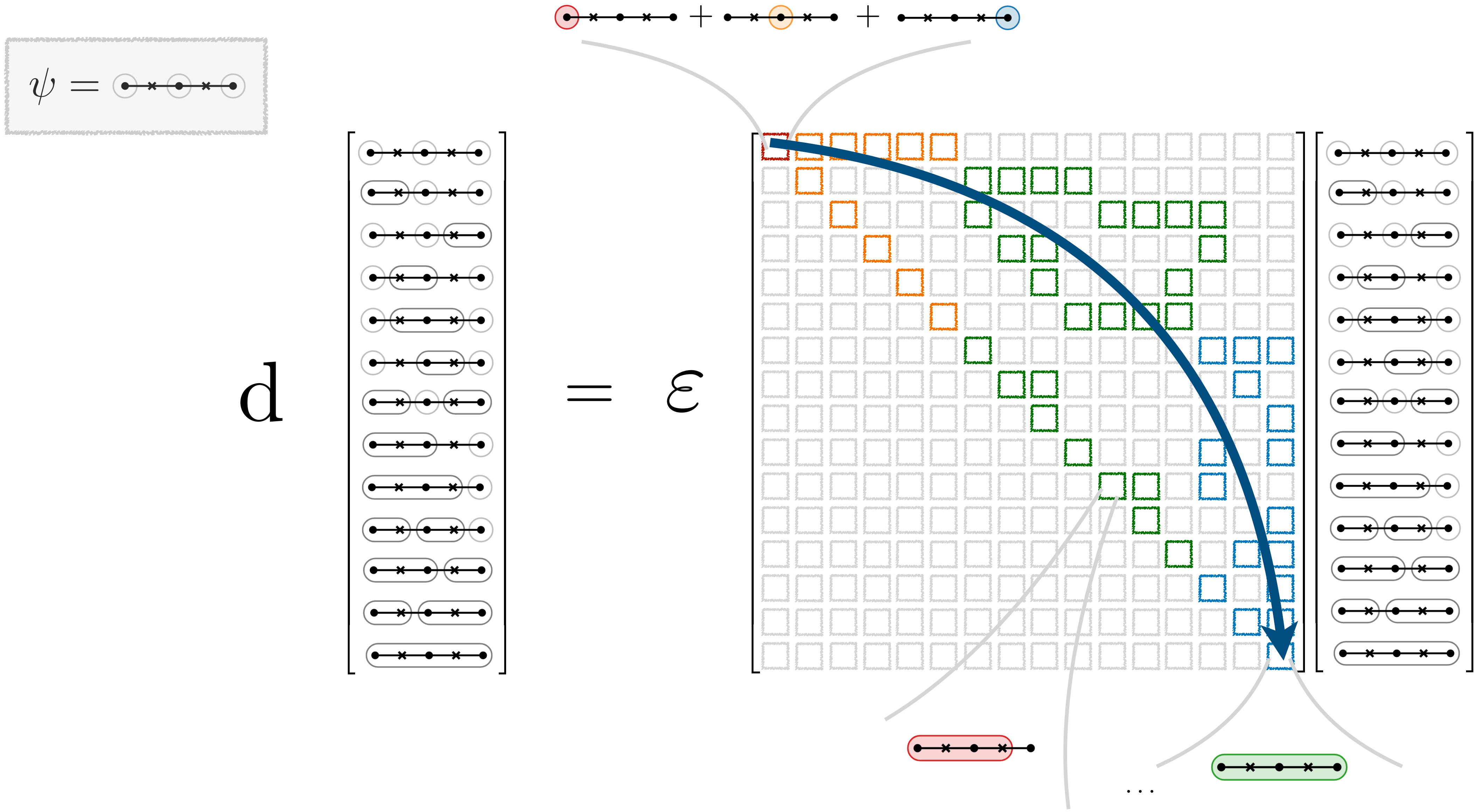
$\equiv \mathrm{d}\log(X_2+Y)\,,$

$\equiv \mathrm{d}\log(X_1+X_2)\,.$

The tubings **grow**, and the **system closes** when all vertices are enclosed.

A Hidden Pattern

The same pattern is found for arbitrary **n-site graphs** at tree level.



Remarkably, we can predict all entries with **simple rules (kinematic flow).**

Graphical Representation

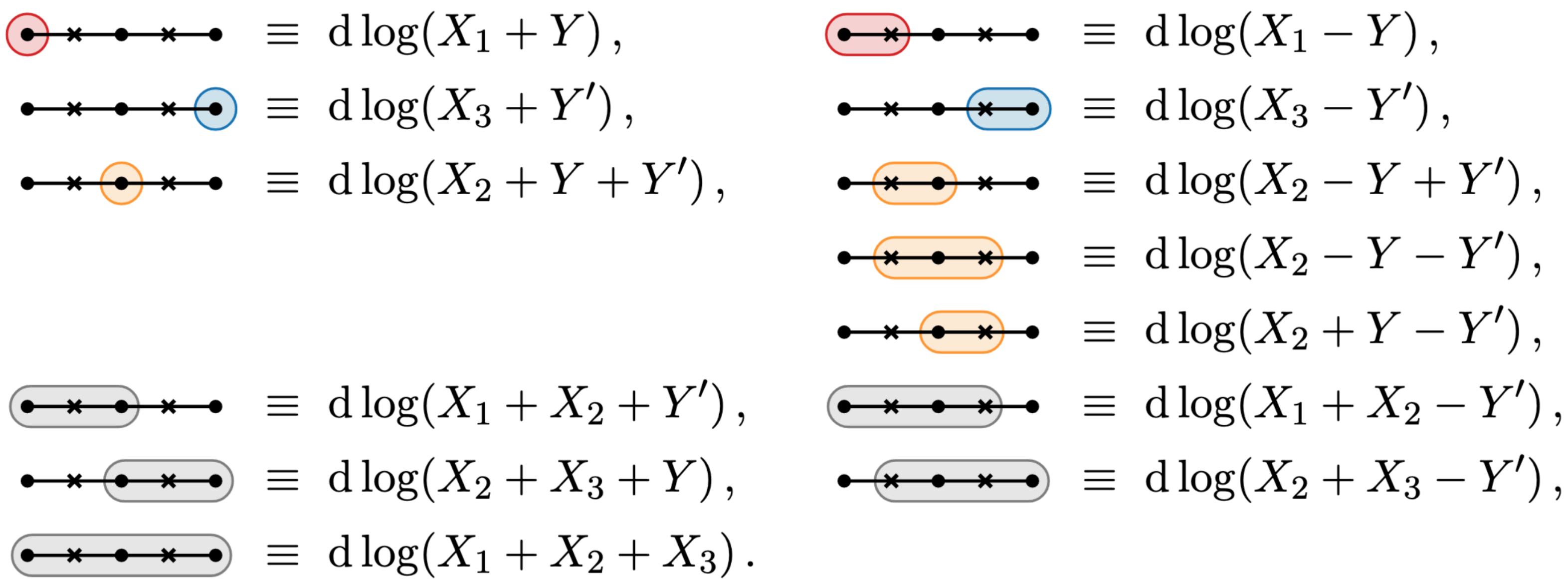
Letters:

connected

(activated)

tubings

13 (~~19~~) letters



$$\equiv \mathrm{d}\log(X_1+Y)\,,$$
$$\equiv \mathrm{d}\log(X_3+Y')\,,$$
$$\equiv \mathrm{d}\log(X_2+Y+Y')\,,$$
$$\equiv \mathrm{d}\log(X_1+X_2+Y')\,,$$
$$\equiv \mathrm{d}\log(X_2+X_3+Y)\,,$$
$$\equiv \mathrm{d}\log(X_1+X_2+X_3)\,.$$
$$\equiv \mathrm{d}\log(X_1-Y)\,,$$
$$\equiv \mathrm{d}\log(X_3-Y')\,,$$
$$\equiv \mathrm{d}\log(X_2-Y+Y')\,,$$
$$\equiv \mathrm{d}\log(X_2-Y-Y')\,,$$
$$\equiv \mathrm{d}\log(X_2+Y-Y')\,,$$
$$\equiv \mathrm{d}\log(X_1+X_2-Y')\,,$$
$$\equiv \mathrm{d}\log(X_2+X_3-Y')\,,$$

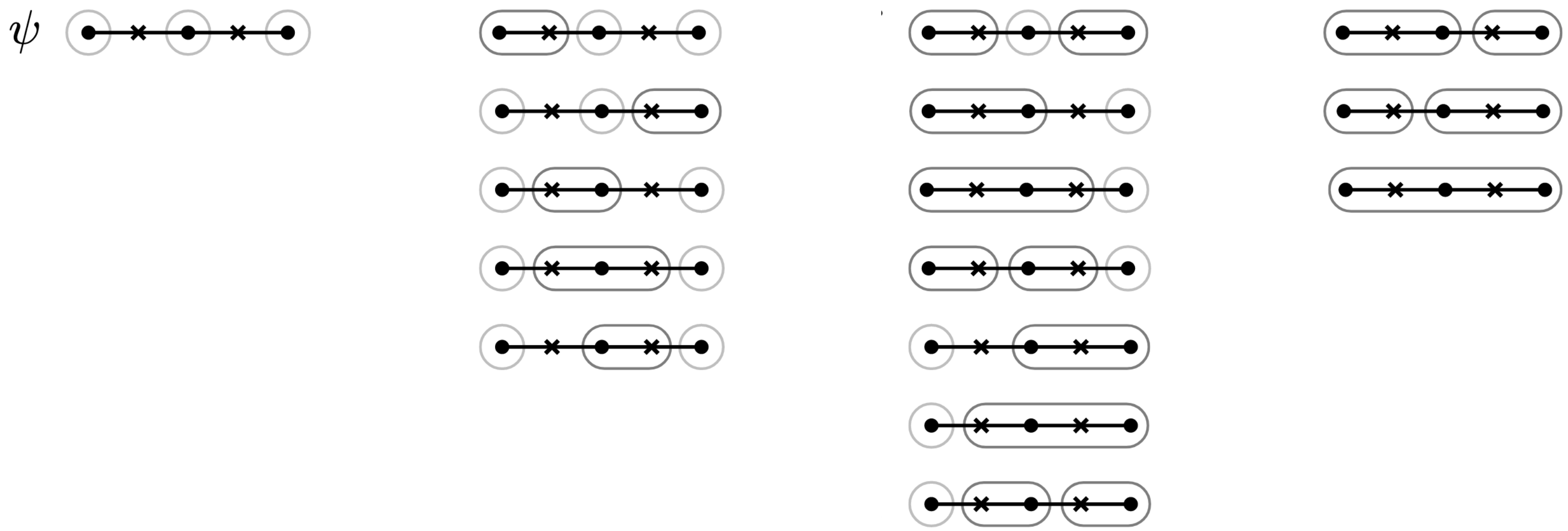
Functions:

complete

(disconnected)

tubings

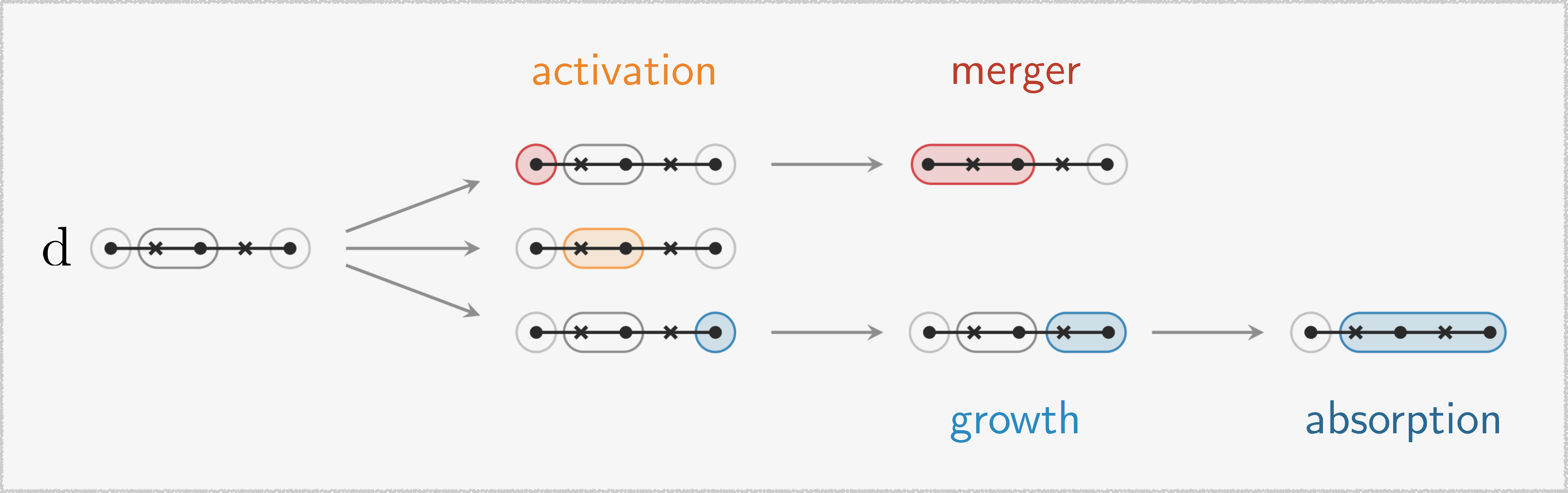
16 (~~25~~) functions



naive counting from twisted cohomology (64 vs. 201 for $n_{\text{site}}=4$)

Kinematic Flow

Upon differentiation, graph tubings evolve according to simple graphical rules.

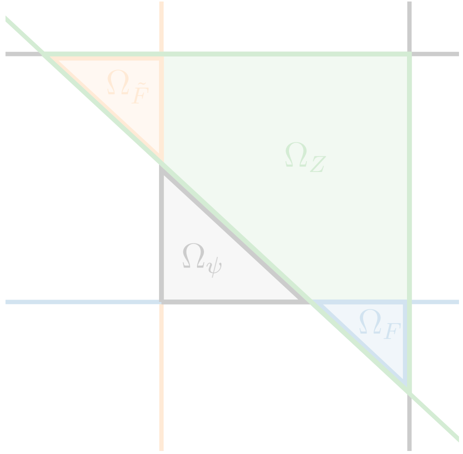


These rules allow us to predict (by hand!) the equations for **all tree graphs**.

This reformulates bulk time evolution as a flow in kinematic space.

I.

Correlators as Twisted Integrals



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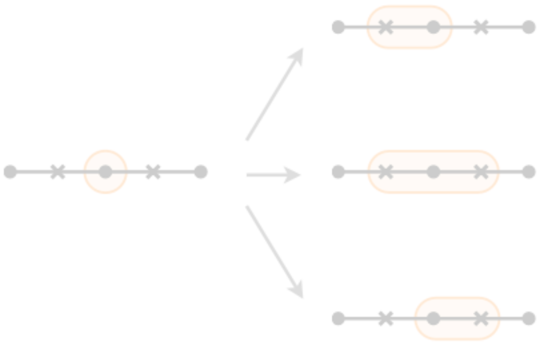

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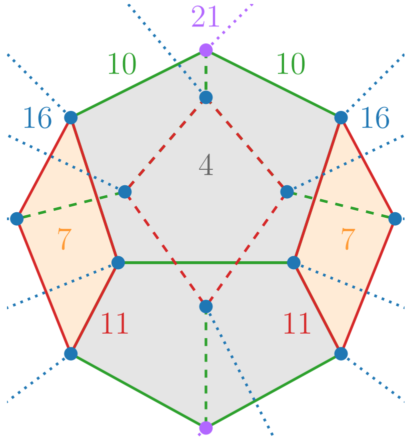
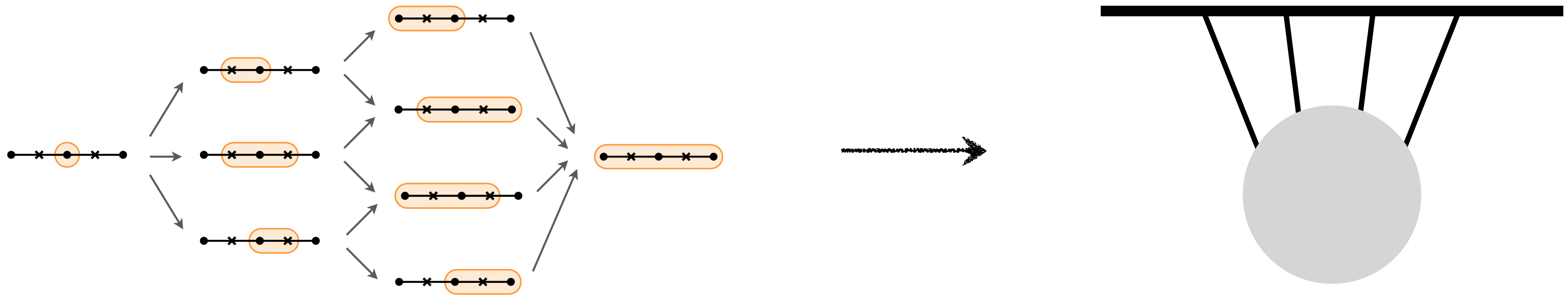


Conclusion

We have found a **hidden pattern** in the differential equations for the tree-level FRW wavefunction of conformally coupled scalars.

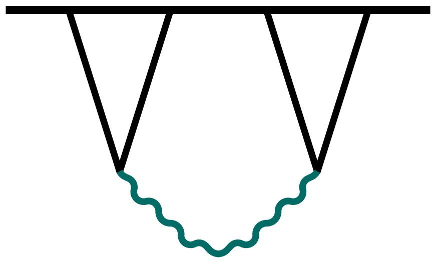


Simple graphical rules allow us to predict the equations for **all tree graphs**.

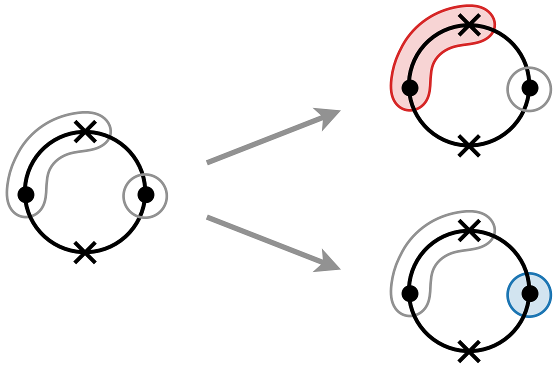
Outlook

Next Frontier: more nontrivial graphs, deeper mathematical structures, ...

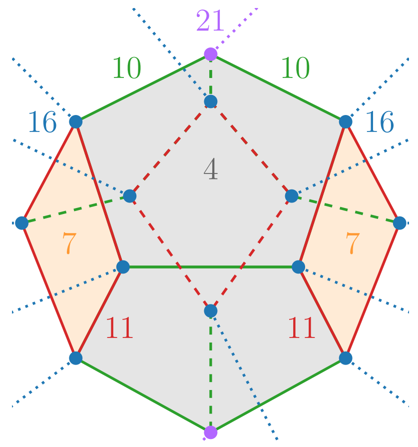
Massive particles



Loops



Positive geometry



This presents a fascinating connection between amplitudes and cosmology.