Enhanced Duality In 4D supergravity

Renata Kallosh, Stanford

Amplitudes 2024, Princeton, June 10, 2024

1. Gauge-fixing local H symmetry in supergravity

RK, Samtleben, Van Proeyen, **KSVP** work in progress

2. The role of Sp(2n, $\mathcal{R}$) duality in quantum theory

RK work in progress

3. Enhanced duality in 4D supergravity

RK arXiv:2405.20275

And earlier work of RK in 2023,2024

Is 4D maximal supergravity special?

Ward Identities for Superamplitudes, JHEP 06(2024)035

Abstract

U-duality imposes strong constraints on the structure of UV divergences in supergravity

But Gaillard-Zumino **symplectic Sp(2n, $\mathcal{R}$) duality in 4D** has more symmetries than **U-duality** (**n** is a number of vectors)

For example, in $\mathcal{N}$=8, **Sp(56)** has a dimension of 1596, whereas the dimension of its **U-duality** subgroup **$E_{7(7)}$ is 133**.

Abstract

U-duality imposes strong constraints on the structure of UV divergences in supergravity

But Gaillard-Zumino **symplectic Sp(2n, $\mathcal{R}$) duality in 4D** has more symmetries than **U-duality** (**n** is a number of vectors)

For example, in $\mathcal{N}$=8, **Sp(56)** has a dimension of 1596, whereas the dimension of its **U-duality** subgroup **$E_{7(7)}$ is 133**.

In comparison, in D > 4 maximal duality symmetry is U-duality, there are no enhanced dualities

We argue that the **extra dualities**, enhancing U-dualities, determine the properties of **perturbative quantum supergravity**, being implemented into a **Hamiltonian path integral**

The presence/absence of enhanced dualities suggests a possible explanation of **known** amplitude loop computations in D-dimensional $\mathcal{N}$ > 4 supergravities and of the special status of D = 4 in this respect

Abstract

U-duality imposes strong constraints on the structure of UV divergences in supergravity

But Gaillard-Zumino **symplectic Sp(2n, $\mathcal{R}$) duality in 4D** has more symmetries than **U-duality** (**n** is a number of vectors)

For example, in $\mathcal{N}$=8, **Sp(56)** has a dimension of 1596, whereas the dimension of its **U-duality** subgroup **$E_{7(7)}$ is 133**.

In comparison, in D > 4 maximal duality symmetry is U-duality, there are no enhanced dualities

We argue that the **extra dualities**, enhancing U-dualities, determine the properties of **perturbative quantum supergravity**, being implemented into a **Hamiltonian path integral**

The presence/absence of enhanced dualities suggests a possible explanation of **known** amplitude loop computations in D-dimensional $\mathcal{N}$ > 4 supergravities and of the special status of D = 4 in this respect

Enhanced duality explains enhanced cancellations in $\mathcal{N}$>4 supergravity in 4D

Abstract

U-duality imposes strong constraints on the structure of UV divergences in supergravity

But Gaillard-Zumino **symplectic Sp(2n, $\mathcal{R}$) duality in 4D** has more symmetries than **U-duality** (**n** is a number of vectors)

For example, in $\mathcal{N}$=8, **Sp(56)** has a dimension of 1596, whereas the dimension of its **U-duality** subgroup **$E_{7(7)}$ is 133**.

In comparison, in D > 4 maximal duality symmetry is U-duality, there are no enhanced dualities

We argue that the **extra dualities**, enhancing U-dualities, determine the properties of **perturbative quantum supergravity**, being implemented into a **Hamiltonian path integral**

The presence/absence of enhanced dualities suggests a possible explanation of **known** amplitude loop computations in D-dimensional $\mathcal{N}$ > 4 supergravities and of the special status of D = 4 in this respect

Enhanced duality explains enhanced cancellations in $\mathcal{N}$>4 supergravity in 4D

More 4D supergravity amplitude computations are desirable. New amplitude computations will show that either perturbative 4D supergravity is as bad as D > 4, or it continues to be special due to 4D enhanced symmetries!

Geometric Superinvariants, candidate counterterms, at the critical loop order

RK, 2023

G/H coset space supergravities

$$S_{cr} = \kappa^{2\,(L_{cr}-1)} \int d^{4\mathcal{N}} d^D x \det E \; \mathcal{L}(x,\theta)$$

Below critical order there is no local H-symmetry, no global G symmetry and no local nonlinear supersymmetry

$$L_{cr} = \frac{2\mathcal{N}+n}{(D-2)} \; , \qquad n \geq 0$$

Geometric Superinvariants, candidate counterterms, at the critical loop order

RK, 2023

G/H coset space supergravities

$$S_{cr} = \kappa^{2\,(L_{cr}-1)} \int d^{4\mathcal{N}} d^D x \det E \;\mathcal{L}(x,\theta)$$

Below critical order there is no local H-symmetry, no global G symmetry and no local nonlinear supersymmetry

$$L_{cr} = \frac{2\mathcal{N}+n}{(D-2)}\;, \qquad n \geq 0$$

D=4 **$\mathcal{N}$ > 4** no UV divergences so far

$D=4,\, L_{cr}=8:$ $\quad \kappa^{14}\int d^4\,x\, D^{10}R^4+\dots$	$n=0$	
$D=5,\, L_{cr}=6:$ $\quad \kappa^{10}\int d^5\,x\, D^{12}R^4+\dots$	$n=2$	$L_{UV}=5<L_{cr}=6$
$D=6,\, L_{cr}=4:$ $\quad \kappa^{6}\int d^6\,x\, D^{10}R^4+\dots$	$n=0$	$L_{UV}=3<L_{cr}=4$
$D=7,\, L_{cr}=4:$ $\quad \kappa^{6}\int d^7\,x\, D^{14}R^4+\dots$	$n=4$	$L_{UV}=2<L_{cr}=4$
$D=8,\, L_{cr}=3:$ $\quad \kappa^{4}\int d^8\,x\, D^{12}R^4+\dots$	$n=3$	$L_{UV}=1<L_{cr}=3$
$D=9,\, L_{cr}=3:$ $\quad \kappa^{4}\int d^9\,x\, D^{15}R^4+\dots$	$n=5$	$L_{UV}=2<L_{cr}=3$

Maximal supergravities

Z. Bern et al

Geometric Superinvariants, candidate counterterms, at the critical loop order

RK, 2023

$$S_{cr} = \kappa^{2\,(L_{cr}-1)} \int d^{4\mathcal{N}} d^D x \det E\ \mathcal{L}(x,\theta)$$

G/H coset space supergravities

Below critical order there is no local H-symmetry, no global G symmetry and no local nonlinear supersymmetry

$$L_{cr} = \frac{2\mathcal{N} + n}{(D-2)}\ , \qquad n \geq 0$$

D=4 **$\mathcal{N}$ > 4** no UV divergences so far

$$D=4,\ L_{cr}=8: \quad \kappa^{14}\int d^4\,x\,D^{10}R^4 + \dots \quad n=0$$

??? $L_{cr} = \mathcal{N}$

$$D=5,\ L_{cr}=6: \quad \kappa^{10}\int d^5\,x\,D^{12}R^4 + \dots \quad n=2$$

$L_{UV} = 5 < L_{cr} = 6$

$$D=6,\ L_{cr}=4: \quad \kappa^{6}\int d^6\,x\,D^{10}R^4 + \dots \quad n=0$$

$L_{UV} = 3 < L_{cr} = 4$

$$D=7,\ L_{cr}=4: \quad \kappa^{6}\int d^7\,x\,D^{14}R^4 + \dots \quad n=4$$

$L_{UV} = 2 < L_{cr} = 4$

$$D=8,\ L_{cr}=3: \quad \kappa^{4}\int d^8\,x\,D^{12}R^4 + \dots \quad n=3$$

$L_{UV} = 1 < L_{cr} = 3$

$$D=9,\ L_{cr}=3: \quad \kappa^{4}\int d^9\,x\,D^{15}R^4 + \dots \quad n=5$$

$L_{UV} = 2 < L_{cr} = 3$

Maximal supergravities

Z. Bern et al

Geometric Superinvariants, candidate counterterms, at the critical loop order

RK, 2023

G/H coset space supergravities

$$S_{cr} = \kappa^{2\,(L_{cr}-1)} \int d^{4\mathcal{N}} d^D x \det E \; \mathcal{L}(x,\theta)$$

Below critical order there is no local H-symmetry, no global G symmetry and no local nonlinear supersymmetry

$$L_{cr} = \frac{2\mathcal{N}+n}{(D-2)}\;, \qquad n \geq 0$$

D=4 $\mathcal{N}$ **> 4** no UV divergences so far

??? $L_{cr} = \mathcal{N}$

$D=4,\; L_{cr}=8:$	$\kappa^{14} \int d^4 x\, D^{10} R^4 + \dots$	$n=0$		
$D=5,\; L_{cr}=6:$	$\kappa^{10} \int d^5 x\, D^{12} R^4 + \dots$	$n=2$		$L_{UV}=5 < L_{cr}=6$
$D=6,\; L_{cr}=4:$	$\kappa^{6} \int d^6 x\, D^{10} R^4 + \dots$	$n=0$		$L_{UV}=3 < L_{cr}=4$
$D=7,\; L_{cr}=4:$	$\kappa^{6} \int d^7 x\, D^{14} R^4 + \dots$	$n=4$		$L_{UV}=2 < L_{cr}=4$
$D=8,\; L_{cr}=3:$	$\kappa^{4} \int d^8 x\, D^{12} R^4 + \dots$	$n=3$		$L_{UV}=1 < L_{cr}=3$
$D=9,\; L_{cr}=3:$	$\kappa^{4} \int d^9 x\, D^{15} R^4 + \dots$	$n=5$		$L_{UV}=2 < L_{cr}=3$

Maximal supergravities

Z. Bern et al

All D>4 UV divergences are at loop order below critical! $\Rightarrow$ Local H-symmetry and G-symmetry must have anomalies!

Cremmer-Julia 1979 gauge-fixed local H=SU(8) of 4D maximal supergravity in the symmetric gauge

They also mention Iwaswa gauge: "**The choice of gauge is up to the user**; 11D people seem to like the "Iwasawa" or triangular gauge best (we have seen that it has a remarkable polynomiality). The canonical or symmetrical gauge is more familiar

In **KSVP** in 2024 we have gauge-fixed different versions of D-dimensional supergravities in various gauges.

Cremmer-Julia 1979 gauge-fixed local H=SU(8) of 4D maximal supergravity in the symmetric gauge

They also mention Iwaswa gauge: "**The choice of gauge is up to the user**; 11D people seem to like the "Iwasawa" or triangular gauge best (we have seen that it has a remarkable polynomiality). The canonical or symmetrical gauge is more familiar

In **KSVP** in 2024 we have gauge-fixed different versions of D-dimensional supergravities in various gauges.

We agree with CJ, the choice of the gauge is up to the user

However, quantum theory is consistent only if these gauges give the same S-matrix: the gauge equivalence of all these versions has to be investigated.

The local H-symmetry must be anomaly-free for the S-matrix to be independent of the user's choice!

A Tale of Two Supergravities in dimension D:

G $E_{8(8)} \supset E_{7(7)} \supset E_{6(6)} \supset E_{5(5)} \supset E_{4(4)} \supset E_{3(3)} \supset E_{2(2)} \supset E_{1(1)} \supset E_{0(0)}$

$D=3 \leftarrow D=4 \leftarrow D=5 \leftarrow D=6 \leftarrow D=7 \leftarrow D=8 \leftarrow D=9 \leftarrow D=10 \leftarrow D=11$

B. Julia
Group
Disintegrations

A Tale of Two Supergravities in dimension D:

G $E_{8(8)} \supset E_{7(7)} \supset E_{6(6)} \supset E_{5(5)} \supset E_{4(4)} \supset E_{3(3)} \supset E_{2(2)} \supset E_{1(1)} \supset E_{0(0)}$

$D=3 \leftarrow D=4 \leftarrow D=5 \leftarrow D=6 \leftarrow D=7 \leftarrow D=8 \leftarrow D=9 \leftarrow D=10 \leftarrow D=11$

B. Julia
Group
Disintegrations

It is not widely recognized that there are two different types of supergravities in dimension D, with the same amount of local supersymmetry.

Supergravity I and supergravity II, KSVP

I: Supergravities with global U-duality symmetry G and local H symmetry, where H is the maximal compact subgroup of G, physical scalars in **$(G/H)_D$ coset space**
all scalars in the action in **symmetric H-gauge are** dilatons,
have **non-polynomial** dependence

4D: Cremmer Julia, 1979 de Wit, Nicolai 1982

6D Tanii,1984 Bergshoeff, Samtleben, Sezgin 2008

A Tale of Two Supergravities in dimension D:

G $E_{8(8)} \supset E_{7(7)} \supset E_{6(6)} \supset E_{5(5)} \supset E_{4(4)} \supset E_{3(3)} \supset E_{2(2)} \supset E_{1(1)} \supset E_{0(0)}$

$D=3 \leftarrow D=4 \leftarrow D=5 \leftarrow D=6 \leftarrow D=7 \leftarrow D=8 \leftarrow D=9 \leftarrow D=10 \leftarrow D=11$

B. Julia
Group
Disintegrations

It is not widely recognized that there are two different types of supergravities in dimension D, with the same amount of local supersymmetry.

Supergravity I and supergravity II, KSVP

I: Supergravities with global U-duality symmetry G and local H symmetry, where H is the maximal compact subgroup of G, physical scalars in **$(G/H)_D$ coset space**
all scalars in the action in **symmetric H-gauge are** dilatons,
have **non-polynomial** dependence

4D: Cremmer Julia, 1979 de Wit, Nicolai 1982

6D Tanii,1984 Bergshoeff, Samtleben, Sezgin 2008

II: Supergravities dimensionally reduced from higher dimensions D+n, **without dualization**. These have less global and local symmetries: higher dimensions have smaller U-dualities and smaller maximal subgroups inherited from higher dimensions

Some of the scalars in the action **necessarily** have **polynomial** dependence: **axions**

$D+1 \rightarrow D$ examples

4D: Andrianopoli, D'Auria, Ferrara and Lledo, 2002

6D Cowdall, 1998

I: Supergravities where **all scalars** in the action in **symmetric gauge (dilatons)** have **non-polynomial** dependence

G/H coset space

$$\mathcal{V} = e^{\phi}$$

any D

Amplitudes!

Marcus, 1985

Global H-symmetry anomalies

natural physical parameterization of the scalar vielbein is where ϕ is in a noncompact part of the algebra

I: Supergravities where **all scalars** in the action in **symmetric gauge (dilatons)** have **non-polynomial** dependence

G/H coset space

$$\mathcal{V} = e^{\phi}$$

any D

Amplitudes!

Marcus, 1985

Global H-symmetry anomalies

natural physical parameterization of the scalar vielbein is where ϕ is in a noncompact part of the algebra

II: Supergravities where some of the scalars in the action (**axions) necessarily** have **polynomial** dependence**.** Dimensionally reduced (D+n) supergravities, no dualization, **less symmetries:**

no global H-symmetry

Marcus anomaly?

I: Supergravities where **all scalars** in the action in **symmetric gauge (dilatons)** have **non-polynomial** dependence

G/H coset space

$$\mathcal{V} = e^{\phi}$$

any D

Amplitudes!

Marcus, 1985

Global H-symmetry anomalies

natural physical parameterization of the scalar vielbein is where ϕ is in a noncompact part of the algebra

II: Supergravities where some of the scalars in the action (**axions) necessarily** have **polynomial** dependence**.** Dimensionally reduced (D+n) supergravities, no dualization, **less symmetries:**

no global H-symmetry

Marcus anomaly?

For every D

Andrianopoli, D'Auria, Ferrara, Fr'e, Minasian, Trigiante, 1996

$D+1 \to D$

Dimension of **abelian nilpotent ideals** = **min number of axionic scalars in any D** in partial Iwasawa gauges

translational symmetries
of the scalar manifolds

4D supergravity I

Cremmer Julia, **1979** de Wit, Nicolai **1982**

$$\frac{E_{7(7)}}{SU(8)}$$

70=133-63

4D supergravity II

Andrianopoli, D'Auria, Ferrara and Lledo, **2002**
Sezgin, Nieuwenhuizen, **1982**
Cremmer, Scherk, Schwarz, **1979**

5D supergravity compactified on a circle,
in the limit of vanishing masses/gaugings

$$\frac{E_{7(7)}}{SU(8)} \sim \left(\frac{E_{6(6)}}{USp(8)}, \sigma, 27_{axions}\right)$$

70 = 78-36 +1 +27

4D supergravity I

Cremmer Julia, **1979** de Wit, Nicolai **1982**

$$\frac{E_{7(7)}}{SU(8)}$$

70=133-63

4D supergravity II

Andrianopoli, D'Auria, Ferrara and Lledo, **2002**
Sezgin, Nieuwenhuizen, **1982**
Cremmer, Scherk, Schwarz, **1979**

5D supergravity compactified on a circle,
in the limit of vanishing masses/gaugings

$$\frac{E_{7(7)}}{SU(8)} \sim \left(\frac{E_{6(6)}}{USp(8)}, \sigma, 27_{axions} \right)$$

70 = 78-36 +1 +27

Both have maximal 4D $\mathcal{N}$=8 local supersymmetry

4D supergravity I

Cremmer Julia, **1979** de Wit, Nicolai **1982**

$$\frac{E_{7(7)}}{SU(8)}$$

70=133-63

4D supergravity II

Andrianopoli, D'Auria, Ferrara and Lledo, **2002**
Sezgin, Nieuwenhuizen, **1982**
Cremmer, Scherk, Schwarz, **1979**

5D supergravity compactified on a circle,
in the limit of vanishing masses/gaugings

$$\frac{E_{7(7)}}{SU(8)} \sim \left(\frac{E_{6(6)}}{USp(8)}, \sigma, 27_{axions}\right)$$

70 = 78-36 +1 +27

Both have maximal 4D $\mathcal{N}$=8 local supersymmetry

4D **gauged** supergravity I

1/8-BPS $\mathcal{N}$=8 extremal black holes: one of the $\mathcal{N}$=8 attractors with finite area of the horizon

4D **gauged** supergravity II

Non-BPS $\mathcal{N}$=8 extremal black holes: one of the $\mathcal{N}$=8 attractors with finite area of the horizon

Spontaneously broken N=8 supergravity
Cremmer, Scherk, Schwarz, **1979**

Symmetric, Iwasawa and partial Iwasawa unitary gauges

In supergravities with physical scalars in G/H coset space, the Lie algebra $\mathfrak{g}$ of a group G can be decomposed into two orthogonal subspaces: the Lie algebra $\mathfrak{h}$ of a group H and a coset space $\mathfrak{k}$. Here H is the maximal compact group in G.

$$\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{k} \qquad [\mathfrak{h}, \mathfrak{h}] \subset \mathfrak{h}\,; \qquad [\mathfrak{h}, \mathfrak{k}] \subset \mathfrak{k}\,; \qquad [\mathfrak{k}, \mathfrak{k}] \subset \mathfrak{h} \oplus \cancel{\mathfrak{k}}$$

1. Symmetric gauges

These correspond to a generalization of the polar decomposition of a linear matrix into a product of the orthogonal and asymmetric matrix and Λ are the generators of the H group and Σ are the coset generators.

$$\mathcal{V} = e^{\phi\cdot\boldsymbol{\Sigma}} e^{\theta\cdot\boldsymbol{\Lambda}}$$

A symmetric gauge is a choice $\theta = 0$

$$\mathcal{V}_{sym}(\phi^r) = e^{\phi^r K_r} \in \exp(\mathfrak{k}) \qquad r = 1, \ldots, n_{sc}$$

where K_r is a basis of the coset algebra

the on shell Lagrangian has global H-invariance

All scalars occur in the action non-polynomially

Coset generators are not in a subalgebra of G

Symmetric, Iwasawa and partial Iwasawa unitary gauges

In supergravities with physical scalars in G/H coset space, the Lie algebra $\mathfrak{g}$ of a group G can be decomposed into two orthogonal subspaces: the Lie algebra $\mathfrak{h}$ of a group H and a coset space $\mathfrak{k}$. Here H is the maximal compact group in G.

$$\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{k} \qquad [\mathfrak{h}, \mathfrak{h}] \subset \mathfrak{h}\,; \qquad [\mathfrak{h}, \mathfrak{k}] \subset \mathfrak{k}\,; \qquad [\mathfrak{k}, \mathfrak{k}] \subset \mathfrak{h} \cancel{\oplus \mathfrak{k}}$$

1. Symmetric gauges

These correspond to a generalization of the polar decomposition of a linear matrix into a product of the orthogonal and asymmetric matrix and Λ are the generators of the H group and Σ are the coset generators.

$$\mathcal{V} = e^{\phi\cdot\mathbf{\Sigma}} e^{\theta\cdot\mathbf{\Lambda}}$$

A symmetric gauge is a choice $\theta = 0$

$$\mathcal{V}_{sym}(\phi^r) = e^{\phi^r K_r} \in \exp(\mathfrak{k}) \qquad r = 1, \ldots, n_{sc}$$

where K_r is a basis of the coset algebra

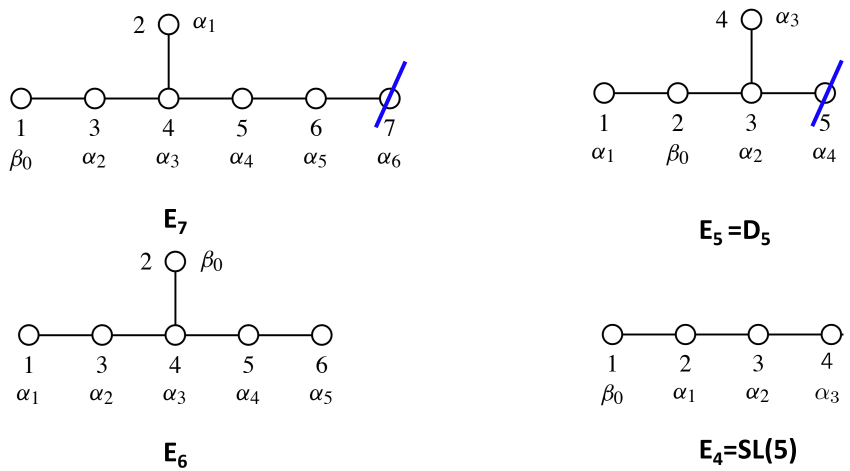
the on shell Lagrangian has global H-invariance

All scalars occur in the action non-polynomially

Coset generators are not in a subalgebra of G

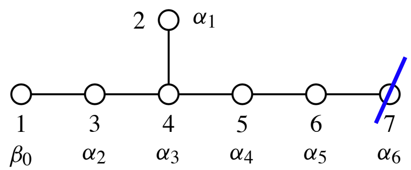
All amplitude computations fit supergravities in symmetric gauges: there is global SU(8) in 4D superamplitudes and global USp(4)xUSp(4) in 6D etc

2. Iwasawa gauge of the local H-symmetry : the right node deleted from the Dynkin diagram of $E_{11\text{-}D}$: in this gauge the theory is related to compactified D+1 supergravity

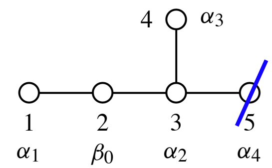


In this gauge there is no global E_7 or E_5, these are broken, at best it is E_6 or E_4

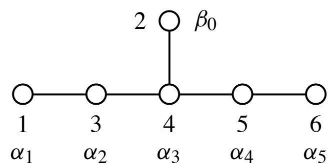
2. Iwasawa gauge of the local H-symmetry : the right node deleted from the Dynkin diagram of $E_{11\text{-}D}$: in this gauge the theory is related to compactified D+1 supergravity



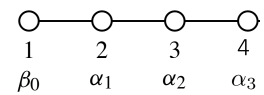
E_7



E_5 =D_5



E_6

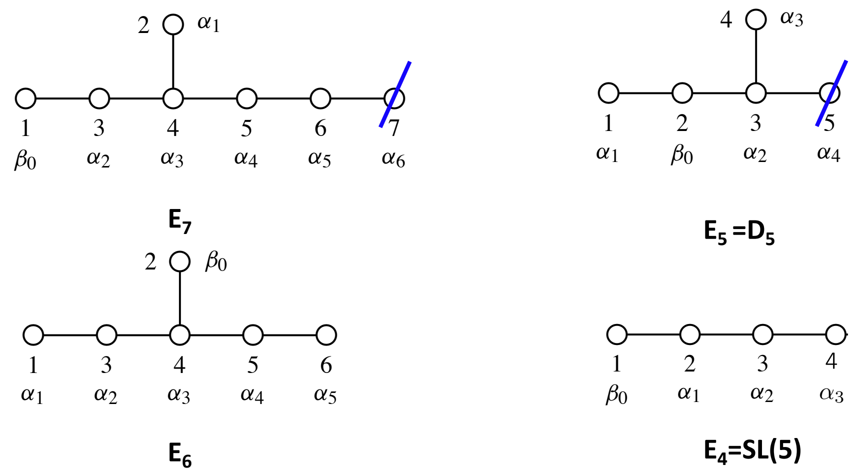


E_4=SL(5)

In this gauge there is no global E_7 or E_5, these are broken, at best it is E_6 or E_4

3. Partial Iwasawa gauge of local H-symmetry, where the related D+1 theory was gauge-fixed in the symmetric gauge for $(G/H)_{D+1}$ before compactification on a circle of the theory from D+1 to D dimensions was performed.

2. Iwasawa gauge of the local H-symmetry : the right node deleted from the Dynkin diagram of $E_{11\text{-}D}$: in this gauge the theory is related to compactified D+1 supergravity



In this gauge there is no global E_7 or E_5, these are broken, at best it is E_6 or E_4

3. Partial Iwasawa gauge of local H-symmetry, where the related D+1 theory was gauge-fixed in the symmetric gauge for $(G/H)_{D+1}$ before compactification on a circle of the theory from D+1 to D dimensions was performed.

Marcus computation of global H-symmetry anomaly is relevant in symmetric gauges, but not in Iwasawa-type gauges, unless on shell the observables in these gauges are equivalent.

Iwasawa gauge with the right node deleted

These gauges are suitable for addressing the relation between D-dimensional supergravity with $(G/H)_D$ coset space with the one in D derived from compactified D+1 dimension where the coset space is $(G/H)_{D+1}$

These gauges are associated with the Iwasawa decomposition of G with respect to H and with a solvable parametrization of the coset space so that the gauge-fixed vielbein $\mathcal{V}$ belongs to a solvable Lie group

$$\mathcal{V}_{Iwasawa}(\varphi^r) = e^{\varphi^r T_r} \in \exp(\mathscr{S})$$

Here $\{T_r\}$ is a basis of $\mathscr{S}$ $(r = 1, ..., n_{sc})$

$\mathscr{S}$ generators are in a subalgebra of G

$$\mathscr{S} = \mathtt{C} \oplus \mathtt{N}$$

Cartan subspace of the coset space

nilpotent subalgebra

When the theory originates from a higher dimensional supergravity, **C** is parametrized by the dilatonic moduli. **N** being nilpotent, is parametrized by axionic moduli.

The axionic scalars occur in the action polynomially

Iwasawa gauge with the right node deleted

These gauges are suitable for addressing the relation between D-dimensional supergravity with $(G/H)_D$ coset space with the one in D derived from compactified D+1 dimension where the coset space is $(G/H)_{D+1}$

These gauges are associated with the Iwasawa decomposition of G with respect to H and with a solvable parametrization of the coset space so that the gauge-fixed vielbein $\mathcal{V}$ belongs to a solvable Lie group

$$\mathcal{V}_{Iwasawa}(\varphi^r) = e^{\varphi^r T_r} \in \exp(\mathscr{S})$$

Here $\{T_r\}$ is a basis of $\mathscr{S}$ $(r = 1, ..., n_{sc})$

$\mathscr{S}$ generators are in a subalgebra of G

$$\mathscr{S} = \mathtt{C} \oplus \mathtt{N}$$

Cartan subspace of the coset space

nilpotent subalgebra

When the theory originates from a higher dimensional supergravity, **C** is parametrized by the dilatonic moduli. **N** being nilpotent, is parametrized by axionic moduli.

The axionic scalars occur in the action polynomially

Partial Iwasawa gauge (not a triangular one)

Example: Sezgin, Nieuwenhuizen, 1982 on spontaneously broken gauged $\mathcal{N}$=8 in 4D, derived by compactification from 5D with local USp(8)

Andrianopoli, D'Auria, Ferrara and Lledo, 2002
Type II 4D ungauged supergravity

Cremmer, Julia 1978

Amplitudes and $E_{7(7)}$

hep-th 0808.1446

What is the simplest quantum field theory?

Nima Arkani-Hamed,[a] Freddy Cachazo[b] and Jared Kaplan[a,c]

… a non-linearly realized $E_{7(7)}$ symmetry. We elucidate how non-linearly realized symmetries are reflected in the more familiar setting of pion scattering amplitudes, and go on to identify the action of $E_{7(7)}$ on amplitudes in SUGRA.

2010

$E_{7(7)}$ constraints on counterterms in $\mathcal{N}=8$ supergravity

N. Beisert[a], H. Elvang[b,c], D. Freedman[d,e], M. Kiermaier[f,*], A. Morales[d], S. Stieberger[g]

Soft Scalar Limit, direct proof that no $E_{7(7)}(\mathcal{R})$ -invariant candidate counterterm exists **below 7-loop order**

$E_{7(7)}(\mathcal{R})$ protects maximal 4D supergravity up to L=6

Cremmer, Julia 1978

Amplitudes and E $_{7(7)}$

hep-th 0808.1446

What is the simplest quantum field theory?

Nima Arkani-Hamed,[a] Freddy Cachazo[b] and Jared Kaplan[a,c]

… a non-linearly realized **E** $_{7(7)}$ symmetry. We elucidate how non-linearly realized symmetries are reflected in the more familiar setting of pion scattering amplitudes, and go on to identify the action of **E** $_{7(7)}$ on amplitudes in SUGRA.

2010

$E_{7(7)}$ constraints on counterterms in $\mathcal{N}=8$ supergravity

N. Beisert[a], H. Elvang[b,c], D. Freedman[d,e], M. Kiermaier[f,*], A. Morales[d], S. Stieberger[g]

Soft Scalar Limit, direct proof that no $E_{7(7)}$ $(\mathcal{R})$ -invariant candidate counterterm exists **below 7-loop order**

$E_{7(7)}$ $(\mathcal{R})$ protects maximal 4D supergravity up to L=6

Freedman, RK, Yamada, SSL, analogous results for $\mathcal{N}$=5,6,8, all **groups of type E7**, **soft scalar limit does <u>not</u> explain $\mathcal{N}$=5, L=4 cancellation of UV infinities in 82 diagrams**

2018

analog of $\mathcal{N}$=8, L=7

$E_{7(7)}(\mathcal{R})$: Amplitudes, single scalar soft limit

RK and Soroush, 2008

Gauge-fixed maximal supergravity in a symmetric gauge:

$$\mathscr{V} = \mathscr{V}^\dagger \qquad \mathscr{V} = \exp\begin{pmatrix} 0 & a\,\phi_{ijkl} \\ a\,\bar{\phi}^{mnpq} & 0 \end{pmatrix} \qquad \phi_{ijkl} = \frac{1}{24}\eta\epsilon_{ijklmnpq}\bar{\phi}^{mnpq} \qquad y_{ij,kl} \equiv \phi_{ijmn}\left(\frac{\tanh(\sqrt{\frac{1}{8}\bar{\phi}\phi})}{\sqrt{\bar{\phi}\phi}}\right)^{mn}_{\ \ kl}$$

non-linearly realized exact continuous $E_{7(7)}(\mathcal{R})$

$$\boxed{\delta y \equiv y' - y = \Sigma + y\bar{\Lambda} - \Lambda y - y\bar{\Sigma}y\ ,}$$

constant shift | SU(8) | nonlinear

linear $E_{7(7)}(\mathcal{R})$ symmetry

$$\delta\phi_{ijkl} = \Sigma_{ijkl}$$

$E_{7(7)}(\mathcal{R})$: Amplitudes, single scalar soft limit

RK and Soroush, 2008

Gauge-fixed maximal supergravity in a symmetric gauge:

$$\mathscr{V} = \mathscr{V}^\dagger \qquad \mathscr{V} = \exp\begin{pmatrix} 0 & a\,\phi_{ijkl} \\ a\,\bar\phi^{mnpq} & 0 \end{pmatrix} \qquad \phi_{ijkl} = \frac{1}{24}\eta\epsilon_{ijklmnpq}\bar\phi^{mnpq} \qquad y_{ij,kl} \equiv \phi_{ijmn}\left(\frac{\tanh(\sqrt{\frac{1}{8}\bar\phi\phi}}{\sqrt{\bar\phi\phi}}\right)^{mn}_{\ \ kl}$$

non-linearly realized exact continuous $E_{7(7)}(\mathcal{R})$

$$\boxed{\delta y \equiv y' - y = \Sigma + y\bar\Lambda - \Lambda y - y\bar\Sigma y\ ,}$$

constant shift SU(8) nonlinear

linear $E_{7(7)}(\mathcal{R})$ symmetry

$$\delta\phi_{ijkl} = \Sigma_{ijkl}$$

Today's talk is about **more symmetries** in maximal 4D supergravity and their role in quantum theory

Gaillard-Zumino 1981

$$Sp(56,\mathbb{R}) \supset E_{7(7)}(\mathbb{R})$$

$$1596 \gg 133$$

$E_{7(7)}(\mathcal{R})$: Amplitudes, single scalar soft limit

RK and Soroush, 2008

Gauge-fixed maximal supergravity in a symmetric gauge:

$$\mathscr{V} = \mathscr{V}^\dagger \qquad \mathscr{V} = \exp\begin{pmatrix} 0 & a\,\phi_{ijkl} \\ a\,\bar\phi^{mnpq} & 0 \end{pmatrix} \qquad \phi_{ijkl} = \frac{1}{24}\eta\epsilon_{ijklmnpq}\bar\phi^{mnpq} \qquad y_{ij,kl} \equiv \phi_{ijmn}\left(\frac{\tanh(\sqrt{\frac{1}{8}\bar\phi\phi}}{\sqrt{\bar\phi\phi}}\right)^{mn}_{kl}$$

non-linearly realized exact continuous $E_{7(7)}(\mathcal{R})$

$$\boxed{\delta y \equiv y' - y = \Sigma + y\bar\Lambda - \Lambda y - y\bar\Sigma y\ ,}$$

constant shift — SU(8) — nonlinear

linear $E_{7(7)}(\mathcal{R})$ symmetry

$$\delta\phi_{ijkl} = \Sigma_{ijkl}$$

Today's talk is about **more symmetries** in maximal 4D supergravity and their role in quantum theory

Gaillard-Zumino 1981

$$Sp(56,\mathbb{R}) \supset E_{7(7)}(\mathbb{R})$$

$$1596 \gg 133$$

now

dimension of the double quotient

1596-133-784=679

enhanced duality

takeaway message

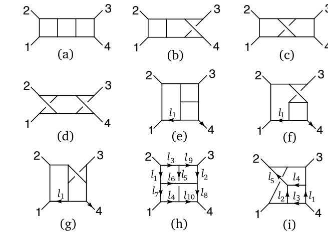
Computational data from amplitudes

4D Three-Loop Superfiniteness of N = 8 Supergravity

Bern, Carrasco, Dixon, Johansson, Kosower , Roiban, 2007

$E_{7(7)}$ $(\mathcal{R})$

Soft scalar limit

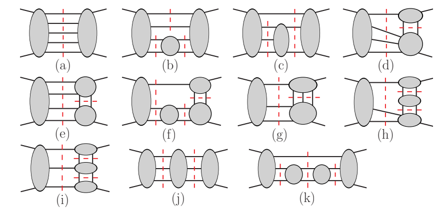


The Ultraviolet Behavior of N = 8 Supergravity at 4D Four Loops,

Bern, Carrasco, Dixon, Johansson, Roiban, 2009

$E_{7(7)}$ $(\mathcal{R})$

Soft scalar limit



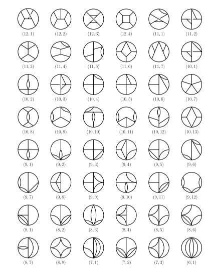
Ultraviolet Properties of N = 8 Supergravity at Five Loops

Bern, Carrasco, Wei-Ming Chen, Edison, Johansson, Parra-Martinez ,
Roiban, Mao Zenga, 2018

the five-loop critical dimension where ultraviolet divergences first occur is $D_c = 24/5 > 4$

$E_{7(7)}$ $(\mathcal{R})$

Soft scalar limit



Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger

Up to L=6 for $\mathcal{N}=8$

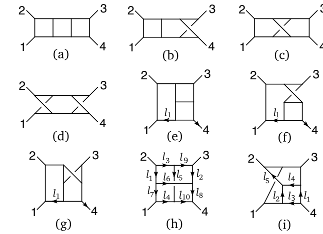
Computational data from amplitudes

4D Three-Loop Superfiniteness of N = 8 Supergravity

Bern, Carrasco, Dixon, Johansson, Kosower , Roiban, 2007

$E_{7(7)}$ $(\mathcal{R})$

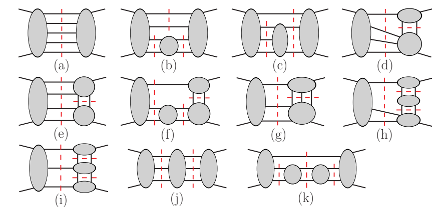
Soft scalar limit



The Ultraviolet Behavior of N = 8 Supergravity at 4D Four Loops, $E_{7(7)}$ $(\mathcal{R})$

Bern, Carrasco, Dixon, Johansson, Roiban, 2009

Soft scalar limit

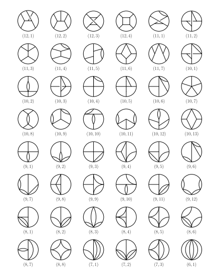


Ultraviolet Properties of N = 8 Supergravity at Five Loops

Bern, Carrasco, Wei-Ming Chen, Edison, Johansson, Parra-Martinez , Roiban, Mao Zenga, 2018

$E_{7(7)}$ $(\mathcal{R})$

Soft scalar limit



Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger

Up to L=6 for $\mathcal{N}$=8

the five-loop critical dimension where ultraviolet divergences first occur is $D_c = 24/5 > 4$

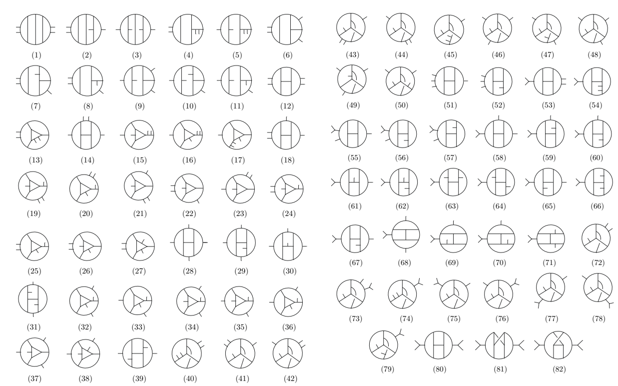
4-loop 4D $\mathcal{N}$ = 5 UV finiteness

Bern, Davies, Dennen, 2014

L_{cr}=5

Soft scalar limit of E7 type group SU(1,5) does not explain the cancellation!

Freedman, RK, Yamada



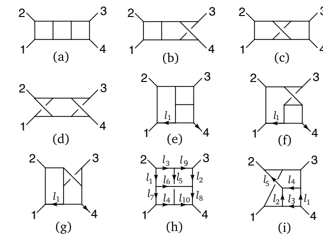
Computational data from amplitudes

4D Three-Loop Superfiniteness of N = 8 Supergravity

Bern, Carrasco, Dixon, Johansson, Kosower , Roiban, 2007

$E_{7(7)}\,(\mathcal{R})$

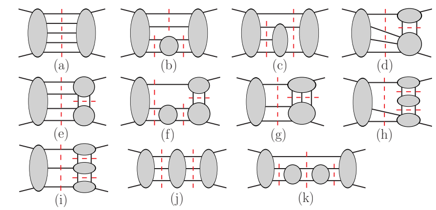
Soft scalar limit



The Ultraviolet Behavior of N = 8 Supergravity at 4D Four Loops, $E_{7(7)}\,(\mathcal{R})$

Bern, Carrasco, Dixon, Johansson, Roiban, 2009

Soft scalar limit



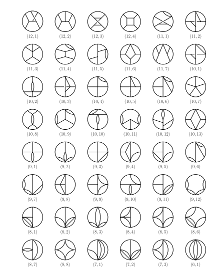
Ultraviolet Properties of N = 8 Supergravity at Five Loops

Bern, Carrasco, Wei-Ming Chen, Edison, Johansson, Parra-Martinez , Roiban, Mao Zenga, 2018

the five-loop critical dimension where ultraviolet divergences first occur is $D_c = 24/5 > 4$

$E_{7(7)}\,(\mathcal{R})$

Soft scalar limit



Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger

Up to L=6 for $\mathcal{N}$=8

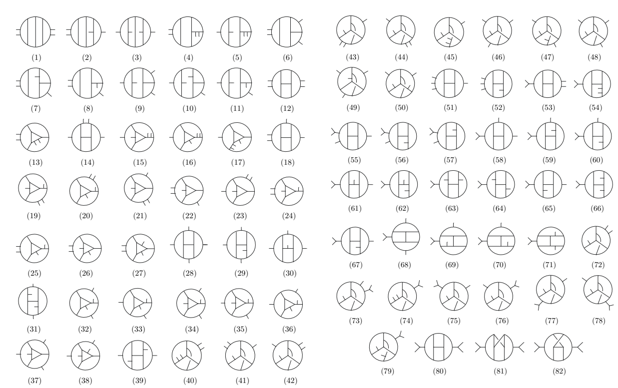
4-loop 4D $\mathcal{N}$ = 5 UV finiteness

Bern, Davies, Dennen, 2014

L_{cr}=5

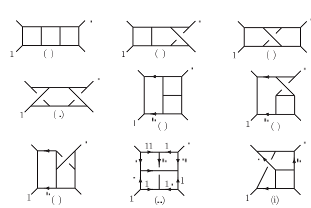
Soft scalar limit of E7 type group SU(1,5) does not explain the cancellation!

Freedman, RK, Yamada



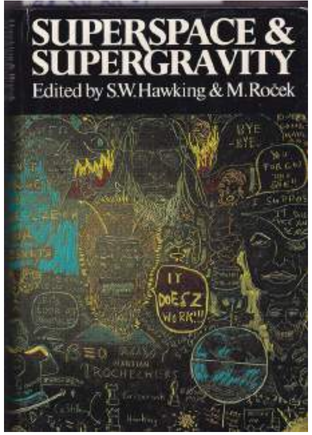
Manifest Ultraviolet Behavior for the Three-Loop Four-Point Amplitude of N=8 6D Supergravity,

Bern, Carrasco, Dixon, Johansson Roiban, 2008



$E_{5(5)}\,(\mathcal{R})$

$$M_4^{\mathcal{N}=(2,2)\ \mathrm{L}=3} = \frac{1}{\epsilon}\frac{5\zeta_3}{(4\pi)^9}\left(\frac{\kappa}{2}\right)^4 \delta^6\left(\sum_{i=1}^4 p_i^{AB}\right)\delta^8\left(\sum_{i=1}^4 q_i^{A,I}\right)\delta^8\left(\sum_{i=1}^4 \tilde{q}^{\hat{I}}_{i,\hat{A}}\right) s_{12}\, s_{23}\, s_{34}$$



1980

Bernard Julia: Group Disintegrations

Chain of U-dualities

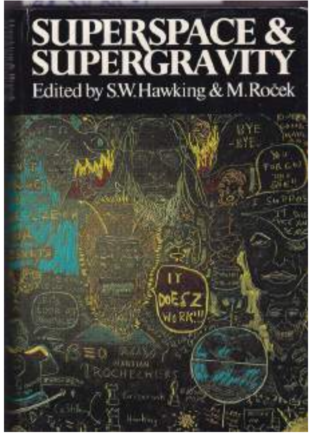
$$E_{8(8)} \supset E_{7(7)} \supset E_{6(6)} \supset E_{5(5)} \supset E_{4(4)} \supset E_{3(3)} \supset E_{2(2)} \supset E_{1(1)} \supset E_{0(0)}$$

$$D=3 \leftarrow D=4 \leftarrow D=5 \leftarrow D=6 \leftarrow D=7 \leftarrow D=8 \leftarrow D=9 \leftarrow D=10 \leftarrow D=11$$

Electro-magnetic dualities are dimension dependent

Gaillard-Zumino, 1981, Tanii, 1984

-- Sp(56) -- $E_{5(5)}$ -- $E_{3(3)}$ -- -- --



1980

Bernard Julia: Group Disintegrations

Chain of U-dualities

$$E_{8(8)} \supset E_{7(7)} \supset E_{6(6)} \supset E_{5(5)} \supset E_{4(4)} \supset E_{3(3)} \supset E_{2(2)} \supset E_{1(1)} \supset E_{0(0)}$$

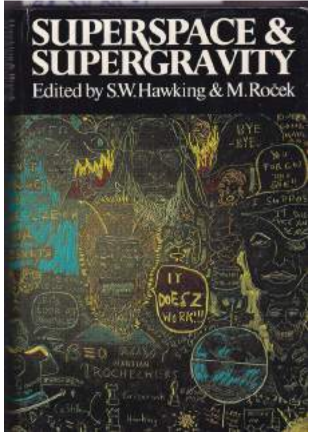
$$D = 3 \leftarrow D = 4 \leftarrow D = 5 \leftarrow D = 6 \leftarrow D = 7 \leftarrow D = 8 \leftarrow D = 9 \leftarrow D = 10 \leftarrow D = 11$$

Electro-magnetic dualities are dimension dependent

Gaillard-Zumino, 1981, Tanii, 1984

--	Sp(56)	--	$E_{5(5)}$	--	$E_{3(3)}$	--	--	--

A **question** is: Why $E_{7(7)}$ symmetry appears to protect, so far, maximal 4D supergravity from UV divergences, whereas $E_{6(6)}$, $E_{5(5)}$, $E_{4(4)}$, $E_{3(3)}$, $E_{2(2)}$ already failed to do so in all D>4 maximal supergravities where there are UV divergences at some loop order?



1980

Bernard Julia: Group Disintegrations

Chain of U-dualities

$$E_{8(8)} \supset E_{7(7)} \supset E_{6(6)} \supset E_{5(5)} \supset E_{4(4)} \supset E_{3(3)} \supset E_{2(2)} \supset E_{1(1)} \supset E_{0(0)}$$

$$D = 3 \leftarrow D = 4 \leftarrow D = 5 \leftarrow D = 6 \leftarrow D = 7 \leftarrow D = 8 \leftarrow D = 9 \leftarrow D = 10 \leftarrow D = 11$$

Electro-magnetic dualities are dimension dependent

Gaillard-Zumino, 1981, Tanii, 1984

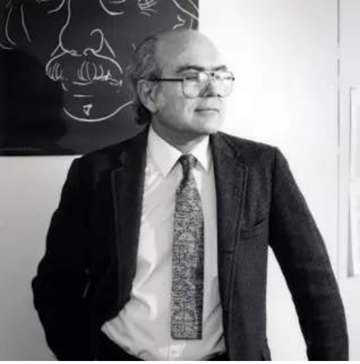
-- Sp(56) -- $E_{5(5)}$ -- $E_{3(3)}$ -- -- --

A **question** is: Why $E_{7(7)}$ symmetry appears to protect, so far, maximal 4D supergravity from UV divergences, whereas $E_{6(6)}$, $E_{5(5)}$, $E_{4(4)}$, $E_{3(3)}$, $E_{2(2)}$ already failed to do so in all D>4 maximal supergravities where there are UV divergences at some loop order?

A **quick answer** is: Only in 4D dimension a maximal duality, including GZ duality, is bigger than that of U-duality:

dim [Sp(56)] >> dim [$E_{7(7)}$]

Only in **4D** one can argue **quantum equivalence** of different gauges in supergravities, using these **extra symmetries**



Was undervalued

DUALITY ROTATIONS FOR INTERACTING FIELDS*

Mary K. GAILLARD
LAPP, Annecy-le-Vieux, France

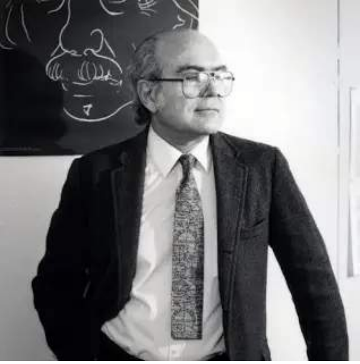
Bruno ZUMINO
CERN, Geneva, Switzerland

Received 26 May 1981

We study the properties of interacting field theories which are invariant under duality rotations which transform a vector field strength into its dual. We consider non-abelian duality groups and find that the largest group for n interacting field strengths is the non-compact Sp($2n$,R), which has U(n) as its maximal compact subgroup. We show that invariance of the equations of motion requires that the lagrangian change in a particular way under duality. We use this property to demonstrate the existence of conserved currents, the invariance of the energy-momentum tensor and the S-matrix, and also in the general construction of the lagrangian.

e. g. 4D, $\mathcal{N}$=8 case n=28: Sp(56) duality, maximal compact subgroup U(28)

1596	784
$E_{7(7)}$	SU(8)
133	63



Was undervalued

DUALITY ROTATIONS FOR INTERACTING FIELDS*

Mary K. GAILLARD

LAPP, Annecy-le-Vieux, France

Bruno ZUMINO

CERN, Geneva, Switzerland

Received 26 May 1981

We study the properties of interacting field theories which are invariant under duality rotations which transform a vector field strength into its dual. We consider non-abelian duality groups and find that the largest group for n interacting field strengths is the non-compact Sp($2n$,R), which has U(n) as its maximal compact subgroup. We show that invariance of the equations of motion requires that the lagrangian change in a particular way under duality. We use this property to demonstrate the existence of conserved currents, the invariance of the energy-momentum tensor and the S-matrix, and also in the general construction of the lagrangian.

e. g. 4D, $\mathcal{N}$=8 case

n=28: Sp(56) duality, maximal compact subgroup U(28)

1596	784
$E_{7(7)}$	SU(8)
133	63

What is new?

GZ duality in **all D + data from amplitudes**

Maximal Gaillard-Zumino (GZ) electro-magnetic duality is available in even dimensions **D=2k**

For **even k** duality group is **symplectic Sp(2n)**, for **odd k** it is **orthogonal SO(n,n)**.

Supergravities with G/H coset spaces have local H symmetry which can be gauge-fixed in

symmetric, or Iwasawa type gauges

In 4D

$$\dim[Sp(56)] \gg \dim[E_{7(7)}] \qquad \mathcal{N}=8$$

$$\dim[Sp(2n)] \gg \dim[G_U] \qquad \mathcal{N}=5,6$$

Maximal Gaillard-Zumino (GZ) electro-magnetic duality is available in even dimensions **D=2k**

For **even k** duality group is **symplectic Sp(2n)**, for **odd k** it is **orthogonal SO(n,n)**.

Supergravities with G/H coset spaces have local H symmetry which can be gauge-fixed in

symmetric, or Iwasawa type gauges

In 4D

$$\dim[Sp(56)] \gg \dim[E_{7(7)}] \qquad \mathcal{N}=8$$

$$\dim[Sp(2n)] \gg \dim[G_U] \qquad \mathcal{N}=5,6$$

These extra symmetries allow to establish on shell equivalence of theories quantized in various gauges in supergravities I and II (to be described).

Maximal Gaillard-Zumino (GZ) electro-magnetic duality is available in even dimensions **D=2k**

For **even k** duality group is **symplectic Sp(2n)**, for **odd k** it is **orthogonal SO(n,n)**.

Supergravities with G/H coset spaces have local H symmetry which can be gauge-fixed in

symmetric, or Iwasawa type gauges

In 4D

$$\dim[Sp(56)] \gg \dim[E_{7(7)}] \qquad \mathcal{N}=8$$

$$\dim[Sp(2n)] \gg \dim[G_U] \qquad \mathcal{N}=5,6$$

These extra symmetries allow to establish on shell equivalence of theories quantized in various gauges in supergravities I and II (to be described).

In 6D and 8D GZ duality groups have the same dimension as U-duality groups G, in odd dimensions there is no GZ duality. Therefore for all D > 4 enhanced symmetries are not available to establish

quantum equivalence

This is consistent with UV divergences below critical loop order in all D > 4 supergravities and absence of these so far in 4D N > 4 supergravities.

Maximal Gaillard-Zumino (GZ) electro-magnetic duality is available in even dimensions **D=2k**

For **even k** duality group is **symplectic Sp(2n)**, for **odd k** it is **orthogonal SO(n,n)**.

Supergravities with G/H coset spaces have local H symmetry which can be gauge-fixed in

symmetric, or Iwasawa type gauges

In 4D

$$\dim[Sp(56)] \gg \dim[E_{7(7)}] \qquad \mathcal{N}=8$$

$$\dim[Sp(2n)] \gg \dim[G_U] \qquad \mathcal{N}=5,6$$

These extra symmetries allow to establish on shell equivalence of theories quantized in various gauges in supergravities I and II (to be described).

In 6D and 8D GZ duality groups have the same dimension as U-duality groups G, in odd dimensions there is no GZ duality. Therefore for all D > 4 enhanced symmetries are not available to establish

quantum equivalence

This is consistent with UV divergences below critical loop order in all D > 4 supergravities and absence of these so far in 4D N > 4 supergravities.

$$D=4,\ L_{cr}=8: \quad \kappa^{14}\int d^4x\, D^{10}R^4+\dots \quad n=0$$

$$D=5,\ L_{cr}=6: \quad \kappa^{10}\int d^5x\, D^{12}R^4+\dots \quad n=2 \qquad L_{UV}=5<L_{cr}=6$$

$$D=6,\ L_{cr}=4: \quad \kappa^{6}\int d^6x\, D^{10}R^4+\dots \quad n=0 \qquad L_{UV}=3<L_{cr}=4$$

$$D=7,\ L_{cr}=4: \quad \kappa^{6}\int d^7x\, D^{14}R^4+\dots \quad n=4 \qquad L_{UV}=2<L_{cr}=4$$

$$D=8,\ L_{cr}=3: \quad \kappa^{4}\int d^8x\, D^{12}R^4+\dots \quad n=3 \qquad L_{UV}=1<L_{cr}=3$$

$$D=9,\ L_{cr}=3: \quad \kappa^{4}\int d^9x\, D^{15}R^4+\dots \quad n=5 \qquad L_{UV}=2<L_{cr}=3$$

Maximal Gaillard-Zumino (GZ) electro-magnetic duality is available in even dimensions **D=2k**

For **even k** duality group is **symplectic Sp(2n)**, for **odd k** it is **orthogonal SO(n,n)**.

Supergravities with G/H coset spaces have local H symmetry which can be gauge-fixed in

symmetric, or Iwasawa type gauges

In 4D

$$\dim[Sp(56)] \gg \dim[E_{7(7)}] \qquad \mathcal{N}=8$$

$$\dim[Sp(2n)] \gg \dim[G_U] \qquad \mathcal{N}=5,6$$

These extra symmetries allow to establish on shell equivalence of theories quantized in various gauges in supergravities I and II (to be described).

In 6D and 8D GZ duality groups have the same dimension as U-duality groups G, in odd dimensions there is no GZ duality. Therefore for all D > 4 enhanced symmetries are not available to establish

quantum equivalence

This is consistent with UV divergences below critical loop order in all D > 4 supergravities and absence of these so far in 4D N > 4 supergravities.

$$
\begin{array}{llll}
D=4,\ L_{cr}=8: & \kappa^{14}\int d^4x\, D^{10}R^4+\dots & n=0 & \\
D=5,\ L_{cr}=6: & \kappa^{10}\int d^5x\, D^{12}R^4+\dots & n=2 & L_{UV}=5<L_{cr}=6\\
D=6,\ L_{cr}=4: & \kappa^{6}\int d^6x\, D^{10}R^4+\dots & n=0 & L_{UV}=3<L_{cr}=4\\
D=7,\ L_{cr}=4: & \kappa^{6}\int d^7x\, D^{14}R^4+\dots & n=4 & L_{UV}=2<L_{cr}=4\\
D=8,\ L_{cr}=3: & \kappa^{4}\int d^8x\, D^{12}R^4+\dots & n=3 & L_{UV}=1<L_{cr}=3\\
D=9,\ L_{cr}=3: & \kappa^{4}\int d^9x\, D^{15}R^4+\dots & n=5 & L_{UV}=2<L_{cr}=3
\end{array}
$$

On Lagrangians and gaugings of maximal supergravities

Bernard de Wit, Henning Samtleben, Mario Trigiante (2002)

We discuss the subtleties in four spacetime dimensions, where the ungauged Lagrangians are not unique and encoded in an E7(7)\Sp(56; R)/GL(28) matrix.

Symplectic Frames and Lagrangians

The measure of **enhanced duality** is a dimension of the **double quotient**

quotient space: G/H

G modulo H

Coset

G/H

where G is a group and

H is the subgroup of G

Double quotient

G\X/Y

where X is a group and

G,Y are subgroups of X

The measure of **enhanced duality** is a dimension of the **double quotient**

de Wit, Samtleben, Trigiante, 2002

$$E^{4D} = G_U(\mathbb{R})\backslash Sp(2n_v,\mathbb{R})/GL(n_v,\mathbb{R})$$

$$E^{4D}_{\mathcal{N}=8} = E_{7(7)}(\mathbb{R})\backslash Sp(56,\mathbb{R})/GL(28,\mathbb{R})$$

quotient space: G/H

G modulo H

Coset

G/H

where G is a group and

H is the subgroup of G

Double quotient

G\X/Y

where X is a group and

G,Y are subgroups of X

The measure of **enhanced duality** is a dimension of the **double quotient**

de Wit, Samtleben, Trigiante, 2002

$$E^{4D} = G_U(\mathbb{R})\backslash Sp(2n_v, \mathbb{R})/GL(n_v, \mathbb{R})$$

$$E^{4D}_{\mathcal{N}=8} = E_{7(7)}(\mathbb{R})\backslash Sp(56, \mathbb{R})/GL(28, \mathbb{R})$$

Non-trivial only in 4D, N=5,6,8

quotient space: G/H

G modulo H

Coset

G/H

where G is a group and

H is the subgroup of G

Double quotient

G\X/Y

where X is a group and

G,Y are subgroups of X

Combining theory with amplitude data one can understand the pattern, explain the data, make predictions

1.Faddeev-Fradkin-Tyutin-Batalin-Vilkovisky-Henneaux, Hamiltonian path integral for gauge theories, 1969

2.Gaillard-Zumino duality, 1981

3.Andrianopoli, D'Auria, Ferrara, Fr'e, Minasian, Trigiante: D+1-->D, abelian ideal, 1996

4.DeWit, Samtleben, Trigiante: symplectic frames, classical equivalence of various supergravities, 2002

6.Henneaux, Julia, Lekeu and Ranjbar: Hamiltonian formulation of duality, 2017

6. Bern et al: amplitude computations in D-dimensional supergravity, 1998

7. Recent RK (with Yamada, Samtleben, Van Proeyen),

and most recent: Sp(2n, $\mathcal{R}$)-invariant path integral, 2023-2024

Combining theory with amplitude data one can understand the pattern, explain the data, make predictions

1.Faddeev-Fradkin-Tyutin-Batalin-Vilkovisky-Henneaux, Hamiltonian path integral for gauge theories, 1969

2.Gaillard-Zumino duality, 1981

3.Andrianopoli, D'Auria, Ferrara, Fr'e, Minasian, Trigiante: D+1-->D, abelian ideal, 1996

4.DeWit, Samtleben, Trigiante: symplectic frames, classical equivalence of various supergravities, 2002

6.Henneaux, Julia, Lekeu and Ranjbar: Hamiltonian formulation of duality, 2017

6. Bern et al: amplitude computations in D-dimensional supergravity, 1998

7. Recent RK (with Yamada, Samtleben, Van Proeyen),

and most recent: Sp(2n, $\mathcal{R}$)-invariant path integral, 2023-2024

We apply classical results in 2, 3,4 to **quantization of supergravity I, II in different gauges using Hamiltonian path integral** based on 1, 4. We argue that the gauge-independence of the on shell S-matrix is possible due to existence of the **non-trivial double quotient in 4D $\mathcal{N}$ > 4**

$$E^{4D} = G_U(\mathbb{R})\backslash Sp(2n_v, \mathbb{R})/GL(n_v, \mathbb{R})$$

all Sp(2n) symmetries, modulo scalar and vector reparametrization

Combining theory with amplitude data one can understand the pattern, explain the data, make predictions

1.Faddeev-Fradkin-Tyutin-Batalin-Vilkovisky-Henneaux, Hamiltonian path integral for gauge theories, 1969

2.Gaillard-Zumino duality, 1981

3.Andrianopoli, D'Auria, Ferrara, Fr'e, Minasian, Trigiante: D+1-->D, abelian ideal, 1996

4.DeWit, Samtleben, Trigiante: symplectic frames, classical equivalence of various supergravities, 2002

6.Henneaux, Julia, Lekeu and Ranjbar: Hamiltonian formulation of duality, 2017

6. Bern et al: amplitude computations in D-dimensional supergravity, 1998

7. Recent RK (with Yamada, Samtleben, Van Proeyen),

and most recent: Sp(2n, $\mathcal{R}$)-invariant path integral, 2023-2024

We apply classical results in 2, 3,4 to **quantization of supergravity I, II in different gauges using Hamiltonian path integral** based on 1, 4. We argue that the gauge-independence of the on shell S-matrix is possible due to existence of the **non-trivial double quotient in 4D $\mathcal{N} > 4$**

$$E^{4D} = G_U(\mathbb{R})\backslash Sp(2n_v, \mathbb{R})/GL(n_v, \mathbb{R})$$

all Sp(2n) symmetries, modulo scalar and vector reparametrization

No evidence for local H-symmetry or global G-symmetry anomaly, and no UV divergences **so far**

For all D > 4 the corresponding quotient is trivial since the dimension of the maximal duality group, Including GZ duality, is the same as dimension of the U-duality group G_U

$$E^{D>4} = \mathrm{I}$$

double quotient is trivial

Combining theory with amplitude data one can understand the pattern, explain the data, make predictions

1.Faddeev-Fradkin-Tyutin-Batalin-Vilkovisky-Henneaux, Hamiltonian path integral for gauge theories, 1969

2.Gaillard-Zumino duality, 1981

3.Andrianopoli, D'Auria, Ferrara, Fr'e, Minasian, Trigiante: D+1-->D, abelian ideal, 1996

4.DeWit, Samtleben, Trigiante: symplectic frames, classical equivalence of various supergravities, 2002

6.Henneaux, Julia, Lekeu and Ranjbar: Hamiltonian formulation of duality, 2017

6. Bern et al: amplitude computations in D-dimensional supergravity, 1998

7. Recent RK (with Yamada, Samtleben, Van Proeyen),

and most recent: Sp(2n, $\mathcal{R}$)-invariant path integral, 2023-2024

We apply classical results in 2, 3,4 to **quantization of supergravity I, II in different gauges using Hamiltonian path integral** based on 1, 4. We argue that the gauge-independence of the on shell S-matrix is possible due to existence of the **non-trivial double quotient in 4D $\mathcal{N} > 4$**

$$E^{4D} = G_U(\mathbb{R})\backslash Sp(2n_v,\mathbb{R})/GL(n_v,\mathbb{R})$$

all Sp(2n) symmetries, modulo scalar and vector reparametrization

No evidence for local H-symmetry or global G-symmetry anomaly, and no UV divergences **so far**

For all D > 4 the corresponding quotient is trivial since the dimension of the maximal duality group, Including GZ duality, is the same as dimension of the U-duality group G_U

$$E^{D>4} = \mathrm{I}$$

double quotient is trivial

No proof available of the gauge-independence of the on shell S-matrix on the choice of the gauge or supergravity type. Evidence that in all D > 4 there are UV divergences below critical loop $\Longrightarrow$ anomalies

How to prove the on shell gauge-independence of the S-matrix?

$$\begin{pmatrix} F \\ G \end{pmatrix}\Big|_{on\ shell} = \begin{pmatrix} d\mathcal{A} \\ d\mathcal{B} \end{pmatrix}$$

Why Sp(2n,$\mathcal{R}$) helps? We use the classical construction of de Wit, Samtleben, Trigianteof 4D **symplectic frames**

Use 4D GZ duality transformation to change the Lagrangian

$$\delta L = \frac{1}{4}(FC\tilde{F} + GB\tilde{G})$$

$$F_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$$

$$\tilde{G}_{\mu\nu} = 2\frac{\partial L}{\partial F^{\mu\nu}}$$

$$\delta_{Sp(2n_v)} \begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix}$$

B=0 **manifest electric subgroup G_e**

How to prove the on shell gauge-independence of the S-matrix?

$$\begin{pmatrix} F \\ G \end{pmatrix}|_{on\ shell} = \begin{pmatrix} d\mathcal{A} \\ d\mathcal{B} \end{pmatrix}$$

Why Sp(2n,$\mathcal{R}$) helps? We use the classical construction of de Wit, Samtleben, Trigianteof 4D **symplectic frames**

Use 4D GZ duality transformation to change the Lagrangian

$$\delta_{Sp(2n_v)} \begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix}$$

$$\delta L = \frac{1}{4}(FC\tilde{F} + GB\tilde{G}) \qquad F_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$$

$$\tilde{G}_{\mu\nu} = 2\frac{\partial L}{\partial F^{\mu\nu}}$$

B=0 **manifest electric subgroup G_e**

Different symplectic frames: different manifest global symmetries of the Lagrangian G_e

Off shell Lagrangians are different in different frames: however, on shell equations of motion and BI are the same and have a U-duality G_U symmetry

$\mathcal{N}$=8 example

The SL(8, R) symplectic frame: G_e= SL(8, R)		Cremmer et al	4D action I
The $E_{6(6)}$ symplectic frame :	G_e= $E_{6(6)}$	5D $\longrightarrow$	4D action II

How to prove the on shell gauge-independence of the S-matrix?

$$\begin{pmatrix} F \\ G \end{pmatrix}|_{on\ shell} = \begin{pmatrix} d\mathcal{A} \\ d\mathcal{B} \end{pmatrix}$$

Why Sp(2n,$\mathcal{R}$) helps? We use the classical construction of de Wit, Samtleben, Trigianteof 4D **symplectic frames**

Use 4D GZ duality transformation to change the Lagrangian

$$\delta_{Sp(2n_v)} \begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix}$$

$$\delta L = \frac{1}{4}(FC\tilde{F} + GB\tilde{G}) \qquad F_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$$

$$\tilde{G}_{\mu\nu} = 2\frac{\partial L}{\partial F^{\mu\nu}}$$

B=0 **manifest electric subgroup G_e**

Different symplectic frames: different manifest global symmetries of the Lagrangian G_e

Off shell Lagrangians are different in different frames: however, on shell equations of motion and BI are the same and have a U-duality G_U symmetry

$\mathcal{N}$=8 example

The SL(8, R) symplectic frame:	G_e= SL(8, R)	Cremmer et al	4D action I
The $E_{6(6)}$ symplectic frame :	G_e= $E_{6(6)}$	5D $\longrightarrow$	4D action II

dWST: A finite transformation of L from one symplectic frame to the other can be performed using a symplectic matrix E defined modulo redefinitions of the scalar and vector fields in the action

double quotient space

$$E^{4D} = G_U(\mathbb{R}) \backslash Sp(2n_v, \mathbb{R}) / GL(n_v, \mathbb{R})$$

$$E^{4D}_{\mathcal{N}=8} = E_{7(7)}(\mathbb{R}) \backslash Sp(56, \mathbb{R}) / GL(28, \mathbb{R})$$

How to prove the on shell gauge-independence of the S-matrix?

$$\begin{pmatrix} F \\ G \end{pmatrix}|_{on\ shell} = \begin{pmatrix} d\mathcal{A} \\ d\mathcal{B} \end{pmatrix}$$

Why Sp(2n,$\mathcal{R}$) helps? We use the classical construction of de Wit, Samtleben, Trigianteof 4D **symplectic frames**

Use 4D GZ duality transformation to change the Lagrangian

$$\delta_{Sp(2n_v)} \begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix}$$

$$\delta L = \frac{1}{4}(FC\tilde{F} + GB\tilde{G}) \qquad F_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$$

$$\tilde{G}_{\mu\nu} = 2\frac{\partial L}{\partial F^{\mu\nu}}$$

B=0 **manifest electric subgroup G_e**

Different symplectic frames: different manifest global symmetries of the Lagrangian G_e

Off shell Lagrangians are different in different frames: however, on shell equations of motion and BI are the same and have a U-duality G_U symmetry

$\mathcal{N}$=8 example

The SL(8, R) symplectic frame: G_e= SL(8, R) Cremmer et al 4D action I

The $E_{6(6)}$ symplectic frame : G_e= $E_{6(6)}$ 5D $\longrightarrow$ 4D action II

dWST: A finite transformation of L from one symplectic frame to the other can be performed using a symplectic matrix E defined modulo redefinitions of the scalar and vector fields in the action

double quotient space

$$E^{4D} = G_U(\mathbb{R})\backslash Sp(2n_v, \mathbb{R})/GL(n_v, \mathbb{R})$$

$$E^{4D}_{\mathcal{N}=8} = E_{7(7)}(\mathbb{R})\backslash Sp(56, \mathbb{R})/GL(28, \mathbb{R})$$

The classical dWST result when applied to quantization of local H-symmetry of the D-dimensional Supergravity I in various gauges, shows that the on-shell S-matrix is gauge-independent. For example it must be the same in symmetric and Iwasawa gauges. Also supergravity I and II are equivalent on shell

How to prove the on shell gauge-independence of the S-matrix?

$$\begin{pmatrix} F \\ G \end{pmatrix}|_{on\ shell} = \begin{pmatrix} d\mathcal{A} \\ d\mathcal{B} \end{pmatrix}$$

Why Sp(2n,$\mathcal{R}$) helps? We use the classical construction of de Wit, Samtleben, Trigianteof 4D **symplectic frames**

Use 4D GZ duality transformation to change the Lagrangian

$$\delta_{Sp(2n_v)} \begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix}$$

$$\delta L = \frac{1}{4}(FC\tilde{F} + GB\tilde{G}) \qquad F_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$$

$$\tilde{G}_{\mu\nu} = 2\frac{\partial L}{\partial F^{\mu\nu}}$$

B=0 **manifest electric subgroup G_e**

Different symplectic frames: different manifest global symmetries of the Lagrangian G_e

Off shell Lagrangians are different in different frames: however, on shell equations of motion and BI are the same and have a U-duality G_U symmetry

$\mathcal{N}$=8 example

The SL(8, R) symplectic frame: G_e= SL(8, R) Cremmer et al 4D action I

The $E_{6(6)}$ symplectic frame : G_e= $E_{6(6)}$ 5D $\longrightarrow$ 4D action II

dWST: A finite transformation of L from one symplectic frame to the other can be performed using a symplectic matrix E defined modulo redefinitions of the scalar and vector fields in the action

double quotient space

$$E^{4D} = G_U(\mathbb{R})\backslash Sp(2n_v, \mathbb{R})/GL(n_v, \mathbb{R})$$

$$E^{4D}_{\mathcal{N}=8} = E_{7(7)}(\mathbb{R})\backslash Sp(56, \mathbb{R})/GL(28, \mathbb{R})$$

The classical dWST result when applied to quantization of local H-symmetry of the D-dimensional Supergravity I in various gauges, shows that the on-shell S-matrix is gauge-independent. For example it must be the same in symmetric and Iwasawa gauges. Also supergravity I and II are equivalent on shell

It was necessary to have dim[Sp($2n_v$)] > dim[G_U] + dim GL(n_v)

To promote classical equivalence due to Sp(2n, $\mathcal{R}$) duality of different versions of supergravities to quantum equivalence one has to address the problem: **duality symmetry** acts on

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Noether-Gaillard-Zumino **Sp(2n$_v$)** **conserved current** in 4D supergravity

and its dual

$$\delta_{Sp(2n_v)} \begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix}$$

To promote classical equivalence due to Sp(2n, $\mathcal{R}$) duality of different versions of supergravities to quantum equivalence one has to address the problem: **duality symmetry** acts on

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Noether-Gaillard-Zumino **Sp(2n$_v$)** **conserved current** in 4D supergravity

and its dual

$$\delta_{Sp(2n_v)} \begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix}$$

But in the Lorentz covariant path integral the integration variable is a vector field A_μ

To promote classical equivalence due to Sp(2n, $\mathcal{R}$) duality of different versions of supergravities to quantum equivalence one has to address the problem: **duality symmetry** acts on

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Noether-Gaillard-Zumino **Sp(2n$_v$) conserved current** in 4D supergravity

and its dual

$$\delta_{Sp(2n_v)} \begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix}$$

But in the Lorentz covariant path integral the integration variable is a vector field A_μ

To claim symplectic duality symmetry via a change of variables in the path integral one has to use a Hamiltonian path integral of the kind studied since 1969 by Faddeev, Fradkin, Vilkoviski, Batalin, Tyutin, Henneaux …

$\Downarrow$

Bona fide Noether symmetry, bona fide Sp(2n, $\mathcal{R}$) current conservation

To promote classical equivalence due to Sp(2n, $\mathcal{R}$) duality of different versions of supergravities to quantum equivalence one has to address the problem: **duality symmetry** acts on

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Noether-Gaillard-Zumino **Sp(2n$_v$)** **conserved current** in 4D supergravity

and its dual

$$\delta_{Sp(2n_v)} \begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix}$$

But in the Lorentz covariant path integral the integration variable is a vector field A_μ

To claim symplectic duality symmetry via a change of variables in the path integral one has to use a Hamiltonian path integral of the kind studied since 1969 by Faddeev, Fradkin, Vilkoviski, Batalin, Tyutin, Henneaux …

⬇

Bona fide Noether symmetry, bona fide Sp(2n, $\mathcal{R}$) current conservation

Classical on shell Sp(2n, $\mathcal{R}$) equivalence was proven by de Wit, Samtleben, Trigiante for Lorentz covariant 4D maximal supergravity

To promote classical equivalence due to Sp(2n, $\mathcal{R}$) duality of different versions of supergravities to quantum equivalence one has to address the problem: **duality symmetry** acts on

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Noether-Gaillard-Zumino **Sp(2n$_v$)** **conserved current** in 4D supergravity

and its dual

$$\delta_{Sp(2n_v)} \begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix}$$

But in the Lorentz covariant path integral the integration variable is a vector field A_μ

To claim symplectic duality symmetry via a change of variables in the path integral one has to use a Hamiltonian path integral of the kind studied since 1969 by Faddeev, Fradkin, Vilkoviski, Batalin, Tyutin, Henneaux …

⬇

Bona fide Noether symmetry, bona fide Sp(2n, $\mathcal{R}$) current conservation

Classical on shell Sp(2n, $\mathcal{R}$) equivalence was proven by de Wit, Samtleben, Trigiante for Lorentz covariant 4D maximal supergravity

Hamiltonian: simplified version of 4D maximal supergravity with vector-scalar action (no gravity and no fermions, which are duality neutral)

Start with 4D **classical** vector-scalar action of DeWit, Hamtleben, Trigiante, any symplectic frame, any gauge

$$e^{-1}\mathcal{L}_{vector} = -\frac{1}{4}\mathcal{I}_{IJ}(\phi)F^I_{\mu\nu}F^{J\mu\nu} + \frac{1}{8}\mathcal{R}_{IJ}(\phi)\,\varepsilon^{\mu\nu\rho\sigma}F^I_{\mu\nu}F^J_{\rho\sigma}$$

Using double quotient and electric subgroups of Sp(56, $\mathcal{R}$) on shell symmetry

$$\mathcal{L} \to \mathcal{L}' + GB\tilde{G}$$

Start with 4D **classical** vector-scalar action of DeWit, Hamtleben, Trigiante,
any symplectic frame, any gauge

$$e^{-1}\mathcal{L}_{vector} = -\frac{1}{4}\mathcal{I}_{IJ}(\phi)F^I_{\mu\nu}F^{J\mu\nu} + \frac{1}{8}\mathcal{R}_{IJ}(\phi)\,\varepsilon^{\mu\nu\rho\sigma}F^I_{\mu\nu}F^J_{\rho\sigma}$$

Using double quotient
and electric subgroups of
Sp(56, $\mathcal{R}$) on shell symmetry

$$\mathcal{L} \to \mathcal{L}' + GB\tilde{G}$$

Henneaux, Julia, Lekeu,
Ranjbar, 2017

Start with 4D **classical** vector-scalar action of DeWit, Hamtleben, Trigiante, any symplectic frame, any gauge

$$e^{-1}\mathcal{L}_{vector} = -\frac{1}{4}\mathcal{I}_{IJ}(\phi)F^{I}_{\mu\nu}F^{J\mu\nu} + \frac{1}{8}\mathcal{R}_{IJ}(\phi)\,\varepsilon^{\mu\nu\rho\sigma}F^{I}_{\mu\nu}F^{J}_{\rho\sigma}$$

Using double quotient and electric subgroups of Sp(56, $\mathcal{R}$) on shell symmetry

$$\mathcal{L} \to \mathcal{L}' + GB\tilde{G}$$




Henneaux, Julia, Lekeu, Ranjbar, 2017

Switch to **canonical variables and the 1st order action** with manifest **off-shell Sp(56, $\mathcal{R}$) symmetry**

$$S= \int d^4x \left(\Omega_{MN}\mathcal{B}^{Mi}\dot{\mathcal{A}}^{N}_{i} - \mathcal{M}_{MN}(\phi)\mathcal{B}^{M}_{i}\mathcal{B}^{Ni}\right)$$

$$\Omega = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$$

Start with 4D **classical** vector-scalar action of DeWit, Hamtleben, Trigiante, any symplectic frame, any gauge

$$e^{-1}\mathcal{L}_{vector} = -\frac{1}{4}\mathcal{I}_{IJ}(\phi)F^I_{\mu\nu}F^{J\mu\nu} + \frac{1}{8}\mathcal{R}_{IJ}(\phi)\,\varepsilon^{\mu\nu\rho\sigma}F^I_{\mu\nu}F^J_{\rho\sigma}$$

Using double quotient and electric subgroups of Sp(56, $\mathcal{R}$) on shell symmetry

$$\mathcal{L} \to \mathcal{L}' + GB\tilde{G}$$

Henneaux, Julia, Lekeu, Ranjbar, 2017

Switch to **canonical variables and the 1st order action** with manifest **off-shell Sp(56, $\mathcal{R}$) symmetry**

$$S= \int d^4x \left(\Omega_{MN}\mathcal{B}^{Mi}\dot{\mathcal{A}}^N_i - \mathcal{M}_{MN}(\phi)\mathcal{B}^M_i\mathcal{B}^{Ni}\right)$$

$$\Omega = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$$

RK, hep-th 2024

Path integral (assuming no gravity and no fermions)

$$\langle \mathrm{out}|S|\mathrm{in}\rangle = \int \exp\left(\frac{i}{h}\int d^4x(P^T_i\dot{Q}^i - P^T\mathcal{M}(\phi)P)\right)\prod_x \delta(P^i - P^i(Q))dP_i(x)dQ^i(t)$$

Start with 4D **classical** vector-scalar action of DeWit, Hamtleben, Trigiante, any symplectic frame, any gauge

$$e^{-1}\mathcal{L}_{vector} = -\frac{1}{4}\mathcal{I}_{IJ}(\phi)F^{I}_{\mu\nu}F^{J\mu\nu} + \frac{1}{8}\mathcal{R}_{IJ}(\phi)\,\varepsilon^{\mu\nu\rho\sigma}F^{I}_{\mu\nu}F^{J}_{\rho\sigma}$$

$$\mathcal{L} \to \mathcal{L}' + GB\tilde{G}$$

Using double quotient and electric subgroups of Sp(56, $\mathcal{R}$) on shell symmetry




Henneaux, Julia, Lekeu, Ranjbar, 2017

Switch to **canonical variables and the 1st order action** with manifest **off-shell Sp(56, $\mathcal{R}$) symmetry**

$$S= \int d^4x \left(\Omega_{MN}\mathcal{B}^{Mi}\dot{\mathcal{A}}^N_i - \mathcal{M}_{MN}(\phi)\mathcal{B}^M_i\mathcal{B}^{Ni}\right)$$

$$\Omega = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$$

RK, hep-th 2024

Path integral (assuming no gravity and no fermions)

$$\langle \mathrm{out}|S|\mathrm{in}\rangle = \int \exp\left(\frac{i}{h}\int d^4x(P_i^T\dot{Q}^i - P^T\mathcal{M}(\phi)P)\right)\prod_x \delta(P^i - P^i(Q))dP_i(x)dQ^i(t)$$

Sp(56, R) symmetry of the path integral: canonical change of variables scanning various supergravities I and II, various symplectic frames, various local H-symmetry gauges

Start with 4D **classical** vector-scalar action of DeWit, Hamtleben, Trigiante, any symplectic frame, any gauge

$$e^{-1}\mathcal{L}_{vector} = -\frac{1}{4}\mathcal{I}_{IJ}(\phi)F^{I}_{\mu\nu}F^{J\mu\nu} + \frac{1}{8}\mathcal{R}_{IJ}(\phi)\,\varepsilon^{\mu\nu\rho\sigma}F^{I}_{\mu\nu}F^{J}_{\rho\sigma}$$

$$\mathcal{L} \to \mathcal{L}' + GB\tilde{G}$$

Using double quotient and electric subgroups of Sp(56, $\mathcal{R}$) on shell symmetry




Henneaux, Julia, Lekeu, Ranjbar, 2017

Switch to **canonical variables and the 1st order action** with manifest **off-shell Sp(56, $\mathcal{R}$) symmetry**

$$S = \int d^4x \left(\Omega_{MN}\mathcal{B}^{Mi}\dot{\mathcal{A}}^{N}_{i} - \mathcal{M}_{MN}(\phi)\mathcal{B}^{M}_{i}\mathcal{B}^{Ni}\right)$$

$$\Omega = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$$

RK, hep-th 2024

Path integral (assuming no gravity and no fermions)

$$\langle \mathrm{out}|S|\mathrm{in}\rangle = \int \exp\left(\frac{i}{h}\int d^4x(P^T_i\dot{Q}^i - P^T\mathcal{M}(\phi)P)\right)\prod_x \delta(P^i - P^i(Q))dP_i(x)dQ^i(t)$$

Sp(56, R) symmetry of the path integral: canonical change of variables scanning various supergravities I and II, various symplectic frames, various local H-symmetry gauges

Local measure of integration: current consensus: one can discount the local measure of integration $\delta^4(0)$ in the functional integrals

Start with 4D **classical** vector-scalar action of DeWit, Hamtleben, Trigiante,
any symplectic frame, any gauge

$$e^{-1}\mathcal{L}_{vector} = -\frac{1}{4}\mathcal{I}_{IJ}(\phi)F^{I}_{\mu\nu}F^{J\mu\nu} + \frac{1}{8}\mathcal{R}_{IJ}(\phi)\,\varepsilon^{\mu\nu\rho\sigma}F^{I}_{\mu\nu}F^{J}_{\rho\sigma}$$

Using double quotient
and electric subgroups of
Sp(56, $\mathcal{R}$) on shell symmetry

$$\mathcal{L} \to \mathcal{L}' + GB\tilde{G}$$

Henneaux, Julia, Lekeu,
Ranjbar, 2017

Switch to **canonical variables and the 1st order action** with manifest **off-shell Sp(56, $\mathcal{R}$) symmetry**

$$S= \int d^4x \left(\Omega_{MN}\mathcal{B}^{Mi}\dot{\mathcal{A}}^{N}_{i} - \mathcal{M}_{MN}(\phi)\mathcal{B}^{M}_{i}\mathcal{B}^{Ni}\right)$$

$$\Omega = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$$

RK, hep-th 2024

Path integral (assuming no gravity and no fermions)

$$\langle \mathrm{out}|S|\mathrm{in}\rangle = \int \exp\left(\frac{i}{h}\int d^4x(P_i^T\dot{Q}^i - P^T\mathcal{M}(\phi)P)\right)\prod_x \delta(P^i - P^i(Q))dP_i(x)dQ^i(t)$$

Sp(56, R) symmetry of the path integral: canonical change of variables scanning various supergravities I and II, various symplectic frames, various local H-symmetry gauges

Local measure of integration: current consensus: one can discount the local measure of integration $\delta^4(0)$
in the functional integrals

Compare with Fujikawa anomaly

How to prove quantum equivalence of the 6D supergravity in different gauges and supergravity I and II?

This would be the proof that a local H=SO(5)xSO(5) and global $E_{5(5)}$ symmetries have no anomalies: a symmetric and Iwasawa gauges give the same S-matrix

There is no non-trivial quotient SO(5,5)/$E_{5(5)}$, so there are no different frames, no extra classical symmetries for different actions which are the same on shell!

$$SO(5,5) = E_{5(5)}$$

How to prove quantum equivalence of the 6D supergravity in different gauges and supergravity I and II?

This would be the proof that a local H=SO(5)xSO(5) and global $E_{5(5)}$ symmetries have no anomalies: a symmetric and Iwasawa gauges give the same S-matrix

There is no non-trivial quotient SO(5,5)/$E_{5(5)}$, so there are no different frames, no extra classical symmetries for different actions which are the same on shell!

$$SO(5,5) = E_{5(5)}$$

Therefore, **no proof of quantum equivalence is available in 6D**

Consistent with 6D $E_{5(5)}$ and local SO(5)xSO(5) anomalies and 3-loop UV divergences

How to prove quantum equivalence of the 6D supergravity in different gauges and supergravity I and II?

This would be the proof that a local H=SO(5)xSO(5) and global $E_{5(5)}$ symmetries have no anomalies: a symmetric and Iwasawa gauges give the same S-matrix

There is no non-trivial quotient SO(5,5)/$E_{5(5)}$, so there are no different frames, no extra classical symmetries for different actions which are the same on shell!

$$SO(5,5) = E_{5(5)}$$

Therefore, **no proof of quantum equivalence is available in 6D**

Consistent with 6D $E_{5(5)}$ and local SO(5)xSO(5) anomalies and 3-loop UV divergences

RK, 2311.10084, 2402.03453

2402.03453

JHEP06(2024)035

Same for D=5,6,7,8,9

"… **in absence of duality and supersymmetry anomalies**, which still require a better understanding, N ≥ 5 perturbative supergravities may be UV finite at higher-loops"

RK, **2019**

"…in the **absence of anomalies**, E7 type duality together with supersymmetry, might protect $\mathcal{N} \geq 5$ supergravity from UV divergences"

M. Gunaydin and RK, **2019**

RK, Yamada**, 2020**

"… **in absence of duality and supersymmetry anomalies**, which still require a better understanding, $N \geq 5$ perturbative supergravities may be UV finite at higher-loops"

RK, **2019**

"…in the **absence of anomalies**, E7 type duality together with supersymmetry, might protect $\mathcal{N} \geq 5$ supergravity from UV divergences"

M. Gunaydin and RK, **2019**
RK, Yamada**, 2020**

by **2024** we learned more about **anomalies**, and about the role of **Sp(2n) duality** in quantum theory, **strong evidence of absence of anomalies in $\mathcal{N} > 4$ supergravities in 4D**

UV divergences in loops below critical order signify local H symmetry, global G symmetry anomaly and local supersymmetry anomaly

"… **in absence of duality and supersymmetry anomalies**, which still require a better understanding, N ≥ 5 perturbative supergravities may be UV finite at higher-loops"

RK, **2019**

"…in the **absence of anomalies**, E7 type duality together with supersymmetry, might protect $\mathcal{N} \geq 5$ supergravity from UV divergences"

M. Gunaydin and RK, **2019**
RK, Yamada**, 2020**

by **2024** we learned more about **anomalies**, and about the role of **Sp(2n) duality** in quantum theory, **strong evidence of absence of anomalies in $\mathcal{N} > 4$ supergravities in 4D**

UV divergences in loops below critical order signify local H symmetry, global G symmetry anomaly and local supersymmetry anomaly

In D > 4 New loop computations are not helpful: there is a local H-symmetry anomaly, perturbative supergravities are inconsistent ✓

"… **in absence of duality and supersymmetry anomalies**, which still require a better understanding, N ≥ 5 perturbative supergravities may be UV finite at higher-loops"

RK, **2019**

"…in the **absence of anomalies**, E7 type duality together with supersymmetry, might protect $\mathcal{N} \geq 5$ supergravity from UV divergences"

M. Gunaydin and RK, **2019**
RK, Yamada**, 2020**

by **2024** we learned more about **anomalies**, and about the role of **Sp(2n) duality** in quantum theory, **strong evidence of absence of anomalies in $\mathcal{N} > 4$ supergravities in 4D**

UV divergences in loops below critical order signify local H symmetry, global G symmetry anomaly and local supersymmetry anomaly

In D > 4 New loop computations are not helpful: there is a local H-symmetry anomaly, perturbative supergravities are inconsistent ✓

In 4D $\mathcal{N} \geq 5$

New loop computations are highly desirable:
if duality enhancement is an explanation of absence of 4D UV divergences so far, we will see UV finiteness at higher loops

Closure or opening?

"… **in absence of duality and supersymmetry anomalies**, which still require a better understanding, N ≥ 5 perturbative supergravities may be UV finite at higher-loops"

RK, **2019**

"…in the **absence of anomalies**, E7 type duality together with supersymmetry, might protect $\mathcal{N} \geq 5$ supergravity from UV divergences"

M. Gunaydin and RK, **2019**
RK, Yamada**, 2020**

by **2024** we learned more about **anomalies**, and about the role of **Sp(2n) duality** in quantum theory, **strong evidence of absence of anomalies in $\mathcal{N} > 4$ supergravities in 4D**

UV divergences in loops below critical order signify local H symmetry, global G symmetry anomaly and local supersymmetry anomaly

In D > 4 — New loop computations are not helpful: there is a local H-symmetry anomaly, perturbative supergravities are inconsistent ✓

In 4D $\mathcal{N} \geq 5$ — **New loop computations are highly desirable**:
if duality enhancement is an explanation of absence of 4D UV divergences so far, we will see UV finiteness at higher loops

Closure or opening?

$$Sp(56,\mathbb{R}) \supset E_{7(7)}, \qquad Sp(32,\mathbb{R}) \supset SO^*(12), \qquad Sp(20,\mathbb{R}) \supset SU(1,5)$$

$\mathcal{N}$=8 $\qquad$ $\mathcal{N}$=6 $\qquad$ $\mathcal{N}$=5

The crucial test of my arguments: **is the double quotient in 4D $\mathcal{N}$=5 supergravity non-trivial?**

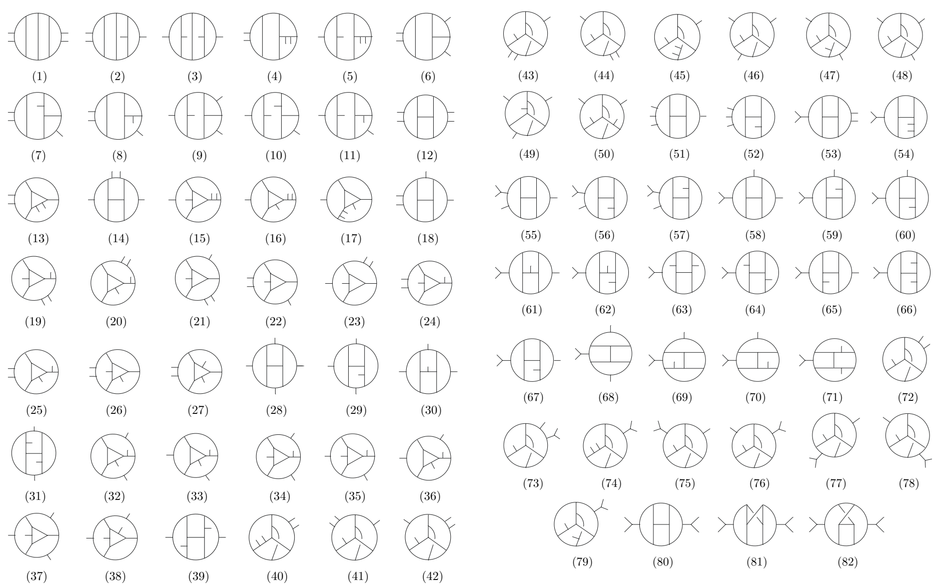
$$E^{4D}_{\mathcal{N}=5} = SU(1,5)(\mathbb{R})\backslash Sp(20,\mathbb{R})/GL(10,\mathbb{R}) : \qquad 210 - 35 - 100 = 75$$

The crucial test of my arguments: **is the double quotient in 4D $\mathcal{N}$=5 supergravity non-trivial?**

$$E^{4D}_{\mathcal{N}=5} = SU(1,5)(\mathbb{R})\backslash Sp(20,\mathbb{R})/GL(10,\mathbb{R}) : \qquad 210 - 35 - 100 = 75$$

Yes! A decent amount of enhanced dualities

Cancellation of 82 diagrams supports “no local H-anomaly ”

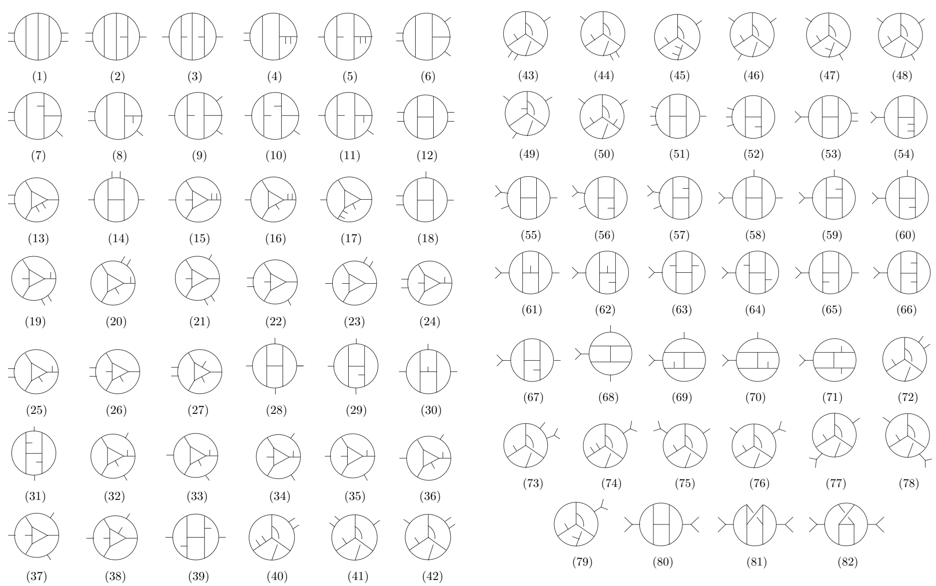


The crucial test of my arguments: **is the double quotient in 4D $\mathcal{N}$=5 supergravity non-trivial?**

$$E^{4D}_{\mathcal{N}=5} = SU(1,5)(\mathbb{R})\backslash Sp(20,\mathbb{R})/GL(10,\mathbb{R}) : \qquad 210 - 35 - 100 = 75$$

Yes! A decent amount of enhanced dualities

Cancellation of 82 diagrams supports "no local H-anomaly "



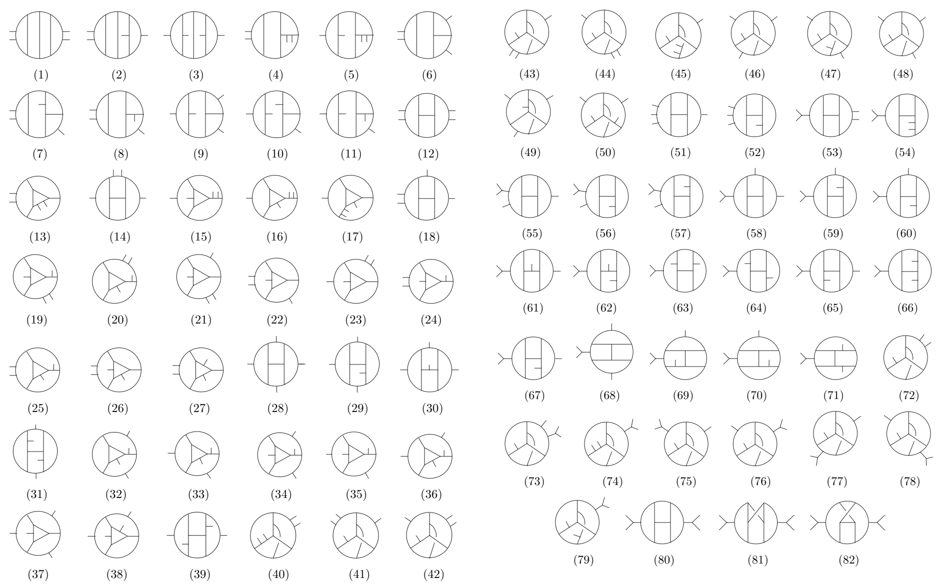
Enhanced duality explains enhanced cancellations in $\mathcal{N}$>4 supergravity in 4D

The crucial test of my arguments: **is the double quotient in 4D $\mathcal{N}$=5 supergravity non-trivial?**

$$E^{4D}_{\mathcal{N}=5} = SU(1,5)(\mathbb{R})\backslash Sp(20,\mathbb{R})/GL(10,\mathbb{R}) : \qquad 210 - 35 - 100 = 75$$

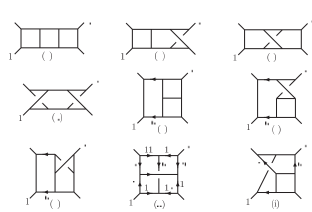
Yes! A decent amount of enhanced dualities

Cancellation of 82 diagrams supports “no local H-anomaly ”



Enhanced duality explains enhanced cancellations in $\mathcal{N}$>4 supergravity in 4D

Absence of enhanced duality consistent with the UV divergence in 6D maximal supergravity



$$M_4^{\mathcal{N}=(2,2)\ \mathrm{L}=3} = \frac{1}{\epsilon}\frac{5\zeta_3}{(4\pi)^9}\left(\frac{\kappa}{2}\right)^4 \delta^6\left(\sum_{i=1}^{4} p_i^{AB}\right)\delta^8\left(\sum_{i=1}^{4} q_i^{A,I}\right)\delta^8\left(\sum_{i=1}^{4} \tilde{q}^{\hat{I}}_{i,\hat{A}}\right) s_{12}\, s_{23}\, s_{34}$$

Back up slides

Paul Ehrenfest,

In what way does it become manifest in the fundamental laws of physics that **space has three dimensions**?

The Royal Netherlands Academy of Arts and Sciences (KNAW), Proceedings, 20 I, 1918, Amsterdam, 1918, pp. 200-209
Communicated by Prof. Dr. H. A. Lorentz

https://dwc.knaw.nl/DL/publications/PU00012213.pdf

$$V(r) = -\kappa \frac{Mm}{(D-3)r^{D-3}}\,, \quad D > 3$$

In R_3 a small disturbance leaves the trajectory finite if the energy is not too great

In R_{D-1} $D > 4$ the planet falls on the attracting centre or flies away infinitely, there is no elliptic motion.
- All trajectories have the character of spirals.

There is **no stable planetary motion at D>4**, therefore **D=4 is special in classical gravity**

Anthropic argument: we leave in D=4 where planetary motion is stable and supports life

String theoretic models of the universe postulate more than three physical space dimensions, but those beyond three are typically small and unobservable.

Amplitudes 2024

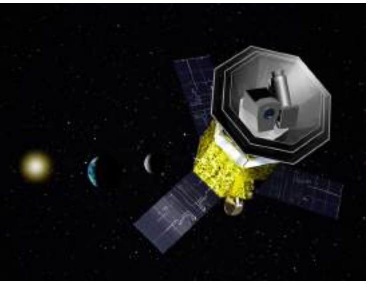
ERC Synergy Project UNIVERSE+

One may wonder why so many experts in amplitudes, quantum gravity, and string theory are interested in cosmology?

Four decades ago, a prediction was made that **galaxies were formed from quantum fluctuations** generated at the universe's first moments of existence. This was the single **most significant experimentally confirmed achievement** that brings together fundamental theoretical particle physics and cosmology.

Theory and experiment: primordial gravitational waves

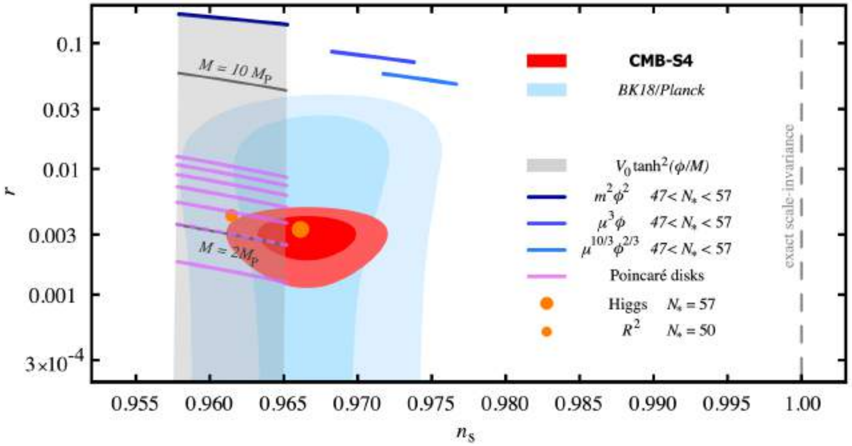
Targets include: cosmological α-attractor inflation models



LiteBIRD

Launch date: **2032**

BICEP



Probing Cosmic Inflation with Cosmic Microwave Background Polarization Survey CMB-S4, South Pole Telescope, Simons Observatory

LISA

Launch date: **2035**

GW from black hole merger

H_0 tension?

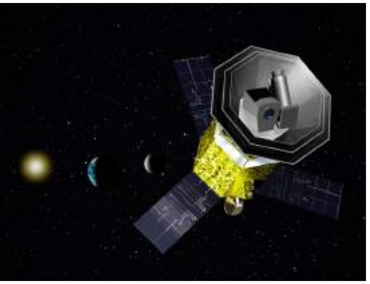
Primordial black holes

Stochastic gravitational waves

from the early evolution of the universe

Theory and experiment: primordial gravitational waves

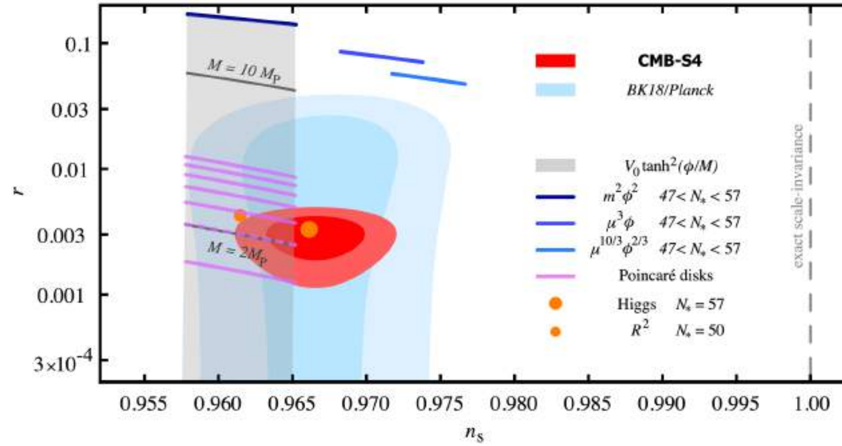
Targets include: cosmological α-attractor inflation models



LiteBIRD

Launch date: **2032**

BICEP



Probing Cosmic Inflation with Cosmic Microwave Background Polarization Survey CMB-S4, South Pole Telescope, Simons Observatory

LISA

Launch date: **2035**

GW from black hole merger

H_0 tension?

Primordial black holes

Stochastic gravitational waves

from the early evolution of the universe

Waiting for new data!

Today's talk

Theory versus the **data: amplitude loop computations**

2007, Lance Dixon call SLAC to Stanford

Hi, Renata, we have found that 3-loop UV divergence in $\mathcal{N}$=8 supergravity cancels

Hi, Lance, in what supergravity gauge you made your computation?

We do not use supergravity: we compute an on-shell S-matrix using unitarity and maximal SYM

= we do not care about the choice of the gauge

But some properties of supergravities might be **behind the scene**

Why UV divergences cancel sometimes?

Table from de Wit, Louis, 1998

	Global	Local	
D	G	H	dim [G] − dim [H]
11	1	1	$0-0=0$
10A	$SO(1,1)/Z_2$	1	$1-0=1$
$E_{1(+1)}=\mathbb{R}$ 10B	$SL(2)$	$SO(2)$	$3-1=2$
$E_{2(+2)}$ 9	$GL(2)$	$SO(2)$	$4-1=3$
8	$E_{3(+3)}\sim SL(3)\times SL(2)$	$U(2)$	$11-4=7$
7	$E_{4(+4)}\sim SL(5)$	$USp(4)$	$24-10=14$
6	$E_{5(+5)}\sim SO(5,5)$	$USp(4)\times USp(4)$	$45-20=25$
5	$E_{6(+6)}$	$USp(8)$	$78-36=42$
4	$E_{7(+7)}$	$SU(8)$	$133-63=70$
3	$E_{8(+8)}$	$SO(16)$	$248-120=128$

Homogeneous scalar manifolds G/H for maximal supergravities in integer dimensions

U-duality: $E_{d+1} = G_{U=} E_{11-D\,(11-D)} = E_{d+1\,(d+1)}$ often called E_{d+1} d=10-D

Table from de Wit, Louis, 1998

		Global	Local	
	D	G	H	dim [G] − dim [H]
	11	1	1	$0-0=0$
	10A	$SO(1,1)/Z_2$	1	$1-0=1$
$E_{1(+1)} = \mathbb{R}$	10B	$SL(2)$	$SO(2)$	$3-1=2$
$E_{2(+2)}$	9	$GL(2)$	$SO(2)$	$4-1=3$
	8	$E_{3(+3)} \sim SL(3)\times SL(2)$	$U(2)$	$11-4=7$
	7	$E_{4(+4)} \sim SL(5)$	$USp(4)$	$24-10=14$
	6	$E_{5(+5)} \sim SO(5,5)$	$USp(4)\times USp(4)$	$45-20=25$
	5	$E_{6(+6)}$	$USp(8)$	$78-36=42$
	4	$E_{7(+7)}$	$SU(8)$	$133-63=70$
	3	$E_{8(+8)}$	$SO(16)$	$248-120=128$

Homogeneous scalar manifolds G/H for maximal supergravities in integer dimensions

U-duality: $E_{d+1} = G_{U} = E_{11-D\,(11-D)} = E_{d+1\,(d+1)}$ often called E_{d+1} d=10-D

We have added blue arrows in D=4,6,8 where GZ type duality is available

I Bosonic **4D** supergravity of Cremmer-Julia-de Wit-Nicolai

$(G/H)_{4D} = E_{7(7)}/SU(8)$

$$\mathcal{L}_1 = e\,R + \tfrac{1}{4} e\,\mathrm{tr}(\partial_\mu \mathcal{M} \partial^\mu \mathcal{M}^{-1}) + \tfrac{1}{8} e\, F^{ab}_{\mu\nu} *G^{\mu\nu}_{ab},$$

a, b=1…8

28-dimensional rep of SL(8,R)

scalar-dependent linear combination of F and *F

28 F and 28 G form a 56-dimensional rep of $E_{7(7)}$

$$\mathcal{M} = \mathcal{V}^T \eta \mathcal{V}$$

In symmetric gauge 70 physical scalars in complex **35** of SU(8)

$$\mathcal{L}_2 = e\,R + \tfrac{1}{4} e\,\mathrm{tr}(\partial_\mu \mathcal{M} \partial^\mu \mathcal{M}^{-1}) - \tfrac{1}{8} H^T_{(2)} \mathcal{M} H_{(2)} \qquad H_{(2)} = \Omega \mathcal{M} * H_{(2)} \qquad H_{(2)} = \begin{pmatrix} F \\ G \end{pmatrix} \qquad \Omega = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$$

silver rule

II Bosonic **4D** supergravity derived by Andrianopoli, D'Auria, Ferrara and Lledo´, 2002 from Sezgin, Nieuwenhuizen 5D supergravity compactified on a circle, 1982, in the limit of vanishing gaugings

$$\mathbf{133} \xrightarrow[\mathrm{E}_{6,6}\times \mathrm{SO}(1,1)]{} \mathbf{78_0} + \mathbf{1_0} + \mathbf{27_{-2}} + \mathbf{27'_{+2}};$$

5D ⟶ 4D

$(G/H)_{5D} = E_{6(6)}/USp(8)$

$$\begin{aligned}\mathcal{L}_{4D} \;=\; & -\frac{1}{4} V R + \frac{3}{2} V \partial_\mu \phi \partial^\mu \phi + \frac{1}{4} V e^{-4\phi} \hat{\mathcal{N}}_{\Lambda\Sigma} \partial_\mu a^\Lambda \partial^\mu a^\Sigma + \frac{1}{24} V P^{abcd}_\mu P^\mu_{abcd} + \\ & + V \Im(\mathcal{N}_{00}) B_{\mu\nu} B^{\mu\nu} + 2V \Im(\mathcal{N}_{0\Lambda}) Z^\Lambda_{\mu\nu} B^{\mu\nu} + V \Im(\mathcal{N}_{\Lambda\Sigma}) Z^\Lambda_{\mu\nu} Z^{\Sigma\mu\nu} + \\ & + \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \left[\Re(\mathcal{N}_{00}) B_{\mu\nu} B_{\rho\sigma} + 2\Re(N_{\Lambda 0}) B_{\mu\nu} Z^\Lambda_{\rho\sigma} + \Re(\mathcal{N}_{\Lambda\Sigma}) Z^\Lambda_{\mu\nu} Z^\Sigma_{\rho\sigma}\right]\end{aligned}$$

Decomposition of $E_{7(7)}$ under the subgroup $\mathrm{E}_{6,6} \times \mathrm{SO}(1,1)$

$$\frac{G_D}{H_D} \sim \left(\frac{G_{D+1}}{H_{D+1}}, r_{D+1}, \mathbf{V}_r^{D+1}\right)$$

$$\mathbf{70} \xrightarrow[\mathrm{USp}(8)]{} \mathbf{42} + \mathbf{27} + \mathbf{1}$$

Same field content, maximal number of local supersymmetries

abelian ideal

$$\mathbf{56} \xrightarrow[\mathrm{E}_{6,6}\times \mathrm{SO}(1,1)]{} \mathbf{27_{+1}} + \mathbf{27'_{-1}} + \mathbf{1_{+3}} + \mathbf{1_{-3}}$$

CJdWN classical action in the SL(8,R) frame in a symmetric gauge

dWST classical action in the $E_{6(6)}$ frame in a parabolic gauge

⟶ AD'AFL 5D ⟶ 4D action

$\mathcal{N}$=8 4D extremal black hole attractors

Ferrara, RK, 2006

Non-BPS black holes $E_{7(7)} \to E_{6(6)} \times SO(1,1)$ Relation to 5D → 4D type II supergravity

Orbits of Exceptional Groups, Duality and BPS States in String Theory

Ferrara, Gunaydin, 1997

Extremal BPS black hole states coming from string and M theory compactifications to 4D and 5D, preserving various fractions of the original $\mathcal{N}$ = 8 supersymmetry, can be invariantly classified in terms of orbits of the fundamental representations of the exceptional groups $E_{7(7)}$ and $E_{6(6)}$

Only 1/8 BPS and non-BPS states have non vanishing entropy and regular horizons, while 1/4 and 1/2 BPS configurations lead to vanishing classical entropy

Ceresole, Ferrara, Gnecchi, Marrani, 2009

4D Non-BPS extremal KK black hole solutions with spontaneously broken $\mathcal{N}$ = 8 supersymmetry are based on solvable Lie algebra

Non-BPS orbit

$$\mathcal{O}_{non-BPS} : \begin{cases} E_{7(7)} \to E_{6(6)} \times SO(1,1)\,; \qquad \mathbf{70} \to \mathbf{42} + \mathbf{27} + \mathbf{1} \\ \mathbf{56} \to (\mathbf{27}, 1) + (\mathbf{1}, 3) + (\mathbf{27}', -1) + (\mathbf{1}', -3) \end{cases}$$

Instead of a standard $\mathcal{N}$ = 8

$\mathbf{56} \to \mathbf{28} + \mathbf{28}'$ of SU(8)

70, 4-fold antisymmetric self-real irrep of SU(8)

$\mathcal{N}$=8 D=4 extremal black holes

RK, Kol 1996
Ferrara, RK, 2006

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$$ BPS and non-BPS

4 complex central charges $z_i = \rho_i e^{i \frac{\varphi}{4}}$

½ BPS $|z_1| = |z_2| = |z_3| = |z_4|$

¼ BPS $|z_1| > |z_2|$ $|z_3| = |z_4| = 0$

1/8 BPS $|z_2| = |z_3| = |z_4| = 0$

Non-BPS $z_i = \rho\, e^{i \frac{\pi}{4}}$

I Bosonic **6D** supergravity of Tanii-Begshoeff-Samtleben-Sezgin

$(G/H)_{6D}= E_{5(5)}/SO(5)\times SO(5)$

$$\mathcal{L} = eR + \tfrac{1}{4}e\,\mathrm{tr}(\partial_\mu \mathcal{M}^{-1}\partial^\mu \mathcal{M}) - \tfrac{1}{24}e\,H^{\mathrm{T}}_{(3)}\mathcal{M}H_{(3)}$$

$$dH_{(3)} = 0, \qquad d*(\mathcal{M}H_{(3)}) = 0,$$

$$H_{(3)} = \Omega\mathcal{M} * H_{(3)}$$

silver rule

There is a local S0(5) x S0(5) H-symmetry and on shell global S0(5,5)

The kinetic terms for the scalars and 2-form potentials can then be written in the manifestly $E_{5(5)}$ =S0(5, 5)-invariant form. In symmetric gauge $\mathcal{M}$ is the S0(5,5)/ S0(5) x S0(5) coset matrix and the action has a global H-symmetry S0(5) x S0(5) = Sp(4) x Sp(4)

Marcus, 1981: 1-loop Sp(4) x Sp(4) anomaly cancels

II Bosonic **6D** supergravity derived by Cowdall, 1998, from 7D supergravity of Pernici, Pilch, van Nieuwenhuizen, 1984, and compactified on a circle, in the limit of vanishing gaugings

It has local SO(5) symmetry and an on shell global SL(5,R) inherited from 7D

$(G/H)_{7D}= SL(5)/SO(5)$

7D → 6D

$$\begin{aligned} e^{-1}\mathcal{L}_6 = R &- \frac{1}{4}e^{-\frac{5\sigma}{\sqrt{10}}}(f_{\mu\nu})^2 - \frac{1}{12}e^{-\frac{2\sigma}{\sqrt{10}}}(\Pi^{-1}{}_i{}^I H_{\mu\nu\rho I})^2 - \frac{1}{4}e^{-\frac{\sigma}{\sqrt{10}}}(\Pi_I{}^i\Pi_J{}^j F^{IJ}_{\mu\nu})^2 \\ &- \frac{1}{4}e^{\frac{3\sigma}{\sqrt{10}}}(\Pi^{-1}{}_i{}^I G_{\mu\nu I})^2 - \frac{1}{2}e^{\frac{4\sigma}{\sqrt{10}}}(\Pi_I{}^i\Pi_J{}^j F^{IJ}_{\mu})^2 - \frac{1}{2}(\partial_\mu\sigma)^2 - P_{\mu ij}P^{\mu ij} \\ &- \frac{e^{-1}}{36\sqrt{2}}\epsilon^{\mu\nu\rho\sigma\lambda\tau}B_0{}^{IJ}H_{\mu\nu\rho I}H_{\sigma\lambda\tau J} - \frac{e^{-1}}{6\sqrt{2}}\epsilon^{\mu\nu\rho\sigma\lambda\tau}H_{\mu\nu\rho I}B_\sigma{}^{IJ}G_{\lambda\tau J} \end{aligned}$$

TBSS classical action in the Iwasawa gauge → Cowdall action

Same scalar field content, but not vectors, maximal number of local supersymmetries

Different symplectic frames constructed by dWST were given in the form preserving local H-symmetry

New: bridge between Sp(56) and SU(8)

Old: bridge between $E_{7(7)}$ and SU(8)

There is a new 56-bein
no longer a group element of $E_{7(7)}$

$$\hat{\mathcal{V}}(x) = \mathsf{E}^{-1}\,\mathcal{V}(x)$$

$$\mathcal{V}(x) = \begin{pmatrix} u^{ij}{}_{IJ}(x) & -v_{klIJ}(x) \\ -v^{ijKL}(x) & u_{kl}{}^{KL}(x) \end{pmatrix}$$

$$E_{7(7)}\backslash \mathrm{Sp}(56;\mathbb{R})/GL(28) \longrightarrow \mathsf{E} = \begin{pmatrix} \mathsf{U}_{IJ}{}^{AB} & \mathsf{V}_{IJCD} \\ \mathsf{V}^{KLAB} & \mathsf{U}^{KL}{}_{CD} \end{pmatrix}$$

$$E_{7(7)} \subset \mathrm{Sp}(56;\mathbb{R})$$

i,j SU(8), I,J in $E_{7(7)}$

Lagrangian in SL(8, R) basis: CJdWN $\mathcal{L}$ **with local SU(8) H-symmetry**

Lagrangian in $E_{6(6)}$ basis: dWST $\mathcal{L}$ **with local SU(8) H-symmetry**, related to CJdWN $\mathcal{L}$ by a change of the symplectic frame

Off shell these two theories are different, but on shell equivalent due to a property of the GZ duality

Take CJdWN $\mathcal{L}$ In SL(8, R) frame and gauge-fix local H-symmetry in a symmetric gauge

Take dWST $\mathcal{L}$ In $E_{6(6)}$ frame and gauge-fix local H-symmetry in a parabolic gauge, get

5D $\longrightarrow$ 4D supergravity

Quantum equivalence between standard 4D supergravity and 5D $\longrightarrow$ 4D follows from classical on shell equivalence of different symplectic frames in 4D supergravity

Gauge-independence

Noether-Gaillard-Zumino $Sp(2n_v)$ conserved current in 4D supergravity

Duality symmetry is different from Noether symmetries by the fact that it acts on doublets of field strength's rather than on vector fields as the standard Noether symmetry

$$\delta_{Sp(2n_v)} \begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix}, \qquad C = C^T, \quad B = B^T, \quad D = -A^T$$

$$F_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu \,, \qquad \tilde{G}_{\mu\nu} = 2\frac{\partial L}{\partial F^{\mu\nu}}$$

The action of duality on vector fields is non-local, why duality symmetry is NOT a Noether symmetry

NGZ identitiy $$\frac{\delta}{\delta F^\Lambda}\Big(S[F', \varphi'] - S[F, \varphi] - \frac{1}{4}\int (\tilde{F}CF + \tilde{G}BG)\Big) = 0$$

$Sp(2n_v)$ conserved current consists of 2 parts: standard Noether current for scalars and Gaillard-Zumino current

$$J^\mu_{\mathcal{V}} = \frac{\partial \mathcal{L}_{\mathcal{V}}}{\partial(\partial_\mu \mathcal{V})}\,\delta\,\mathcal{V} \qquad \hat{J}^\mu_{GZ} \equiv \frac{1}{2}\Big(\tilde{G}^{\mu\nu} A\,\mathcal{A}_\nu - \tilde{F}^{\mu\nu} C\mathcal{A}_\nu + \tilde{G}^{\mu\nu} B\mathcal{B}_\nu - \tilde{F}^{\mu\nu} D\mathcal{B}_\nu\Big)$$

The classical Lagrangian provides the conservation of the total current, the Noether current of the scalars and the Gaillard-Zumino current of vectors

$$\partial_\mu J^\mu_{NGZ} = \partial_\mu \hat{J}^\mu_{GZ} + \partial_\mu J^\mu_{\mathcal{V}} = 0$$

The proof that the Sp(2n, R) current conservation requires that scalar and vector field equations are satisfied follows from NGZ identity

Why **E&M GZ duality** is available only in even dimensions and why symplectic or orthogonal

Only in even dimensions D=2k there are both electric and magnetic k-forms

4D, 2-forms
6D, 3-forms
8D 4-forms

$$\text{electric } F_{\mu_1\ldots\mu_k} \qquad \text{magnetic } \tilde{F}^{\mu_1\ldots\mu_k} = \frac{1}{k!} e^{-1} \epsilon^{\mu_1\ldots\mu_k\nu_1\ldots\nu_k} F_{\nu_1\ldots\nu_k}$$

Only electric forms are in the Lagrangian $\frac{1}{2k!} F \cdot {}^*G + L(\phi)$ **=$\mathbf{L_1}$** (F, *F)

the expression for the k-form G (F, *F) can be solved in terms of electric k-forms using a constraint imposed on a k-form doublet

$$H = \begin{pmatrix} F \\ G \end{pmatrix} \qquad \delta_X \begin{pmatrix} F \\ G \end{pmatrix} = X \begin{pmatrix} F \\ G \end{pmatrix}$$

$$\Omega_{k=2p} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \qquad \Omega_{k=2p+1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$^{**}H = (-1)^{k-1} H \qquad X^T \Omega = -\Omega X$$

$$H = \Omega \mathcal{M}\, {}^*H$$

silver rule

$$-\frac{1}{4k!} H^T \mathcal{M} H + L(\phi)$$ **=$\mathbf{L_2}$**(H)

$$^*H = \Omega\mathcal{M}\, {}^{**}H = (-1)^{k-1} \Omega\mathcal{M}\, H, \quad H = (\Omega\mathcal{M})^2 (-1)^{k-1} H, \quad (\Omega\mathcal{M})^2 = (-1)^{k-1} I$$

even k=D/2 : Sp(2n)

n is the number of k-forms

odd k=D/2 : SO(n,n)