Beyond the Helicity Lamppost

Continuous Spin Particles and their Interactions

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based mainly on 2303.04816 (JHEP) with P. Schuster, **K. Zhou,** 2308.16218 with Schuster
(see also 1302.1198, 1302.1577, 1404.0675 with Schuster)

Simple Question

Why do massless particles have Lorentz-invariant helicities?

Simple Questions

Why do massless particles have Lorentz-invariant helicities? Because we said so!

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Helicity operator $H = \mathbf{J} \cdot \hat{\mathbf{p}}$ is ***not*** Lorentz-invariant

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Helicity operator $H = \mathbf{J} \cdot \hat{\mathbf{p}}$ is ***not*** Lorentz-invariant

It is part of a ***covariant*** 4-vector $W^\mu = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}J_{\nu\rho}p_\sigma$ whose 3 independent components generate the little group and whose square is invariant (massive: $p = (m, \mathbf{0})$, $W = (0, m\mathbf{J})$)

The other two independent components combine transverse boost and rotation, *e.g. for* $\mathbf{p} \propto \hat{\mathbf{z}}$: $W_1 \propto J_x + K_y$ and $W_2 \propto -J_y + K_x$. *They commute* $\Rightarrow$ *"translations" of ISO(2) group structure.*

$W^2 = -W_1^2 - W_2^2 \leq 0$ is an invariant, but **independent of helicity**!

$W^2 = -\rho^2$ determines a particle's spin-scale ρ, with units of momentum.

Representations

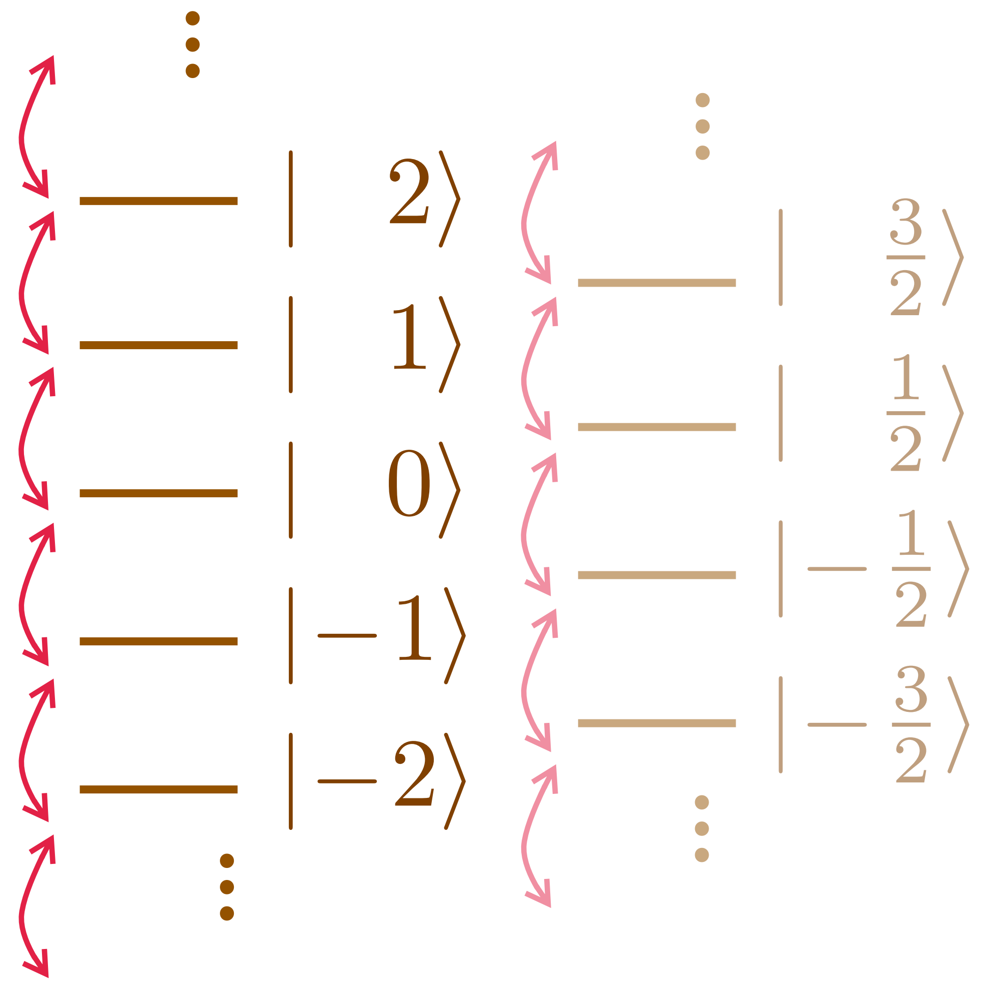
It's convenient to work in a helicity eigenstate basis: $\mathbf{J}\cdot\hat{\mathbf{p}}\,|p,\sigma\rangle = \sigma\,|p,\sigma\rangle$,

Eigenvalues σ must be (half-)integer so that 4π rotation returns state to itself, since Lorentz group is doubly connected.

Can group "translations" into raising/lowering operators $W_\pm = W_1 \pm iW_2$, which act as $W_\pm\,|p,\sigma\rangle = \rho\,|p,\sigma\pm 1\rangle$ (Coefficient fixed by $W^2 = -\rho^2$).

σ-independent coefficient $\Rightarrow$ generic massless irrep has ***infinite*** ladder of helicity states that mix under Lorentz

Called "continuous spin" particles or "CSPs"



Exception: if $\rho = 0$ the states decouple. Each $|\sigma\rangle$ is a singlet representation, related only to $|-\sigma\rangle$ by CPT.

This ***choice*** is the way to get Lorentz-invariant helicity.

Simple Questions

Why do massless particles have Lorentz-invariant helicities?

Because we have *always* assumed $\rho = 0$ – it's all we know how to do

Is there a deep reason why this assumption was required?

Or have we confused local custom with physical law?

Why can we write massless amplitudes in spinor helicity variables?

Spinor helicity's job is encoding Little Group covariance of amplitudes

$\lambda, \tilde{\lambda}$ rephase under rotations generated by H. But they are ***annihilated*** by $W_{\pm}$, so cannot encode general little group transformations.

Spinor helicity enforces the $\rho = 0$ orthodoxy.

Why has this possibility been ignored?

Many counter-arguments sound serious, but don't survive scrutiny.

Massless high spin is sick → continuous spin is sick too?

Robust constraints(e.g. Weinberg soft theorems, Weinberg-Witten) rely deeply on boost-invariance of helicity states, don't transfer to $\rho \neq 0$. (CSP soft factors a la Weinberg: Schuster+NT 1302.1198+1577)

(Note: massive high spin can be consistent — *e.g.* nuclei and string theory)

Incompatible with field theory?

Early analyses didn't allow for gauge redundancy — would have excluded QED!

Gauge theory exists! (Schuster+NT 1404.0675)

Infinitely many states at fixed energy ⇒ divergent cross-sections & thermodynamics?

Lorentz symmetry heavily constrains interactions — yields **calculable** and **well-behaved** results:

Interactions at energies $\gg \rho$ can be "scalar-like", "vector-like", or "tensor-like"

Other helicities' interactions suppressed by powers of ρ/E

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This basic structure intrigued us a decade ago. New (2023) current-coupling framework enables more general calculations that support this picture.

Simple Questions

Why do massless particles have Lorentz-invariant helicities?

Why can we write massless amplitudes in spinor helicity variables?

Because ~~we said so~~!

deviations are controlled by a small parameter, ρ/E!

Why explore $\rho \neq 0$?

- Theorist: "Because it's there"

  Falls out simply from postulates of relativity and quantum mechanics $\Rightarrow$ worth understanding!

- Phenomenologist: "Because it might **really** be there"

  Can think about experimental measurements/constraints on the spin-scale of photons and gravitons

  All SM fields are either fundamentally massless (before EWSB) or unnaturally light.

  Thinking about models with non-zero spin scales may illuminate new approaches to many SM problems.

Outline

- Invariant helicity is a special case – in general, helicities mix under Lorentz – controlled by ***spin scale*** ρ
- **Coupling to matter particles: a predictive and (so far) well-behaved IR deformation of familiar theories**
  - Gauge theory of CSPs [1404.0675]
  - Coupling matter particles to CSP fields, classically [Schuster, NT, Zhou 2303.04816] and for scattering amplitudes [Schuster, NT 2308.16218]
- An invitation to CSP scattering amplitudes

A Field Theory for All Helicities

- Intuition: As $\rho \to 0$, CSP contains all integer helicity modes
  - helicities $\pm h$ usually described by rank-h symmetric gauge field
  - CSP field $\Psi(\eta, x) \equiv \phi^{(0)}(x) + \eta^\mu \phi^{(1)}_\mu(x) + \eta^\mu \eta^\nu \phi^{(2)}_{\mu\nu}(x) + \ldots$

Lorentz acts as $x \to \Lambda x, \eta \to \Lambda \eta$

- Action:

$$\mathscr{L} = \frac{1}{2}\int_\eta \delta'(\eta^2+1)(\partial_x \Psi)^2 + \frac{1}{2}\delta(\eta^2+1)(\Delta\Psi)^2 \quad \text{with } \Delta\Psi \equiv \partial_\eta \cdot \partial_x + \rho$$

- Naively divergent integral – but fully fixed (up to "volume" factor) by symmetry & can be regulated by analytically continuing η^0.

introduced in 1404.0675 – complementary pedagogical discussion in 2303.04816

Intuition for the Free Action

$$\mathscr{L} = \frac{1}{2}\int_\eta \delta'(\eta^2+1)(\partial_x\Psi)^2 + \frac{1}{2}\delta(\eta^2+1)(\Delta\Psi)^2 \qquad \Delta\Psi \equiv \partial_\eta \cdot \partial_x + \rho$$

Decompose Ψ into tensor fields

$\rho = 0$: Decomposes as sum of familiar and Fronsdal actions for integer helicities

$\rho \neq 0$: Adds $\mathcal{O}(\rho)$ and $\mathcal{O}(\rho^2)$ rank-mixing terms

Fix gauge: $\Delta\Psi = 0$

EOM: $\delta'(\eta^2+1)\,\square\,\Psi = 0$

Solutions are functions of η with arbitrary dependence on "transverse" components, i.e. $\eta \cdot k$ and $\eta \cdot \epsilon_\pm$.

1) Only the function at unit-norm η is dynamical

2) $\eta \cdot k$-dependence is pure gauge

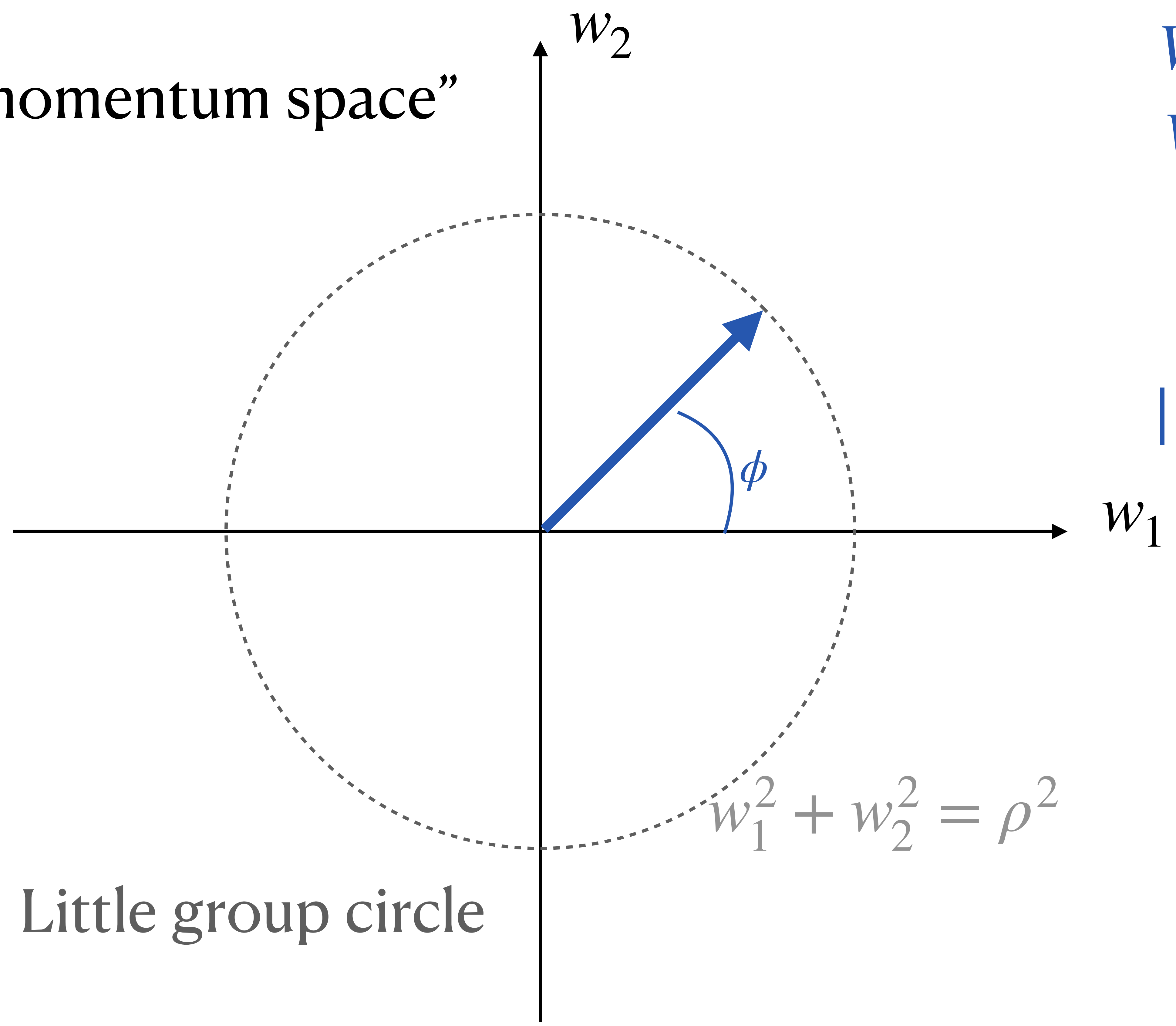
So **physical** content is **a function on unit η-circle transverse to k**

This circle has a nice relationship to the little group "translation eigenstate" basis, and helicity modes are Fourier components of this function.

Another Basis

Instead of diagonalizing helicity $\mathbf{J}\cdot\hat{\mathbf{p}}$, can instead diagonalize both "translation" generators $W_{1,2}$ – the "momentum" operators if we think of ISO(2) as 0+2D Poincare

"momentum space"



$$W_x|\phi\rangle = \rho\cos\phi\,|\phi\rangle$$
$$W_y|\phi\rangle = \rho\sin\phi\,|\phi\rangle$$
$$R|\phi\rangle = -i\partial_\phi|\phi\rangle$$

$$|\Psi\rangle = \int\frac{d\phi}{2\pi}\boxed{\psi(\phi)}|\phi\rangle$$

Fourier-conjugate to helicity basis

$$|h\rangle = \int\frac{d\phi}{2\pi}e^{ih\phi}|\phi\rangle$$ -- simpler physics

$$|\phi\rangle = \sum_h e^{-ih\phi}|h\rangle$$ -- simpler math

Note that labeling of states and their transformation properties in *any* basis depends on chosen reference directions $\hat{\mathbf{x}}, \hat{\mathbf{y}}$ – or equivalently the real and imaginary parts of a null, transverse reference vector ϵ_+^μ

Field Theory…what about interactions?

Free theory doesn't guarantee consistent interactions — but gives us a framework to look for appropriately conserved matter current

(This is where helicity ≥ 3 fail – BBVD type currents not conserved at non-zero coupling)

Current must have $\delta(\eta^2+1)(\partial_\eta \cdot \partial_x + \rho)J(\eta, x) = 0$ for gauge invariance

Once found, sourced EOM in suitable gauge is simply $\Box\,\Psi(\eta, x) = J(\eta, x)$ & can use familiar machinery to compute physical quantities [2303.04816, 2308.16218]

For technical reasons, easier to build such a current out of matter particle worldline than matter fields.

General Solutions

Assume current is local function on worldline $\int d\tau\, f(x - z(\tau), \dot{z}(\tau), \eta)$

Most general conserved solution can be written in momentum-space as

$$f(k, \dot{z}, \eta) = e^{-i\rho\frac{\eta\cdot\dot{z}}{k\cdot\dot{z}}}\hat{g}(k\cdot\dot{z}) + \mathcal{O}X(k, \dot{z}, \eta)$$

"Shape" term proportional to EOM operator $\mathcal{O}$
⇒ doesn't source radiation (like e.g. charge-radius)
Can affect CSP-exchange forces and space-time support of current

$\hat{g}$ **fully** determines worldline interactions with on-shell CSP radiation.

Expanding $\hat{g}$ in Taylor series gives "universality classes" of currents:

$$\hat{g} = \begin{cases} g & \text{scalar-like current} \\ \frac{e}{\rho} k\cdot\dot{z} & \text{vector-like current} \\ (k\cdot\dot{z})^n/\Lambda^n & \text{non-minimal currents*} \end{cases}$$

Scalar-like and vector-like current: Classical results in these cases are main focus of 2303.04816

Non-minimal currents*: graviton-like if all worldlines couple equally

Limiting Behavior: $k^0 \gg \rho$

Look at small-ρ behavior of current:

$$f(k, \eta, \dot{z}) = -\frac{e}{\rho} k \cdot \dot{z}(\tau)\, e^{-i\rho \frac{\eta \cdot \dot{z}}{k \cdot \dot{z}}}$$

$$= \boxed{-\frac{e}{\rho} k \cdot \dot{z}(\tau)} + \boxed{ie\, \eta \cdot \dot{z}(t)} + \mathcal{O}(\rho)$$

Physically irrelevant
(∝ total τ-derivative)

η-space form of usual vector current

⇒ Leading physical effects should be QED-like!

$$J(\eta, x) = \int d\tau\, d^4k\; e^{ik\cdot(z(\tau)-x)} f(k, \dot{z}, \eta)$$

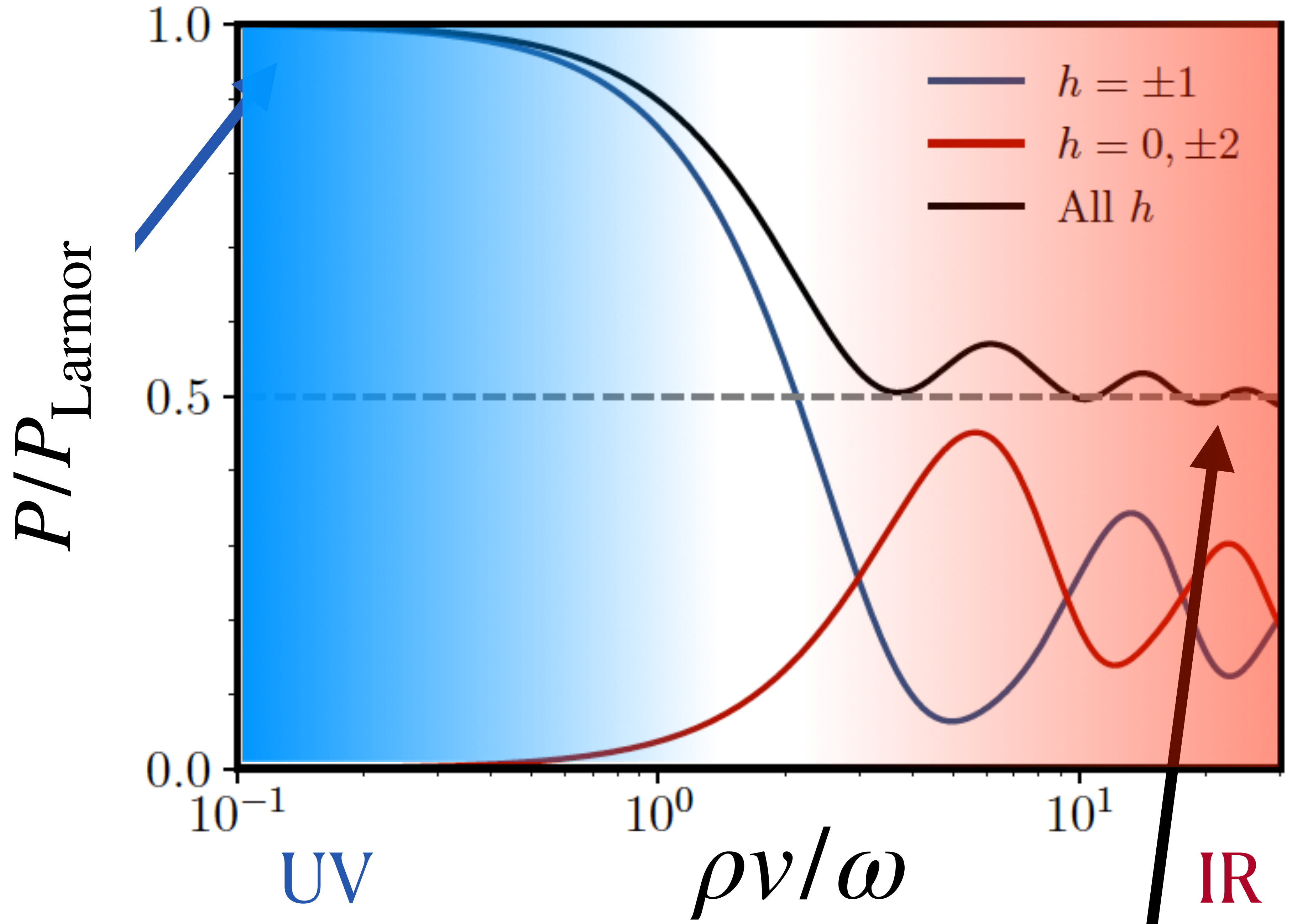
Radiation from a Moving Particle

For example, for vector-like coupling to oscillating charge:

$$P = \frac{e^2\omega^2 v_0^2}{12\pi}\left(1 - \frac{9}{80}\frac{\rho^2 v_0^2}{\omega^2} + \dots\right)$$

Standard Larmor power

For small $\rho v/\omega$, power **matches Larmor** and **dominated by h=±1 modes**



At large $\rho v/\omega$, power spread among many modes, harmonics **but total power emitted has finite limit.**

Compton-Like Amplitudes

Worldline path-integral calculated as for QED, but with η-dependent vertex operator yields "angle-basis" amplitudes:

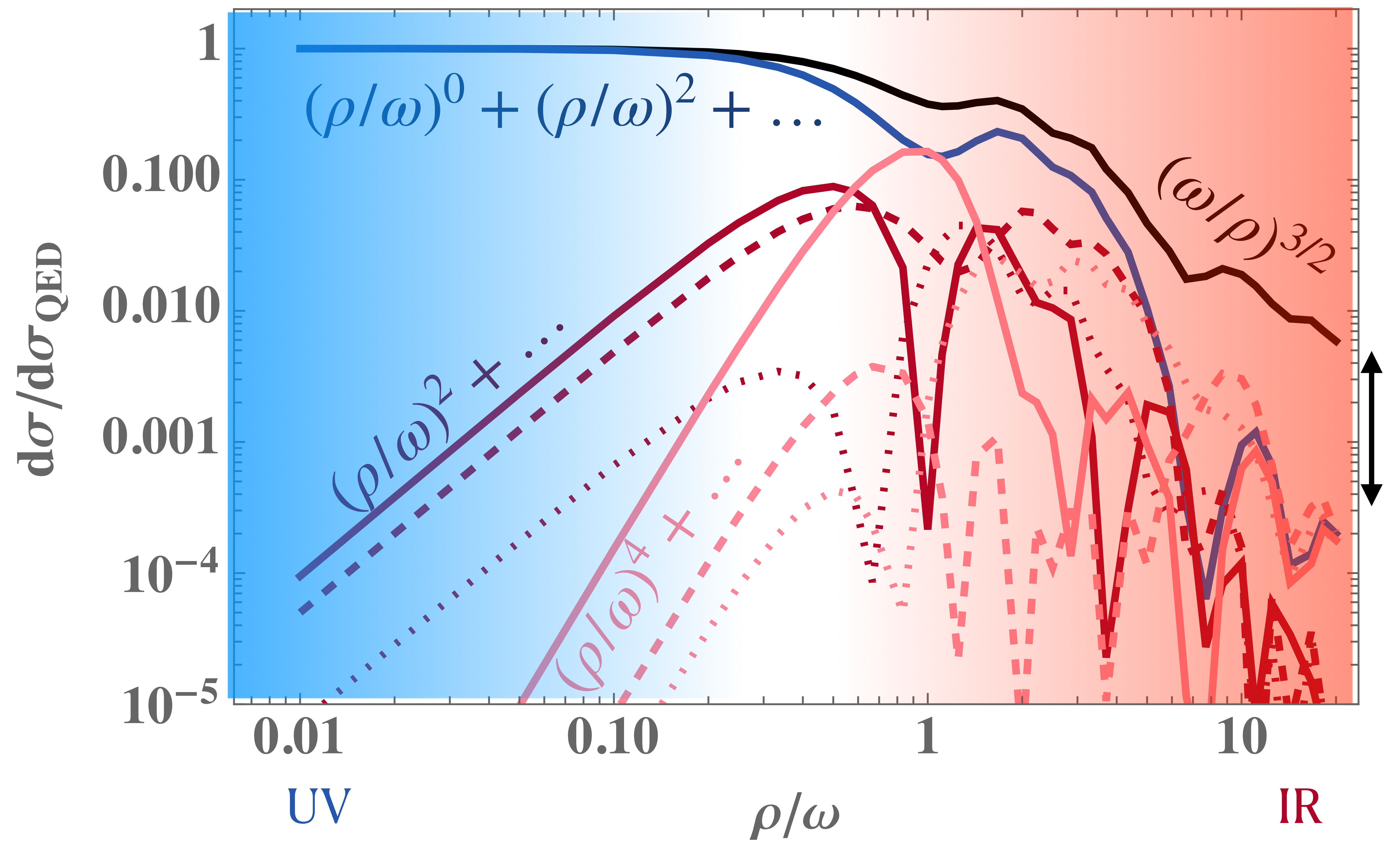
$$M(p_0, p_3, \{k_1, \eta_1\}, \{k_2, \eta_2\}) = 2\int_{-1}^{1} dx \left(\eta_1 - \frac{\eta_1 \cdot P_1(x)}{k_1 \cdot P_1(x)} k_1\right) \cdot \left(\eta_2 - \frac{\eta_2 \cdot P_2(x)}{k_2 \cdot P_2(x)} k_2\right) e^{-i\rho \frac{\eta_1 \cdot P_1(x)}{k_1 \cdot P_1(x)} - i\rho \frac{\eta_2 \cdot P_2(x)}{k_2 \cdot P_2(x)}.}$$

$P_{1,2}(x) = p_3 - p_0 \pm x\, k_{2,1}$ → at endpoints $x = \pm 1$, these are momenta appearing in $s(u)$-channel photon vertex

Helicity-basis amplitudes via Fourier transform

(1) no unphysical singularities,
(2) sensible at physical singularities,
(3) finite kinematics-differential cross-section at all energies,
(4) $\rho \to 0$ limit is Feynman-parametrization of standard scalar-QED result.

Compton-Like Cross-Section: UV to IR



Modes with $|h| \lesssim \rho/\omega$ are relevant

Spinor Helicity for CSPs

State labeling depends on reference null ϵ_+ orthogonal to k $\Longleftrightarrow$ choice of $|\mu\rangle$ $\quad \epsilon_+ = \frac{1}{[\tilde{\lambda}\tilde{\mu}]}|\mu\rangle|\tilde{\lambda}].$

Label angle-basis states as $|\lambda\rangle_\mu$ (phase encodes angle) $\quad \langle\lambda|\lambda'\rangle = (2\pi)^4\delta^{2|2}(\lambda - \lambda'$

Generators act as $T_+|\lambda\rangle_\mu = \rho\frac{\langle\lambda\mu\rangle}{[\tilde{\lambda}\tilde{\mu}]}|\lambda\rangle_\mu$ and $R|\lambda\rangle_\mu = -\frac{1}{2}\langle\lambda\partial_\lambda\rangle|\lambda\rangle_\mu.$

Amplitudes are Lorentz-invariant functions of $\lambda, \tilde{\lambda}, \mu, \tilde{\mu}$ satisfying

(1) $\langle\mu\partial_\mu\rangle A = [\tilde{\mu}\partial_{\tilde{\mu}}]A = 0$ (projective in μ)

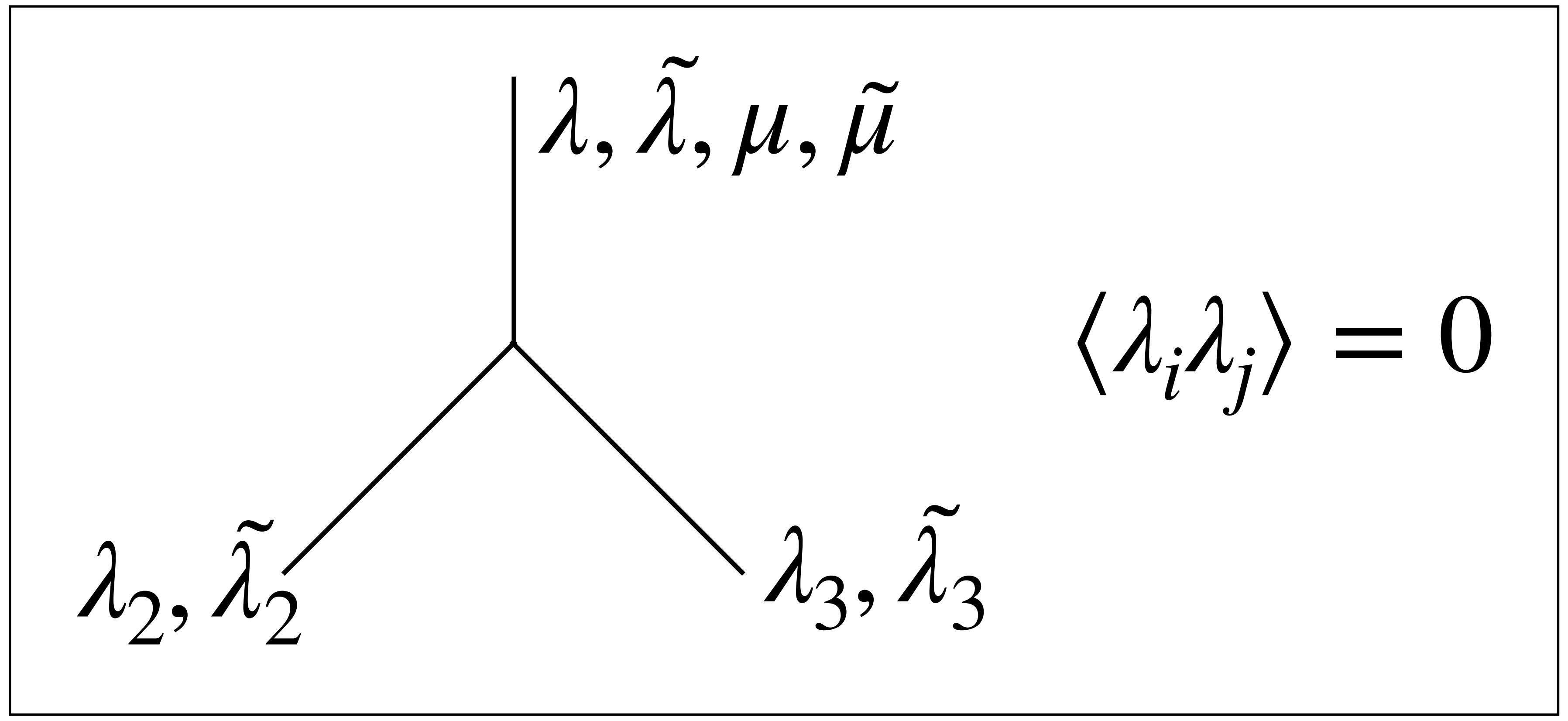
(2) **In**homogeneous $\left(i\langle\lambda\mu\rangle\langle\lambda\partial_\mu\rangle + \rho\right)A = \left(i[\tilde{\lambda}\tilde{\mu}][\tilde{\lambda}\partial_{\tilde{\mu}}] + \rho\right)A = 0$

For $\rho = 0$, simplify to
$\partial_\mu A = \partial_{\tilde{\mu}}A = 0$
i.e. amplitude depends only on λ's

Helicity amplitudes are obtained by Fourier transform $\int d\phi\, e^{ih\phi}|\lambda e^{i\phi/2}\rangle_\mu.$

Complex-Momentum 3-particle Amplitudes?

One CSP, two scalar matter legs:



T_+ covariance requirement $\left(i[\tilde{\lambda}\tilde{\mu}][\tilde{\lambda}\partial_{\tilde{\mu}}] + \rho \right) A = 0$

implies "phase" structure e.g. $A = e^{i\rho \frac{[\tilde{\mu}\,\tilde{\lambda}_2]}{[\tilde{\lambda}\,\tilde{\lambda}_2][\tilde{\mu}\,\tilde{\lambda}]}} f(\lambda_i, \tilde{\lambda}_i)$.

But $T\_$ covariance $\left(i\langle \lambda \mu \rangle \langle \lambda \partial_\mu \rangle + \rho \right) A = 0$ **cannot** be satisfied in this kinematics – all Lorentz-scalars built from this kinematic data are annihilated by $\langle \lambda \partial_\mu \rangle$.

This seems more like a limitation to the method rather than a problem with CSPs…

Complex-Momentum 3-particle Amplitudes?

Another perspective: 4-particle amplitudes contain phase factors like $e^{-i\rho\frac{\eta\cdot P}{k\cdot P}}$! In approach to 3-particle factorization kinematics, the phase oscillates rapidly because $k \cdot P \to 0$ (faster than $\eta \cdot P$).

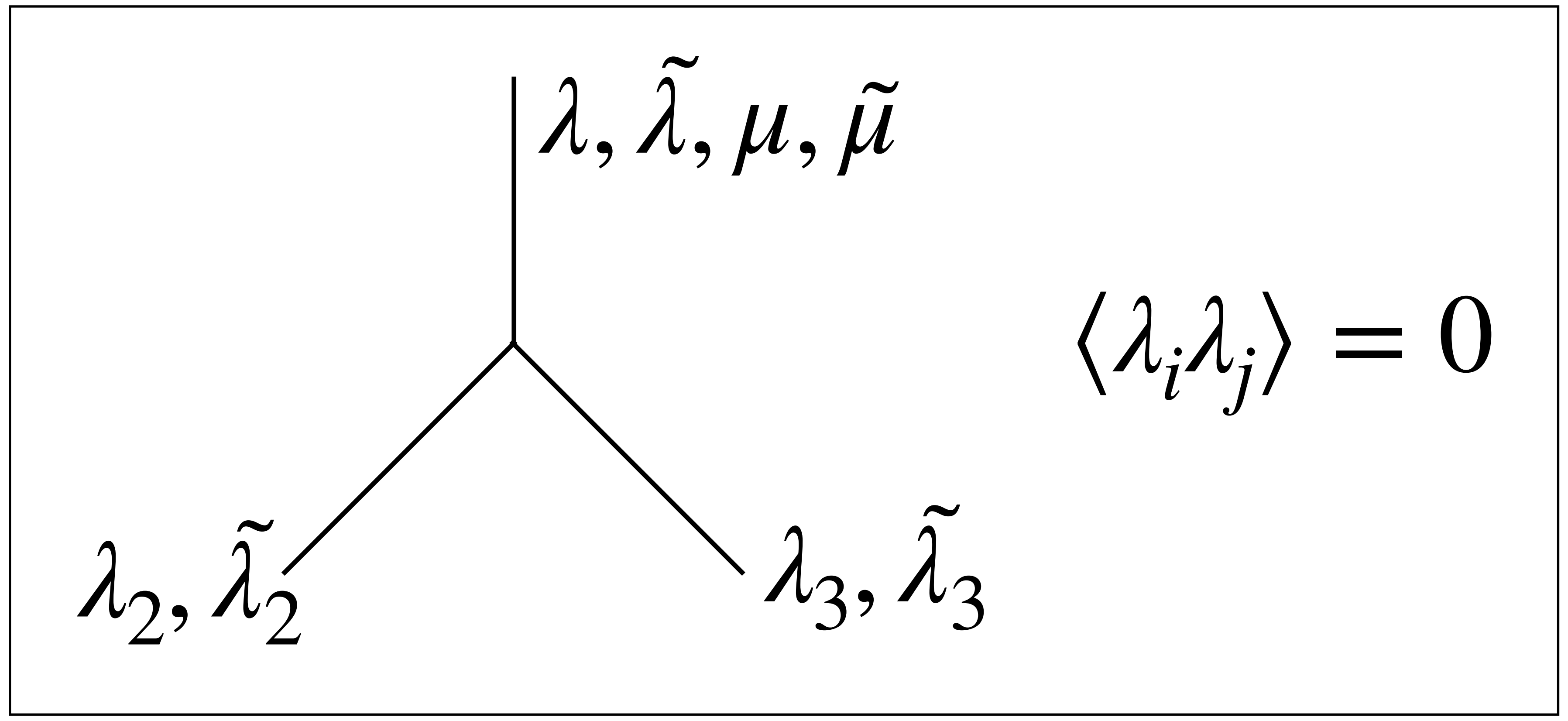
Real-momentum collinear limit ⇒ diverging real phase
⇒ rapid oscillation "damps" the amplitude

But in complex $k \cdot P$-plane, phase is an essential singularity overlapping the 3-particle factorization pole. On some approaches to this kinematics, it diverges exponentially.

This is what obstructs defining 3-point amplitudes involving CSPs.

Complex-Momentum 3-particle Amplitudes?

Complex momentum → complexify LG



W_+ and W_- are no longer conjugate, but independent
→ new "right-CSP" representations with

$$W_+ |\lambda\rangle_\mu = \rho \frac{\langle \lambda \mu \rangle}{[\tilde{\lambda}\tilde{\mu}]} |\lambda\rangle_\mu \text{ but } T_- |\lambda\rangle_\mu = 0.$$

These **do** admit 3-point amplitudes $A = e^{i\rho\left(\frac{[\tilde{\mu}2]}{[\tilde{\lambda}2][\tilde{\mu}\tilde{\lambda}]} + \frac{[\tilde{\mu}3]}{[\tilde{\lambda}3][\tilde{\mu}\tilde{\lambda}]}\right)} \left(\frac{[2\tilde{\lambda}][3\tilde{\lambda}]}{[23]}\right)^a.$

Can construct higher-point amplitudes (e.g. 3 matter legs + one right-CSP) that factorize appropriately at 3-point. For $a = 1$ (photon-like), consistency requires charge conservation just as it does for $\rho = 0$.

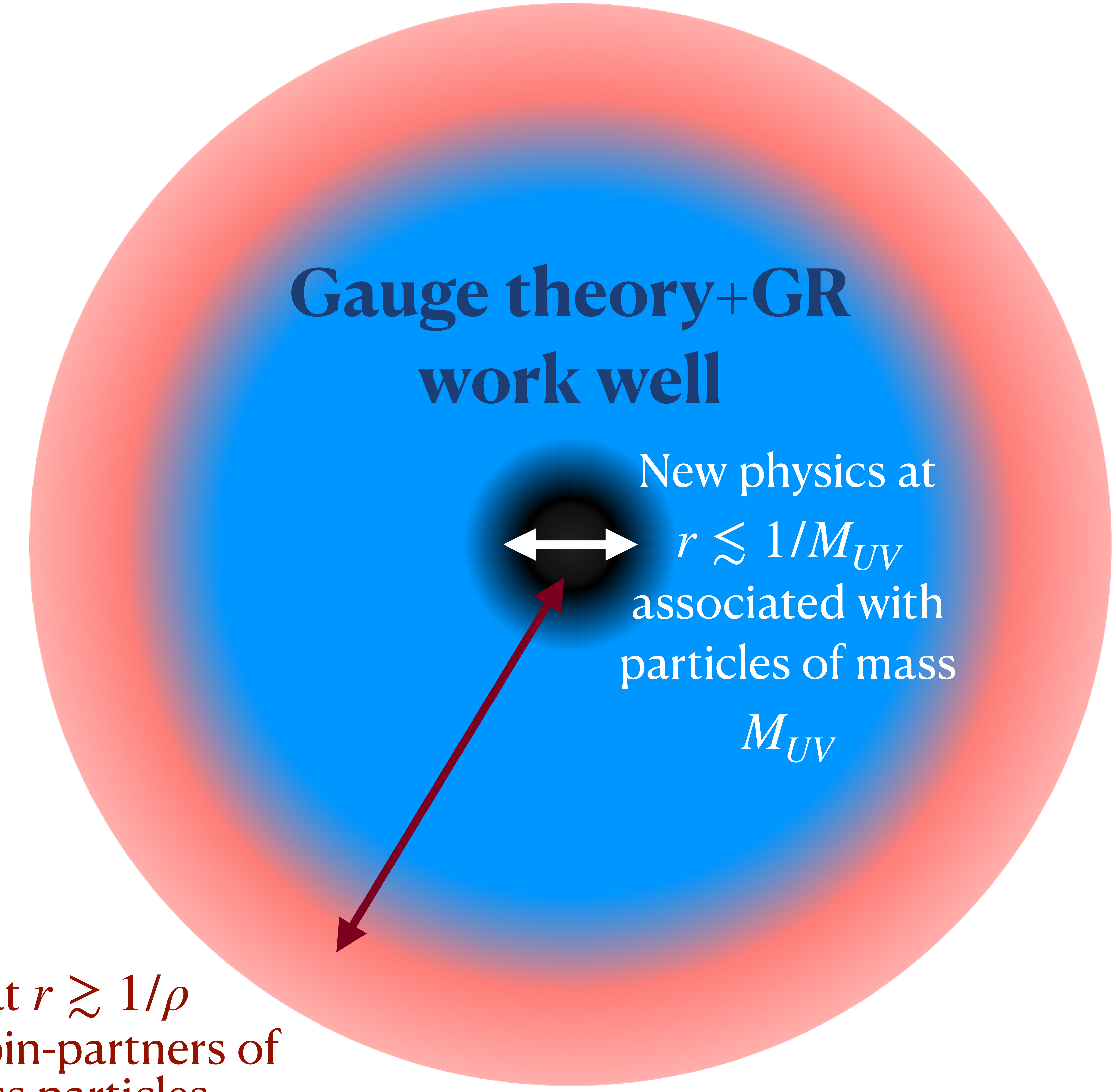
Key Questions

- Physics of on-shell CSPs avoids "shape" ambiguity in worldline current. But predictions for processes involving **intermediate CSPs** require finding the right shape – or another means of inferring from on-shell CSP results.
- All known particles are either massless or unnaturally light – could they all be CSPs? Raises many more theoretical questions
  - CSP self-interactions (e.g. Yang-Mills-like)
  - Coupling to (extension of) GR?
  - CSP Massive phase? What happens to partners?
  - Is there a symmetry protecting the mass of scalar-like CSPs?

These questions beg for an Amplitudes approach!

Conclusions

- Lorentz invariance → massless particles have a spin-scale. **Is it zero or non-zero?**
- The non-zero option makes more sense than previously thought, and has testable consequences
- **If inconsistent, deserves a proper burial**
- If viable, we should think of the Standard Model as an effective theory with both UV and IR completions –– and many questions remain to understand the IR.



New physics at $r \gtrsim 1/\rho$ associated with spin-partners of known massless particles