From Quantum to Classical Scattering of Kerr Black Holes

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Modelling Black Hole Scattering

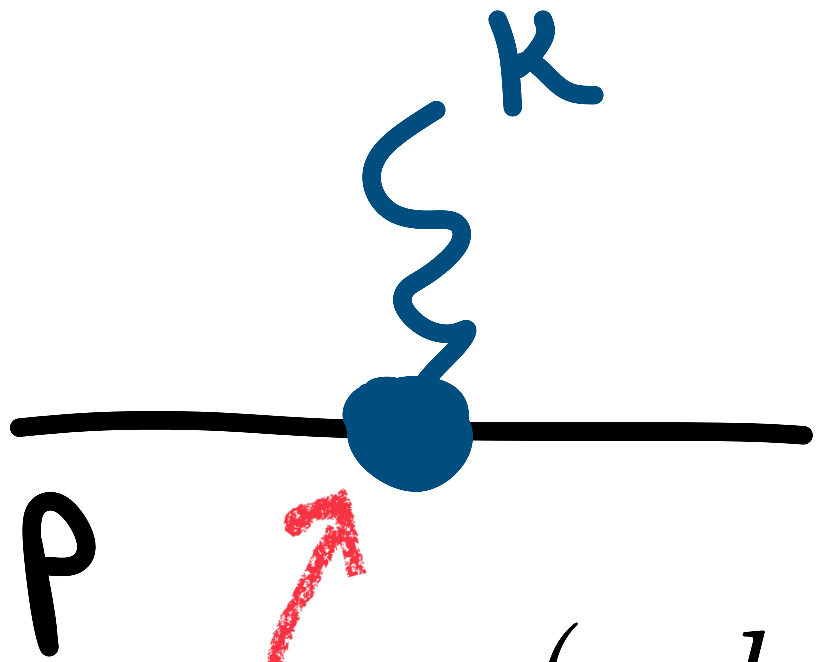
classical worldline:
[Levi, Steinhoff ’15] [Chung, Huang, Kim, Lee ’18] [Scheopner, Vines ’23]
worldline QFT:
[Jakobsen, Mogull, Plefka, Steinhoff ’21] [Ben-Shahar ‘23]

Generic Spinning Compact Objects

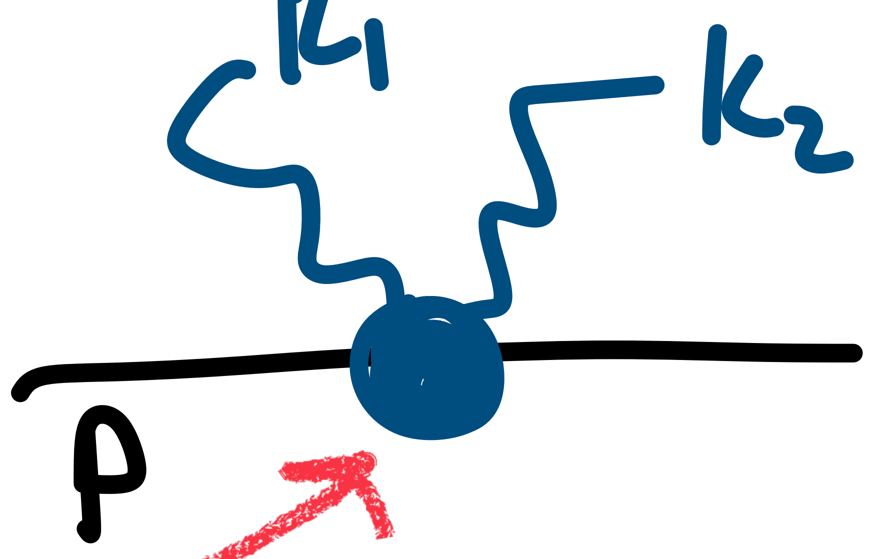
[Bern, Luna, Roiban, Shen, Zeng ’20]

$$\mathcal{L} = \nabla^\mu \phi_s \nabla_\mu \phi_s - m^2 \phi_s^2 + C_{ES^2} R_{\mu\nu\rho\sigma} \nabla^\nu \phi_s \mathbb{S}^{(\mu} \mathbb{S}^{\rho)} \nabla^\sigma \phi_s + \mathcal{O}(R\mathbb{S}^2) + \mathcal{O}(R^2\mathbb{S}^4)$$

- new scale $S^\mu = m a^\mu$
- multipole expansion amplitudes $M_{cl} = \sum_{n=0}^{\infty} c_{\mu_1 \ldots \mu_n} a^{\mu_1} \ldots a^{\mu_n}$



$c(p, k, \varepsilon_k, C_{ES^2}, \ldots)$



$c(p, k_1, k_2, \varepsilon_{k_1}, \varepsilon_{k_2}, C_{ES^2}, C_{E^2S^4}, \ldots)$

Modelling Black Hole Scattering

classical EFT for Kerr black holes

- linear in Riemann — fixed

$$M_{cl.3pt} \sim \varepsilon_{k,\mu\nu} T^{\mu\nu}_{Kerr} \sim (p \cdot \varepsilon_k)^2 e^{k \cdot a}$$

[Vines '17]

- quadratic in Riemann — contribute to Compton @ $\mathcal{O}(S^4)$

- attempts to fix via:

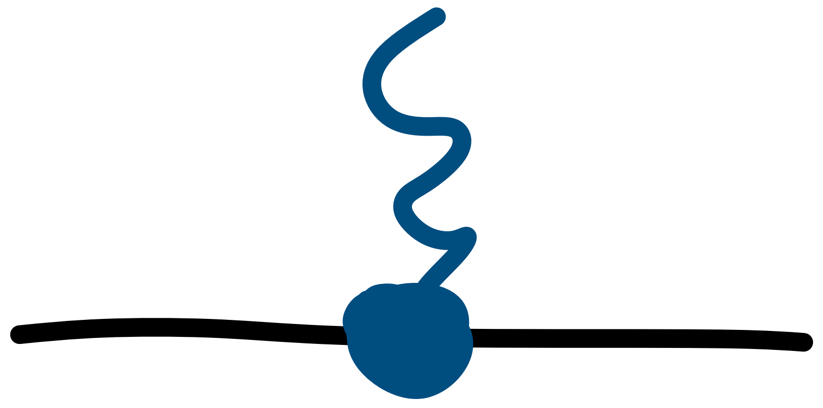
  — classical symmetry: spin-shift symm. [Aoude, Haddad, Helset '22] [Bern, Kosmopoulos, Luna, Roiban, Teng '22]

  — Teukolsky equation: finite orders in spin, open Qs [Bautista, Guevara, Kavanagh, Vines '22] [Bautista, Bonelli, Iossa, Tanzini, Zhou '23]

Modelling Black Hole Scattering

quantum EFT for Kerr black holes

- 3pt q. amp are known:



$$M(1^s, 2^s, k^+) = (p_1 \cdot \varepsilon_k^+)^2 \left(\frac{\langle \mathbf{12} \rangle}{m} \right)^{2s} \longrightarrow (p \cdot \varepsilon_k^+)^2 e^{k \cdot a}$$

[Arkani-Hamed, Huang, Huang '17]

[Guevara, Ochirov, Vines '18]
[Chung, Huang, Kim, Lee '18]

Pipeline from Quantum to Cl. Amps

1. expand in basis of spin operators $M \sim \sum_{n=0}^{2s} \tilde{c}_{\mu_1 \ldots \mu_n} \langle \hat{a}^{\mu_1} \ldots \hat{a}^{\mu_n} \rangle$

$$(\hat{a}^\mu)_{\vec{a}}{}^{\vec{b}} := \frac{1}{2m^2} \left(\langle 1_{a_1} | \sigma^\mu | 1^{(b_1}] \delta_{a_2}^{b_2} \ldots \delta_{a_{2s}}^{b_{2s})} + [1_{a_1} | \bar{\sigma}^\mu | 1^{(b_1} \rangle \delta_{a_2}^{b_2} \ldots \delta_{a_{2s}}^{b_{2s})} \right)$$

2. go to classical regime $k^\mu \sim \hbar$

   relate $\tilde{c}_{\mu_1 \ldots \mu_n}$ to cl. spin multipoles $c_{\mu_1 \ldots \mu_n}$

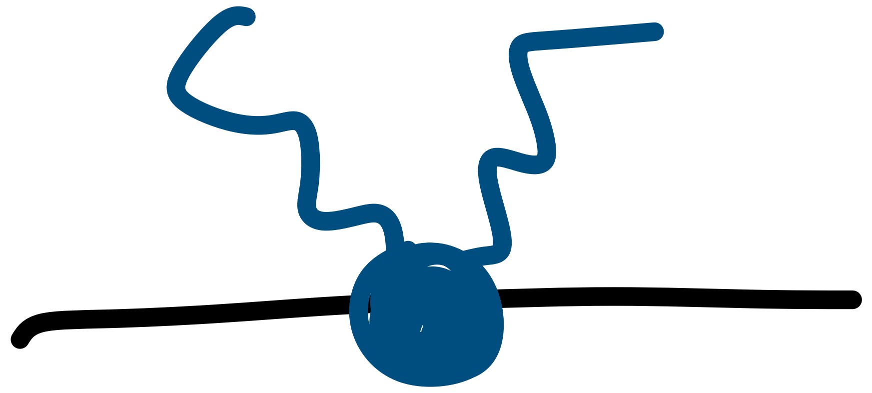
Modelling Black Hole Scattering

quantum EFT for Kerr black holes

- 3pt q. amp are known: $M(1^s, 2^s, k^+) = (p_1 \cdot \varepsilon_k^+)^2 \left(\frac{\langle \mathbf{12} \rangle}{m}\right)^{2s}$

  [Arkani-Hamed, Huang, Huang '17]

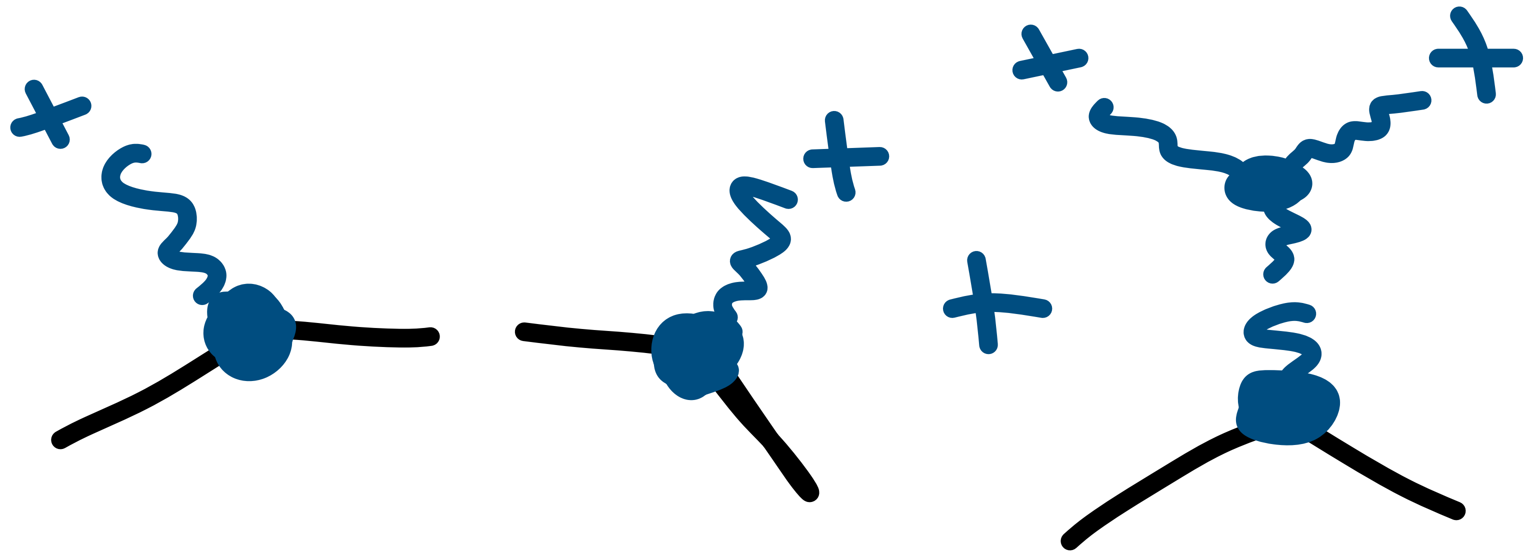
- Higher point?



  - inspired by Schwarzschild - consider minimally coupled spins

    $s \leq 2$ ✓ $s > 2$ minimal coupling breaks down ?

  - on shell recursion techniques (BCFW)



$\Rightarrow$ $M(1^s, 2^s, 3^+, 4^+) = \frac{[34]^4}{s_{12} t_{13} t_{14}} \left(\frac{\langle \mathbf{12} \rangle}{m}\right)^{2s}$ ✓

[Arkani-Hamed, Huang, Huang '17]

Modelling Black Hole Scattering

quantum EFT for Kerr black holes

- 3pt q. amp are known: $M(1^s, 2^s, k^+) = (p_1 \cdot \varepsilon_k^+)^2 \left(\frac{\langle \mathbf{12} \rangle}{m} \right)^{2s}$

[Arkani-Hamed, Huang, Huang '17]

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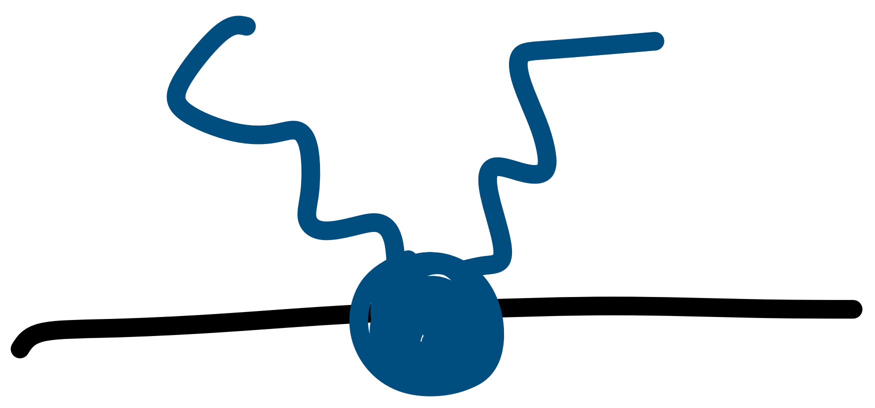

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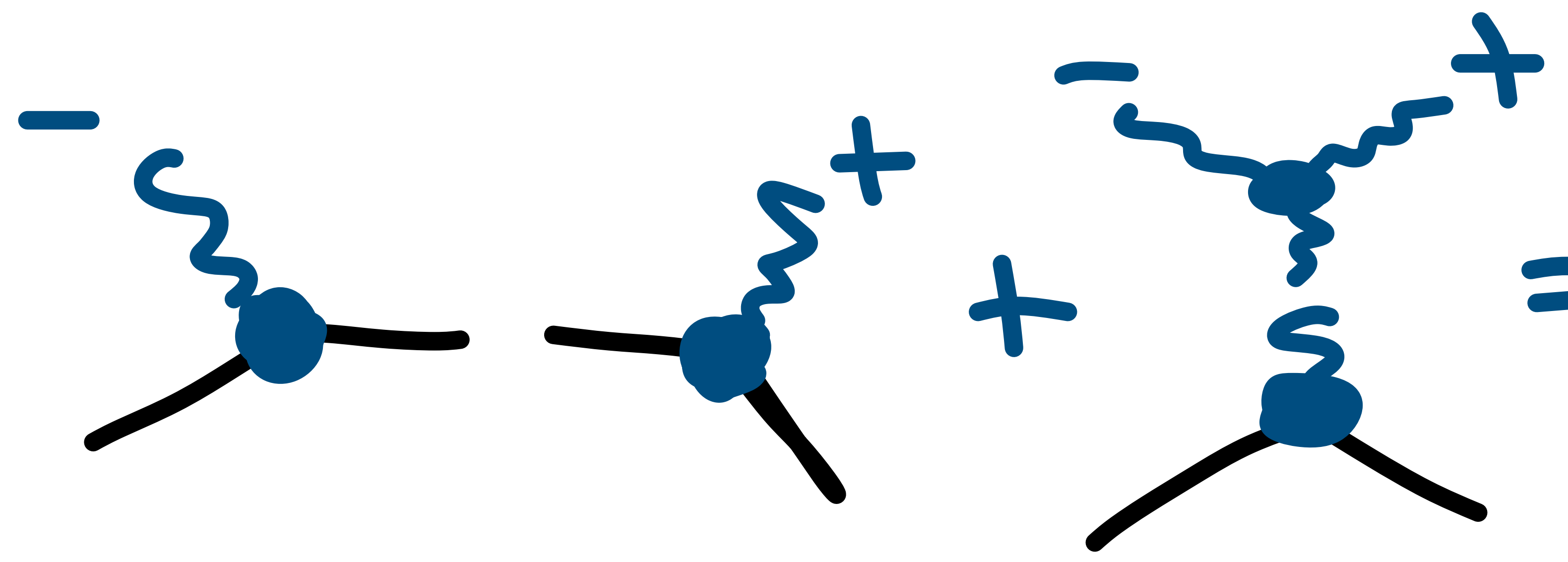

$\Rightarrow M(1^s, 2^s, 3^-, 4^+) = \frac{\langle 3|1|4]^4}{s_{12} t_{13} t_{14}} \left(\frac{\langle \mathbf{13} \rangle [\mathbf{42}] + \langle \mathbf{23} \rangle [\mathbf{41}]}{\langle 3|1|4]} \right)^{2s}$

[Arkani-Hamed, Huang, Huang '17]

ambiguous contact terms for $s > 2$

Higher Spin Construction

Our approach

- construct quantum EFT for $M(1^s, 2^s, k^+) = (p_1 \cdot \varepsilon_k^+)^2 \left(\frac{\langle \mathbf{12} \rangle}{m}\right)^{2s}$

  — **higher spin constraints**

- propose $M(1^s, 2^s, 3^-, 4^+)$ for generic s

  — **introduce classical constraints**

- classical amplitude defined as

$$M_{cl}(1, 2, 3^-, 4^+) \overset{*}{=} \lim_{\hbar \to 0} M(1^s, 2^s, 3^-, 4^+)$$

  — result is to **all-orders-in-spin** — comparison to Teukolsky

[Bautista, Guevara, Kavanagh, Vines ’22]

Higher Spin Construction

Free theory [Zinoviev '01]

- tower of symmetric, double traceless fields $\{\Phi_{\mu(s)}, \Phi_{\mu(s-1)} \dots \Phi_{\mu(0)}\}$
- massive gauge symmetry

$$\delta_0 \Phi_{\mu(k)} = \partial_{(\mu_1} \xi_{\mu_2 \dots \mu_k)} + m\alpha_k \xi_{\mu_1 \dots \mu_k} + m\beta_k \eta_{(\mu_1 \mu_2} \xi_{\mu_3 \dots \mu_k)}$$

- free Lagrangian

$$\mathcal{L}^{(s)} = -\sum_{k=0}^{s} (-1)^{k+1} \left[\bar{\Phi}_{\mu(k)} (\Box + m^2) \Phi^{\mu(k)} - \frac{k(k-1)}{4} \bar{\tilde{\Phi}}_{\mu(k-2)} (\Box + m^2) \tilde{\Phi}^{\mu(k-2)} \right]$$
$$- \sum_{k=0}^{s-1} (-1)^k (k+1) \overline{G_{\mu(k)}} G^{\mu(k)} \,,$$

$$G^{\mu(k)} := \partial_\lambda \Phi^{\lambda\mu(k)} - \frac{k}{2} \partial^\mu \tilde{\Phi}^{\mu(k-1)} + m\alpha_k \Phi^{\mu(k)} - m\alpha_{k+1} \frac{k+1}{2} \tilde{\Phi}^{\mu(k)} - m\alpha_k \frac{k-1}{4} \eta^{\mu\mu} \tilde{\Phi}^{\mu(k-2)}$$

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Free theory [Zinoviev '01]

- tower of symmetric, double traceless fields $\{\Phi_{\mu(s)}, \Phi_{\mu(s-1)} \ldots \Phi_{\mu(0)}\}$
- massive gauge symmetry $\delta_0 \Phi_{\mu(k)}$

adding interactions

- minimal coupling —> breaks gauge symmetry $\delta_0 \mathcal{L}_2 \neq 0$
- add non-min. interactions $\mathcal{L}_3 \sim (\nabla)^{k+k'-2} \Phi_{\mu(k)} \Phi_{\mu(k')} R$
  - — fix using H.S. constraints: Ward ids., power counting, current constraints

Higher Spin Construction

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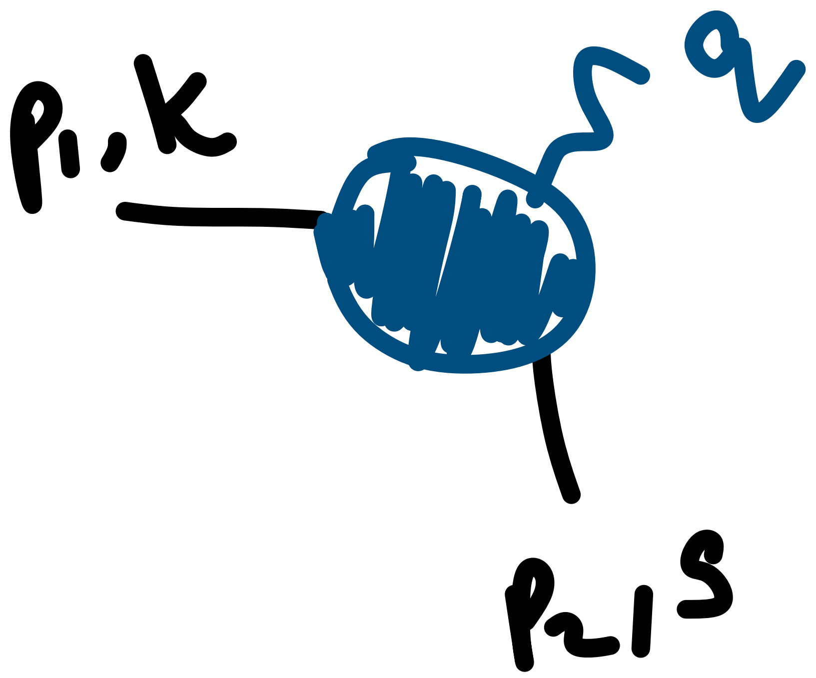
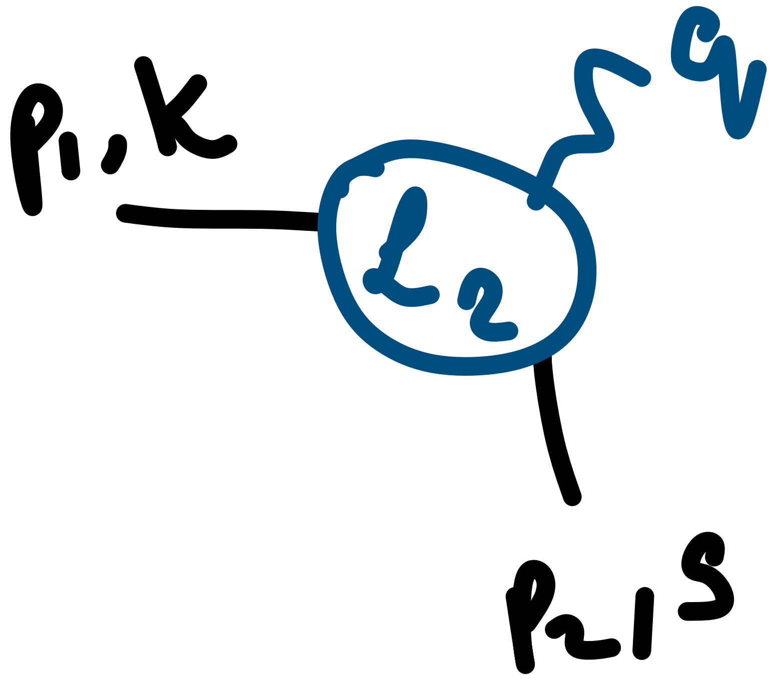
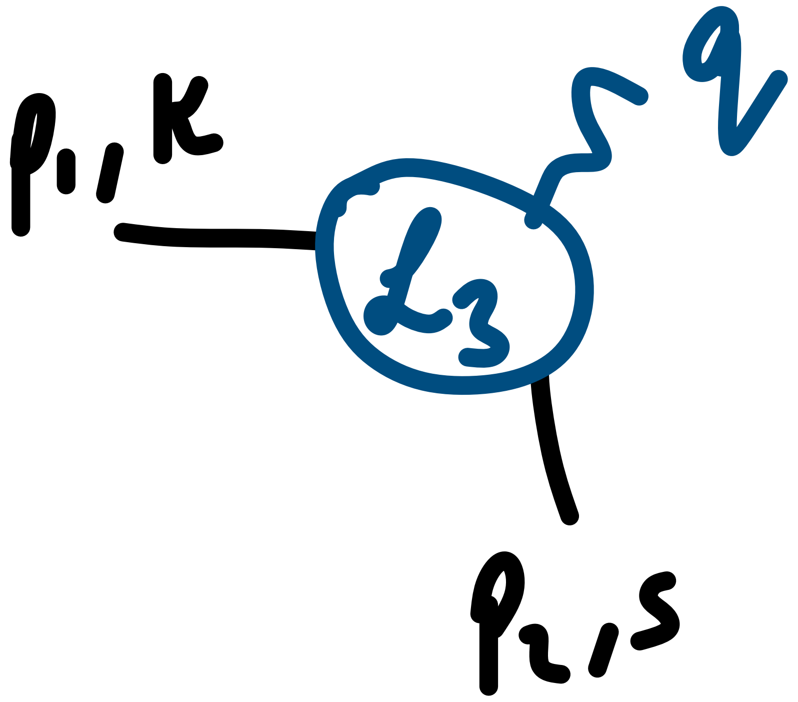
full cubic vertex



$=$ min. vertex



$+$ non-min. vertex



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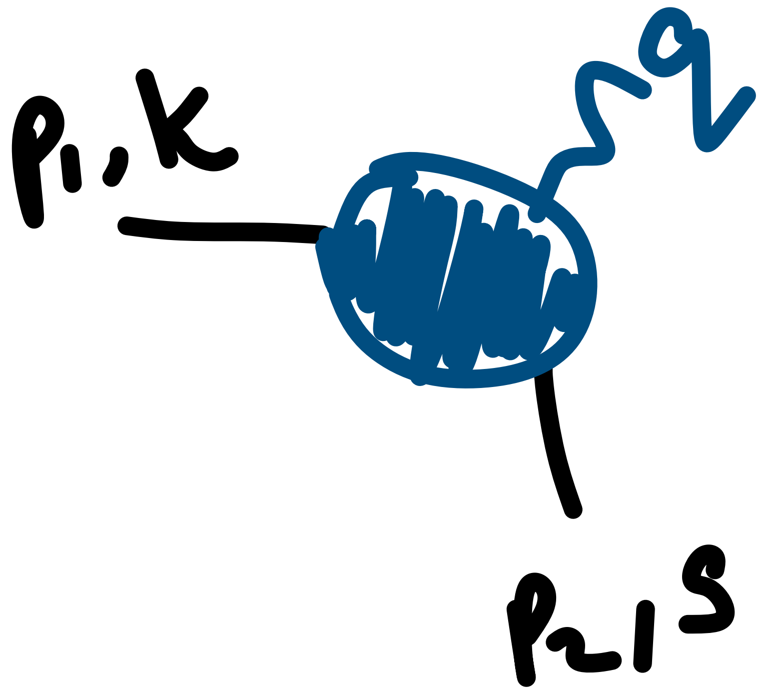
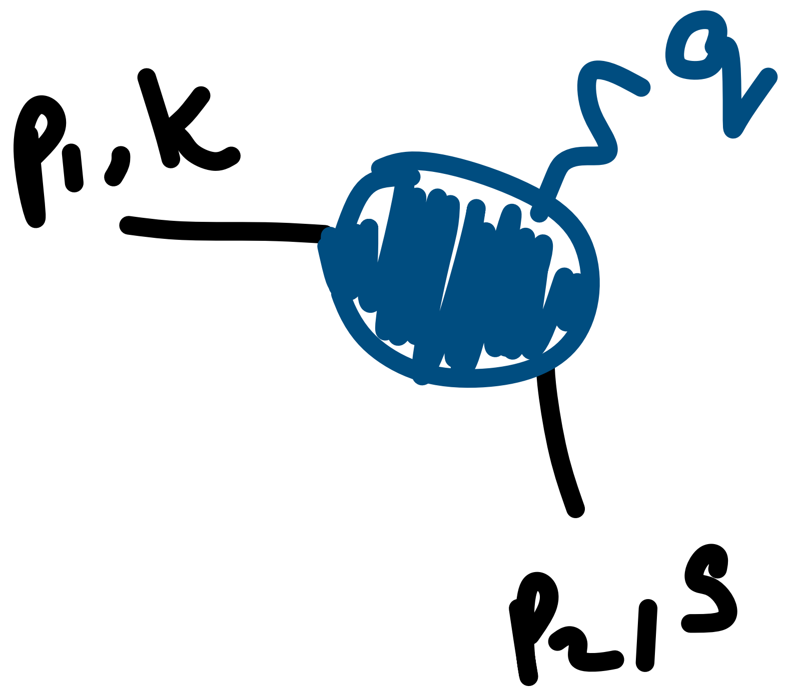


H.S. constraints

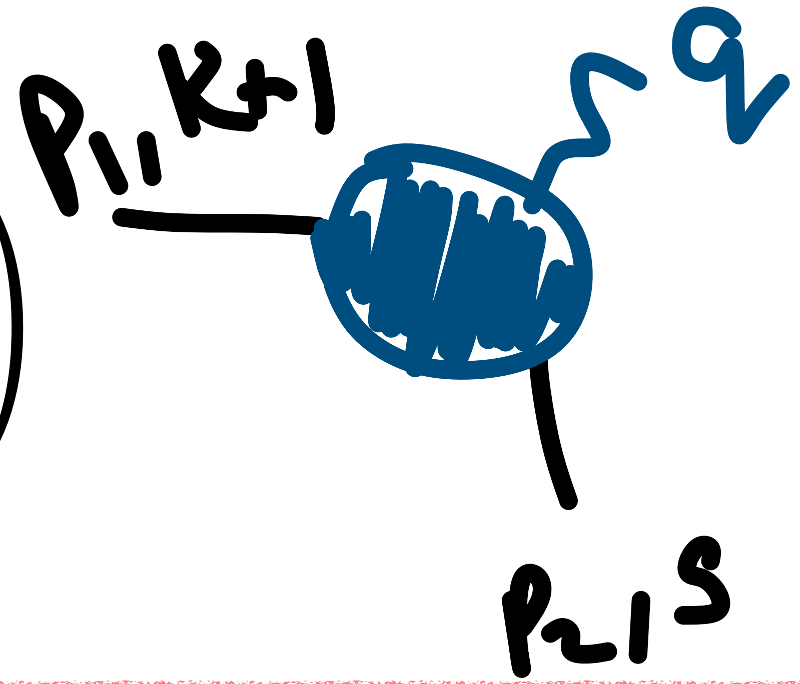
- power counting
- **Ward identity** (from $\delta_0 \Phi_{\mu(k)}$)
- current constraint

Ward Id.

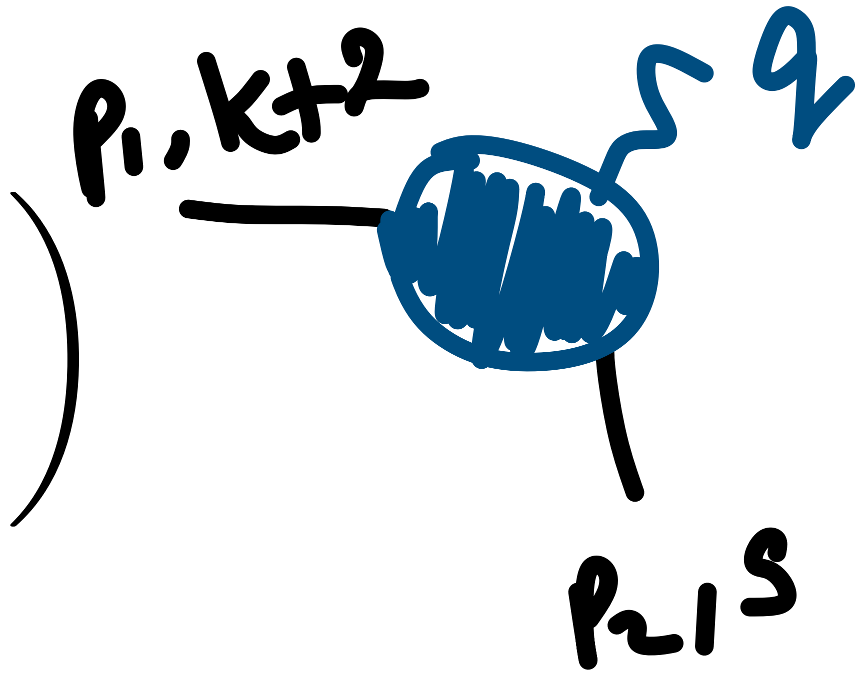
$0 =$



$+ \left(p_1 \cdot \frac{\partial}{\partial \epsilon_1} \right)$



$+ \left(\frac{\partial}{\partial \epsilon_1} \cdot \frac{\partial}{\partial \epsilon_1} \right)$



Higher Spin Construction

Free theory [Zinoviev '01]

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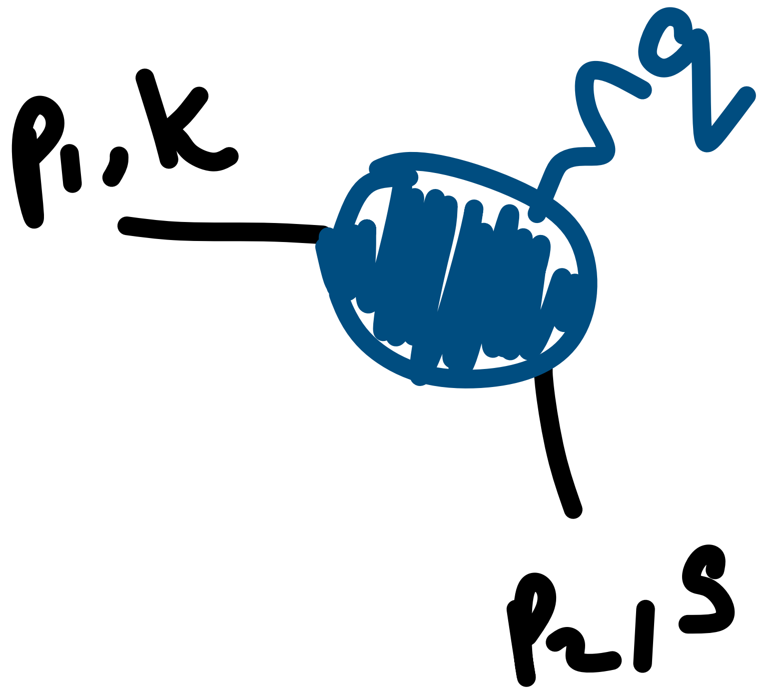
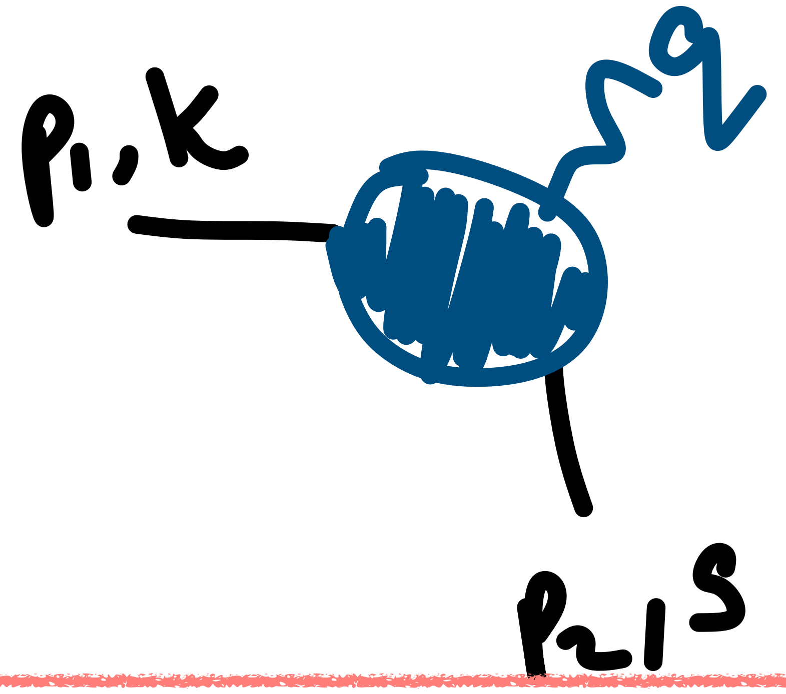


H.S. constraints

- power counting
- Ward identity (from $\delta_0 \Phi_{\mu(k)}$)
- **current constraint**

current constraint

$$\mathcal{O}(m) = \left(p_1 \cdot \frac{\partial}{\partial \epsilon_1} \right)$$



Higher Spin Construction: Results

Kerr

- HS constraints: uniquely fix three point amp.

$$M(1^s, 2^s, k^+) = (p_1 \cdot \varepsilon_k^+)^2 \left(\frac{\langle \mathbf{12} \rangle}{m}\right)^{2s}$$

- Quartic order: $C^{(s<3)} = 0$

root-Kerr* *theory of massive higher spins coupled to electromag.*
— 3pt amps related by double copy

- HS constraints + (near-diag.): uniquely fix three point amp.

$$A(1^s, 2^s, k^+) = (p_1 \cdot \varepsilon_k^+) \left(\frac{\langle \mathbf{12} \rangle}{m}\right)^{2s}$$

- Quartic order:

$$C^{(s)} = \begin{cases} 0 & \text{free params for } s < 2 \\ 3 & \text{free params at } s = 2 \\ 21 & \text{free params at } s = 3 \end{cases}$$

additional constraints needed to fix quartic

Higher Spin Construction: Chiral fields

[Ochirov, Skvortsov ’22]

- single chiral $(2s, 0)$ field $|\Phi\rangle := \Phi_{\alpha(2s)}$ + gravity/electromag.
- minimal coupling breaks parity —> need non-min. interactions

root-Kerr cubic Lag.

anti-selfdual field strength

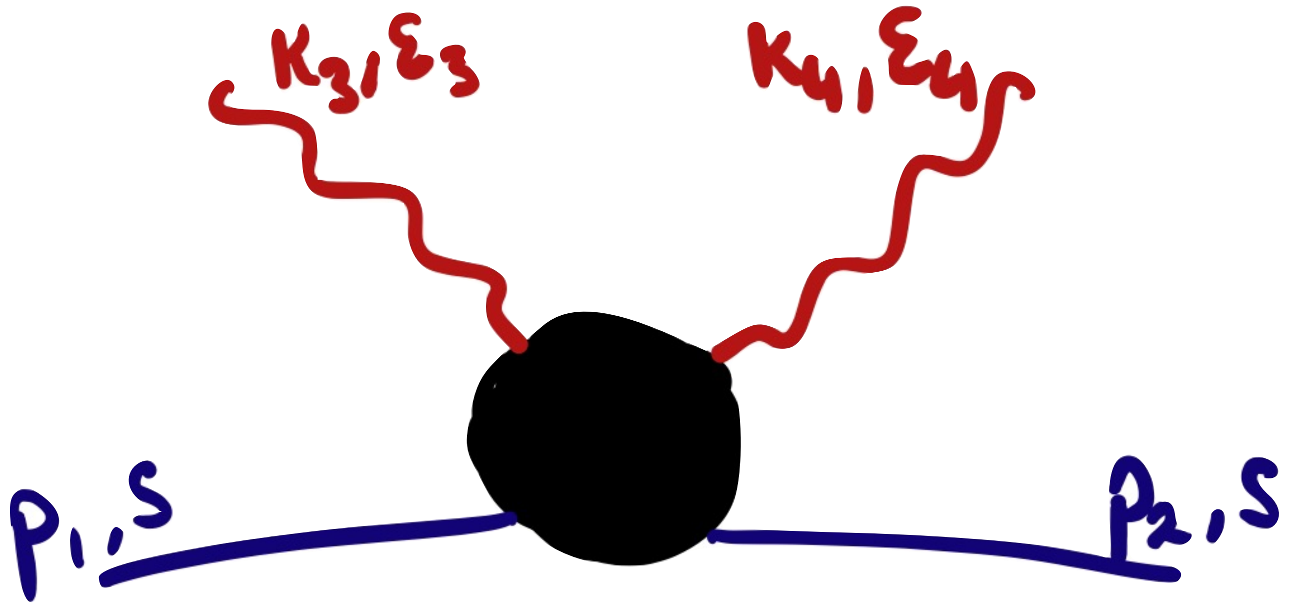
$$\mathcal{L}_{\sqrt{\text{Kerr}}} = \frac{1}{2}\langle D_\mu \Phi | D^\mu \Phi \rangle - \frac{m^2}{2}\langle \Phi | \Phi \rangle + \frac{1}{2}\sum_{k=0}^{2s-1} \frac{iQ}{m^{2k}} \langle \Phi | \left\{ |\overleftarrow{D}|\overrightarrow{D}|^{\odot k} \otimes |F_-| \right\} |\Phi\rangle$$

Kerr cubic Lag.

anti-selfdual Weyl tensor

$$\mathcal{L}_{\text{Kerr}} = \sqrt{-g}\left\{ \frac{1}{2}\langle \nabla_\mu \Phi | \nabla^\mu \Phi \rangle - \frac{m^2}{2}\langle \Phi | \Phi \rangle - \frac{1}{4}\sum_{k=0}^{2s-2} \frac{2s-k-1}{m^{2k}} \langle \Phi | \left\{ |\overleftarrow{\nabla}|\overrightarrow{\nabla}|^{\odot k} \odot |R_-| \right\} |\Phi\rangle \right\}$$

+ (curvature)^2 corrections



Quantum Amplitudes

root-Kerr (electromag.)

$$A_{\sqrt{\mathrm{Kerr}}}(\mathbf{1}^s,\mathbf{2}^s,3^-,4^+) = \frac{\langle 3|1|4]^2}{t_{13}t_{14}}P_1^{(2s)} - \frac{\langle \mathbf{13}\rangle\langle 3|1|4][4\mathbf{2}]}{m^2 t_{13}}P_2^{(2s)} + \frac{\langle \mathbf{13}\rangle\langle 3\mathbf{2}\rangle[\mathbf{1}4][4\mathbf{2}]}{m^4}\left[P_2^{(2s-1)} - \varsigma_3\varsigma_4 P_4^{(2s-1)}\right] + C^{(s)}$$

- photon helicity $\varepsilon_3^-, \varepsilon_4^+$
- massive channels $t_{14} = 2p_1\cdot k_4 \quad t_{13} = 2p_1\cdot k_3$
- spin-dependence:

$$\varsigma_1 := \langle\mathbf{1}|1{+}4|\mathbf{2}]/m, \qquad \varsigma_3 := \langle\mathbf{21}\rangle/m,$$
$$\varsigma_2 := -\langle\mathbf{2}|1{+}4|\mathbf{1}]/m, \quad \varsigma_4 := [\mathbf{21}]/m,$$

symmetric homogeneous **polynomials**, deg. $k+1-n$

$$P_n^{(k)} = \frac{\varsigma_1^k}{(\varsigma_1-\varsigma_2)(\varsigma_1-\varsigma_3)\dots(\varsigma_1-\varsigma_n)} + \mathrm{cyc}(\varsigma_1,\varsigma_2\dots\varsigma_n)$$

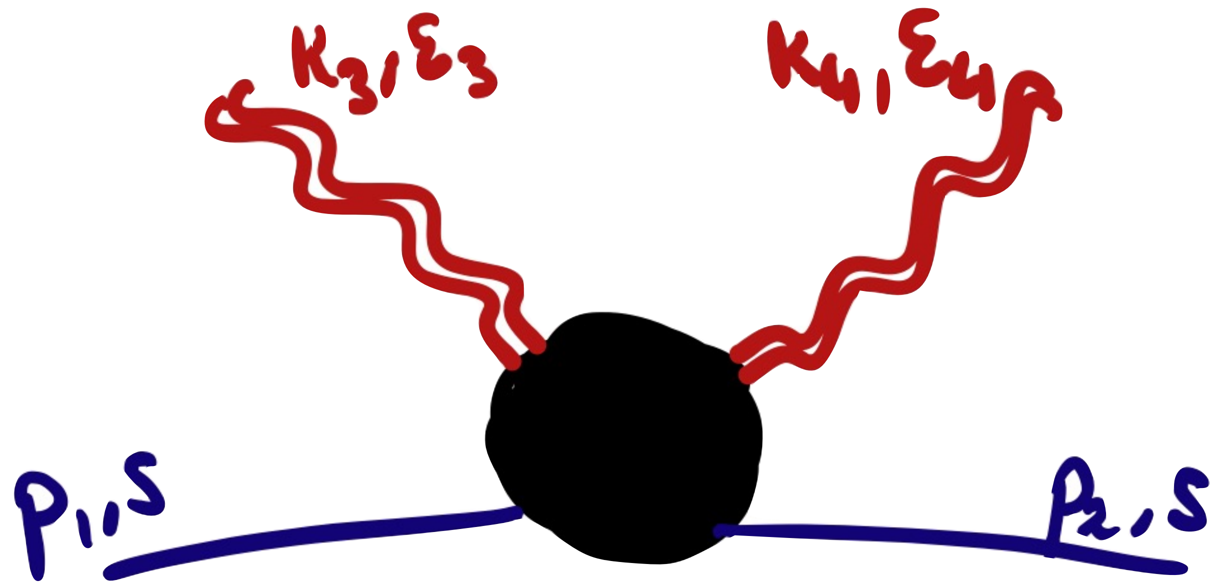
- matches A_{AHH} for $s = 0, \frac{1}{2}, 1$

  [Arkani-Hamed, Huang, Huang ’17]
- matches $s = 3/2$ amp. in

  [Chiodaroli, Johansson, Pichini, ’21]
- contact term constraints

$$C^{(s)} = \begin{cases} 0 & \text{free params for } s < 2 \\ 3 & \text{free params at } s = 2 \\ 21 & \text{free params at } s = 3 \end{cases}$$

Quantum Amplitudes



Kerr (gravity)

$$M(\mathbf{1}^s, \mathbf{2}^s, 3^-, 4^+) = \frac{\langle 3|1|4]^4}{s_{12}t_{13}t_{14}} P_1^{(2s)} - \frac{\langle \mathbf{1}3\rangle[4\mathbf{2}]\langle 3|1|4]^3}{s_{12}t_{13}m^2} P_2^{(2s)}$$

$$+ \frac{\langle \mathbf{1}3\rangle\langle 3\mathbf{2}\rangle[\mathbf{1}4][4\mathbf{2}]}{m^4 s_{12}} \left(\langle 3|1|4]^2 P_2^{(2s-1)} + \langle 3|\rho|4]^2 P_4^{(2s-1)}\right)$$

$$+ \frac{\langle \mathbf{1}3\rangle\langle 3\mathbf{2}\rangle[\mathbf{1}4][4\mathbf{2}]}{m^4 s_{12}} \langle 3|1|4]\langle 3|\rho|4] \left(P_2^{(2s-2)} - \varsigma_3\varsigma_4 P_4^{(2s-2)}\right) + C^{(s)}$$

- graviton helicity $\varepsilon_3^-, \varepsilon_4^+$
- massless channel $s_{12} = (k_3 + k_4)^2 =: q^2$
- spin-dependence:
  variables $\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4$, polynomials $P_n^{(k)}$
  and $\rho^\mu = \frac{1}{2}(\langle \mathbf{1}|\sigma^\mu|\mathbf{2}] + \langle \mathbf{2}|\sigma^\mu|\mathbf{1}])$

- matches $M_{\rm AHH}$ for $s = 0, \frac{1}{2}, 1, \frac{3}{2}, 4$
  [Arkani-Hamed, Huang, Huang '17]
- matches $s = 5/2$ amp. in
  [Chiodaroli, Johansson, Pichini, '21]

Polynomial basis

$$P_n^{(k)} = \frac{\varsigma_1^k}{(\varsigma_1 - \varsigma_2)(\varsigma_1 - \varsigma_3)\dots(\varsigma_1 - \varsigma_n)} + \mathrm{cyc}(\varsigma_1, \varsigma_2 \dots \varsigma_n)$$

- symmetric homogeneous polynomials, degree $k + 1 - n$
- naturally arise from chiral Lagrangian

  eg. cubic non-min term. $\mathcal{L}_{\sqrt{\mathrm{Kerr}}}$:

$$\sum_{k=0}^{2s-1} \frac{1}{m^{2s}} [\mathbf{21}]^k \langle \mathbf{21} \rangle^{2s-k-1} = P_1^{(2s)}(\varsigma_3, \varsigma_4)$$

- construct $C^{(s)}[P_n^{(k)}]$ and **impose classical constraints**

Classical limits with spin — subtleties

Pipeline from Quantum to Cl. Amps

1. expand in basis of spin operators $M \sim \sum_{n=0}^{2s} \tilde{c}_{\mu_1 \ldots \mu_n} \langle \hat{a}^{\mu_1} \ldots \hat{a}^{\mu_n} \rangle$
2. go to classical regime $k^\mu \sim \hbar$

relate $\tilde{c}_{\mu_1 \ldots \mu_n}$ to cl. spin multipoles $c_{\mu_1 \ldots \mu_n}$

- **finite spin?** [Vaidya, '14] spin-s amplitude —> $2s$ classical multipoles

✓ root-Kerr and Kerr @ 3pts —**spin-universality** $\tilde{c}_n = \frac{1}{n!}$

✗ **not in general** — e.g. scattering leading Regge states of open str. [LC, Pichini '22]

$$\lim_{\hbar \to 0} \mathcal{A}^{(s=2)}_{\text{open str.}} \sim 1 + (k_3 \cdot a) + \frac{1}{2}(k_3 \cdot a)^2 + \mathcal{O}(a^3)$$

$$\lim_{\hbar \to 0} \mathcal{A}^{(s=3)}_{\text{open str.}} \sim 1 + (k_3 \cdot a) + \frac{9}{30}(k_3 \cdot a)^2 + \mathcal{O}(a^3)$$

$$\tilde{c}_2^{(s)} = \frac{4s^2 - 7s + 4}{2s(2s-1)}$$

spin-dependent!

Classical limits with spin — subtleties

Pipeline from Quantum to Cl. Amps

1. expand in basis of spin operators $M \sim \sum_{n=0}^{2s} \tilde{c}_{\mu_1 \dots \mu_n} \langle \hat{a}^{\mu_1} \dots \hat{a}^{\mu_n} \rangle$
2. go to classical regime $k^\mu \sim \hbar$

relate $\tilde{c}_{\mu_1 \dots \mu_n}$ to cl. spin multipoles $c_{\mu_1 \dots \mu_n}$

- **large spin limit:** $s \sim \hbar^{-1}$

— expectation values behave classically $\langle \prod_{i=1}^{n} \hat{a}^{\mu_i} \rangle - \prod_{i=1}^{n} \langle \hat{a}^{\mu_i} \rangle \xrightarrow{\hbar \to 0} 0$

— **well defined classical multipoles** — $c_n = \lim_{\substack{\hbar \to 0 \\ s \to \infty}} \tilde{c}_n$

e.g. scattering leading Regge states of open str. [LC, Pichini '22]

$$c_2 = \lim_{s \to \infty} \frac{4s^2 - 7s + 4}{2s(2s-1)} = \frac{1}{4}$$ ✓

Classical limits with spin — subtleties

Pipeline from Quantum to Cl. Amps

1. expand in basis of spin operators $M \sim \sum_{n=0}^{2s} \tilde{c}_{\mu_1 \ldots \mu_n} \langle \hat{a}^{\mu_1} \ldots \hat{a}^{\mu_n} \rangle$
2. go to classical regime $k^\mu \sim \hbar$

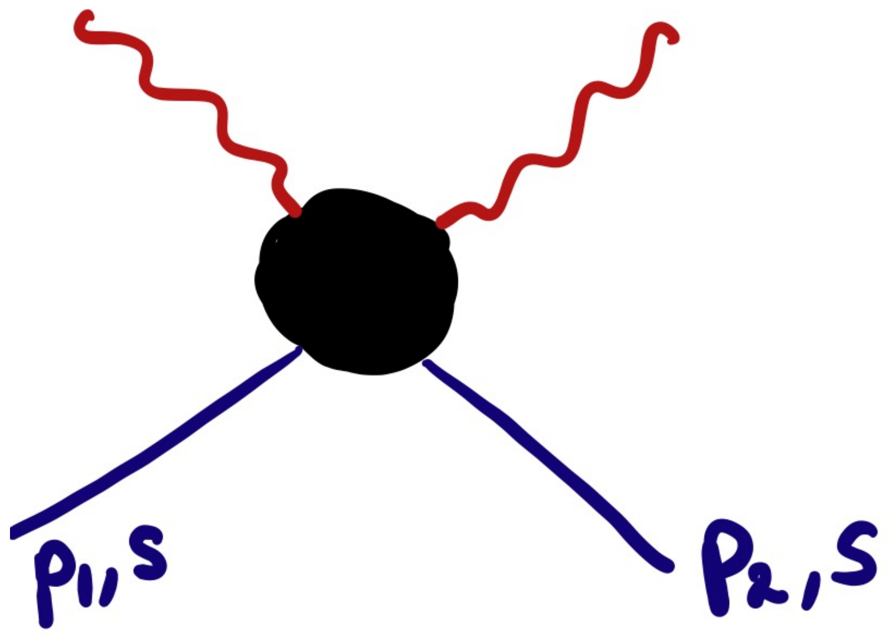
   relate $\tilde{c}_{\mu_1 \ldots \mu_n}$ to cl. spin multipoles $c_{\mu_1 \ldots \mu_n}$

- **coherent spin limit:** [Aoude, Ochirov '21]

  — scatter coherent states: $|\text{coherent}\rangle = \sum_{2s=0}^{\infty} \frac{e^{-|z|^2/2}}{\sqrt{(2s)!}} |p, \{z\}\rangle$

  — infinite spin sum

$$M \sim \lim_{\hbar \to 0} \sum_{2s=0}^{\infty} \frac{e^{-|z|^2}}{(2s)!}$$



equivalent to large spin limit

Classical limits with spin

Closer look at the variables and polynomials

- cl. spin variables $q = k_3 + k_4 \quad q_\perp = k_4 - k_3 \quad \chi = \langle 3|\sigma^\mu|4]$ for a specific gauge choice $\chi \sim \varepsilon_3^- \sim \varepsilon_4^+$

$$x = q_\perp \cdot a, \qquad y = q \cdot a, \qquad z = |a|\frac{p\cdot q_\perp}{m}, \qquad w = \frac{a\cdot\chi}{p\cdot\chi} p\cdot q_\perp$$

- polynomials resum to **entire fns** in the cl. limit

$$\lim_{\hbar\to 0}\sum_{2s=0}^{\infty}\frac{e^{-2m|a|}}{(2s)!}P_1^{(2s)} = e^{x+z}$$

generate **inf. tower of cl. spin multipoles**

$$\lim_{\hbar\to 0}\sum_{2s=0}^{\infty}\frac{e^{-2m|a|}}{(2s)!}P_4^{(2s)} = \frac{1}{2y}\frac{e^y - e^x\cosh z + (x-y)e^x \operatorname{sinhc} z}{(x-y)^2 - z^2} + (y\to -y) =: \tilde{E}(x,y,z)$$

Classical Compton Amplitudes

root-Kerr (electromag.)

$$\mathcal{A}_{\sqrt{\text{Kerr}}}(\mathbf{1},\bar{\mathbf{2}},3^-,4^+) = \mathcal{A}_0\left(e^x\cosh z - w\,e^x \text{sinhc}\, z + \frac{w^2-z^2}{2}E(x,y,z)\right),$$

with entire fn $E(x,y,z) = \frac{e^y - e^x\cosh z + (x-y)e^x\text{sinhc}\,z}{(x-y)^2-z^2} + (y\to -y)$

- **H.S. constraints:** massive gauge invar. + soft high E behaviour

$$C^{(s<2)} = 0$$

$$C^{(2)}_{-+} \sim \langle\mathbf{1}3\rangle\langle 3\mathbf{2}\rangle[\mathbf{1}4][4\mathbf{2}]\left(c_1(\langle\mathbf{12}\rangle+[\mathbf{12}])^2 + c_2(\langle\mathbf{12}\rangle-[\mathbf{12}])^2\right) + c_3(\dots)$$

- **classical constraints:** finite classical limit + $s=1$ quadrupole

$$\lim_{\hbar\to 0}\sum_{2s=0}^{\infty}\frac{e^{-|z|^2}}{(2s)!}\left(C^{(s)}_{\mathcal{L}_3} + C^{(s)}\right) = (1-\delta)\frac{w^2-z^2}{2}E(x,y,z)$$

$\Longrightarrow$ **resulting contact term**

$c_1 = c_2 = 1/4$

$c_3 = 0$

$$C^{(s)} = -\frac{\langle\mathbf{1}3\rangle\langle 3\mathbf{2}\rangle[\mathbf{1}4][4\mathbf{2}]}{2m^4}(\varsigma_3+\varsigma_4)\left(P_4^{(2s)} - P_2^{(2s-2)}\right)$$

Classical Compton Amplitudes

Kerr (gravity)

$$\mathcal{M}(\mathbf{1},\mathbf{2},3^-,4^+) = \mathcal{M}_0\Big[e^x \text{cosh} z - w\, e^x \text{sinhc} z + \frac{w^2-z^2}{2}E(x,y,z)$$
$$+(w^2-z^2)(x-w)\tilde{E}(x,y,z) - \frac{m^2q^2(w^2-z^2)^2}{2(p\cdot q_\perp)^2}\frac{\partial}{\partial x}\tilde{E}(x,y,z)\Big]$$

- **H.S. constraints:** massive gauge invar. $C^{(s<3)}=0$

  massless limit — finite for $s\leq 2$, otherwise $\sim m^{-4s+4}$

- **classical constraints:** $s=2$ fixes a^4 multipole

$\Longrightarrow$ **resulting contact term** $C^{(s)} = \dfrac{\langle\mathbf{1}3\rangle^2\langle 3\mathbf{2}\rangle^2[\mathbf{1}4]^2[4\mathbf{2}]^2}{2m^6}\varsigma_3\varsigma_4\left[P^{(2s-2)}_{5|\varsigma_1}+P^{(2s-2)}_{5|\varsigma_2}\right]$

Comparisons to Teukolsky

- [Bautista et al. '22] computed $\mathcal{M}_{Teuk}$ up to a^7 from BHPT techniques

—book-keeping variables: η tags terms **sensitive to boundary conditions**

α tags terms generated from **expanding polygamma functions**

- match $\mathcal{M} - \mathcal{M}_{Teuk} = 0$, for $\eta = \alpha = 0$ ✓

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— introduce contact term to fix $\eta a^5 \Longrightarrow$ modified $C^{(s)}$

$$C^{(s)} = \frac{\langle \mathbf{1}3\rangle^2 \langle 3\mathbf{2}\rangle^2 [\mathbf{1}4]^2 [4\mathbf{2}]^2}{2m^6} \varsigma_3 \varsigma_4 \left[(1+\eta) P^{(2s-2)}_{5|\varsigma_1} + (1-\eta) P^{(2s-2)}_{5|\varsigma_2} \right]$$

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$$\mathcal{M}(\mathbf{1}, \mathbf{2}, 3^-, 4^+) = \mathcal{M}_0 \Big[e^x \cosh z - w\, e^x \mathrm{sinhc} z + \frac{w^2 - z^2}{2} E + (w^2 - z^2)(x - w)\tilde{E} - \frac{m^2 q^2 (w^2 - z^2)^2}{2(p \cdot q_\perp)^2} \Big(\frac{\partial \tilde{E}}{\partial x} + \eta \frac{\partial \tilde{E}}{\partial z} \Big) + \alpha C_\alpha^{(\infty)} \Big]$$

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Summary of Results

- **constructed consistent higher spin** th. for Kerr and root-Kerr
- **quantum** Compton amplitudes — Kerr + $\sqrt{\text{Kerr}}$
- studied classical limit with spin
- **classical all-order-in spin** Compton amplitudes — Kerr + $\sqrt{\text{Kerr}}$
  - Kerr Compton: **matches** up to a^7

    compatible with a^8

    predicts $a^{n>8}$ for $\alpha = 0$

Work in progress with M. Ben-Shahar & H. Johansson

Matching to worldline QFT formalism [Mogull, Plefka, Steinhoff '20]

Using action in [Ben-Shahar '23]

$$S = \int d\tau \left(p_\mu \dot{x}^\mu + \frac{1}{2} S_{ab} \Omega^{ab} - \frac{1}{2m}(p^2 - \mathcal{M}^2) - \frac{1}{p}\frac{Dp_\mu}{d\tau} S^{\mu\nu} \Lambda_{0\nu} \right)$$

dynamical mass: $\mathcal{M}^2 = m^2 + \frac{C_{ES^2}}{m^2} E_{\mu\nu} S^\mu S^\nu + \ldots$ — linear in Riemann fixed

$E_{\mu\nu} = R_{\mu\rho\nu\sigma} \hat{p}^\rho \hat{p}^\sigma$

$+ \frac{C_{E^2 S^4}}{m^6} (E_{\mu\nu} S^\mu S^\nu)^2 + \ldots$ — quadratic in Riemann fixed up to $\mathcal{O}(R^2 S^4)$ in [Ben-Shahar '23]

- using $\mathcal{M}$, we fix higher order $C_{R^2 S^{n>4}}$ — **non-zero**, need **infinitely** many
- no nice structure — even for the same-helicity

Outlook

- study **non-conservative effects** — e.g. EFTs with multiple physical spins
- **near-zone vs far-zone** effects — how to interpret α terms?
  [Bautista, Bonelli, Iossa, Tanzini, Zhou ‘23]
- **sub-extremal** vs **super-extremal** Kerr
- **loops** — Teukolsky solution is all orders in G

  — first steps for scalar wave scattering
  [Ivanov, Li,Parra-Martinez, Zhou ’24]

Thank you!