Five parton scattering in the high energy limit

Based on: Phys.Rev.D 109 (2024) 9, 094025, arXiv:2311.09870 with B. Agarwal, F. Buccioni, G. Gambuti, A. Von Manteuffel, L. Tancredi
+ ongoing work with F. Buccioni, F. Caola, G. Gambuti

Federica Devoto, 06/10/2024

Amplitudes 2024, IAS

Outline

- (Brief) review of calculation of full color two-loop five-point scattering amplitudes in massless QCD

  Why   How

- Five parton scattering in the high energy limit

  Regge and Multi-Regge kinematics

  Regge poles vs Regge cuts

  Factorization beyond NLL

  NEW!

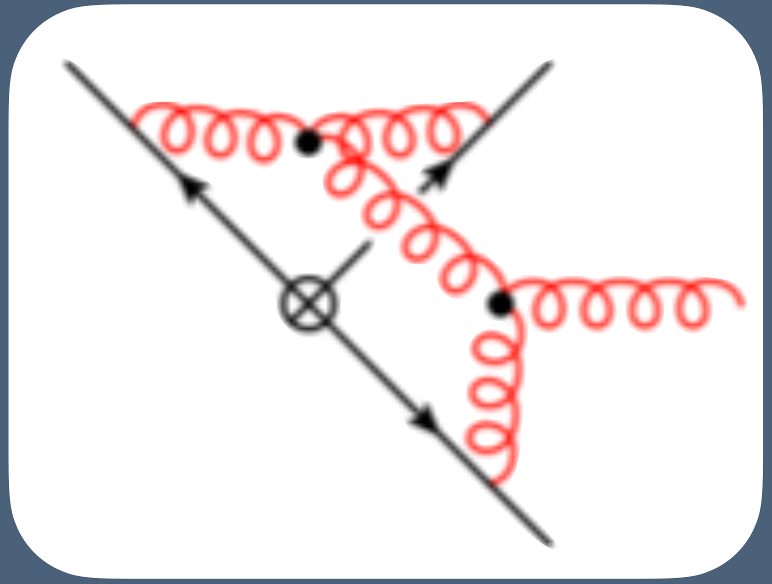
  "Cut contamination"

  2-loop central emission vertex

Why "Pain is inevitable, suffering is ~~optional~~"

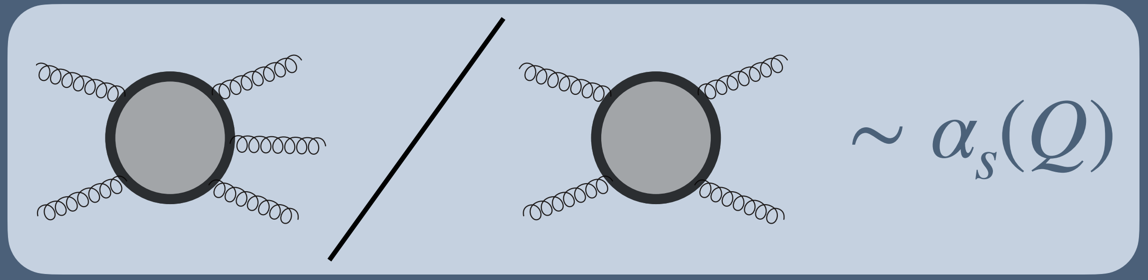
Worth it!

Simplest playground to study non planar sectors of QCD

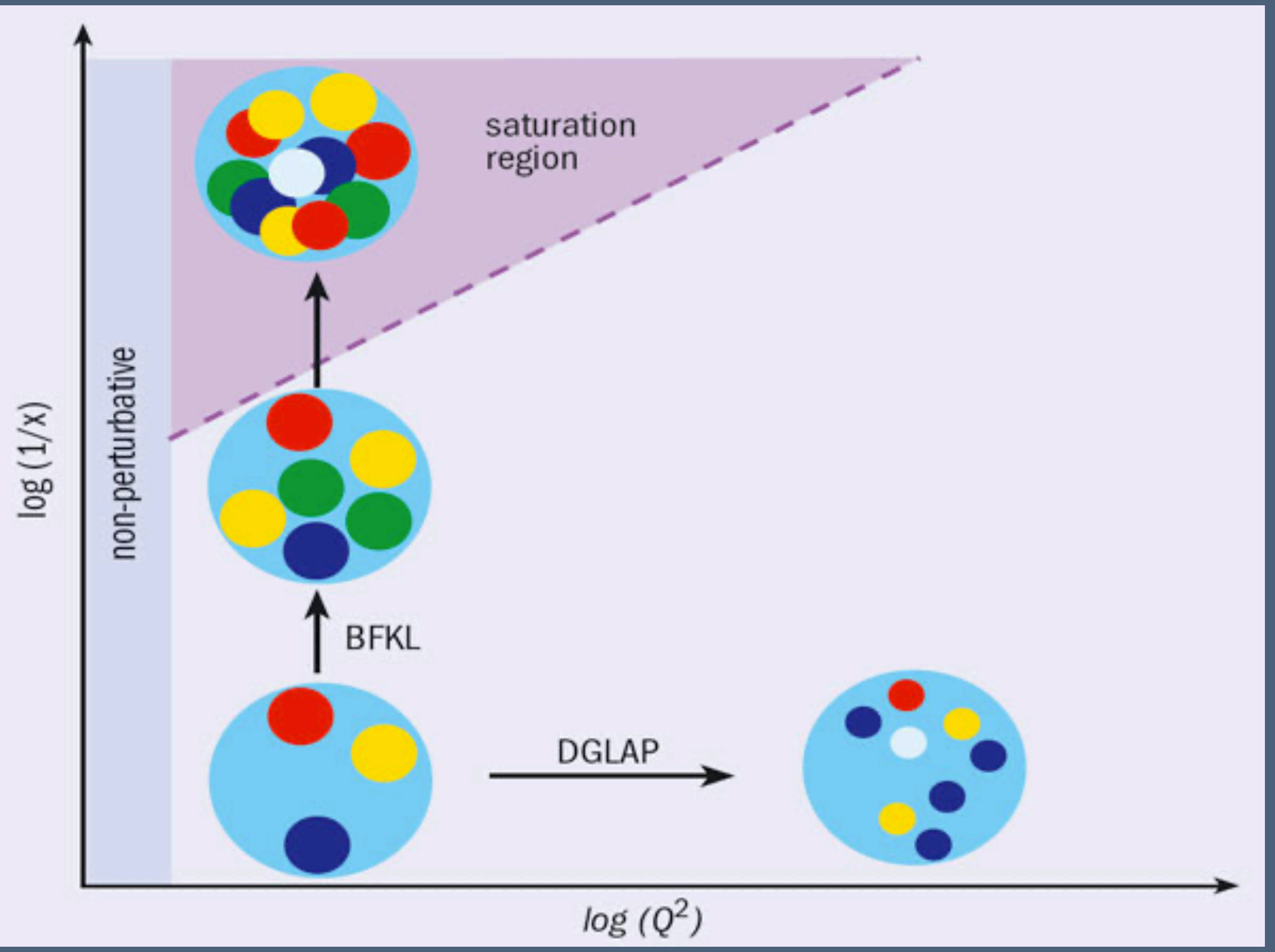


Infrared

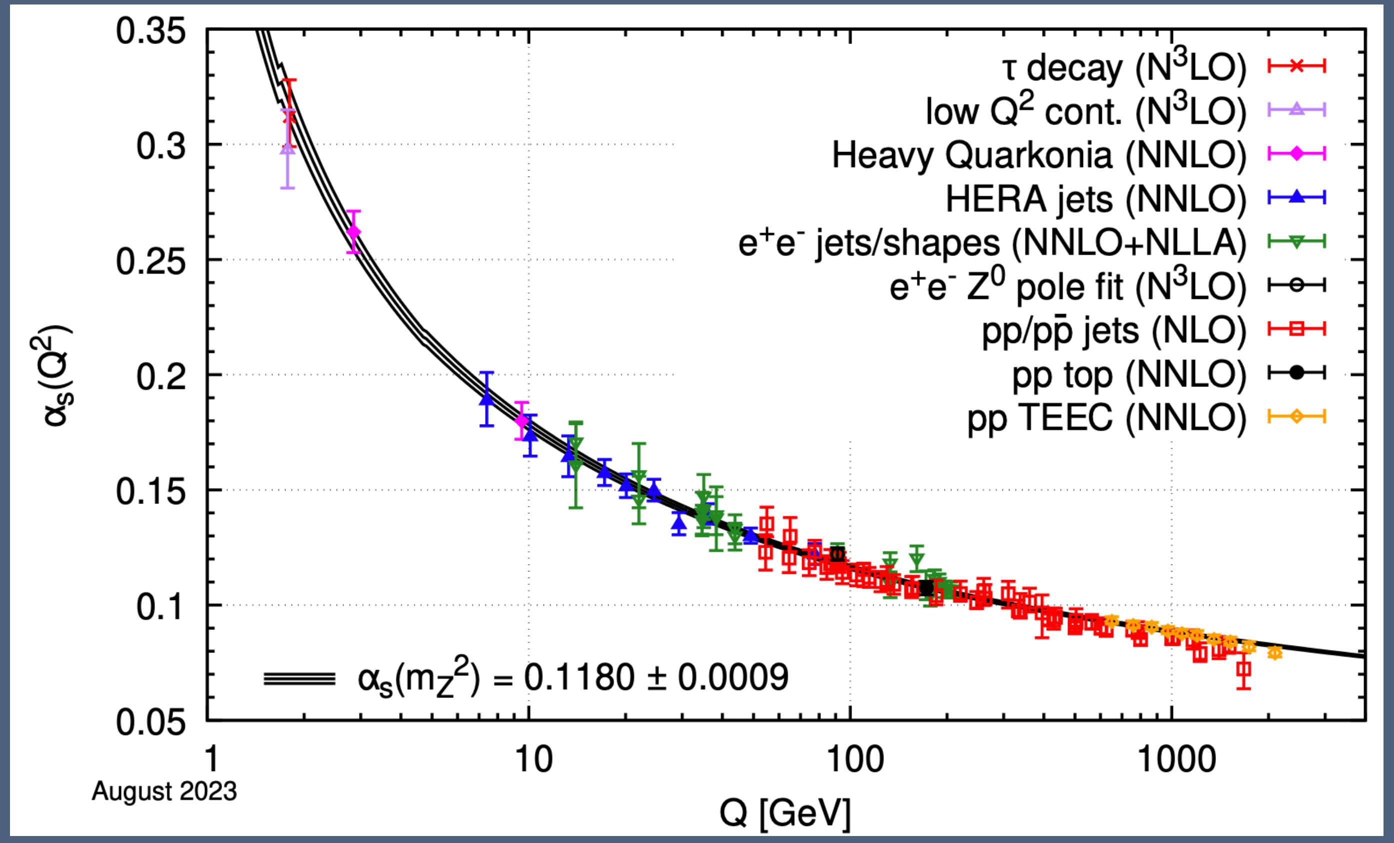
[Dixon et al 1912.09370]



High energy: Regge limit and factorization, BFKL evolution, PDFs@small x etc..



Testing running of strong coupling at TeV scale

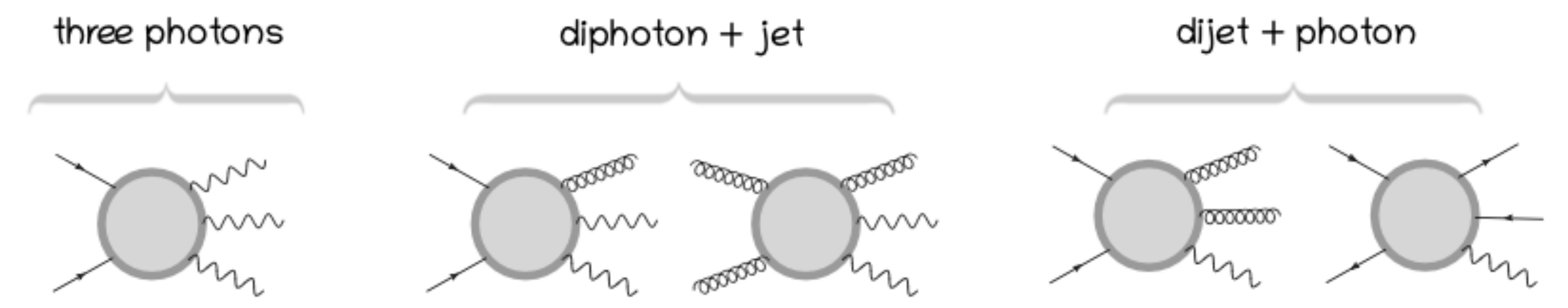


[PDG]

Input for higher order jet cross sections

e.g. 3-jet XS [Czakon, Mitov, Poncelet 2106.05331]

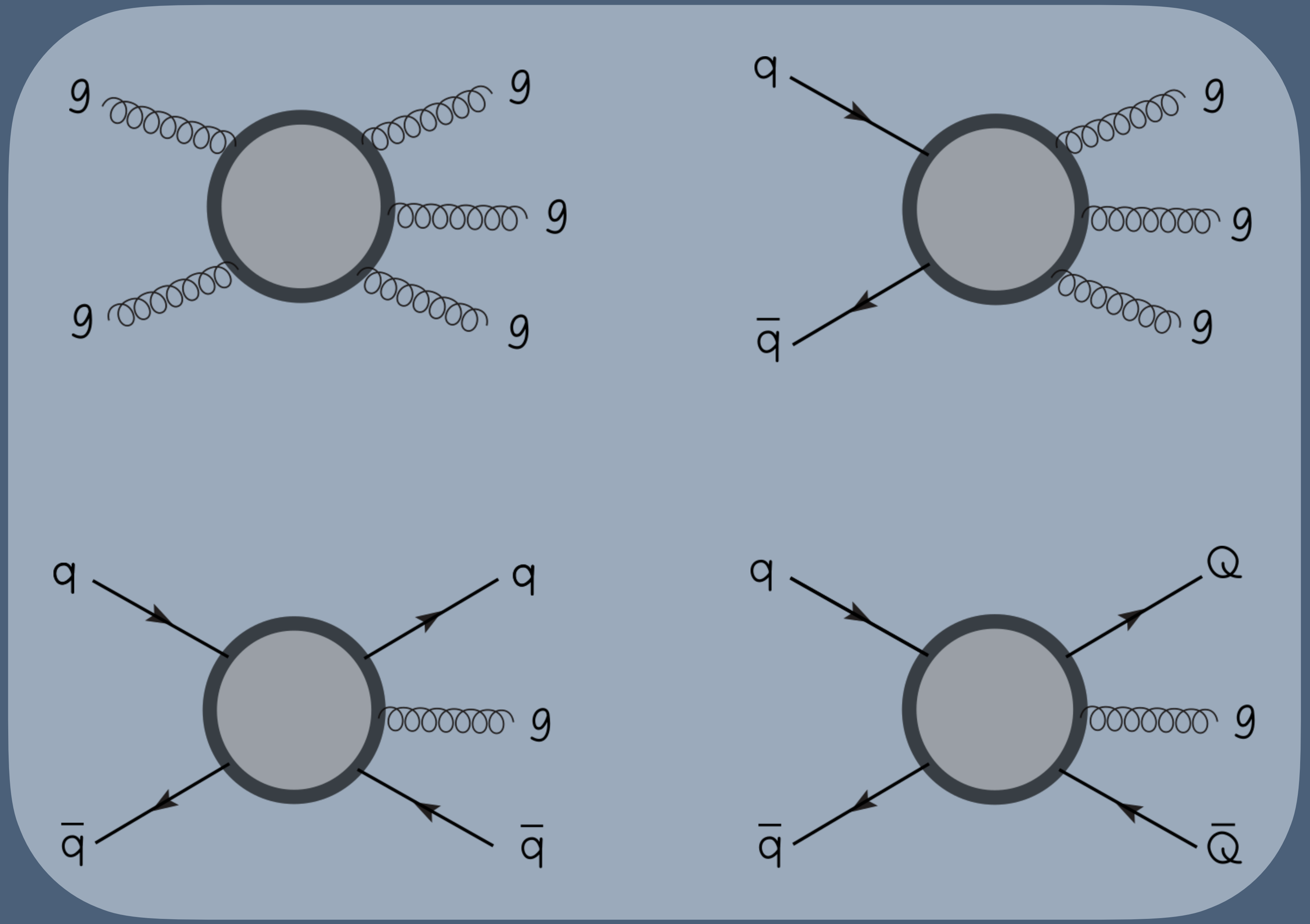
Brief history of 5-point 2-loop QCD amplitudes



Contributors: [Abreu, Agarwal, Badger, Buccioni, Chawhdry, Chicherin, Czakon, Cordero Febres, De Laurentis, Gehrmann, Brønnum-Hansen, Hartanto, Henn, Ita, Klinkert, Kryś, Marcoli, Mitov, Moodie, Page, Pascual, Peraro, Poncelet, Sotnikov, Tancredi, Manteuffel von, Zoia, ...]

This talk

Leading color by Abreu et al in [2102.13609]



Full color: A. Agarwal, F. Buccioni, FD, G. Gambuti, A. von Manteuffel and L. Tancredi [2311.09870] + G. De Laurentis, H. Ita, M. Klinkert, V. Sotnikov in [2311.10086] & [2311.18752]

How

Old but gold

$$\mathscr{A} = \sum \text{Feynman diagrams}$$

> 40k

Helicity projection

Tancredi, Peraro:
1906.03298 & 2012.00820

$$\mathscr{A} = \sum H_h T_h$$

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$$H = \sum_c H_c C_c$$

Color decomposition

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> 40k

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$$\mathscr{A} = \sum H_h T_h$$

$$H = \sum_c H_c C_c$$

Polynomial in N_c, n_f

Color decomposition

C_c	$ggggg$	$q\bar{q}ggg$	$q\bar{q}Q\bar{Q}g$
Tree level	$\mathrm{Tr}(T^{a_1}T^{a_2}T^{a_3}T^{a_4}T^{a_5}) -$ $\mathrm{Tr}(T^{a_5}T^{a_4}T^{a_3}T^{a_2}T^{a_1})$ + permutations	$(T^{a_1}T^{a_2}T^{a_3})_{ji}$ + permutations	$T^a_{ij}\delta_{kl}$ $T^a_{ik}\delta_{jl}$
Beyond tree	$\mathrm{Tr}(T^{a_1}T^{a_2}) \times$ $(\mathrm{Tr}(T^{a_3}T^{a_4}T^{a_5}) - \mathrm{Tr}(T^{a_5}T^{a_4}T^{a_3}))$ + permutations	$\mathrm{Tr}(T^{a_1}T^{a_2})T^{a_3}_{ij}$ $(\mathrm{Tr}(T^{a_3}T^{a_4}T^{a_5}) - \mathrm{Tr}(T^{a_5}T^{a_4}T^{a_3}))\delta_{ij}$ $(\mathrm{Tr}(T^{a_3}T^{a_4}T^{a_5}) + \mathrm{Tr}(T^{a_5}T^{a_4}T^{a_3}))\delta_{ij}$	Same as tree

Optimal choice of color basis depends on problem at hand (see later in MRK)

Old but gold

$$\mathscr{A} = \sum \text{Feynman diagrams}$$

> 40k

Helicity projection

Tancredi, Peraro: 1906.03298 & 2012.00820

$$\mathscr{A} = \sum H_h T_h$$

Color decomposition

$$H = \sum_c H_c C_c$$

Integration by parts identities

$$H = \sum_{m,c} R^{mc} M_m C_c$$

Master integrals & Pentagon functions

Chicherin, Sotnikov: 2009.07803

$$\frac{s_{23}s_{51}s_{45} + (d-4)s_{12}s_{13}s_{25}}{s_{12}s_{23}s_{34}}$$

$\mathcal{O}(10^6)$ Feynman integrals

$\mathcal{O}(10^3)$ master integrals

(Only ~500 indipendent functions)

Finite fields reconstruction
Syzygy techniques

FinRed
(von Manteuffel)

Papadopoulos, Tommasini, Wever, Gehrmann, Henn, Lo Presti, Chicherin, Wasser, Zhang, Mitev, Böhm, Georgoudis, Larsen, Schönemann, Abreu, Page, Zeng,...

$$\frac{s_{23}s_{51}s_{45} + (d-4)s_{12}s_{13}s_{25}}{s_{12}s_{23}s_{34}}$$

Instagram

$$H = \sum_{m,c} R^{mc} M_m C_c$$

Reality


```
id INT(TB,7,247,7,4,1,1,1,-4,1,1,1,1,0,0,0) = ((-19440*s12+18468*q8*s12^2+9720*s23+45036*q1*s23^2+47628*q9*s23^2+16200*q1*q5*s23^3+47628*q1*q9*
s23^3+29160*s34-16524*q8*s12*s34-36936*q8^2*s12^2*s34-9720*q1*s23*s34-9720*q1*s34^2-12636*q8*s34^2+91368*q8^2*s12*s34^2+18468*q8^3*s12^2*s34^2+
10692*q1*q8*s34^3-71928*q8^2*s34^3-55404*q8^3*s12*s34^3+17496*q1*q8^2*s34^4+55404*q8^3*s34^4-18468*q1*q8^3*s34^5+184680*s45-104976*q8*s12*s45+3
0132*q8^2*s12^2*s45+233928*q1*s23*s45+208656*q9*s23*s45+64152*q1*q5*s23^2*s45+156168*q1*q9*s23^2*s45+135756*q5*q9*s23^2*s45-39528*q9^2*s23^2*s4
5-174960*q1*s34*s45-13608*q8*s34*s45+69984*q8^2*s12*s34*s45-30132*q8^3*s12^2*s34*s45-39528*q9^2*s23*s34*s45+108864*q1*q8*s34^2*s45-191484*q8^2*
s34^2*s45+34992*q8^3*s12*s34^2*s45+91368*q1*q8^2*s34^3*s45+20412*q8^3*s34^3*s45-25272*q1*q8^3*s34^4*s45+148716*q1*s45^2+411480*q8*s45^2+60444*q
9*s45^2-99792*q8^2*s12*s45^2+46332*q8^3*s12^2*s45^2-9504*q1*q5*s23*s45^2+99684*q1*q9*s23*s45^2+64440*q5*q9*s23*s45^2-97020*q9^2*s23*s45^2-22680
0*q1*q8*s34*s45^2-10368*q8^2*s34*s45^2-113868*q9^2*s34*s45^2-48600*q8^3*s12*s34*s45^2-34992*q5*q9^2*s23*s34*s45^2-14580*q9^3*s23*s34*s45^2+9072
0*q1*q8^2*s34^2*s45^2+13608*q8^3*s34^2*s45^2-14580*q9^3*s34^2*s45^2-11340*q1*q8^3*s34^3*s45^2-10368*q1*q5*s45^3+414720*q17*q7*s45^3-414720*q17*
q8*s45^3-3240*q8^2*s45^3+84456*q5*q9*s45^3+37080*q9^2*s45^3-156816*q17^2*q7*s12*s45^3+156816*q17^2*q8*s12*s45^3+156816*q17*q8^2*s12*s45^3+62208
*q8^3*s12*s45^3+67068*q17^3*q7*s12^2*s45^3-67068*q17^3*q8*s12^2*s45^3-67068*q17^2*q8^2*s12^2*s45^3-67068*q17*q8^3*s12^2*s45^3-13608*q8^3*s34*s4
5^3-37080*q5*q9^2*s34*s45^3+9720*q9^3*s34*s45^3-4860*q5*q9^3*s34^2*s45^3-16848*q17^2*q7*s45^4+60480*q1*q5*q7*s45^4+16848*q17^2*q8*s45^4+16848*q
17*q8^2*s45^4+13608*q8^3*s45^4+92664*q17^3*q7*s12*s45^4-8424*q17^2*q7^2*s12*s45^4-92664*q17^3*q8*s12*s45^4-84240*q17^2*q8^2*s12*s45^4-75816*q17
*q8^3*s12*s45^4-75816*q17^4*q7*s12^2*s45^4+25272*q17^3*q7^2*s12^2*s45^4+75816*q17^4*q8*s12^2*s45^4+50544*q17^3*q8^2*s12^2*s45^4+25272*q17^2*q8^
3*s12^2*s45^4+13608*q17^3*q7*s45^5-13608*q17^3*q8*s45^5-13608*q17^2*q8^2*s45^5-13608*q17*q8^3*s45^5-40824*q17^4*q7*s12*s45^5+13608*q17^3*q7^2*s
12*s45^5+40824*q17^4*q8*s12*s45^5+27216*q17^3*q8^2*s12*s45^5+13608*q17^2*q8^3*s12*s45^5+27216*q17^5*q7*s12^2*s45^5-13608*q17^4*q7^2*s12^2*s45^5
+4536*q17^3*q7^3*s12^2*s45^5-27216*q17^5*q8*s12^2*s45^5-13608*q17^4*q8^2*s12^2*s45^5-4536*q17^3*q8^3*s12^2*s45^5+116136*s51-4032*q4*s12*s51-222
12*q8*s12*s51+9072*q1*s23*s51+16128*q4*s23*s51+377568*q9*s23*s51-65448*q1*q9*s23^2*s51-9072*q9^2*s23^2*s51+108864*q1^2*q19*s23^3*s51+108864*q1^
2*q9*s23^3*s51-4536*q1*q9^2*s23^3*s51-12960*q1*s34*s51-54648*q8*s34*s51-193536*q9*s34*s51+8064*q4*q8*s12*s34*s51-33192*q8^2*s12*s34*s51-4536*q9
^2*s23*s34*s51-6156*q1*q8*s34^2*s51+193536*q11*q8*s34^2*s51-16128*q4*q8*s34^2*s51+39600*q8^2*s34^2*s51+193536*q11*q9*s34^2*s51-4032*q4*q8^2*s12
*s34^2*s51+55404*q8^3*s12*s34^2*s51-26568*q1*q8^2*s34^3*s51+16128*q4*q8^2*s34^3*s51-110808*q8^3*s34^3*s51+55404*q1*q8^3*s34^4*s51+100440*q1*s45
*s51+360864*q8*s45*s51+71172*q9*s45*s51-23976*q8^2*s12*s45*s51-154872*q1*q9*s23*s45*s51-134784*q9^2*s23*s45*s51-162648*q1*q9^2*s23^2*s45*s51-14
580*q9^3*s23^2*s45*s51-229392*q1*q8*s34*s45*s51-193536*q11*q8*s34*s45*s51+182880*q8^2*s34*s45*s51-193536*q11*q9*s34*s45*s51-205200*q9^2*s34*s45
*s51-34992*q8^3*s12*s34*s45*s51-29160*q9^3*s23*s34*s45*s51+120960*q11^2*q8*s34^2*s45*s51-152280*q1*q8^2*s34^2*s45*s51+24192*q11*q8^2*s34^2*s45*
s51-40824*q8^3*s34^2*s45*s51+120960*q11^2*q9*s34^2*s45*s51+96768*q11*q9^2*s34^2*s45*s51-14580*q9^3*s34^2*s45*s51-24192*q11^3*q8*s34^3*s45*s51-1
2096*q11^2*q8^2*s34^3*s45*s51+75816*q1*q8^3*s34^3*s45*s51-24192*q11^3*q9*s34^3*s45*s51-12096*q11^2*q9^2*s34^3*s45*s51+6912*q1*q7*s45^2*s51+4361
76*q17*q7*s45^2*s51+223128*q1*q8*s45^2*s51-436176*q17*q8*s45^2*s51+56016*q8^2*s45^2*s51+113868*q9^2*s45^2*s51-34272*q17^2*q7*s12*s45^2*s51+3427
2*q17^2*q8*s12*s45^2*s51+34272*q17*q8^2*s12*s45^2*s51+48600*q8^3*s12*s45^2*s51+14580*q9^3*s23*s45^2*s51-192024*q1*q8^2*s34*s45^2*s51-27216*q8^3
*s34*s45^2*s51+29160*q9^3*s34*s45^2*s51+34020*q1*q8^3*s34^2*s45^2*s51-14256*q1*q17*q7*s45^3*s51+66816*q17^2*q7*s45^3*s51-13824*q17*q7^2*s45^3*s
51+14256*q1*q17*q8*s45^3*s51-66816*q17^2*q8*s45^3*s51+10584*q1*q8^2*s45^3*s51-52992*q17*q8^2*s45^3*s51+27216*q8^3*s45^3*s51-4860*q9^3*s45^3*s51
-14400*q17^3*q7*s12*s45^3*s51+38304*q17^2*q7^2*s12*s45^3*s51+14400*q17^3*q8*s12*s45^3*s51-23904*q17^2*q8^2*s12*s45^3*s51-62208*q17*q8^3*s12*s45
^3*s51+10584*q1*q17^2*q7*s45^4*s51-21168*q17^3*q7*s45^4*s51+24192*q17^2*q7^2*s45^4*s51-10584*q1*q17^2*q8*s45^4*s51+21168*q17^3*q8*s45^4*s51-105
84*q1*q17*q8^2*s45^4*s51-3024*q17^2*q8^2*s45^4*s51-27216*q17*q8^3*s45^4*s51-13608*q17^3*q7^2*s12*s45^4*s51+13608*q17^2*q7^3*s12*s45^4*s51+13608
*q17^3*q8^2*s12*s45^4*s51+13608*q17^2*q8^3*s12*s45^4*s51+12096*q4*s51^2+84888*q8*s51^2+12528*q9*s51^2-9072*q1*q9*s23*s51^2+80640*q4*q9*s23*s51^
2-126468*q9^2*s23*s51^2-90720*q1^2*q19*s23^2*s51^2-90720*q1^2*q9*s23^2*s51^2-70308*q1*q9^2*s23^2*s51^2-14580*q9^3*s23^2*s51^2-90720*q1^3*q19*s2
3^3*s51^2-18144*q1^2*q19^2*s23^3*s51^2-90720*q1^3*q9*s23^3*s51^2-72576*q1^2*q9^2*s23^3*s51^2-4860*q1*q9^3*s23^3*s51^2-7128*q1*q8*s34*s51^2-2963
52*q11*q8*s34*s51^2-12096*q4*q8*s34*s51^2+32328*q8^2*s34*s51^2-296352*q11*q9*s34*s51^2-36288*q4*q9*s34*s51^2-47772*q9^2*s34*s51^2-14580*q9^3*s2
3*s34*s51^2+36288*q11*q4*q8*s34^2*s51^2+648*q1*q8^2*s34^2*s51^2-16128*q4*q8^2*s34^2*s51^2+55404*q8^3*s34^2*s51^2+36288*q11*q4*q9*s34^2*s51^2-48
60*q9^3*s34^2*s51^2-55404*q1*q8^3*s34^3*s51^2+2592*q1*q7*s45*s51^2+151740*q17*q7*s45*s51^2+119232*q1*q8*s45*s51^2+193536*q11*q8*s45*s51^2-15174
0*q17*q8*s45*s51^2+8604*q8^2*s45*s51^2+193536*q11*q9*s45*s51^2+205200*q9^2*s45*s51^2+29160*q9^3*s23*s45*s51^2-241920*q11^2*q8*s34*s45*s51^2+304
56*q1*q8^2*s34*s45*s51^2-48384*q11*q8^2*s34*s45*s51^2+20412*q8^3*s34*s45*s51^2-241920*q11^2*q9*s34*s45*s51^2-193536*q11*q9^2*s34*s45*s51^2+2916
0*q9^3*s34*s45*s51^2+72576*q11^3*q8*s34^2*s45*s51^2+36288*q11^2*q8^2*s34^2*s45*s51^2-75816*q1*q8^3*s34^2*s45*s51^2+72576*q11^3*q9*s34^2*s45*s51
^2+36288*q11^2*q9^2*s34^2*s45*s51^2-4968*q1*q17*q7*s45^2*s51^2+37800*q17^2*q7*s45^2*s51^2-5400*q1*q7^2*s45^2*s51^2+7848*q17*q7^2*s45^2*s51^2+49
68*q1*q17*q8*s45^2*s51^2-37800*q17^2*q8*s45^2*s51^2+111888*q1*q8^2*s45^2*s51^2-45648*q17*q8^2*s45^2*s51^2+13608*q8^3*s45^2*s51^2-14580*q9^3*s45
^2*s51^2-34020*q1*q8^3*s34*s45^2*s51^2+10584*q1*q17^2*q7*s45^3*s51^2-21168*q17^3*q7*s45^3*s51^2+10584*q1*q17*q7^2*s45^3*s51^2+10584*q17^2*q7^2*
```

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2009.07803

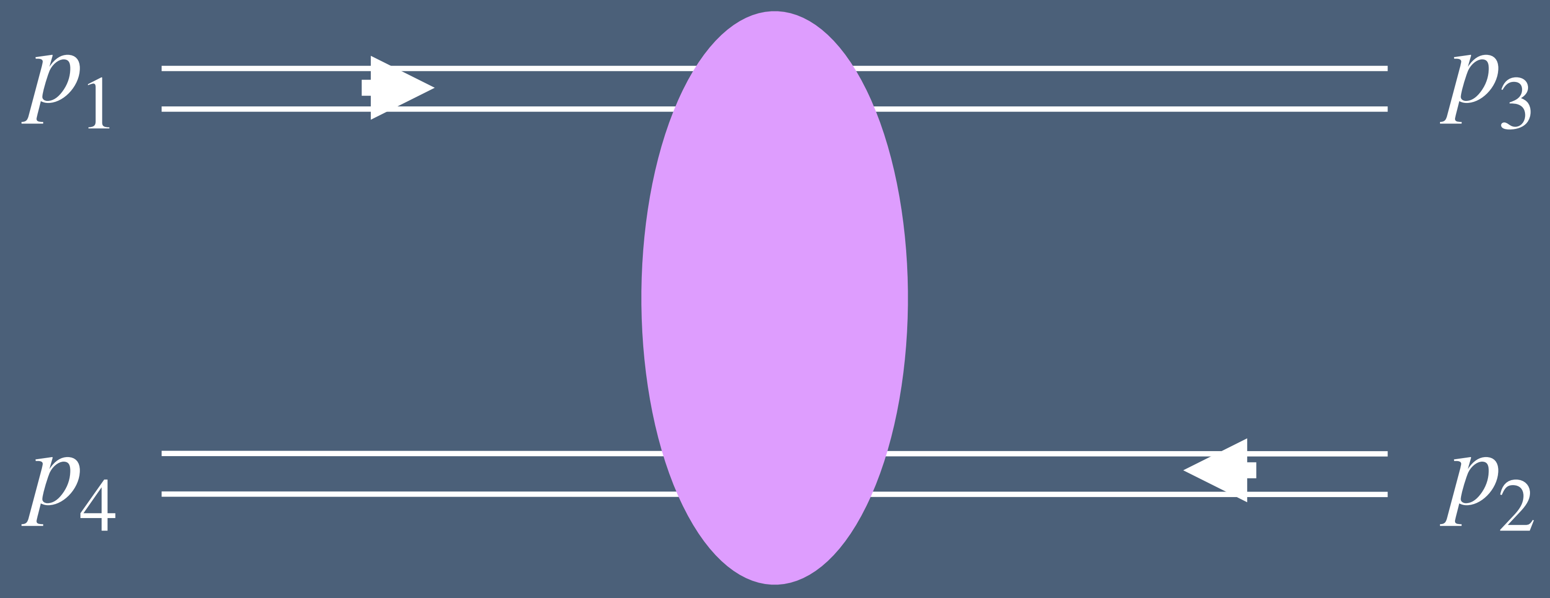
$$\frac{s_{23}s_{51}s_{45} + (d-4)s_{12}s_{13}s_{25}}{s_{12}s_{23}s_{34}}$$

Final result

High energy limit

In collaboration with

Warm-up: $2 \to 2$ amplitudes in Regge kinematics



$$s \sim |u| \gg -t$$

+ Large rapidity gap

$$x = \frac{-t}{s}$$

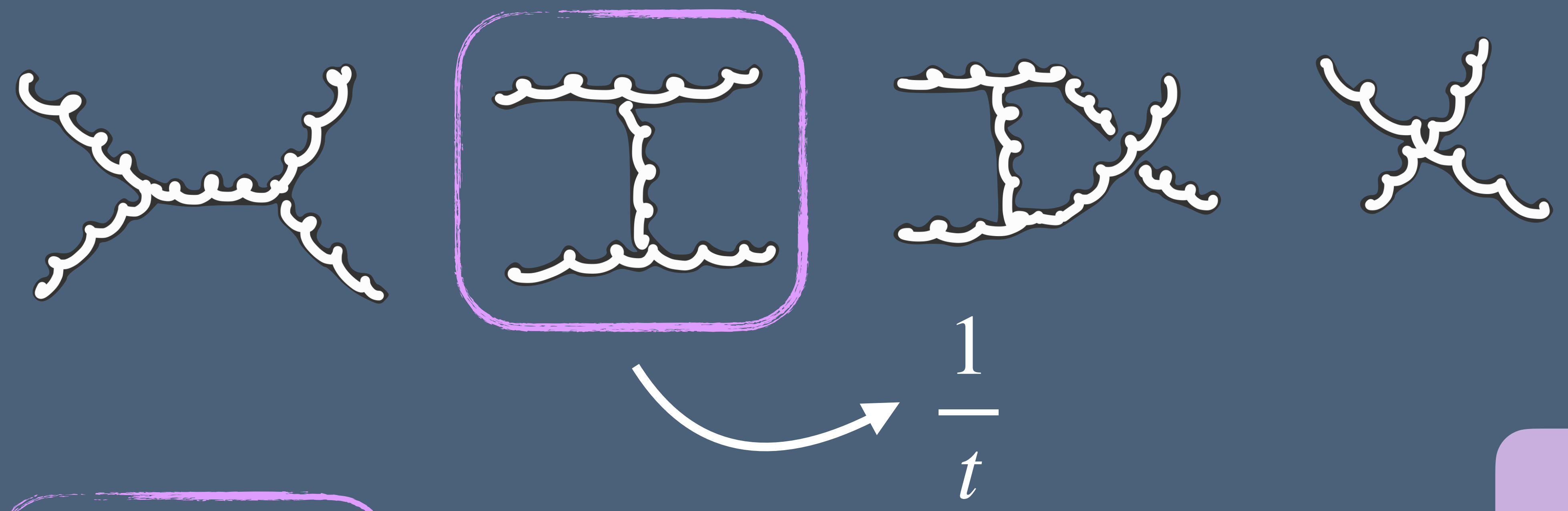
Light-cone components

$$p_1^\mu \sim p_1^+ \quad p_2^\mu \sim p_2^-$$

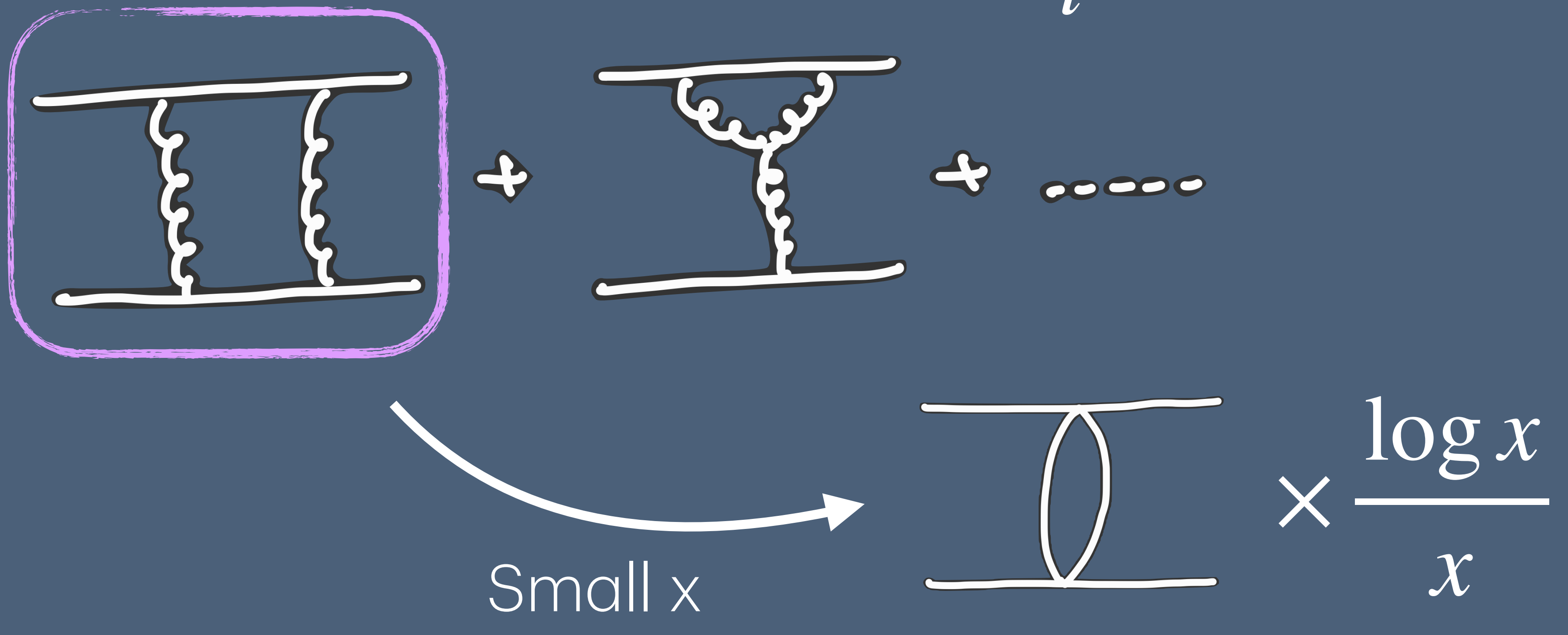
$$p_3^+ \gg |p_3^\perp| \quad p_4^- \gg |p_4^\perp|$$

$$s_{12} = p_1^+ p_2^- \quad t \sim -k_\perp^2$$

Tree level:



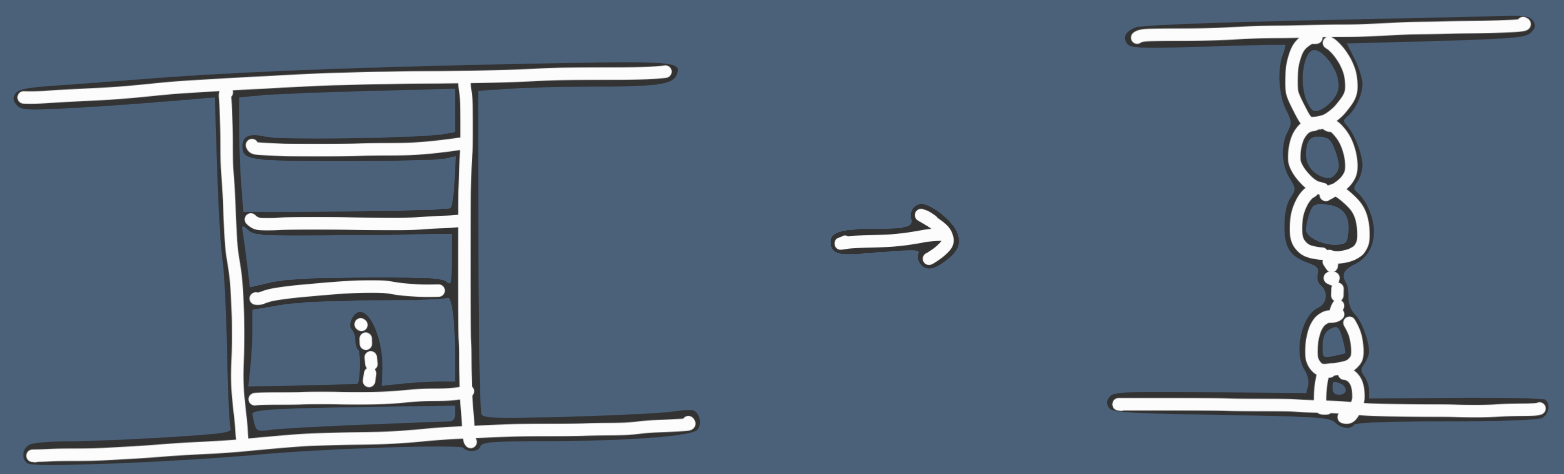
One loop:



Dominant contribution (LL): $\mathscr{A} \sim \tau_g(t) \frac{\log x}{x}$

Warm-up: $2 \to 2$ amplitudes in Regge kinematics

Structure repeated to all orders: generalized ladder topologies



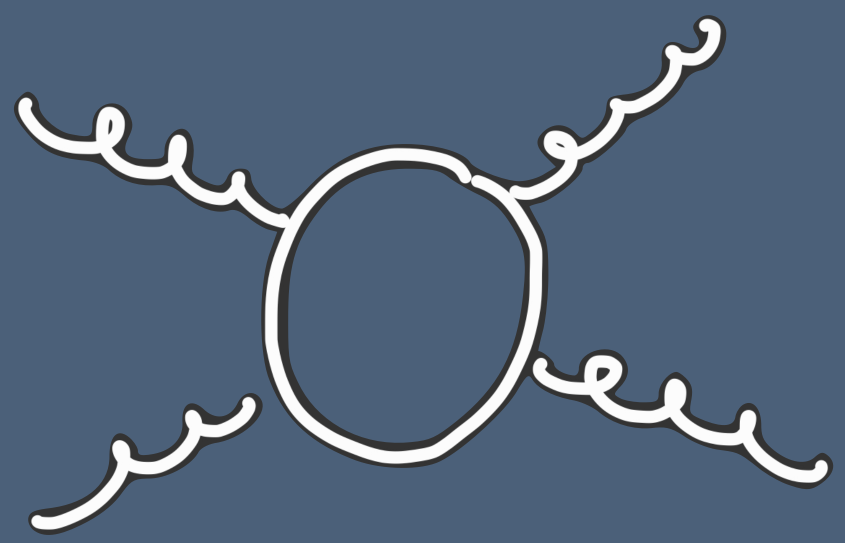
NB: this picture is schematic, in QCD things are more complicated

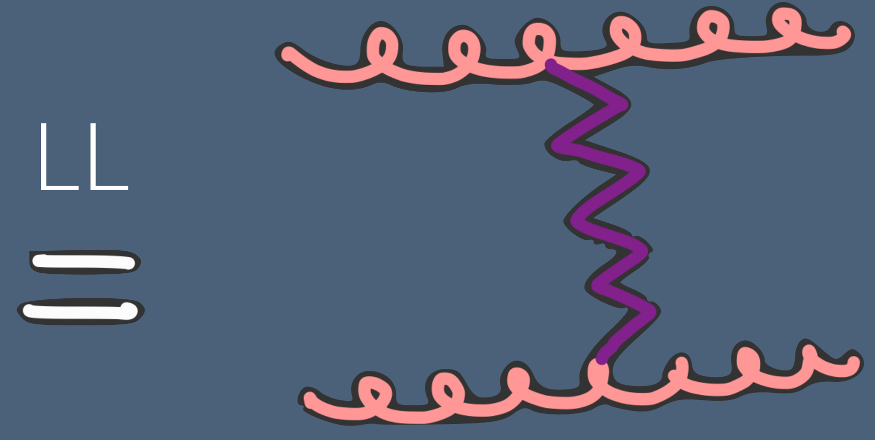
Factorization at LL

$$\mathscr{A}(s,t) \simeq \mathscr{A}^{(0)}(s,t)\left(\frac{s}{-t}\right)^{C_A\,\alpha_s\,\tau_g(t)}$$

τ_g gluon Regge trajectory

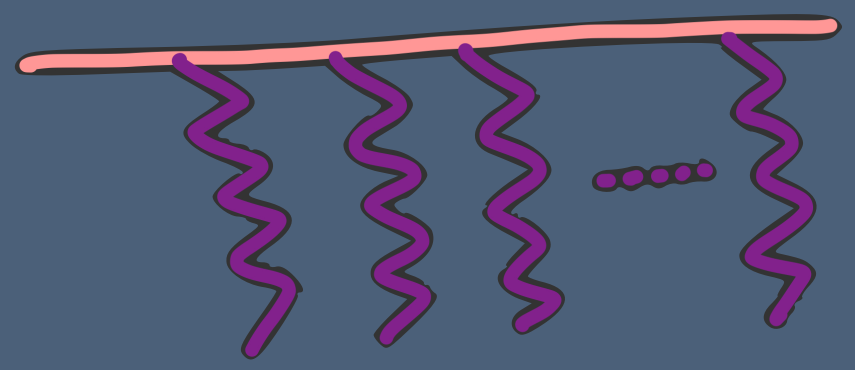
Gluon "reggeization" at LL





Universal, does not depend on partonic nature of projectiles

Beyond LL: effective theory of "reggeons"



Regge pole is in the "odd amplitude": define signature eigenstates

$$\mathscr{A}^{\pm} = \frac{1}{2}\left(\mathscr{A}(s,t) \pm \mathscr{A}(-s-t,t)\right)$$

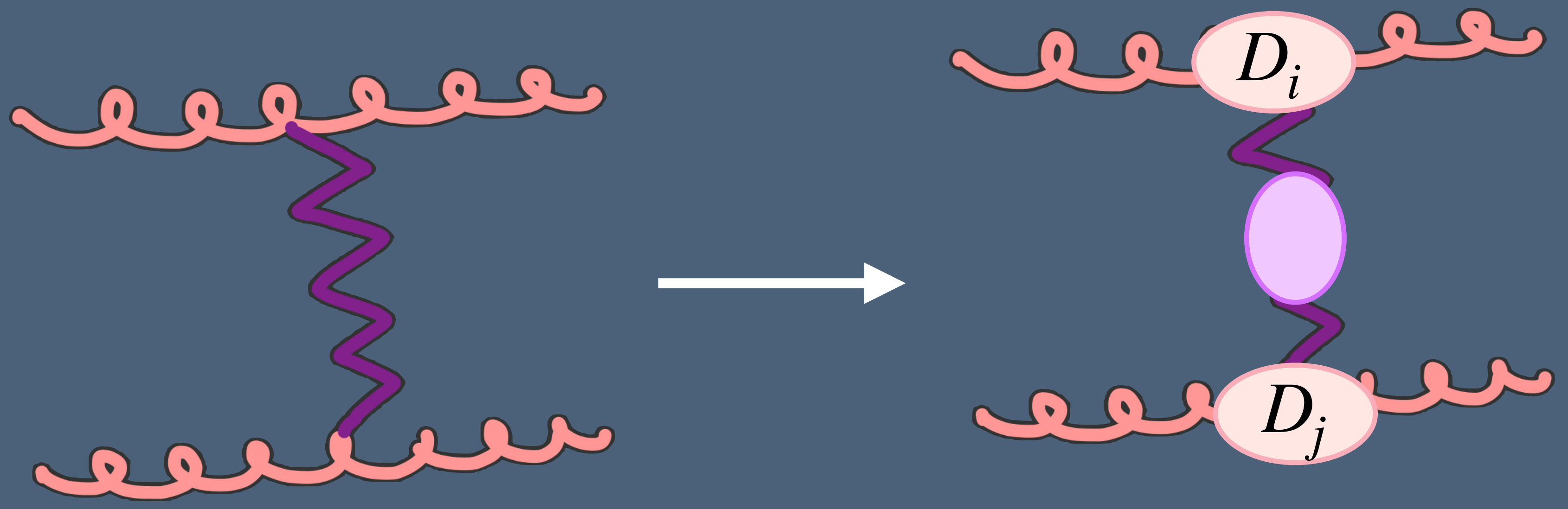
$\mathscr{A}^{+}$: "even" → Starts with cut @1-loop

$\mathscr{A}^{-}$: "odd" → Contains Regge pole

NLL factorization [Fadin, Lipatov hep-ph/9802290]

$$\text{LL} \sim \left(\frac{\alpha_s}{2\pi}\right)^n \log^n x$$

$$\text{NLL} \sim \left(\frac{\alpha_s}{2\pi}\right)^n \log^{n-1} x$$



$$\mathscr{A}(s,t) \simeq \mathscr{A}^{(0)}(s,t) D_i(t) D_j(t) \left(\frac{s}{-t}\right)^{C_A \alpha_s \tau_g(t,\alpha_s)}$$

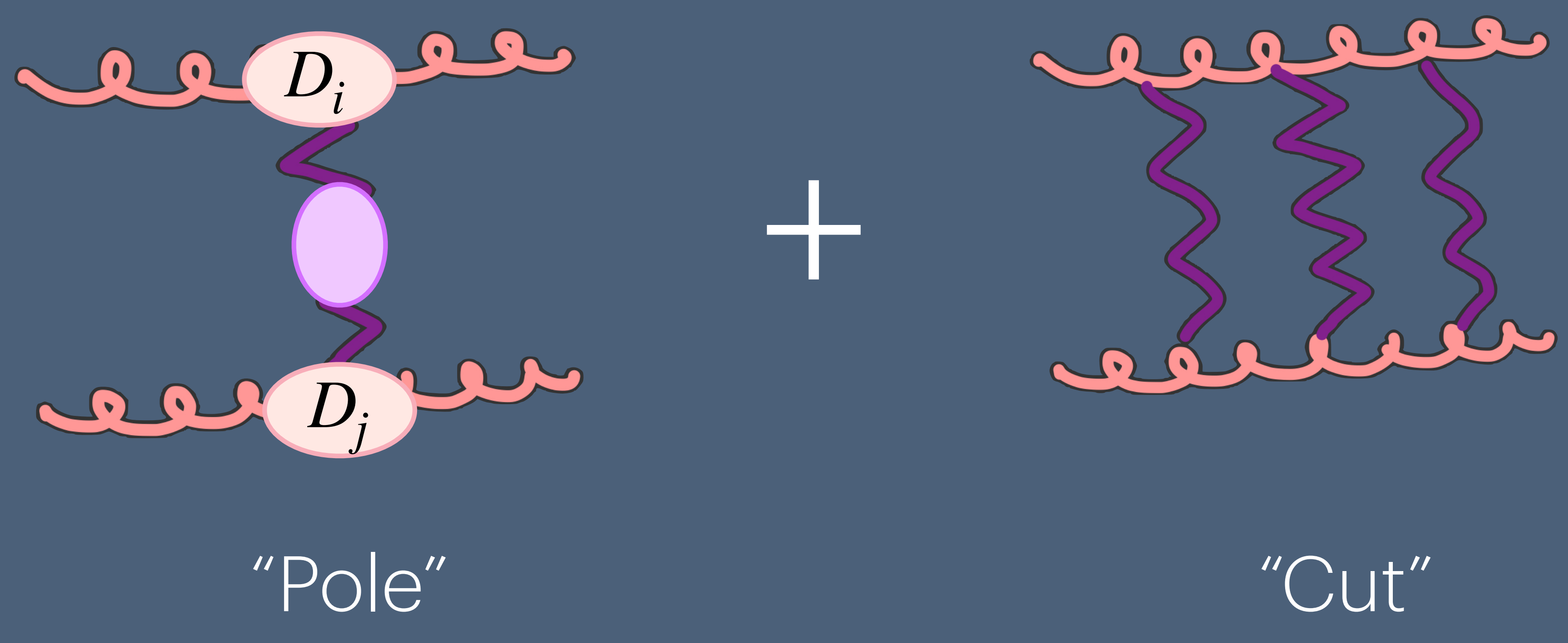
Impact factors

Corrections to Regge trajectory

Factorization still holds (in $\mathscr{A}^{-}$)

NNLL factorization

$$\text{NNLL} \sim \left(\frac{\alpha_s}{2\pi}\right)^n \log^{n-2} x$$



Regge cuts responsible for violation of factorization at NNLL

Factorization is restored once “cut contamination” is removed

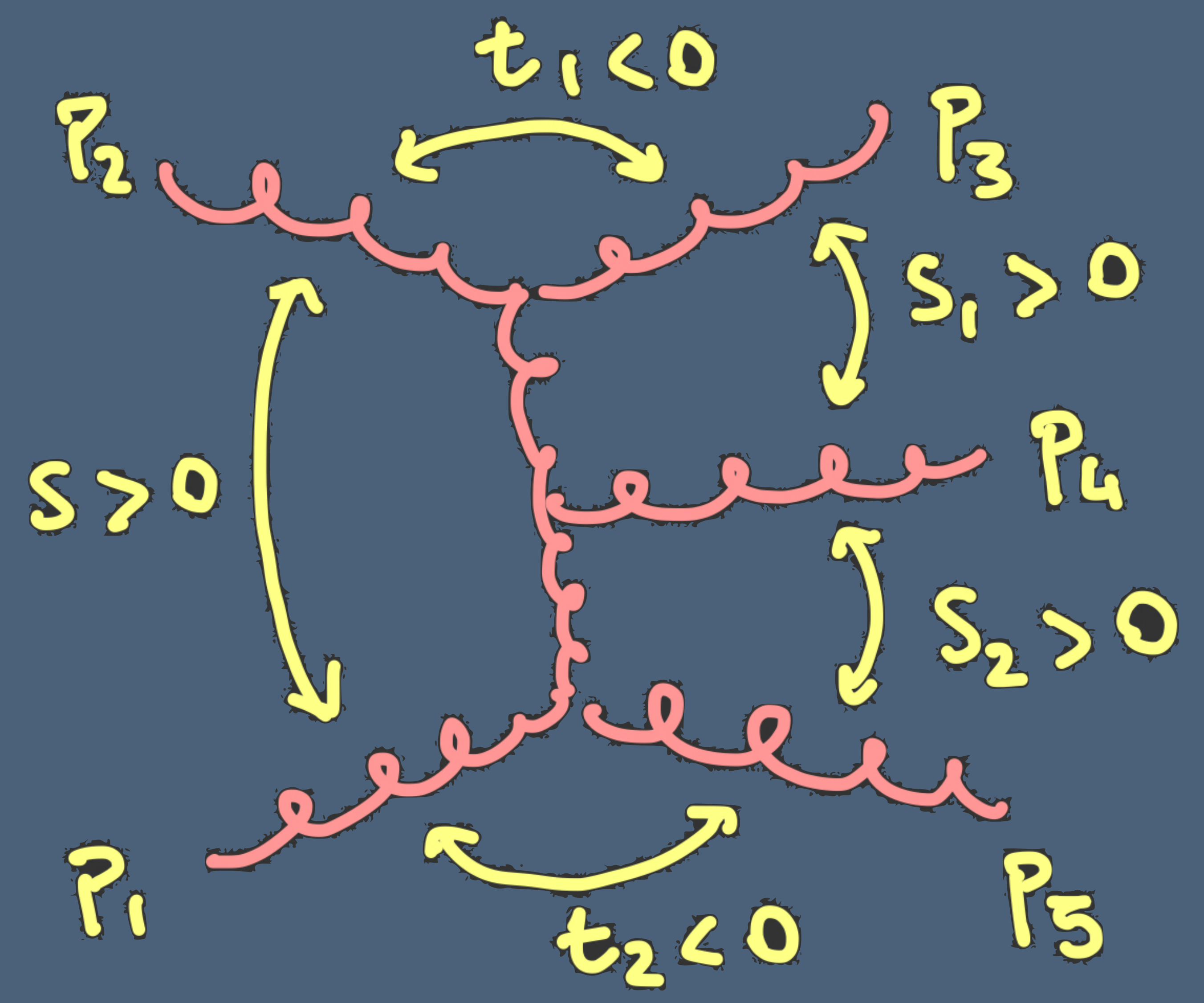
How?

Wilson line approach

[Balitsky/JIMWLK + Caron-Huot 1309.6521, Caron-Huot, Gardi, Vernazza 1701.05241]

$2 \to 3$ amplitudes: Multi-Regge kinematics

$$A^{(h_A)}(p_1)\, B^{(h_B)}(p_2) \to B'^{(h_{B'})}(p_3)\, g^{(h_g)}(p_4)\, A'^{(h_{A'})}(p_5)$$



[Caron-Huot, Chicherin, Henn, Zhang, Zoia 2003.03120]

Strong rapidity ordering, no ordering in transverse components, gluon centrally emitted

$$p_5^+ \gg p_4^+ \gg p_3^+\,, \quad p_3^- \gg p_4^- \gg p_5^-\,, \quad p_4^\pm \sim |p_{3,\perp}| \sim |p_{4,\perp}| \sim |p_{5,\perp}|$$

Longitudinal and transverse dynamics completely factorized

MRK limit: $x \to 0$

$$p_1^+, p_5^+, p_2^-, p_3^- \sim 1/x\,, \quad p_4^+, p_4^- \sim 1\,, \quad p_2^+, p_3^+, p_1^-, p_5^- \sim x$$

$$s_{12} = \frac{s}{x^2}, \quad s_{23} = -\frac{s_1 s_2}{s} z\bar{z}, \quad s_{34} = \frac{s_1}{x}$$

$$s_{45} = \frac{s_2}{x}, \quad s_{51} = -\frac{s_1 s_2}{s}(1-z)(1-\bar{z})$$

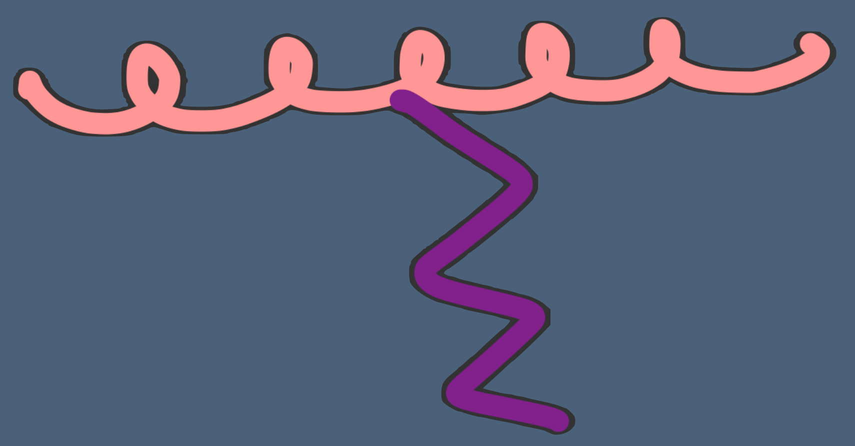
Signature and color decomposition

$$\mathscr{A}^{(\sigma_a,\sigma_b)}(p_1,p_2,p_3,p_4,p_5) \sim \mathscr{A}(p_1,p_2,p_3,p_4,p_5) + \sigma_a \mathscr{A}(p_5,p_2,p_3,p_4,p_1)$$
$$+\sigma_b \mathscr{A}(p_1,p_3,p_2,p_4,p_5) + \sigma_a\sigma_b \mathscr{A}(p_5,p_3,p_2,p_4,p_1)$$

$\mathscr{A}^{(-,-)}$ odd, odd
$\mathscr{A}^{(+,+)}$ even, even
$\mathscr{A}^{(+,-)}$ even, odd
$\mathscr{A}^{(-,+)}$ odd, even

Contains pole contributions, relevant part for factorization
Beyond NLL, receives cut contamination

Color: t-channel in irreducible representation



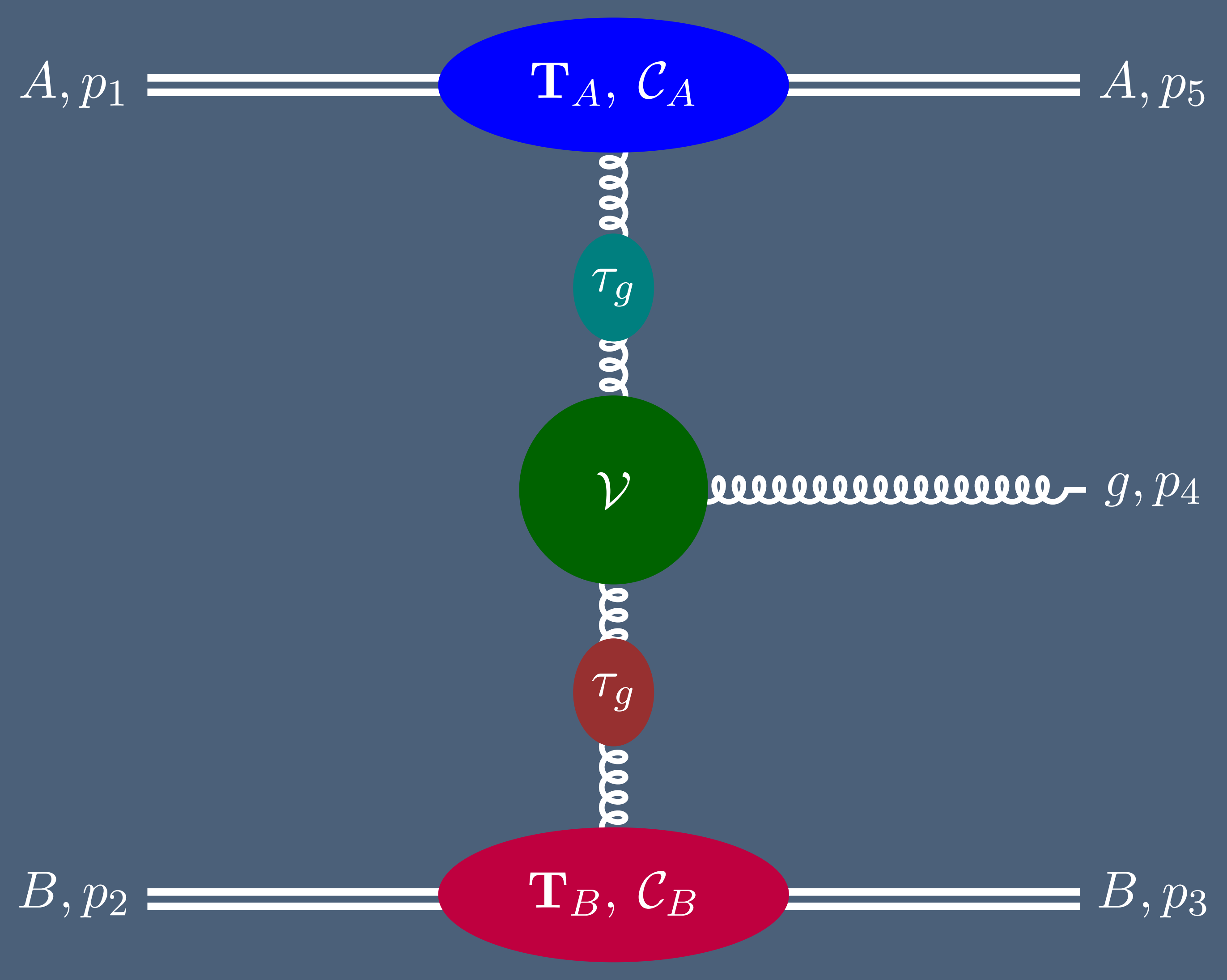
symmetric

$$8 \otimes 8 = 0 + 1 + 8_a + 8_s + 10 + \overline{10} + 27$$

antisymmetric

Only $(8_a,8_a)$ amplitude receives pole contribution

$2 \to 3$ amplitudes: factorization at NLL and beyond



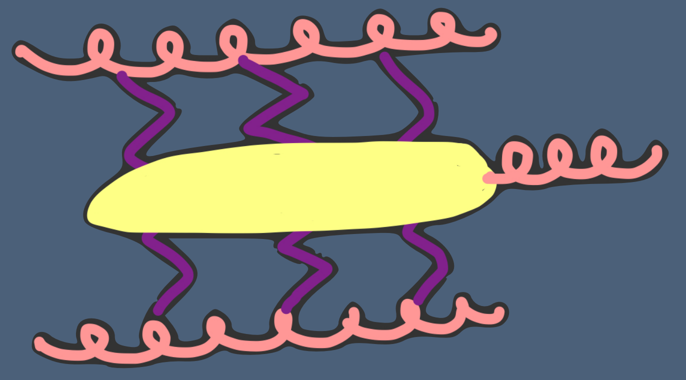
$\mathscr{A}^{(-,-)}$: Factorization at NLL works as expected, only pole contributions

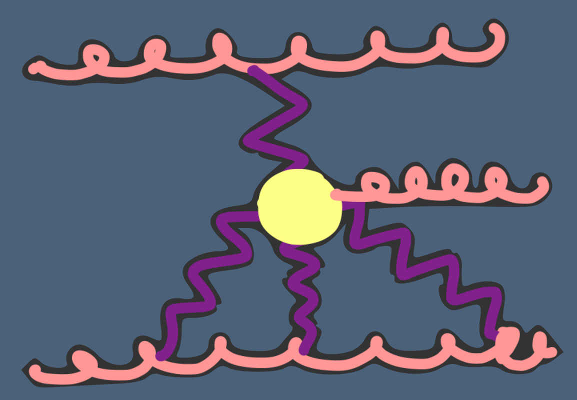
$$\mathscr{A}^{\lambda}(s) = s_{12} \left[\mathbf{T}_A^a \mathscr{C}_A^{\lambda_A}(s_{51})\right] \frac{\mathscr{R}(s_{45}, s_{51})}{s_{51}} \left[f^{aba_4} \mathscr{V}^{h_g}\right] \frac{\mathscr{R}(s_{34}, s_{23})}{s_{23}} \left[\mathbf{T}_B^b \mathscr{C}_B^{\lambda_B}(s_{23})\right]$$

$$\mathscr{R}(s,t) = \frac{1}{2}\left[\left(\frac{s}{t}\right)^{\tau_g(t)} + \left(\frac{-s}{t}\right)^{\tau_g(t)}\right]$$

New ingredient: central emission vertex (Lipatov vertex)

At NNLL: cut contamination in $\mathscr{A}^{(-,-)}$, they enter $(8_a, 8_a)$





Strategy: compute the cut contributions from Wilson-line approach and subtract it to expansion of 5-point amplitudes to get universal structures

Need expansion of amplitude: rational + trascendental

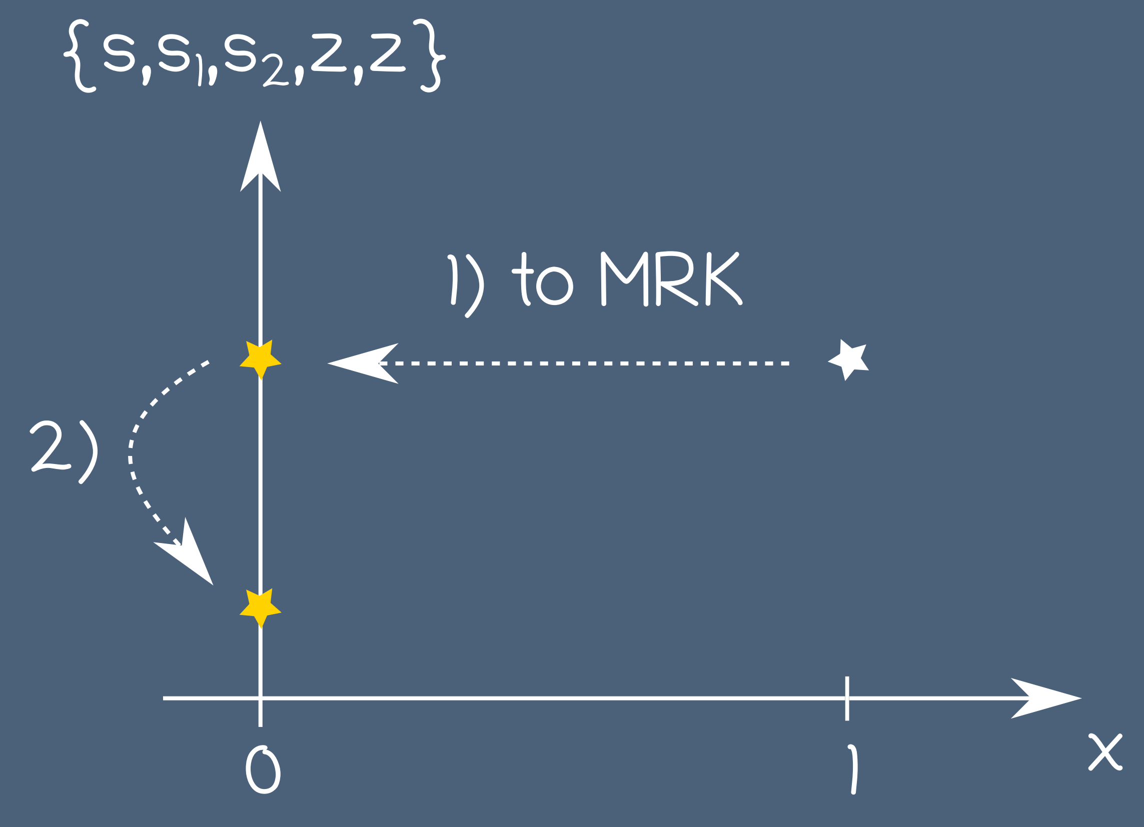
Expansion of the amplitude in MRK

Rational — Transcendental

$$\{s_{12}, s_{23}, s_{34}, s_{45}, s_{51}\} \to \{s, s_1, s_2, z, \bar{z}\} + x$$

Scaling parameter
$x = 1$: "physical point",
$x = 0$:MRK

Set of PF and BC from [Chicherin, Sotnikov: 2009.07803]



$$dI_i(\mathbf{s}) = \epsilon \, dA_{ij}(\mathbf{s}) \, I_j(\mathbf{s}) \qquad dA_{ij}(\mathbf{s}) = \sum_{n=1} a^n_{ij} \, d\log(W_n)$$

$$\begin{cases} \dfrac{\partial}{\partial x}\mathbf{f}(x,y,\epsilon) = \epsilon \, A_x(x,y)\, \mathbf{f}(x,y,\epsilon) \\ \dfrac{\partial}{\partial y}\mathbf{f}(x,y,\epsilon) = \epsilon \, A_y(x,y)\, \mathbf{f}(x,y,\epsilon) \end{cases}$$

$$W_n \to W_n(x)$$

[Caron-Huot, Chicherin, Henn, Zhang, Zoia 2003.03120]

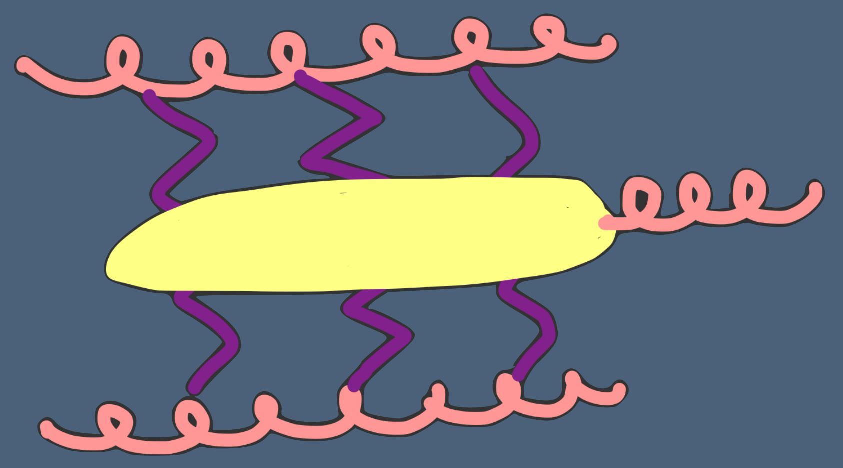
$$\{x\},\ \{\frac{s_1 s_2}{s}\},\ \{s_1, s_2, s_1 - s_2, s_1 + s_2\},$$
$$\{z, \bar{z}, 1 - z, 1 - \bar{z}, z - \bar{z}, 1 - z - \bar{z}\}$$

$$f^{(w)}(\mathbf{s}; x) = \sum_{n=0} \sum_{m=0}^{w} f^{(w)}_{mn}(\mathbf{s}) \, x^n \log^m x$$

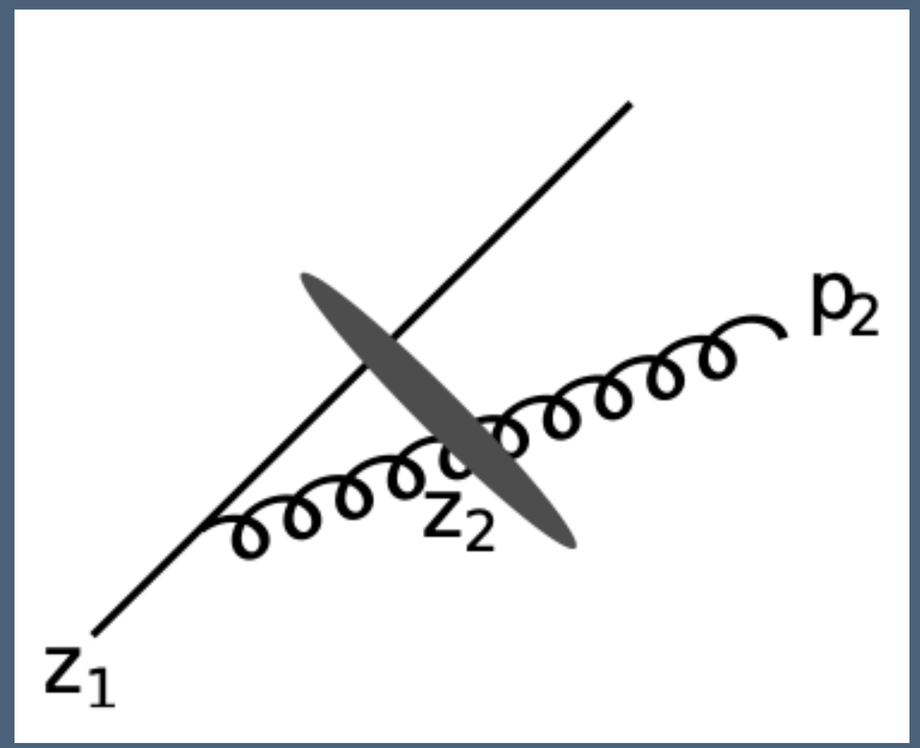
Cut contributions: Wilson-line EFT approach

$$U = e^{i g_s T\cdot W}$$

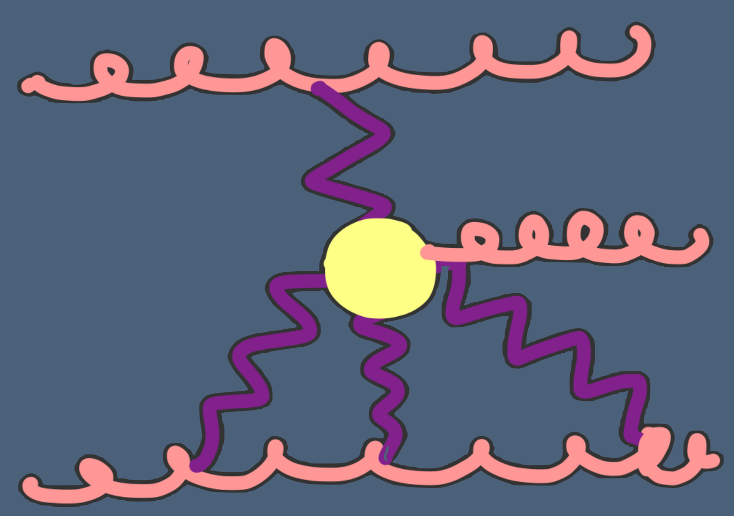
$$U(p_1)a^a(p_2) \sim -2g\int [dz_1][dz_2]\left[U^{ab}_{\rm adj}(z_2)\hat{T}^b_{R,1} - \hat{T}^a_{L,1}\right]U(z_1)e^{-ip_1\cdot z_1 - ip_2\cdot z_2}\int [dq]\frac{\epsilon\cdot q}{q^2}e^{iq\cdot(z_2-z_1)}$$



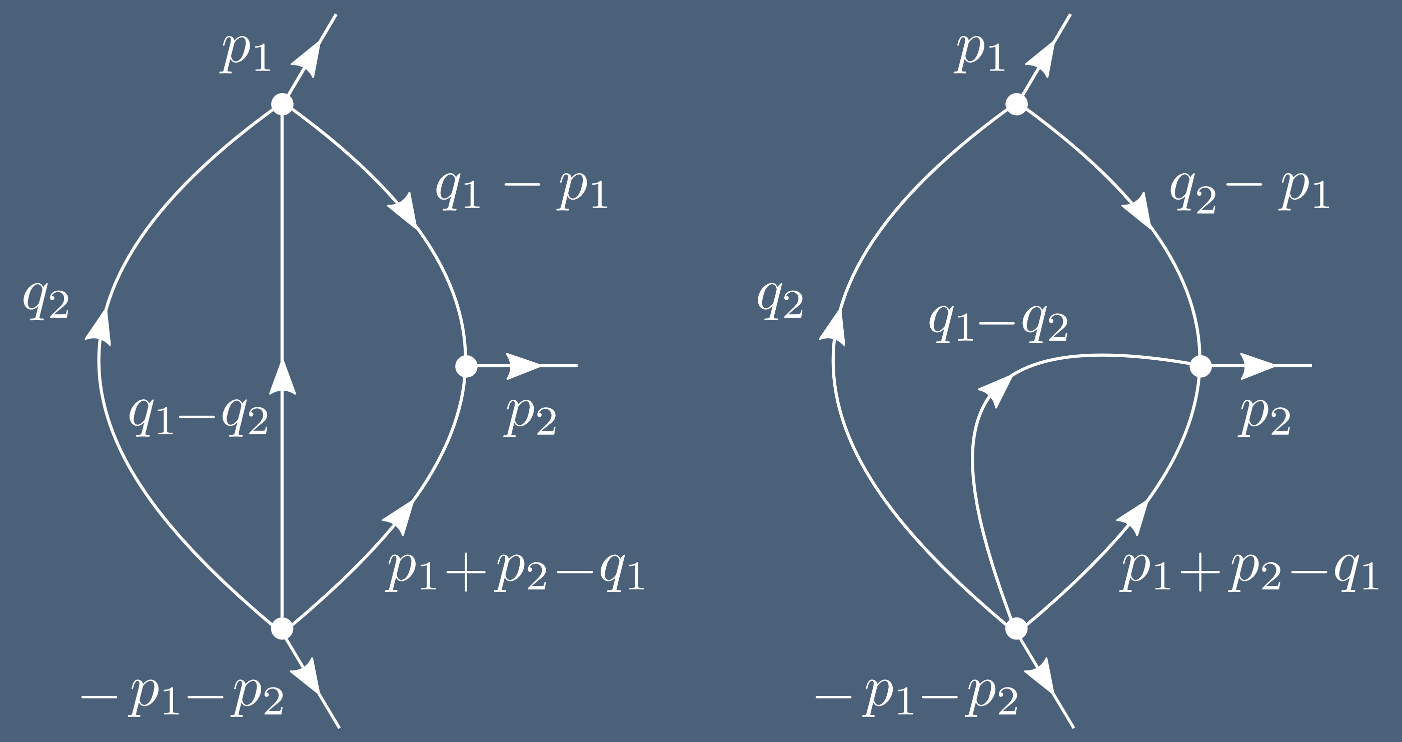
$$W^a_1 W^b_1 W^c_1 a^d \sim 2g_s[F^{a_1}]^{da}\int [dq_1][dq_2]W^{a_1}(p_1+p_2-q_1)W^b(q_1-q_2)W^c(q_2)\times$$
$$\times\left[\frac{\epsilon\cdot p_2}{p_2^2} - \frac{\epsilon\cdot(q_1-p_1)}{(q_1-p_1)^2}\right] + (a\leftrightarrow b) + (a\leftrightarrow c) + \mathcal{O}(g_s^2)$$



Caron-Huot [1309.6521]

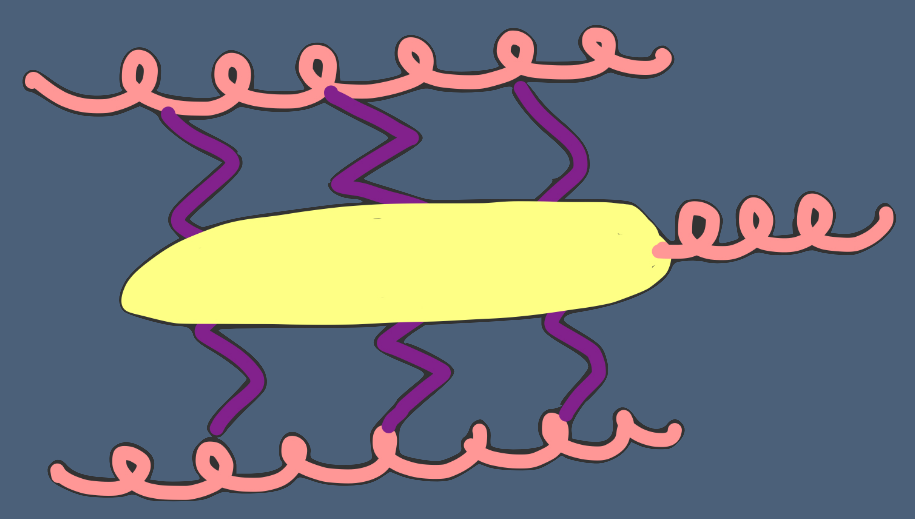


$$W^a_1 a^b \sim -2g[\ldots F\ldots]^{ba} = 2g_s[F^{a_1}]^{ba}W^{a_1}(p_1+p_2)\left[\frac{\epsilon\cdot p_1}{p_1^2} + \frac{\epsilon\cdot p_2}{p_2^2}\right]$$
$$+ig_s^2[F^{a_1}F^{a_2}]^{ba}\int [dq_1]W^{a_1}(p_1+p_2-q_1)W^{a_2}(q_1)\left[\frac{\epsilon\cdot p_1}{p_1^2} + \frac{\epsilon\cdot(q_1-p_1)}{(q_1-p_1)^2}\right]+$$
$$+g_s^3[F^{a_1}F^{a_2}F^{a_3}]^{ba}\int [dq_1][dq_2]W^{a_1}(p_1+p_2-q_1)W^{a_2}(q_1-q_2)W^{a_3}(q_2)\times$$
$$\times\left[\frac{1}{6}\left(\frac{\epsilon\cdot(q_1-p_1)}{(q_1-p_1)^2}\right) - \frac{1}{2}\left(\frac{\epsilon\cdot(q_2-p_1)}{(q_2-p_1)^2}\right) - \frac{1}{3}\left(\frac{\epsilon\cdot p_1}{p_1^2}\right)\right] + \mathcal{O}(g_s^4)$$



Our focus is (8,8)

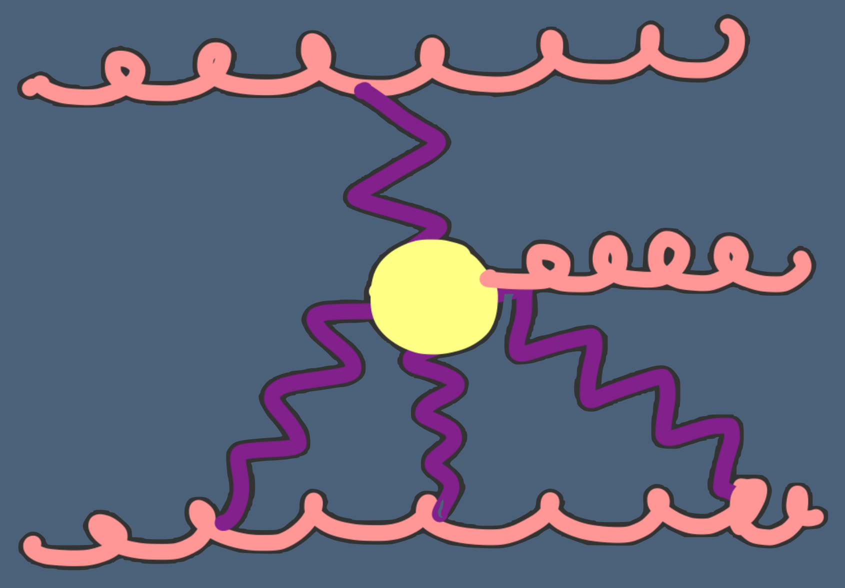
Cut contributions in (8,8) amplitude: Wilson-line EFT approach



$$\mathscr{A}^{2,\mathrm{cut}}_{ab,[8,8]} = \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{i\pi}{3}\right)^2 \left(\frac{\mu^2}{|\mathbf{p}_4|^2}\right)^{2\epsilon} \left[\mathscr{F}_{\mathrm{LC}}(z,\bar{z},\epsilon) + \mathscr{F}_{ab}(z,\bar{z},\epsilon)\right]$$

$$\mathscr{F}_{\mathrm{LC}}(z,\bar{z},\epsilon) = N_c^2\Bigg(\frac{2}{\epsilon^2} - \frac{\log(z\bar{z}) + \log((1-z)(1-\bar{z}))}{\epsilon} + 6iD_2(z,\bar{z}) - 2\zeta_2 + \frac{5}{2}\log^2(z\bar{z}) + \frac{5}{2}\log^2((1-z)(1-\bar{z})) - \log(z\bar{z})\log((1-z)(1-\bar{z}))\Bigg)$$

LC is universal



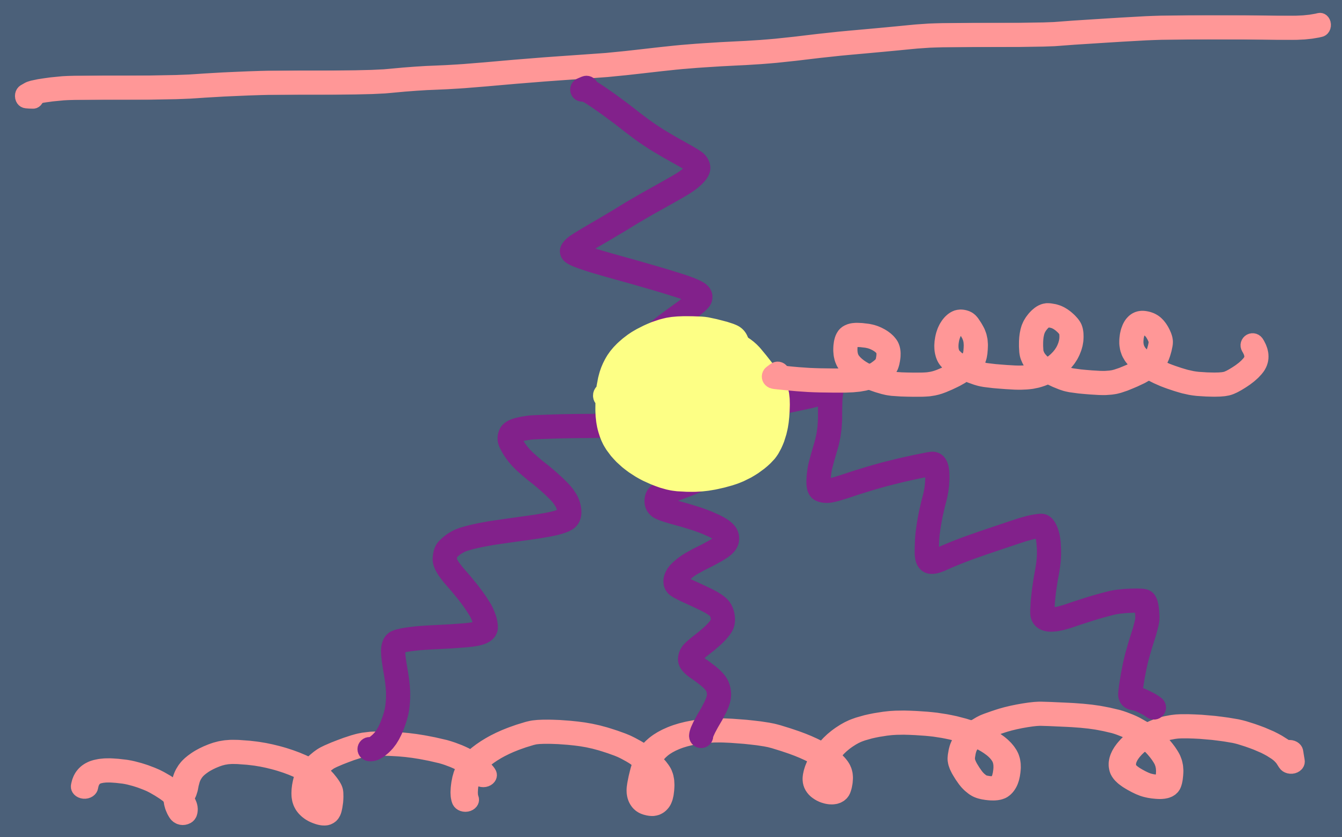
$$\mathscr{F}_{qq}(z,\bar{z},\epsilon) = \frac{18}{\epsilon}\big(\log((1-z)(1-\bar{z})) + \log(z\bar{z})\big) - 9\big(\log((1-z)(1-\bar{z})) + \log(z\bar{z})\big)^2 + 108iD_2(z,\bar{z}) + \frac{1}{N_c^2}\Bigg[\frac{18}{\epsilon^2} - \frac{36}{\epsilon}\big(\log((1-z)(1-\bar{z})) + \log(z\bar{z})\big) - 108iD_2(z,\bar{z}) - 18\zeta_2 + 36\log^2((1-z)(1-\bar{z})) + 36\log^2(z\bar{z}) + 18\log((1-z)(1-\bar{z}))\log(z\bar{z})\Bigg]$$

$$\mathscr{F}_{qg}(z,\bar{z},\epsilon) = \frac{27}{\epsilon^2} + \frac{1}{\epsilon}\big(54\log((1-z)(1-\bar{z})) - 36\log(z\bar{z})\big) + 216iD_2(z,\bar{z}) - 27\zeta_2 + 45\log^2(z\bar{z}) - 36\log(z\bar{z})\log((1-z)(1-\bar{z}))$$

SLC is non-universal

$$D_2(z,\bar{z}) = -i\left(\frac{\log(z\bar{z})}{2}\big(\log(1-z) - \log(1-\bar{z})\big) + \mathrm{Li}_2(z) - \mathrm{Li}_2(\bar{z})\right)$$

In qg $1 \to 3$ reggeon interaction also contributes to $(8_a, 10 + \overline{10})$: we can check it against the MRK expansion of the amplitude!



We find agreement

Odd-Odd: [3→1] in (8,10+10b)

```
(* result from expansion of two-loop amplitudes *)
Do[
  TwoLoopExp["+", "2q3g", "oo", col] =
   A2["2q3g", "MRK", "ppmpm", col] /. MRKpar /. sgn[_] → 1 /. G ⧴ tG /. {tG[0, x_] ⧴ Log[x], tG[1, x_] ⧴ Log[1 - x]} /. tG → G /. ss → s /.
    OddOdd;
  TwoLoopExp["+", "2q3g", "oo", col] = Collect[TwoLoopExp["+", "2q3g", "oo", col], {ep, Nc, π, _G, _Log}, Expand],
  {col, 1, 11}];

In[51]:= (* Prediction for 3→1 *)
PredictionCut2L["oo", 9] = (48 Nc π^4) Cut2L["oo", "3->1"];

In[52]:= (* check *)
Collect[(TwoLoopExp["+", "2q3g", "oo", 9] - PredictionCut2L["oo", 9] // PowerExpand // GToHPL // HPLConvertToKnownFunctions),
    ep, Factor] /. LisToGs // Expand // ShuffleG

Out[52]= 0
```

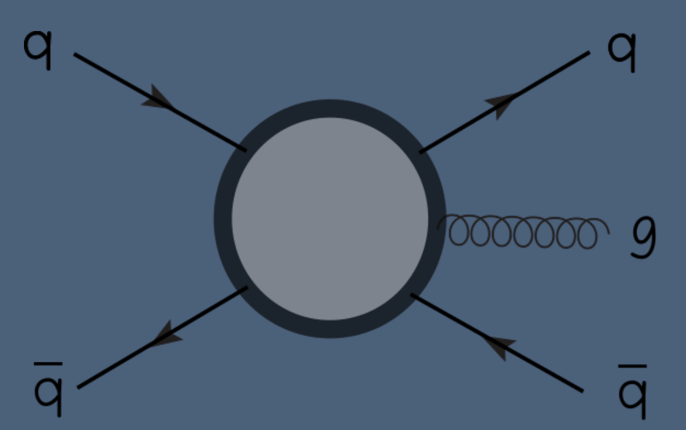
$$\mathcal{A}^{2,\mathrm{cut}}_{qg,[8,10+\overline{10}]} = \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{i\pi}{3}\right)^2 \left(\frac{\mu^2}{|\mathbf{p}_4|^2}\right)^{2\epsilon} \Bigg[\frac{9}{2\epsilon^2} + \frac{18\log((1-z)(1-\bar{z})) - 9\log(z\bar{z})}{\epsilon}$$

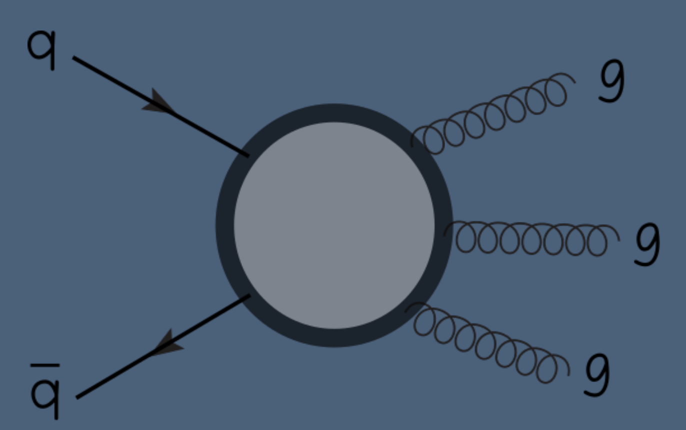
$$+54 i D_2(z,\bar{z}) - \frac{9}{2}\log^2((1-z)(1-\bar{z})) + 9\log^2(z\bar{z}) - 9\log(z\bar{z})\log((1-z)(1-\bar{z})) - \frac{3\zeta_2}{4}\Bigg]$$

2-loop central emission vertex

NEW!!!

- Three-loop Regge trajectory
- Two-loop impact factors





2-loop Lipatov vertex

Universality at NNLL!!!

Final formula not instagrammable yet BUT

- Weight drop in finite remainder (weight 4 $\sim$ prod of lower weights)
- Spurious $z - \bar{z}$ cancels in finite remainder (seen at symbol level for now)

Summary and outlook

- Review of the computation of full color 5-point 2-loop QCD massless amplitudes
- Exploration of high energy regime: MRK kinematics
- Beyond NLL factorization in the Regge pole is broken by multi-reggeon exchanges
- We computed the cuts with EFT approach and subtracted from expansion of the amplitude
- Preliminary results for Lipatov vertex @2-loops from qqggg & qqqqg amplitudes

TO DO:

- Complete check of universality of vertex with ggggg channel
- What about other color structures?, check with N=4?, ...

Thank you!

Backup slides

Old but gold

$\mathscr{A} = \sum$ Feynman diagrams

> 40k

Helicity projection

Tancredi, Peraro: 1906.03298 & 2012.00820

$$\mathscr{A} = \sum H_h T_h$$

Color decomposition

$$H = \sum_c H_c C_c$$

Integration by parts identities

$$H = \sum_{m,c} R^{mc} M_m C_c$$

Key step for simplification

$$R^{mc} M_m = \sum_{k=1}^{M} r_k^c(s_{ij}, d) M(s_{ij}, d)$$

$$a_k(s_{ij}, d) = \frac{N(s_{ij}, d)}{Q(d) D(s_{ij})} \longrightarrow a_k(s_{ij}, d) = \sum_l g_l(d)\, R_l(s_{ij})$$

Uni (d) +multivariate PF

MultivariateApart [Heller, von Manteuffel 2101.08283]

Computation of master integrals + algebraic simplification

Final result

$$R_k = \sum_{m_1+\ldots+m_{30}\leq p} a_{k,m_1\ldots m_{30}} M_{m_1\ldots m_{30}}$$

$$M_{m_1\ldots m_{30}} = q_1^{m_1}\ldots q_{25}^{m_{25}} s_{12}^{m_{26}}\ldots s_{51}^{m_{30}}$$

Look for linear relations amongst rational functions

$$\sum_k R_k b_k = 0 \qquad \sum_k a_{k,m_1\ldots m_{30}} b_k = 0$$

$$H = \sum_{m,c} R^{mc} M_m C_c$$

Key step for simplification

$$R^{mc} M_m = \sum_{k=1}^{M} r_k^c(s_{ij}, d) M(s_{ij}, d)$$

$$r_k^c(s_{ij}, d) = \frac{N(s_{ij}, d)}{Q(d) D(s_{ij})} \longrightarrow r_k^c(s_{ij}, d) = \sum_l g_l(d)\, R_l^c(s_{ij})$$

Uni (d) +multivariate PF

MultivariateApart [Heller, von Manteuffel 2101.08283]

Not independent!

Final result

Absolutely crucial step (see later)

Computation of master integrals + algebraic simplification

2-loop central emission vertex

NEW!!!

With cuts under control we extract the 2-loop Lipatov vertex from the qqggg amplitude and check against the 4q1g channel ✓

Universality at NNLL!!!

Other ingredients are known:

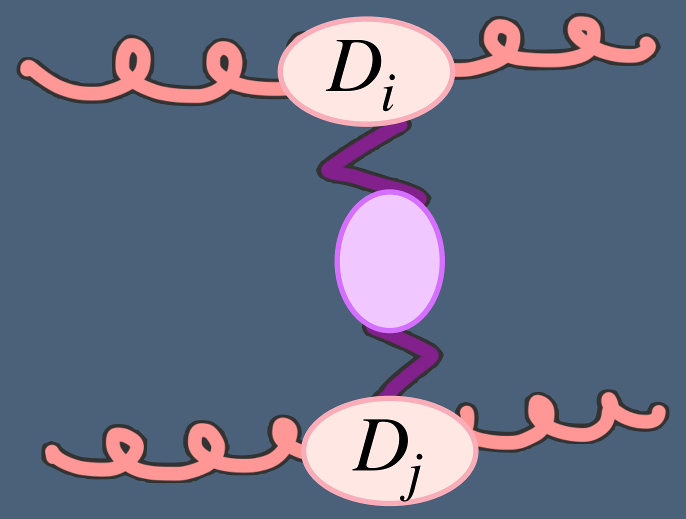
- Three-loop Regge trajectory
- Two-loop impact factors

Final formula not instagrammable yet BUT

- Weight drop in finite remainder (weight 4 $\sim$ prod of lower weights)
- Spurious $z - \bar{z}$ cancels in finite remainder (seen at symbol level for now)

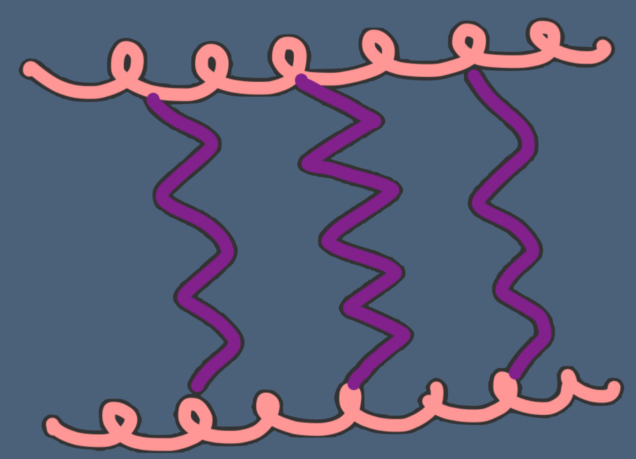
NNLL factorization

$$\text{NNLL} \sim \left(\frac{\alpha_s}{2\pi}\right)^n \log^{n-2} x$$



"Pole"

+


"Cut"

Regge cuts responsible for violation of factorization at NNLL

Factorization is restored once "cut contamination" is removed

How?

Wilson line approach

[Balitsky/JIMWLK + Caron-Huot 1309.6521, Caron-Huot, Gardi, Vernazza 1701.05241]

Leading color: universal → Matched to amplitudes in a given theory

Sub leading color: non-universal → "LO" in the Wilson line correlators

[Falcioni et al 2112.11098]

Expansion of the amplitude in MRK

Rational

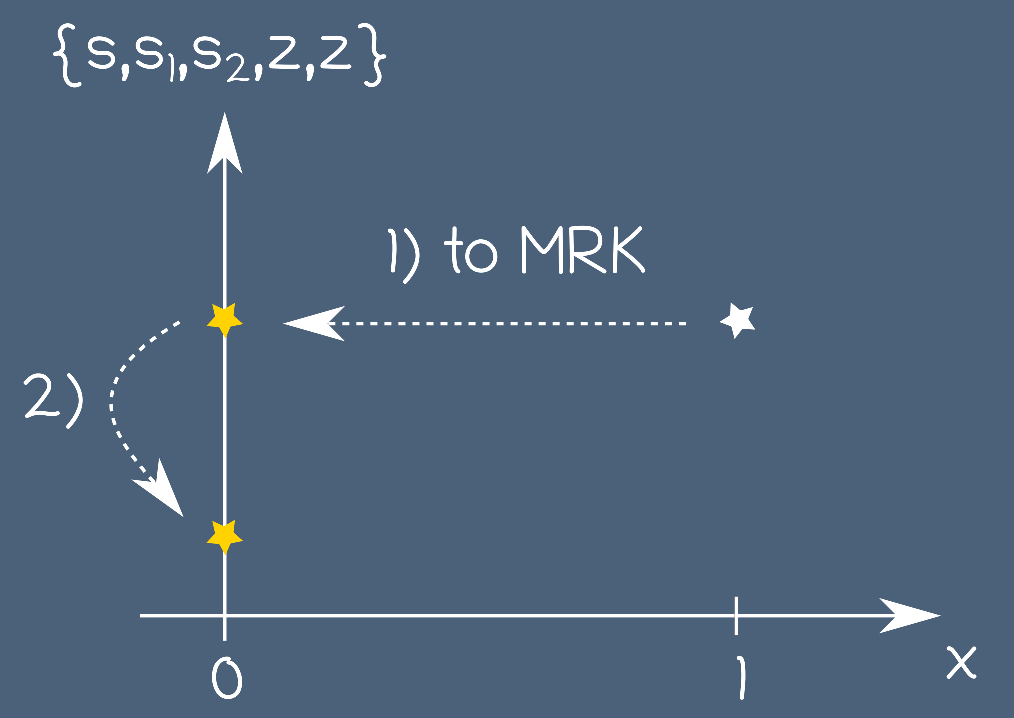
Transcendental

$$\{s_{12}, s_{23}, s_{34}, s_{45}, s_{51}\} \to \{s, s_1, s_2, z, \bar{z}\} + x$$

Scaling parameter

$x = 1$: "physical point",

$x = 0$: MRK



$$W_n \to W_n(x)$$

[Caron-Huot, Chicherin, Henn, Zhang, Zoia 2003.03120]

$$dI_i(\mathbf{s}) = \epsilon\, dA_{ij}(\mathbf{s})\, I_j(\mathbf{s}) \qquad dA_{ij}(\mathbf{s}) = \sum_{n=1} a^n_{ij}\, d\log(W_n)$$

$$\{x\},\ \{\frac{s_1 s_2}{s}\},\ \{s_1, s_2, s_1 - s_2, s_1 + s_2\},$$
$$\{z, \bar{z}, 1 - z, 1 - \bar{z}, z - \bar{z}, 1 - z - \bar{z}\}$$

$$\begin{cases} \dfrac{\partial}{\partial x}\mathbf{f}(x, y, \epsilon) = \epsilon\, A_x(x, y)\, \mathbf{f}(x, y, \epsilon) \\ \dfrac{\partial}{\partial y}\mathbf{f}(x, y, \epsilon) = \epsilon\, A_y(x, y)\, \mathbf{f}(x, y, \epsilon) \end{cases}$$

$$\mathbf{f}(x, y, z) = x^{\epsilon A_0} P \exp\left[\epsilon \int_{y_0}^{y} A_y(0, y')dy'\right] \mathbf{g}_0(\epsilon)$$

We need NNLP

$$A_x(x, y) = \frac{A_0}{x} + \sum_{k\geq 0} x^k A_{k+1}(y)$$

Solution at LP (x^0)

$$f^{(w)}(\mathbf{s}; x) = \sum_{n=0} \sum_{m=0}^{w} f^{(w)}_{mn}(\mathbf{s})\, x^n \log^m x$$

Balitsky/JMWLK equation

$$-\frac{d}{d\eta}U(z_1)...U(z_n) = HU(z_1)...U(z_n),$$

$$-\frac{d}{d\eta} = H = \frac{\alpha_{s,b}}{2\pi^2}\int [dz_0][dz_i][dz_j]K_{ij,0}\times \\ \left\{\left[\hat{T}^a_{i,L}\hat{T}^a_{j,L} + (L \leftrightarrow R)\right] - U^{ab}_{\text{adj}}(z_0)\left[\hat{T}^a_{i,L}\hat{T}^b_{j,R} + (i \leftrightarrow j)\right]\right\} + \mathcal{O}(\alpha^2_{s,b}),$$

$$H = \int [dz_0][dz_i]K_{ii,0}\left\{-\frac{\alpha_{s,b}}{2\pi^2}C_A W^a_{0i}\frac{\delta}{\delta W^a_i} + \frac{\alpha^2_{s,b}}{3\pi}\text{Tr}\left[W_{0i}W_0W_{0i}\frac{\delta}{\delta W_i}\right] + ...\right\} + \\ + \int [dz_0][dz_i][dz_j]K_{ij,0}\left\{-\frac{\alpha_{s,b}}{2\pi^2}[W_{0i}W_{0j}]^{xy} + \\ + \frac{\alpha^2_{s,b}}{6\pi}[W_{0i}W_0W_0W_{0j} - W_{0i}W_0W_{0j}W_j - W_iW_{0i}W_0W_{0j}]^{xy} + ...\right\}\frac{\delta^2}{\delta W^x_i \delta W^y_j}.$$

Poles and cuts

$$\mathcal{A}^{(-)} = \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{dj}{\sin(\pi j)} \sin\left(\frac{\pi j}{2}\right) a_j^{(-)}(t) e^{jL}$$

$$a_j^{(-)}(t) = \frac{1}{j-1-\alpha(t)}$$ → "Regge trajectory"

$$\mathcal{A}^{(-)}(s,t)|_{\text{Regge pole}} = \frac{\pi}{\sin\left(\frac{\pi\alpha(t)}{2}\right)} \frac{s}{t} e^{L\alpha(t)} + \text{sub-leading}$$

--"Regge Pole"

$$a_j^{(-)}(t) = \frac{1}{[j-1-\alpha(t)]^{1+\beta(t)}}$$

$$\mathcal{A}^{(-)}(s,t)|_{\text{Regge cut}} = \frac{\pi}{\sin\left(\frac{\pi\alpha(t)}{2}\right)} \frac{s}{t} \frac{1}{\Gamma(1+\beta(t))} L^{\beta(t)} e^{L\alpha(t)} + \text{sub-leading}$$

--"Regge cut"

Credits to Fabrizio :)

Some jargon…

$$\mathcal{A}(s,t) = \frac{1}{\pi}\int_0^\infty \frac{d\hat{s}}{\hat{s}-s-i\epsilon}\Delta_s(\hat{s},t) + \frac{1}{\pi}\int_0^\infty \frac{d\hat{u}}{\hat{u}+s+t-i\epsilon}\Delta_u(\hat{u},t)$$

Mellin moments

$$a_j^s(t) = \frac{1}{\pi}\int_0^\infty \frac{d\hat{s}}{\hat{s}}\Delta_s(\hat{s},t)\left(\frac{\hat{s}}{-t}\right)^{-j} \qquad \Delta_s(s,t) = \frac{1}{2i}\int_{\gamma-i\infty}^{\gamma+i\infty} dj\, a_j^s(t)\left(\frac{s}{-t}\right)^j$$

Signature eigenstates

$$\mathcal{A}^{(\pm)}(s,t) = \frac{1}{2}\left(\mathcal{A}(s,t) \pm \mathcal{A}(-s-t,t)\right) \qquad a_j^{(\pm)}(t) = \frac{1}{2}(a_j^s(t) \pm a_j^u(t))$$

$$\mathcal{A}^{(+)} = i\int_{\gamma-i\infty}^{\gamma+i\infty} \frac{dj}{\sin(\pi j)} \cos\left(\frac{\pi j}{2}\right) a_j^{(+)}(t) e^{jL} \qquad L = \frac{1}{2}\left(\ln\frac{-s-i\epsilon}{-t} + \ln\frac{-u-i\epsilon}{-t}\right)$$

$$\mathcal{A}^{(-)} = \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{dj}{\sin(\pi j)} \sin\left(\frac{\pi j}{2}\right) a_j^{(-)}(t) e^{jL} \qquad = \ln|s/t| - i\frac{\pi}{2}$$

Expansion of pentagon functions in MRK

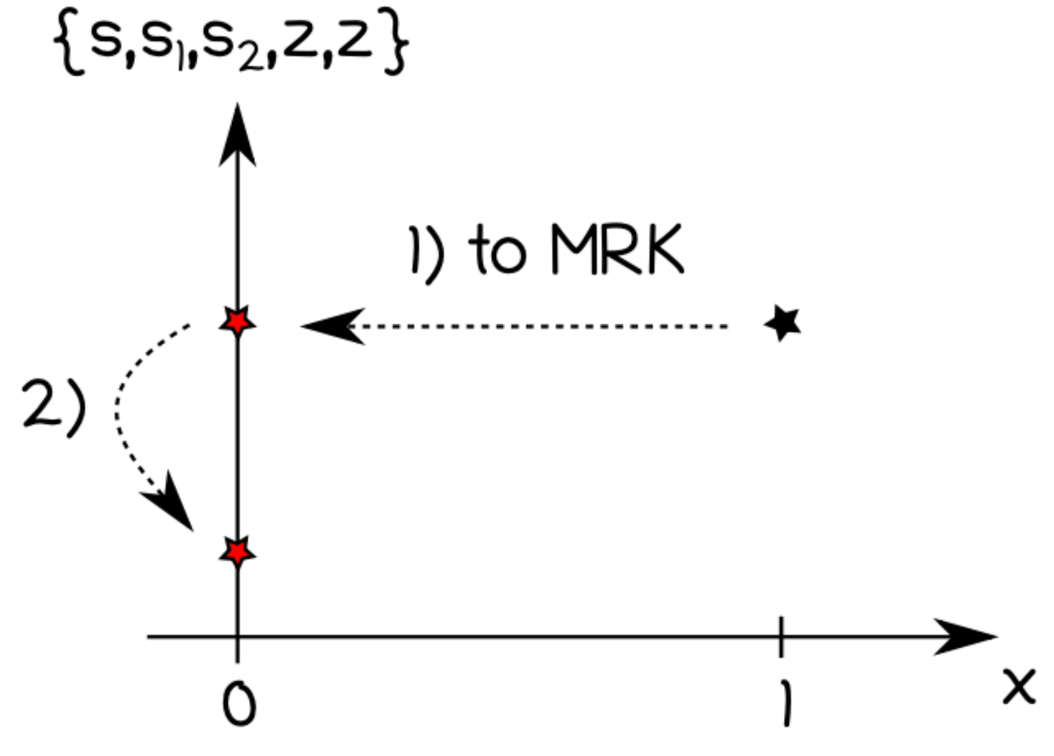
$$\mathrm{d}I_i(\vec{s}) = \epsilon\, \mathrm{d}A_{ij}(\vec{s}) I_j(\vec{s}) \qquad \mathrm{d}A_{ij}(\vec{s}) = \sum_{n=1} a_{ij}^n \mathrm{d}\log(W_n)$$

$W_n \to W_n(x)$ [Caron-Huot, Chicherin, Henn, Zhang, Zoia 2003.03120]

For fixed $\{s_{ij}\} \sim y$, one gets a 1-d differential equation in x

$$\begin{cases} \dfrac{\partial}{\partial x}\vec{f}(x,y,\epsilon) = \epsilon A_x(x,y)\vec{f}(x,y,\epsilon) \\ \dfrac{\partial}{\partial y}\vec{f}(x,y,\epsilon) = \epsilon A_y(x,y)\vec{f}(x,y,\epsilon) \end{cases}$$

$$A_x(x,y) = \frac{A_0}{x} + \sum_{k\geq 0} x^k A_{k+1}(y)$$



$$\vec{f}(x,y,\epsilon) = x^{\epsilon A_0}\, \mathbb{P}\exp\left[\epsilon \int_{y_0}^{y} A_y(0,y')dy'\right] \vec{g}_0(\epsilon)$$

pentagon functions at LP

Nice property of MRK at LP:
Gram-determinant a perfect square
all square-roots in letters rationalized

$$\Delta = \epsilon_5^2 \underset{x\to 0}{\sim} \frac{s_1^2 s_2^2 (z-\bar{z})^2}{x^4} + \mathcal{O}\left(\frac{1}{x^3}\right)$$

$$\{x\}\,,$$
$$\left\{\frac{s_1 s_2}{s}\right\},$$
$$\{s_1, s_2, s_1 - s_2, s_1 + s_2\}\,,$$
$$\{z, \bar{z}, 1-z, 1-\bar{z}, z-\bar{z}, 1-z-\bar{z}\}$$

Alphabet in MRK much simpler than in full kinematics (35→12 letters)

Credits to Federico :)