The S-Matrix and boundary correlators in flat space

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Difference: In AdS ϕ_0 couples to a boundary operator of dimension Δ and the corresponding path integral has independent description in dual theory.

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[Kim,Kraus, Monten, Myers '23]

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- For most of the calculations, we consider the boundary cut-off surface to be a union of two spacelike slices, one in the far past (at time -T) and another in the far future (at time +T).

- However, most of our results can be generalised to arbitrary boundary surface (We consider null boundary as another example and make some comments about relation to CCFT).

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- We conjecture that the flat space wave functional and the S-matrix are related by analytic continuation.
- We analysed the analytic structure of G_{bdry} in position space both for massive and massless particles.
- For massless particles, G_{bdry} exhibits features like bulk point singularity (and it's generalisations) whose coefficient encode the flat space S-matrix.

S-matrix as boundary observable

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$$S(\{p_i\},\{q_j\}) = \prod_{i=1}^{n}\int_M d^{d+1}x_i\sqrt{g(x_i)}\, f_{p_i}(x_i)(\nabla_i^2 - m^2)\prod_{j=1}^{m}\int_M d^{d+1}y_j\sqrt{g(y_j)}\, \bar{f}_{q_j}(y_j)(\nabla_j^2 - m^2)G(\{x_i, y_i\})$$

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$$\int_M d^{d+1}x\sqrt{g(x)}\, f_p(x)(\nabla^2 - m^2)\phi(x) = \int_M d^{d+1}x\,\partial_\mu\left(f_p(x)\sqrt{g(x)}\partial_\mu\phi(x) - \phi(x)\sqrt{g(x)}\partial_\mu f_p(x))\right)$$

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$$G_{\text{bdry}}(\{x_i\}, \{y_i\}) = \prod_{i=1}^{n} \frac{\delta}{\delta\beta_0(x_i)} \prod_{j=1}^{m} \frac{\delta}{\delta\beta_0(y_j)} Z[\beta_0]\Big|_{\beta_0=0}$$

Using Hamilton-Jacobi:

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$$S_E(\{p_i\},\{q_j\}) = \int \prod_{i=1}^{n} d^d x_i f_{p_i}(x_i) \prod_{j=1}^{m} d^d y_j \bar{f}_{q_j}(y_j)\, G_{\text{bdry}}(\{x_i\},\{y_j\})$$

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- We conclude that the Dirichlet path integral serves as a generating functional for S-matrices.

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[Arefeva, Faddeev & Slavnov '74]

- But they considered Lorentzian path integral as a functional of "positive energy data in the past and negative energy data in future"— "In-Out Problem".

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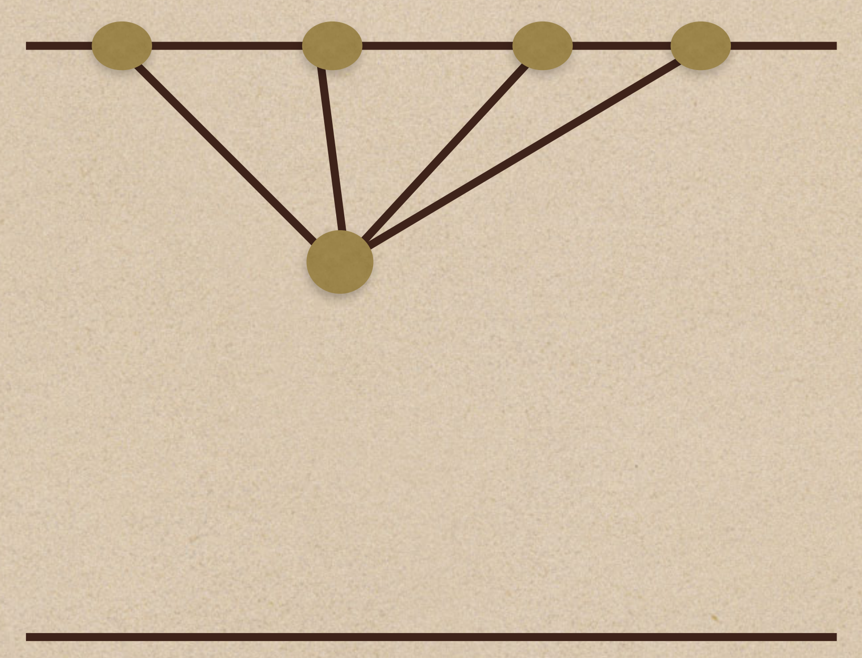

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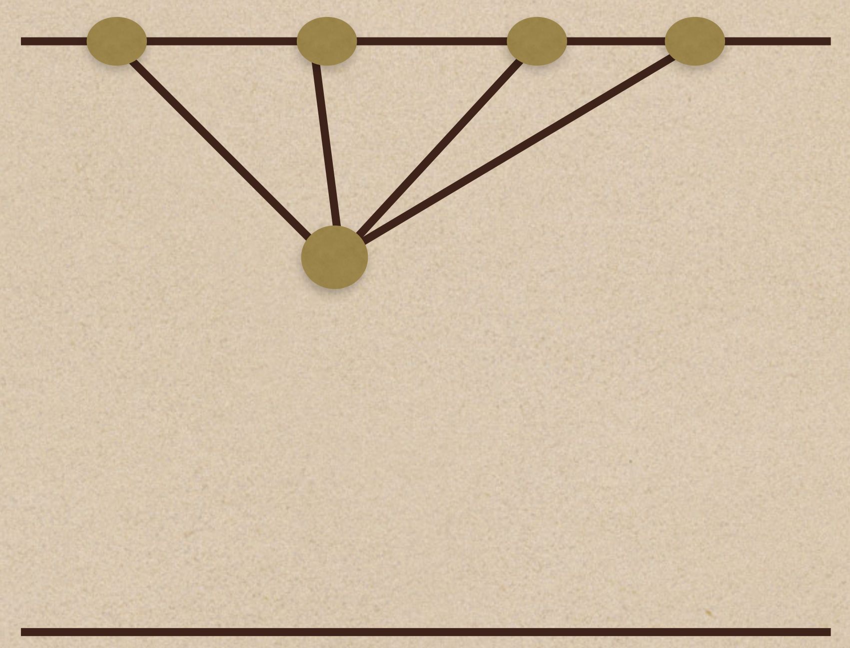

- Such diagrams are different in Dirichlet and In-Out problem. In the Dirichlet case, we obtain vacuum wavefunction in x-basis and in the latter case, in coherent state basis.

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[P. Benincasa '18,…]

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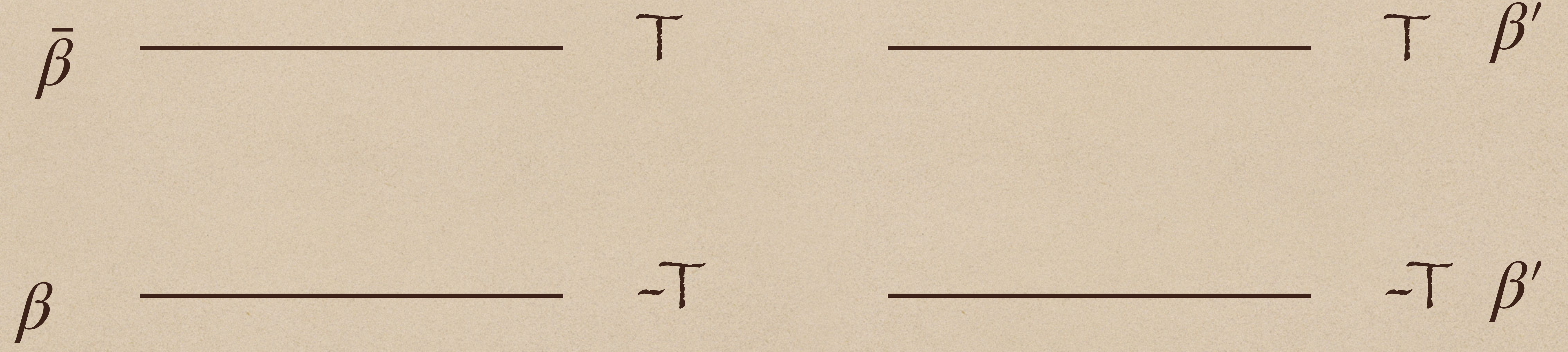

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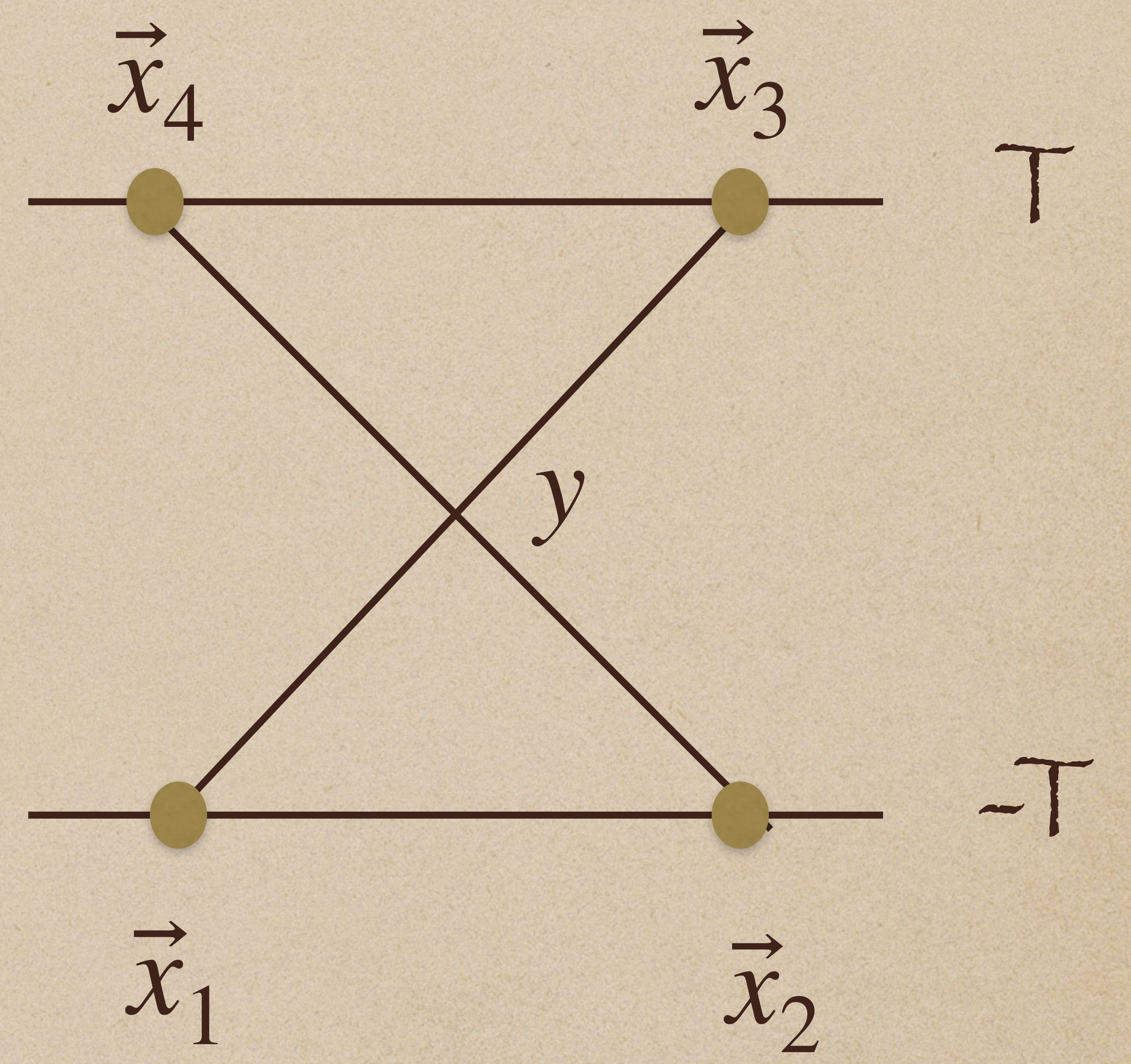

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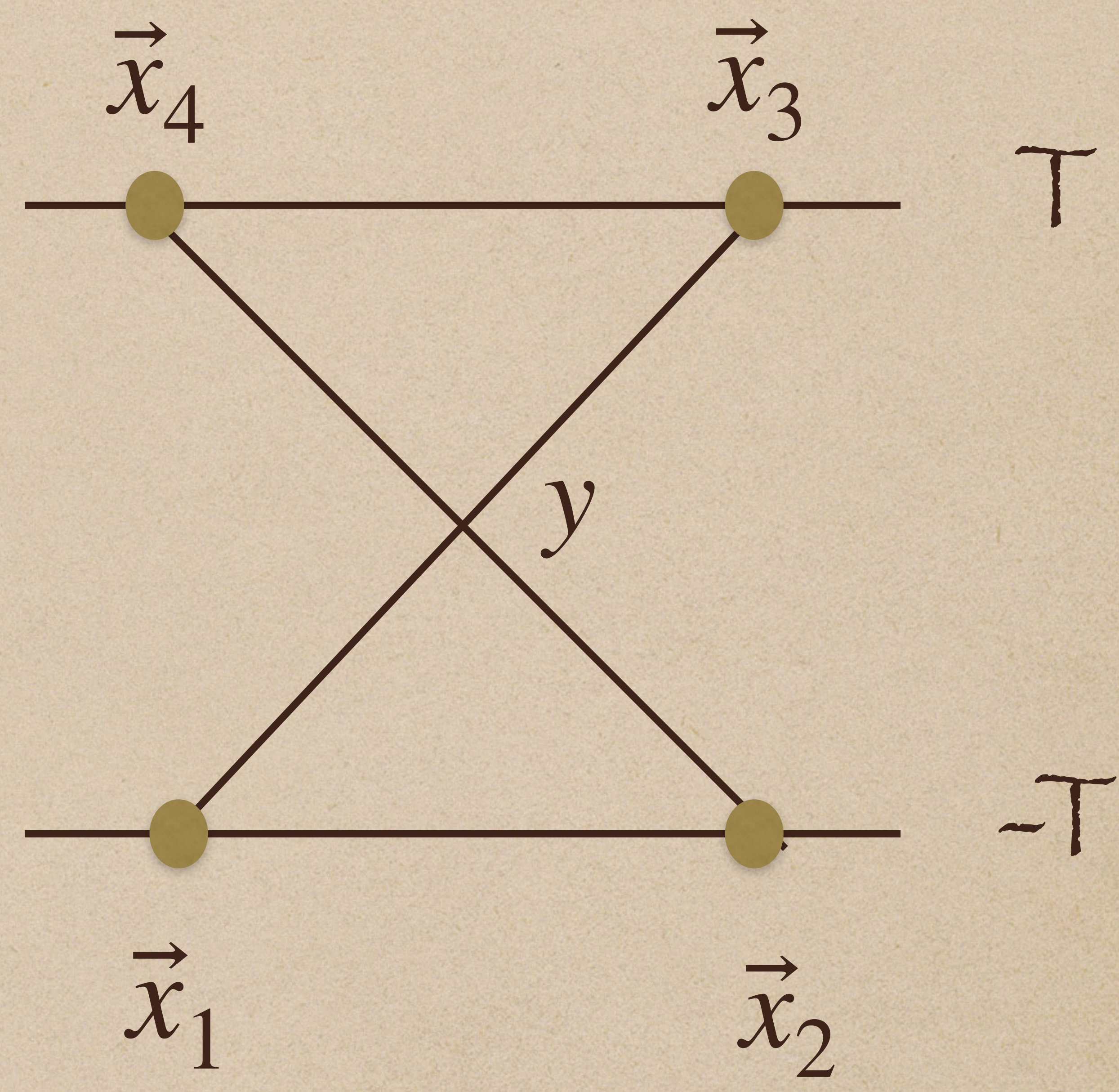

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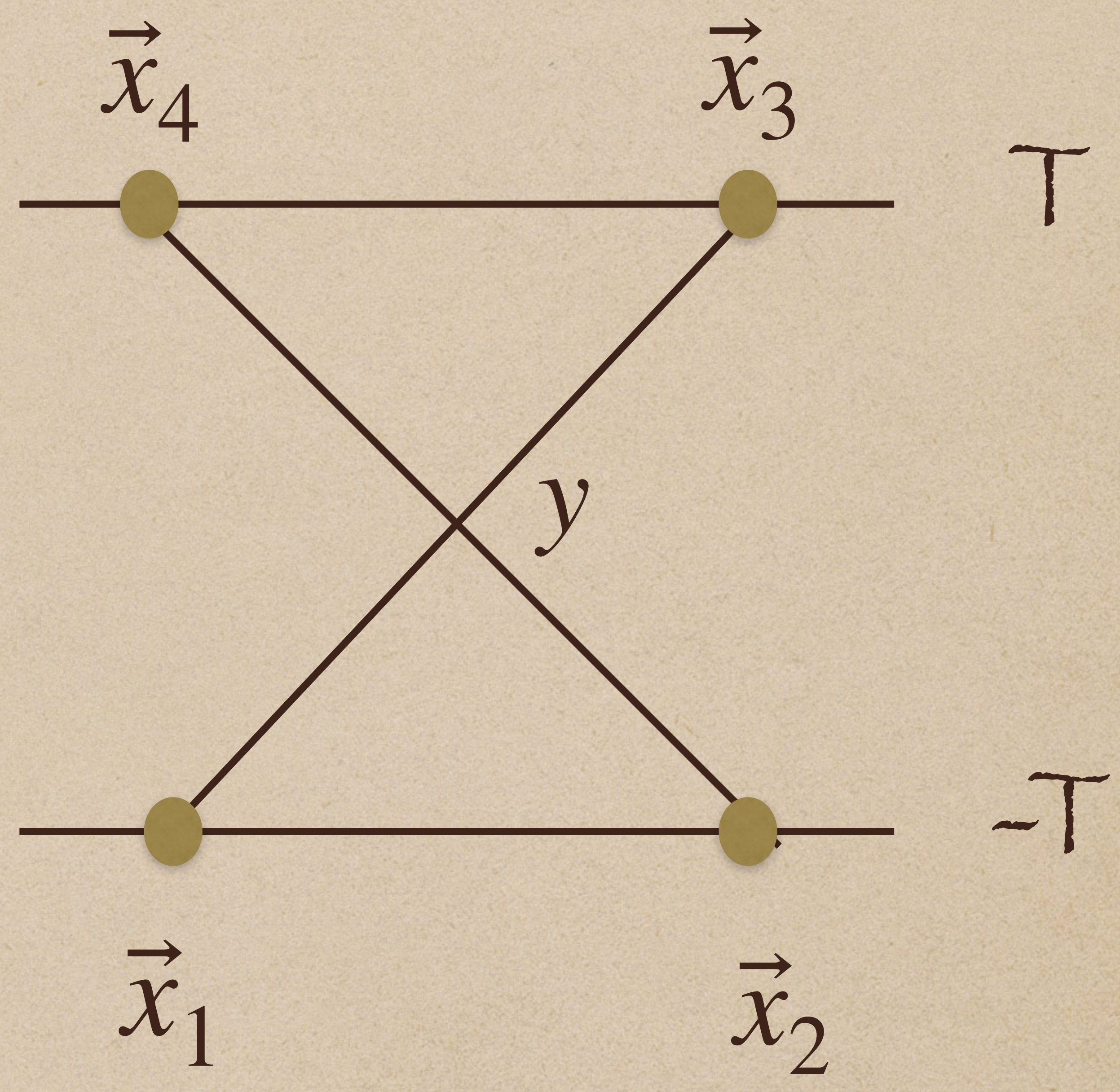

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$$G_{\partial B}(x_i, y) = (2n \,.\, \nabla G(x, y))\Big|_{x \to x_i}$$

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[J. Maldacena, D. Simmons-Duffin and A. Zhiboedov '17]

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Singularity appears when N_{ij} has a zero eigenvalue with a positive eigenvector.

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- Momentum conservation.

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Q1: Given a generic set of $\{x_i\}$, how many "tunings" will one need to perform in order to obtain a singular G_{bdry}?

- Intersection of light cones.
- Momentum conservation.

$$\begin{aligned} c &= 1 && \text{if} \quad \mathrm{m} \leq \mathrm{D}+1 \\ c &= m - D && \text{if} \quad \mathrm{m} > \mathrm{D}+1 \end{aligned}$$

Q2: Given a set of boundary points $\{x_i\}$ such that G_{bdry} is singular for those insertions, does G_{bdry} receive contributions from one S-matrix or many?

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Same as co-dimension of singularity.

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- The location of these singularities can be characterised in terms of zero eigenvalues of the boundary distance matrix: $N_{ij} = (x_i - x_j)^2$.
- The residue at these singularities contain the information about flat space S-matrix.

Two ways to extract S-matrix from G_{bdry}:

1. Multiply with mode functions and integrate (essentially Fourier transform).
2. The coefficient of singularity of G_{bdry} is the S- matrix.

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[S. Banerjee '24]

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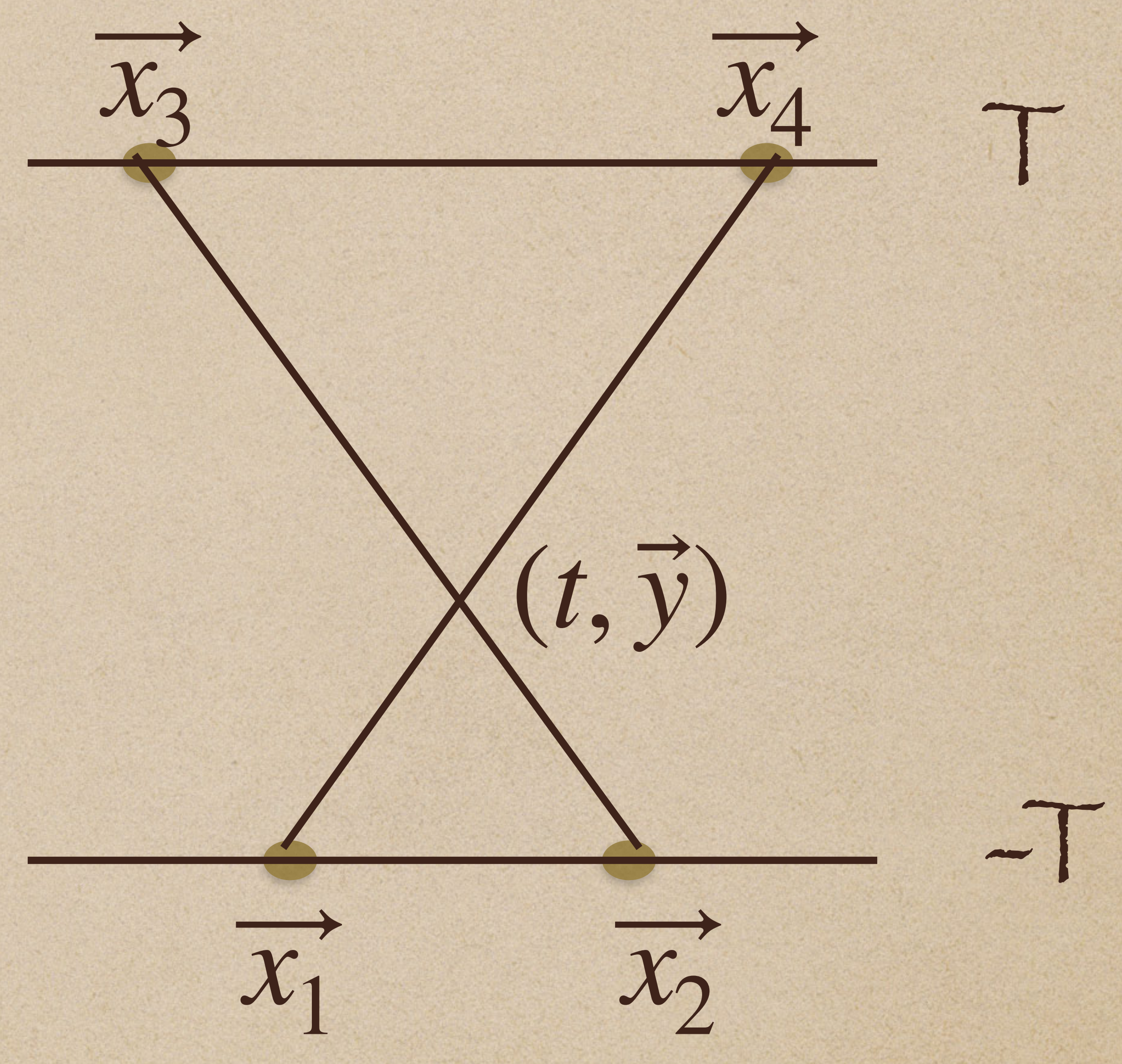

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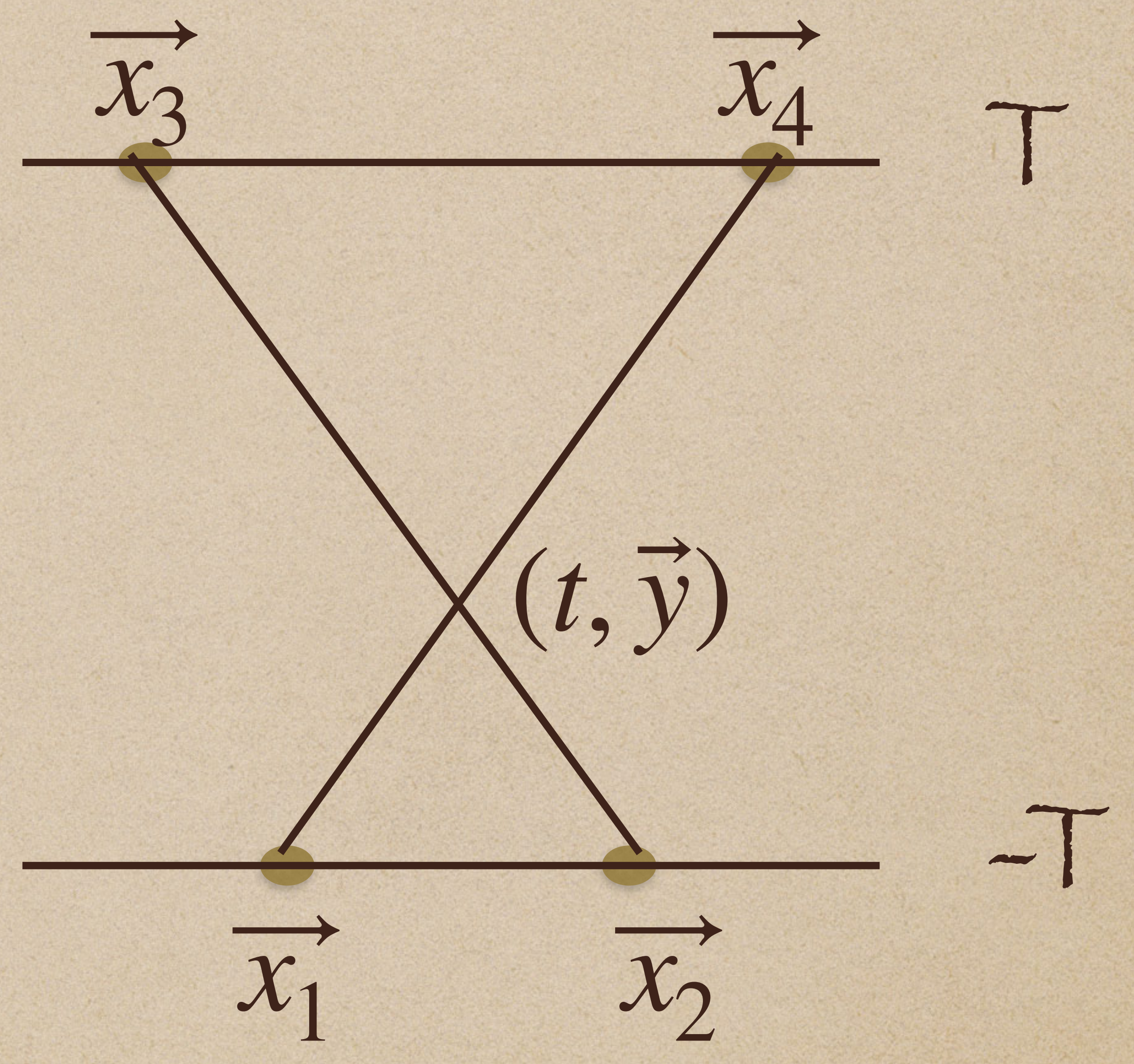

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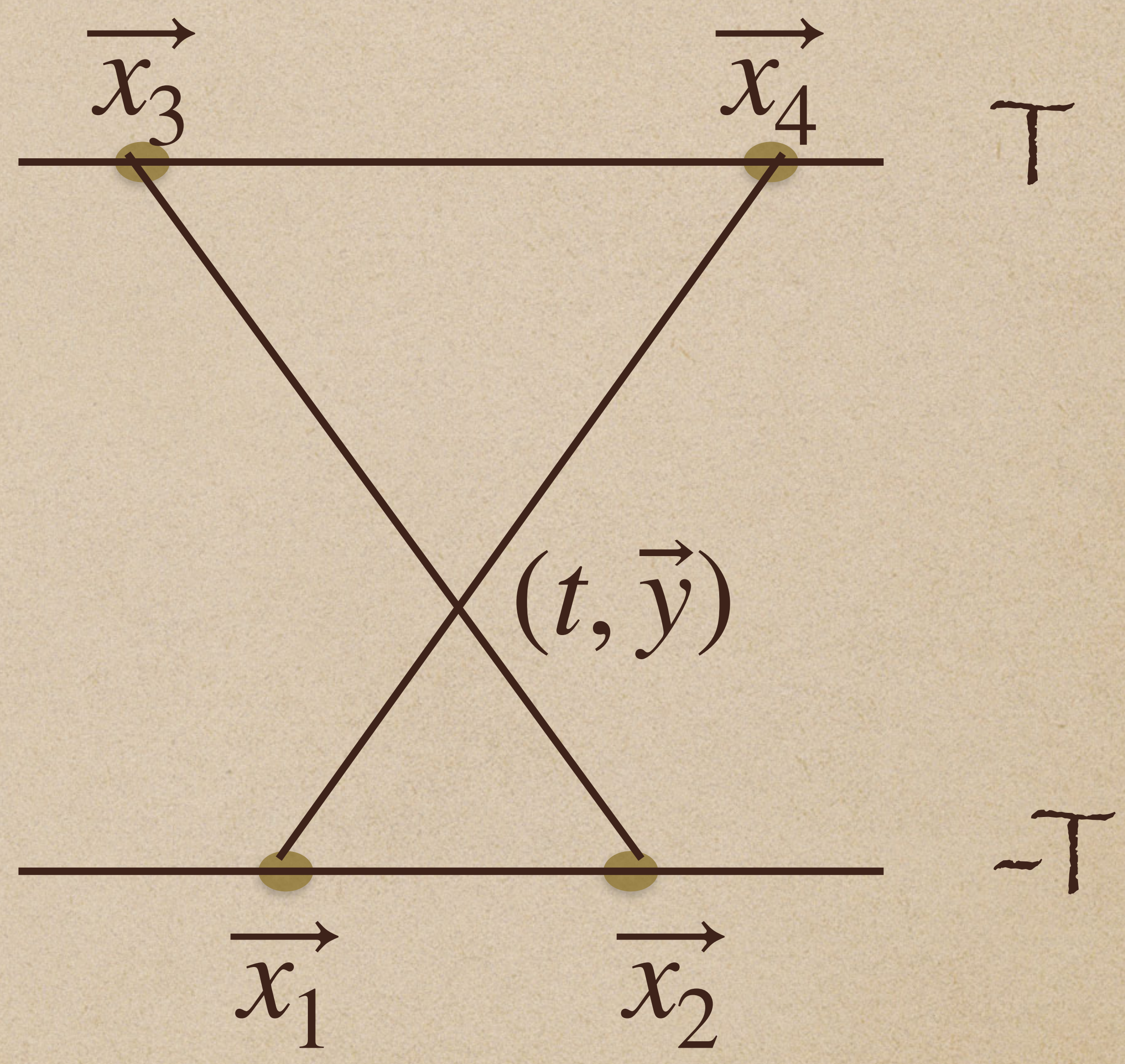

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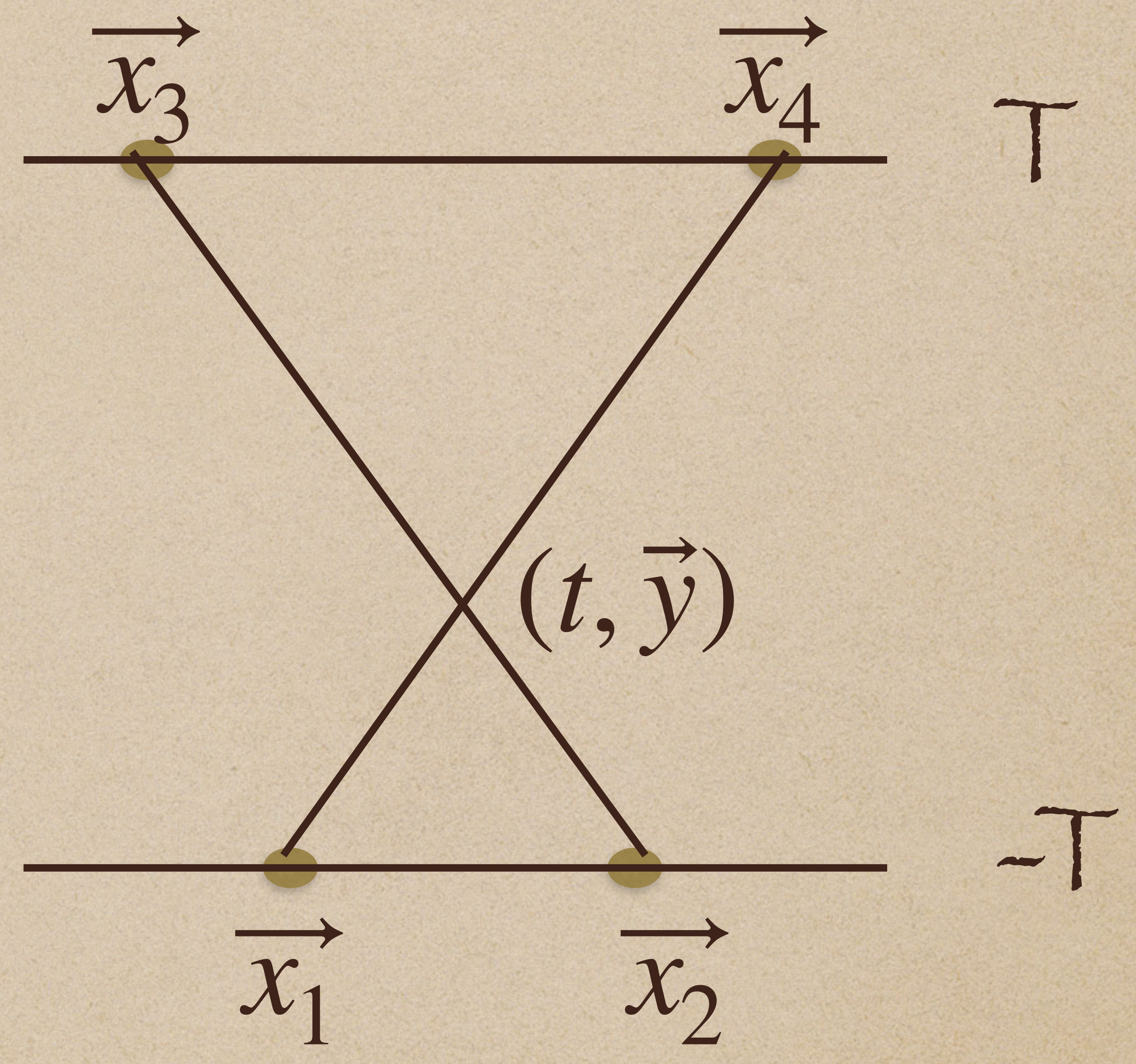
$$G(x, y) = C\frac{e^{-im(x-y)^2}}{((x-y)^2)^{\frac{D-1}{4}}}$$



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$$G_{\text{bdry}} \approx \prod_{i=1}^{n} \left(\left(\frac{m_i}{2\pi} \right)^{\frac{D-1}{2}} \frac{(-T - t)}{d_i^{in\,\frac{D+1}{2}}} e^{-im_i d_i^{in}} \right) \prod_{i=1}^{m} (\text{out})\, S \left(\frac{m_i(\vec{x}_i^{in} - \vec{y})}{d_i^{in}}, \frac{m_i(\vec{x}_i^{out} - \vec{y})}{d_i^{out}} \right)$$

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Holographic Renormalization is non-local!!

Results/Outline

- S-matrix can be thought of as a boundary observable and can be computed using "Path integral as a functional of boundary values".
- S-matrix unitarity provides non-trivial constraint on this path integral.
- We argue that the flat space wave functional and the S-matrix are related by analytic continuation.
- We also analyse properties of G_{bdry} in position space both for massive and massless particles.
- For massless particles, G_{bdry} exhibits features like bulk point singularity whose coefficient encode the flat space S-matrix.

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ਤੁਹਾਡਾ ਧੰਨਵਾਦ

நன்றி നന്ദി

ধন্যবাদ Thank you

ధన్యవాదములు धन्यवाद