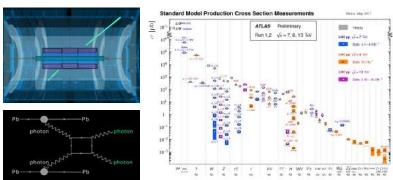


Exclusive Processes

 Tremendous progress in the understanding of exclusive scattering processes: analytic structure, multi-loop perturbative data, amplituhedron, S-matrix bootstrap,...

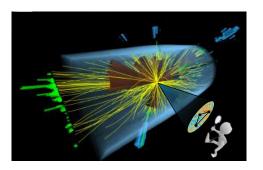


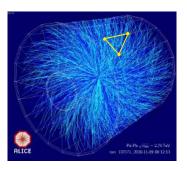
Practical Outcome: Ability to accurately describe complicated SM scattering processes.

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The High Multiplicity Regime

- A complementary regime: high multiplicity
 - Collisions with $E\gg m_{\rm gap}$
 - Conformal Field Theories



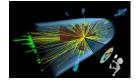


• Good observables are correlations in fluxes at (null) infinity.

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Motivation

- How can we characterize a theory using asymptotic data?
- Theoretical motivation:
 - What is the space of observables at null infinity?
 - How are they related to (C)FT data?
 - How do we constrain theories in the absence of S-matrix and/ or local ops (e.g. CFT coupled to gravity)
- Phenomenological motivation:
 - Can we relate asymptotic measurements to parameters of the underlying theory? (couplings, transport coefficients,)
 - Can we identify universal features that can be computed to high precision?



• Wealth of collider data provides a practical testing ground.

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Detector Operators: History

• Expectation value of energy flux, $\langle \mathcal{E}(\vec{n}_1) \rangle$, at specific angles on the celestial sphere is calculable in perturbation theory: $\varepsilon_{(\vec{n})}$

Sterman 1975:

To make this idea made quantitative we define for any state \underline{a} , as

"angular energy current" in the e⁴e⁷ CM frame

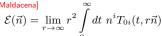
$$\beta_{\mu}(\mathcal{O}) + \frac{\alpha_{\mu}}{\alpha_{\mu+1}} \, \pi_{i} \delta(\mathcal{O} - \alpha_{i}) \tag{O}$$

where the sum is over the n_a massless particles on g_a with energies $\{r_i\}$ and expection disjections $\{z_i\}$ $\{r_i\}$ whende for engles θ_i and $\sigma_i\}$.

[Korchemsky, Sterman] [Sveshnikov, Tkachov] [Hofman, Maldacena]



Light Transform



[Kravchuk, Simmons Duffin]

$$\mathcal{E}(\vec{n}) = 2 \left. \mathbf{L}[T](\infty, z) \right|_{z = (1 \ \vec{n})}$$

$$\mathbf{L}[\mathcal{O}](x, z) = \int_{-\infty}^{\infty} d\alpha (-\alpha)^{-\Delta - J} \mathcal{O}\left(x - \frac{z}{\alpha}, z\right)$$

Energy Correlations in electron - Positron Annihilation: Testing QCD

C.Louis Basheri (Washington U., Seattle), Lowel S. Brown (Washington U., Seattle), Stephen D. Ellis (Washington U., Seattle), Shewin T. Love (Washington U., Seattle) Aug. 1978

13 pages

Published in: Phys.Rev.Lett. 41 (1978) 1585

DOt 10.1103/PhysRevLatt.41.1585 Report number: RLO-1388-759

View in: OSTI Information Bridge Server, ADB Abstract Service







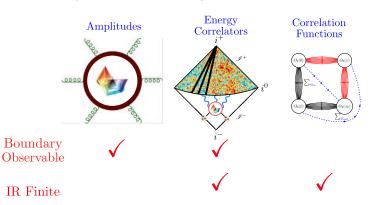
416 citations

R selecence swarch

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Energy Correlators

• Correlation functions $\langle 0|\mathcal{O}^{\dagger}\mathcal{E}(\vec{n}_1)\cdots\mathcal{E}(\vec{n}_k)\mathcal{O}|0\rangle$ take an interesting intermediate position between amplitudes and correlation functions.

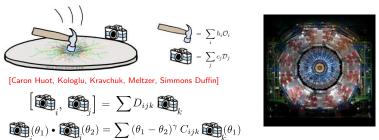


 Provide an interesting example of observables that are well defined at weak coupling, strong coupling, in a CFT, with gravity,

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Detectorology

- Are now known to be part of a wide class of "detector operators".
- Significant recent progress in understanding the classification of detector operators, and their algebras and OPEs.



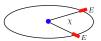
- Exciting from the perspective of phenomenology:
 - 1 Experiment Informing Theory: Correlators can be directly measured to reveal interesting behavior.
 - 2 Theory Informing Experiment: Use refined understanding to better study the SM at colliders.

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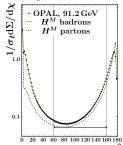
Energy Correlators at LEP

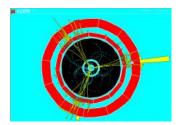
 \bullet Conceptually simplest case is to study energy correlators in some state produced by a local operator, ${\cal O}$

$$\langle 0|\mathcal{O}^{\dagger}\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\mathcal{O}|0\rangle$$



 This has an exact experimental analogue, where 𝒪 is the electromagnetic current ⇒ Two-point correlator measured at LEP.



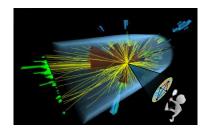


 χ° [QCD at NLO by Dixon, Luo, Shtabovenko, Yang, Zhu] [$\mathcal{N}=4$ analog at NLO by Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov] [and NNLO by Henn, Sokatchev, Yan, Zhiboedov]

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Conformal Colliders Meet the LHC

Transition from GeV→ TeV provides access to a new regime of QCD!



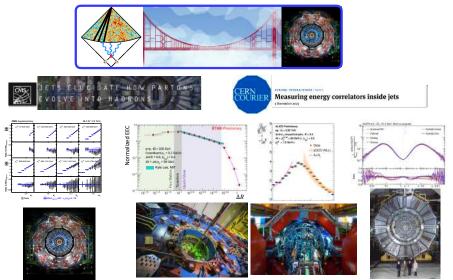
- Can we use this amazing dataset to:
 - Test the lightray OPE in a quantitative way, and measure the spectrum of lightray operators?
 - Measure the full shape of higher point correlators?
 - Measure correlators on more complicated states (top quarks, finite temperature, in a confining theory, ...)

*Also interesting to try in condensed matter systems, but not done...colliders are interesting experiments for generic properties of QFT.

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Energy Correlators in Data!

• Spectacular recent progress bridging theory and experiment!



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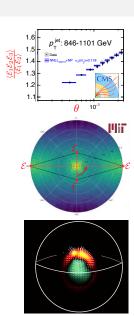
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Outline

• Scaling Behavior

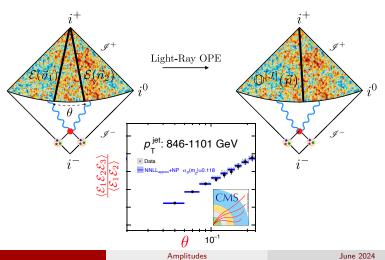
• Higher Point Functions of Energy Flux

 Phenomenological Applications of Energy Correlators



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Scaling Behavior

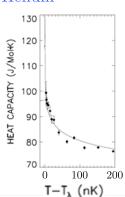


Scaling Behavior in QFT

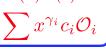
• Euclidean scaling behavior are well understood.

λ -point of Helium





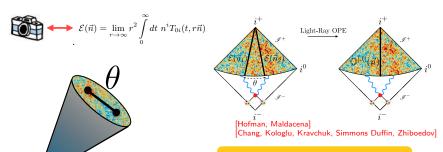




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The OPE Limit of Lightray Operators

• Energy flow operators admit a Lorentzian OPE: "the lightray OPE"



- Scaling can be derived in generic (non-conformal) theories using factorization theorems. [Dixon, Moult, Zhu] See early work by [Konishi, Ukawa, Veneziano]
- Predicts universal scaling behavior in correlations of energy flux at energies $E\gg \Lambda_{\rm QCD}$.

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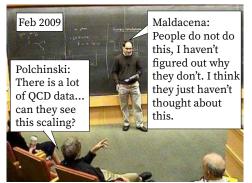
 $\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) \sim \sum \theta^{\tau_i - 4} \mathbb{O}_i(\hat{n}_1)$

Theory-Experiment Gap

Conformal collider physics: Energy and charge correlations

Diego M. Hofman^a and Juan Maldacena^b

^a Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544, USA
^b School of Natural Sciences, Institute for Advanced Study
Princeton, NJ 08540, USA

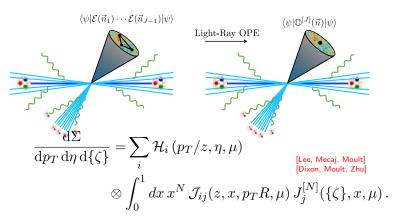


Still true as of 2023...

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Factorization Theorem at the LHC

• Can derive a factorization theorem in the LHC environment using the proofs of Collins-Soper-Sterman for inclusive fragmentation:



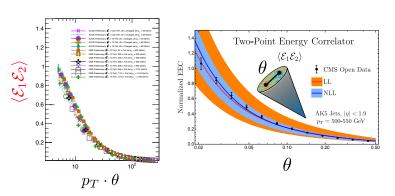
• Incorporates all experimental realities (jet clustering) into the state.

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Scaling Behavior in Jets

 Scaling measured inside jets by STAR, ALICE and CMS from 15 GeV to 1784 GeV:

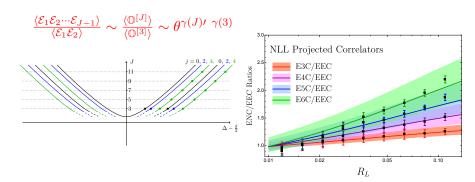
An experimental realization of the detector OPE!



 Can we accurately extract anomalous exponents of different detectors?

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The Spectrum of a Jet

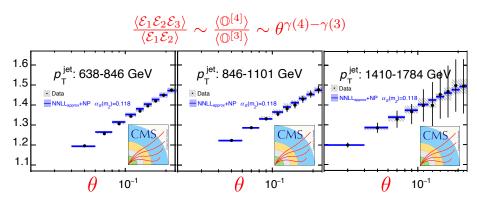


• Directly probes the spectrum of (twist-2) lightray operators from asymptotic energy flux.

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Anomalous Scaling

 Precision measurements reveal anomalous scaling: Universal quantity in complicated hadronic environment.

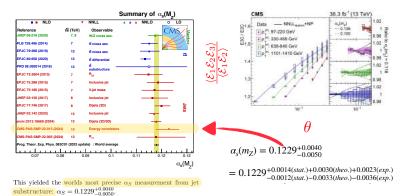


- Uses scaling anomalous dimensions at three-loop order.
- Beautiful quantitative test of QFT!
 Using [Dixon, Moult, Zhu], [Chen, Gao, Li, Xu, Zhang, Zhu]

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The Strong Coupling

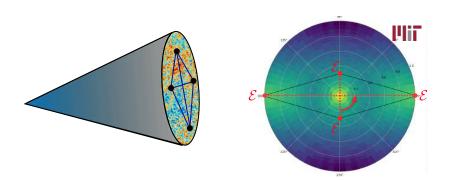
• Use scaling to extract value of the strong coupling constant α_s at 4% accuracy.



 • Very clear target for improved perturbative calculations. e.g. NNLO $2 \to 3$ hard functions not yet included.

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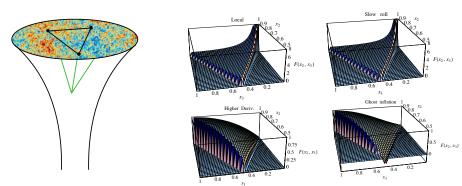
Higher Point Functions in Energy Flux



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Multipoint Correlators

 Higher-point correlators probe detailed aspects of the underlying microscopic interactions. e.g. CMB three-point functions allow to distinguish models of inflation.



• What is the structure of higher-point functions of energy flux?

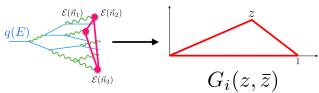
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Multipoint Correlators

• The only explicit results for correlators with N>2 were the remarkable strong coupling results of Hofman and Maldacena:

$$\langle \mathcal{E}(\vec{n}_1) \cdots \mathcal{E}(\vec{n}_n) \rangle = \left(\frac{q}{4\pi}\right)^n \left[1 + \sum_{i < j} \frac{6\pi^2}{\lambda} [(\vec{n}_i \cdot \vec{n}_j)^2 - \frac{1}{3}] + \frac{\beta}{\lambda^{3/2}} [\sum_{i < j < k} (\vec{n}_i \cdot \vec{n}_j) (\vec{n}_j \cdot \vec{n}_k) (\vec{n}_i \cdot \vec{n}_k) + \cdots] + o(\lambda^{-2}) \right]$$

• The wealth of techniques developed to compute perturbative scattering amplitudes can be applied to multi-point correlators at weak coupling.



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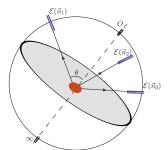
Correlators in Perturbation Theory

- Two approaches to calculate energy correlators:
 - 1 Light transforming N-point functions of stress tensors:

$$\langle 0|\mathcal{O}^{\dagger}T\cdots T\mathcal{O}|0\rangle \rightarrow \langle 0|\mathcal{O}^{\dagger}\mathcal{E}(\vec{n}_1)\cdots \mathcal{E}(\vec{n}_k)\mathcal{O}|0\rangle$$

Two Point NLO in $\mathcal{N}=4$: [Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov] Two Point NNLO in $\mathcal{N}=4$: [Henn, Sokatchev, Yan, Zhiboedov] LO Charge-Charge Correlator in QCD: [Chicherin, Henn, Sokatchev, Yan]

2 Perturbative phase space integrals using (squared) form factors:



$$\frac{\langle \Psi | \mathcal{E}(\vec{n}_1) \cdots \mathcal{E}(\vec{n}_k) | \Psi \rangle}{\langle \Psi | \Psi \rangle} = \sum_{i_1, \dots, i_k} \int d\sigma \prod_{j=1}^k E_{i_j} \delta(\vec{n}_j - \vec{p}_{i_j}/p_{i_j}^0)$$

Two Point LO in QCD: [Basham, Ellis, Brown, Love] Two Point NLO in QCD: [Dixon, Luo, Shtabovenko, Yang, Zhu] Three Point Collinear LO in QCD: [Chen, Luo, Moult, Yang, Zhang, Zhu] Three Point General Angle LO in $\mathcal{N}=4$: [Yan, Zhang] Three Point General Angle LO in QCD: [Yang, Zhang] Four Point Collinear LO in $\mathcal{N}=4$: [Chicherin, Moult, Sokatchev, Yan, Zhu]

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Correlators in Perturbation Theory

• For generic angles, the correlator depends on the cross ratios $\zeta_{ij}=rac{1\prime\ \cos heta_{ij}}{2}$, and the source.

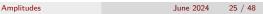


- In the collinear (OPE) limit, $\zeta_{ij} \to 0$, it becomes a function of 2(N-2) variables that is independent of the source.
- ullet The LO contribution to the N-point function is given by a *finite* integral in (N-1) dimensional projective space of the *universal* splitting functions:

$$\mathbf{E}^{\mathbf{N}}\mathbf{C} \stackrel{\text{coll.}}{=} \int_{0}^{1} dx_{1} \cdots dx_{N} \, \delta(1 - \sum_{i} x_{i}) \, (x_{1} \cdots x_{N})^{2} \, \mathcal{P}_{1 \to N}^{(0)}$$

$$\xrightarrow{P_{5}} \qquad \bigotimes_{P_{4}} \qquad \Longrightarrow_{z_{2} = z} \qquad \Longrightarrow_{z_{3} = 1}$$

 This limit can be physically measured inside high energy jets at the LHC.



Three-Point Correlator at Weak Coupling

- First non-trivial correlator: tree level three-point correlator in the collinear limit $G(z,\bar{z})$. [Chen, Luo, Moult, Yang, Zhang, Zhu]
- ullet Turns out to have an elegant perturbative structure. e.g. in ${\cal N}=4$



$$\begin{split} G_{\mathcal{N}=4}(z) &= \frac{1+u+v}{2uv}(1+\zeta_2) - \frac{1+v}{2uv}\log(u) - \frac{1+u}{2uv}\log(v) \\ &- (1+u+v)(\partial_u + \partial_v)\Phi(z) + \frac{(1+u^2+v^2)}{2uv}\Phi(z) + \frac{(z-\bar{z})^2(u+v+u^2+v^2+u^2v+uv^2)}{4u^2v^2}\Phi(z) \\ &+ \frac{(u-1)(u+1)}{2uv^2}D_2^+(z) + \frac{(v-1)(v+1)}{2u^2v}D_2^+(1-z) + \frac{(u-v)(u+v)}{2uv}D_2^+\left(\frac{z}{z-1}\right) \end{split}$$

• where Φ and D_2^+ are

$$\begin{split} \Phi(z) &= \frac{2}{z - \bar{z}} \left(\mathrm{Li}_2(z) - \mathrm{Li}_2(\bar{z}) + \frac{1}{2} \left(\log(1 - z) - \log(1 - \bar{z}) \right) \log(z\bar{z}) \right) \\ D_2^+(z) &= \mathrm{Li}_2(1 - |z|^2) + \frac{1}{2} \log(|1 - z|^2) \log(|z|^2) \end{split}$$

• Provides important perturbative data for the development of the lightray OPE.

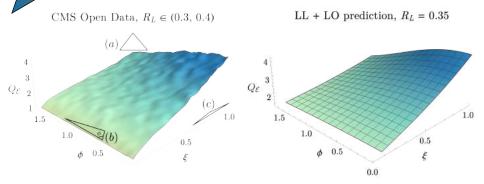
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Shape Dependence of Non-Gaussianities

• Multipoint correlators can be directly measured in high energy jets: Simple analytic functions for the *actual measured observable!*



• Non-Gaussianities inside high energy jets at the LHC:



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- Simple structure makes energy correlators a nice playground for exploration of *physical observables* in perturbation theory.
- Four point correlator computed in $\mathcal{N}=4$ SYM by direct integration in parameter space, using simple form of $1 \to 4$ splitting function.

$$\mathbf{E}^{\mathbf{N}}\mathbf{C} \stackrel{\text{coll.}}{=} \int_{0}^{1} dx_{1} \cdots dx_{N} \, \delta(1 - \sum_{i} x_{i}) \, (x_{1} \cdots x_{N})^{2} \, \mathcal{P}_{1 \to N}^{(0)}$$

$$\stackrel{P_{2}}{\longrightarrow} P_{1} \longrightarrow \sum_{z_{2} = z} \sum_{z_{3} = 1}^{z_{1} = w}$$

- Compact result expressed in terms of weight three polylogarithms: much structure still to be explored.
- \bullet Would be interesting to extend to QCD using known $1 \to 4$ splitting functions. [Del Duca, Duhr, Haindl, Lazopoulos, Michel]

• Can one push to higher points or make general statements?.

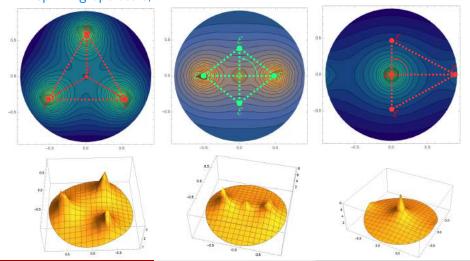


Kai Yan

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The Four Point Correlator

• Intricate view of correlations of energy flow. Access to OPE limits, spinning operators, ...

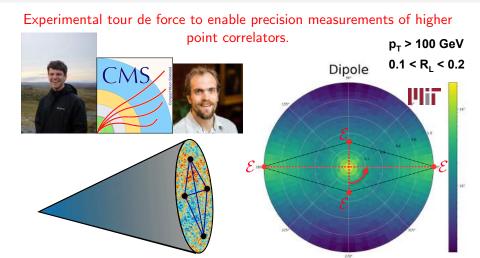


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The Four Point Correlator

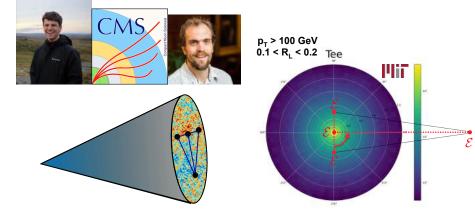


Thanks to Simon Rothman and Phil Harris + Kyle Lee

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The Four Point Correlator

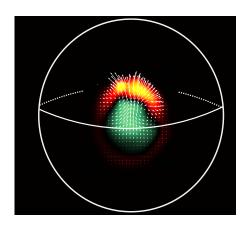
Experimental tour de force to enable precision measurements of higher point correlators.



Thanks to Simon Rothman and Phil Harris + Kyle Lee

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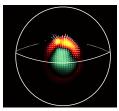
Phenomenological Applications



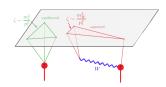
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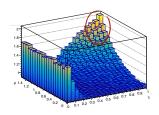
Applications

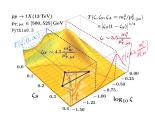
- Measurements on more complicated states:
 - Imaging the Quark Gluon Plasma



• Weighing the Top Quark



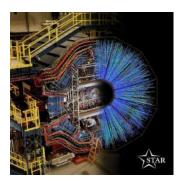


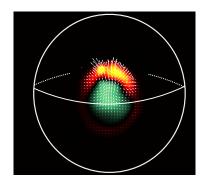


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Application 1: Imaging the Quark Gluon Plasma

 Heavy ion collisions provide opportunity to study energy correlators in thermal states.

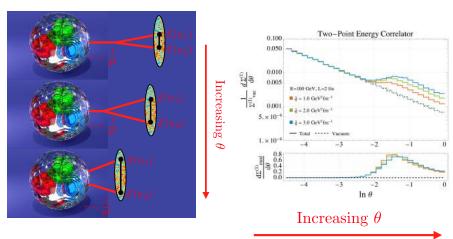




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Application 1: Imaging the Quark Gluon Plasma

• QGP scales cleanly imprinted in two-point correlation.

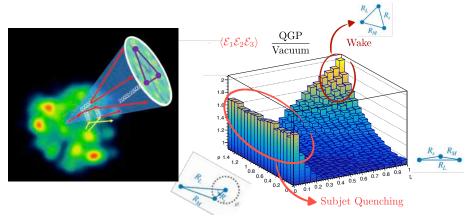


[Andres, Dominguez, Holguin, Kunnawalkam Elayavalli, Marquet, Moult]

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Application 1: Imaging the Quark Gluon Plasma

• Higher point correlators allow the "shape" of the medium response to be imaged.



• Measurements coming soon!

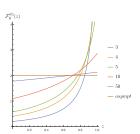
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Application 1: Imaging the Quark Gluon Plasma

- Motivates understanding of asymptotic observables in thermal or large charge states.
- Two recent approaches:
 - Heavy half-BPS operators in $\mathcal{N}=4$:

$$O_K(x) = \operatorname{tr}[\phi^K(x)]\,, \qquad \quad \phi(x) = \sum_{I=1}^6 Y^I X^I\,.$$

[Chicherin, Korchemsky, Sokatchev, Zhiboedov]



• Large charge states using semi-classics:

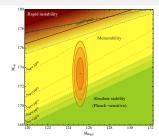
$$\langle \mathcal{E}(\mathbf{n}_1)\mathcal{E}(\mathbf{n}_2)\rangle \approx \left(\frac{E}{\Omega_{d-2}}\right)^2 \left(1 + O(1/\Delta_Q)\right) \quad (Q \to \infty).$$

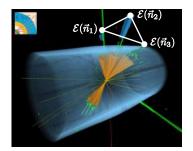
[Firat, Monin, Rattazzi, Walters]

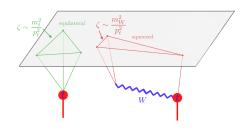
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Application 2: Weighing the Top Quark

- The top quark mass is one of the most important parameters of the SM. e.g. electroweak vacuum stability/criticality, electroweak fits, etc.
- Need simple observables with top mass sensitivity that can be computed from first principles field theory.





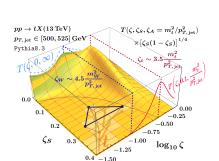


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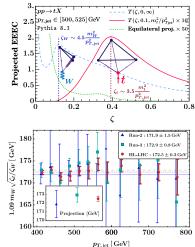
Application 2: Weighing the Top Quark

• Extract the mass ratio between the W and top quark from the shape of the three-point correlator.

3. $pp \to iX$ $\uparrow r(c, b, \infty)$



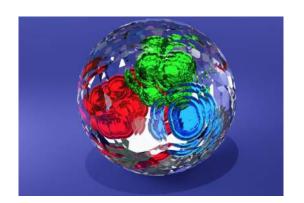
[Holguin, Moult, Pathak, Procura, Schofbeck, Schwarz] See also: [Xiao, Ye, Zhu]



• Requires precision calculations of correlators on top decays.

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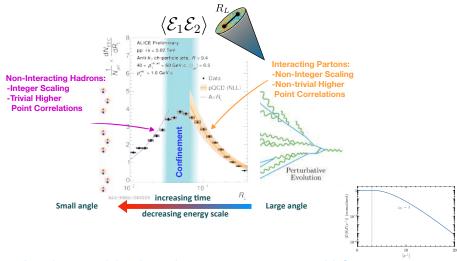
Conformal Colliders Meet Confinement



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Confinement

Figure: Wenqing Fan

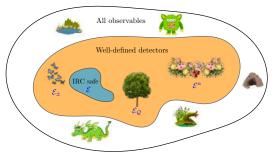


Are there models where this transition is computable?
 IR brane: [Csaki, Ismail]

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The Space of Detectors

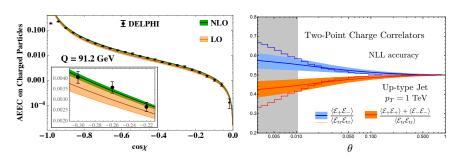
- Details of the hadronization process are encoded in the quantum numbers (charge, flavor, ...): By definition, energy flux is insensitive!
- What is the space of detectors over which we can gain theoretical control?



• More general observables can be calculated by combining factorization into universal non-perturbative detector matrix elements, with the Renormalization Group.

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- Two examples of experimental interest involving electromagnetic charge:
 - $\langle \Psi | \mathcal{E}_{\mathsf{charged}}(\hat{n}_1) \mathcal{E}_{\mathsf{charged}}(\hat{n}_2) | \Psi \rangle$
 - $\langle \Psi | \mathcal{E}_{+}(\hat{n}_1) \mathcal{E}_{-}(\hat{n}_2) | \Psi \rangle$, $\langle \Psi | \mathcal{E}_{+}(\hat{n}_1) \mathcal{E}_{+}(\hat{n}_2) | \Psi \rangle$

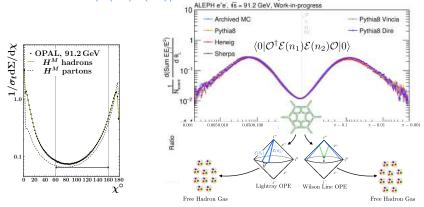


• Confinement generates enhanced small angle correlations between opposite sign hadrons, relative to like sign hadrons.

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Revisiting Old Data with New Resolution

• Reanalysis of ALEPH data has measured the two-point energy correlator $\langle \mathcal{E}(n_1)\mathcal{E}(n_2)\rangle$ on tracks with spectacular resolution.



 Combined with precision track based calculations, opens up a new playground for precision studies of QFT.

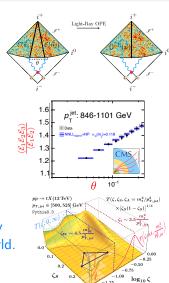
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Summary

 Recent progress understanding correlation functions of detector operators from explicit perturbative calculations, and the light-ray OPE.

 Multi-point correlators of lightray operators can be directly measured at the LHC.

 Provides the opportunity to use theoretically beautiful objects to learn about the real world.



0.4 - 1.50

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Open Questions: "Phenomenologist"

- General structure of N-point energy correlators in perturbation theory? Efficient IBP for finite integrals in Feynman parameter space.
- What is the "light transform of the correlahedron"?
 [Eden, Heslop, Mason] [He, Huang, Kuo]
- What other matrix elements of lightray operators are well defined?
 Relation to IR finite S-matrices?
- What are the implications of asymptotic symmetry algebras?
 [Cordova, Shao] [Korchemsky, Sokatchev, Zhiboedov]
- What is the structure of the lightray OPE in non-conformal theories?
- How to incorporate data from (multiparticle) S-matrix bootstrap?
 [Guerrieri, Homrich, Vieira]
- What other systems have interesting detector operators and how can we experimentally access them?
- Correlators in thermal states? transport? confining theories? [Csaki, Ismail]
- How to interpolate between weak and strong coupling? Integrability?
- How to relate colllider measurements of detector operators to other Lorentzian singularities (Pomeron,...)?
 [Caron-Huot, Kologlu, Kravchuk, Meltzer, Simmons Duffin]

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Open Questions: "Conformal Field Theorist"

• Open Questions from David Simmons Duffin "Strings 2024" Talk:

Some open questions

- How do you measure a general detector operator at a collider?
- · Can we measure detectors in a condensed matter system?
- Can we formulate EFT running and matching for detectors? (Could help organize understanding of confinement effects in $\langle \mathcal{E}\cdots\mathcal{E}\rangle$ [Jaarsma, Li, Moult, Waalewijn, Zhu '23; Csaki, Ismail '24].)
- What does the Chew-Frautschi plot look like at finite λ and finite N?
- Can we find positivity/rigidity conditions for light-ray operators? Can we formulate bootstrap conditions?
- What behavior in the deep Regge limit is possible? Transparency vs. chaos? [Stanford '15; Murugan, Stanford, Witten '17; Caron-Huot, Gobeil, Zahree '20]
- Are other Lorentzian singularities described by other types of operators?
- Does "factorization theorem" = OPE? [Chen '23]
- · What is the general form of the light-ray OPE?
- · How are light-ray operators and conformal line defects related?
- Do light-ray operators participate in interesting algebras? [Casini, Teste, Torroba '17: Cordova, Shao '18: Korchemsky, Sokatchev, Zhiboedov '21: Korchemsky, Zhiboedov '21: Faulkner, Sperana: '29:

Conformal Colliders have met the LHC!

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