From Amplitudes to Gravitational Waves and Back

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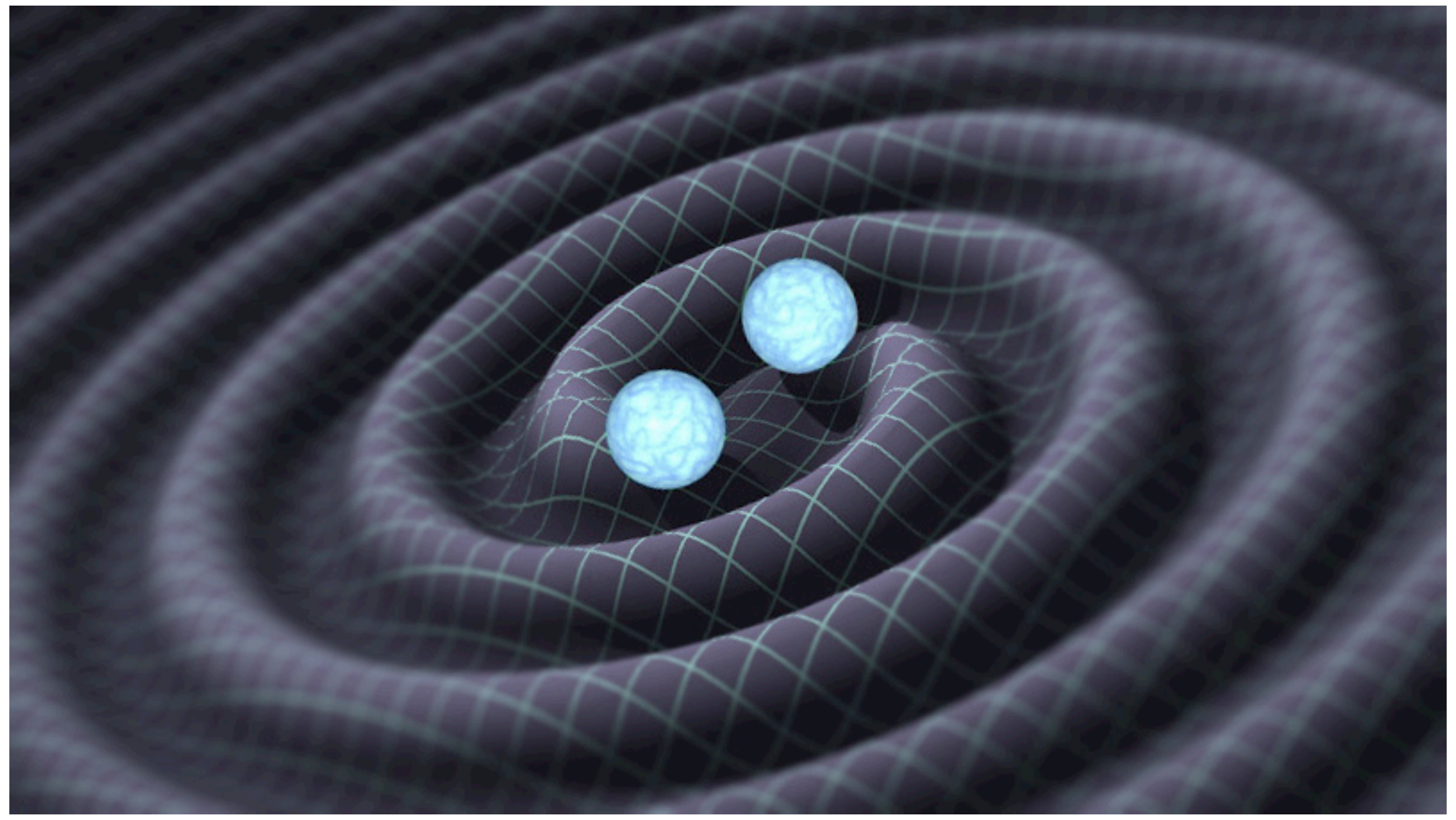
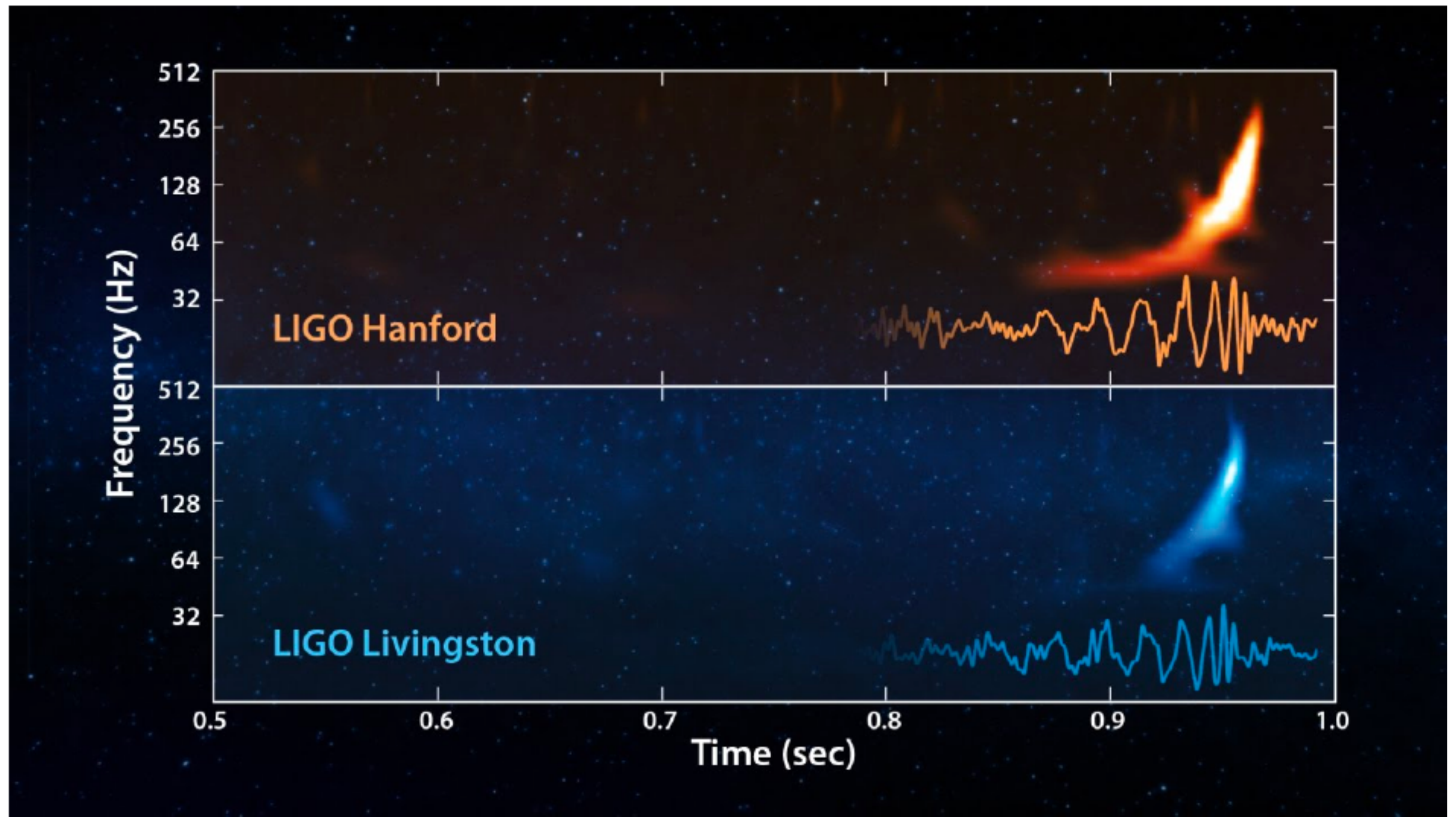
Outline

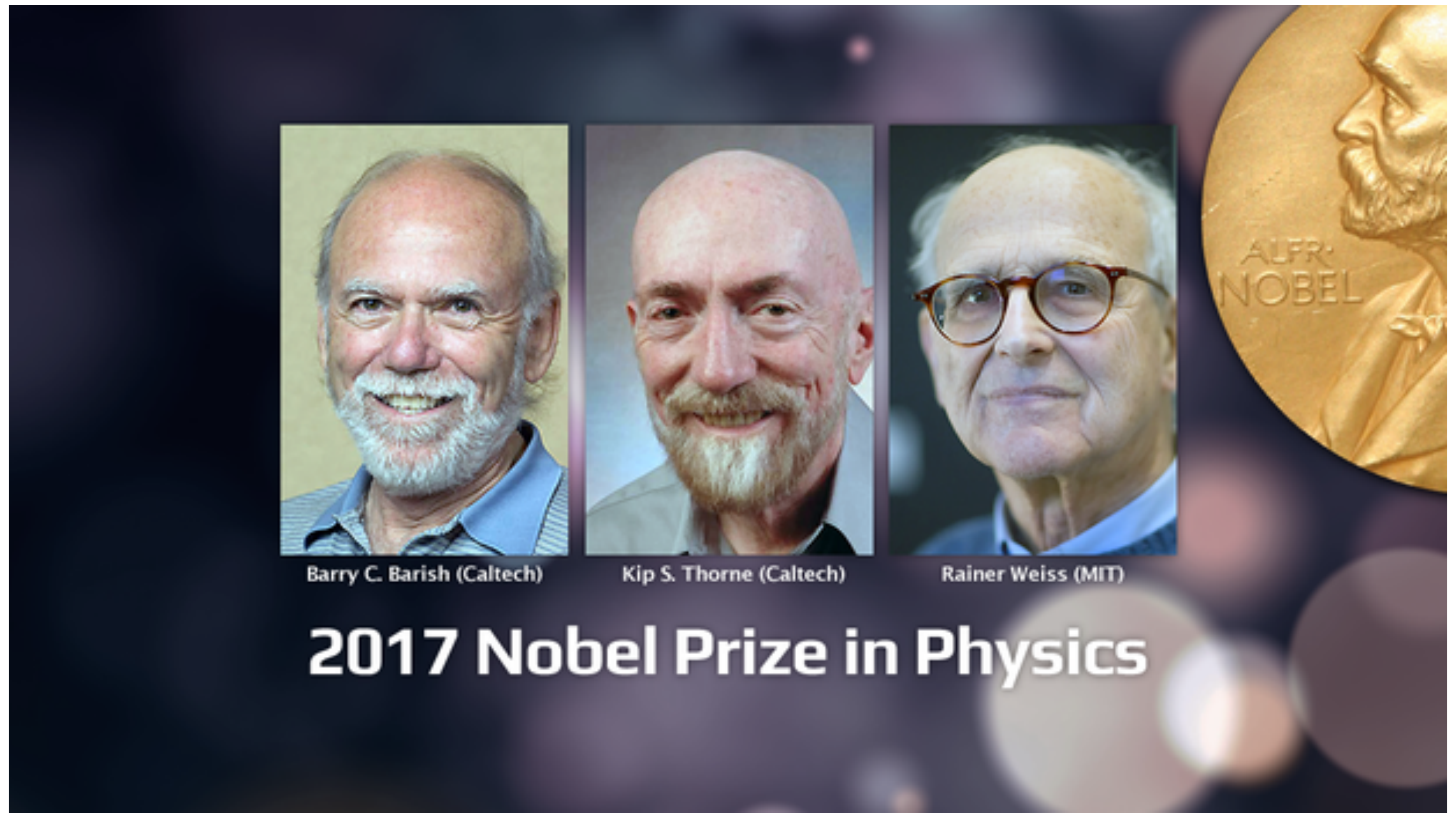
1. Brief review of basics.
   — Amplitudes approach to gravity
   — Amplitudes and gravitational waves
2. Some recent progress and puzzles in gravitational waves
   — Progress at O(G^5)
   — Radiation
   — Spin
3. Feedback into amplitudes
   — Improved integration programs
   — Non-planar integrand bases
   — Double copy to all loop orders
4. Outlook.



Outline

Era of gravitational-wave astronomy has begun.





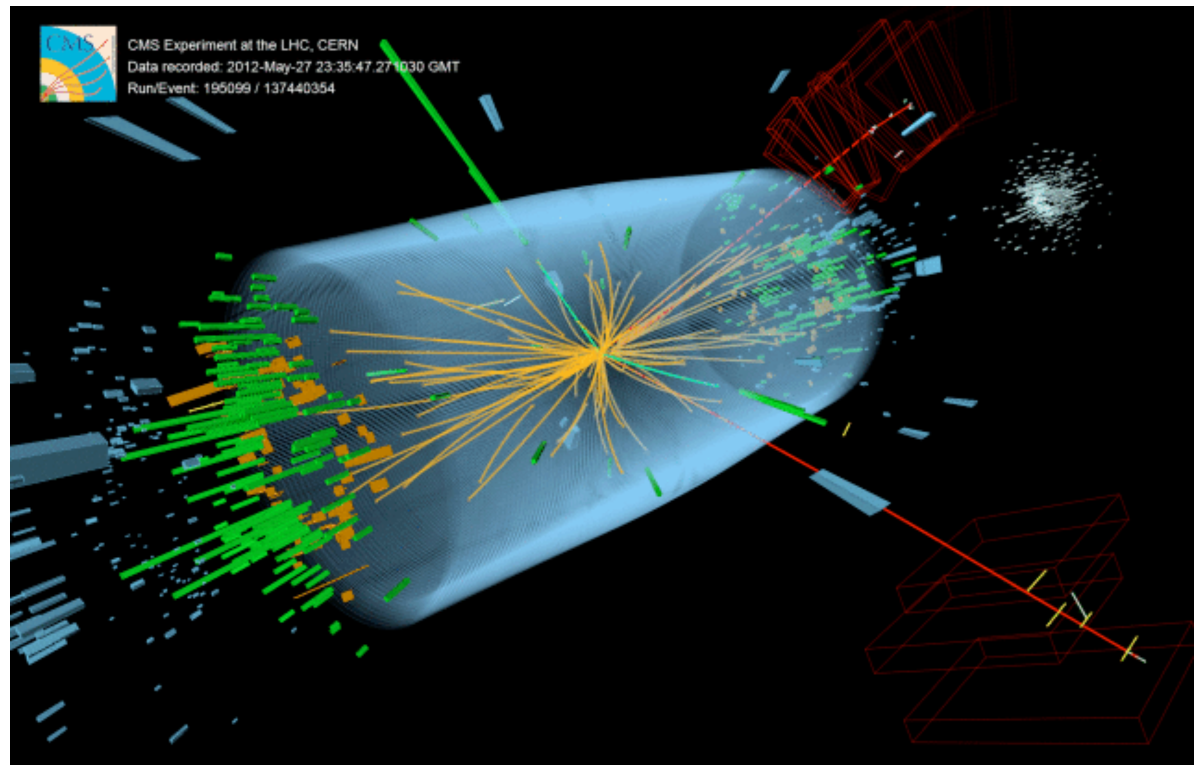
How can we, in the amplitudes community, help out?

See also talks from Buonanno, Cangemi, De Angelis, Ivanov

Can Amplitudes Help with Gravitational Waves?

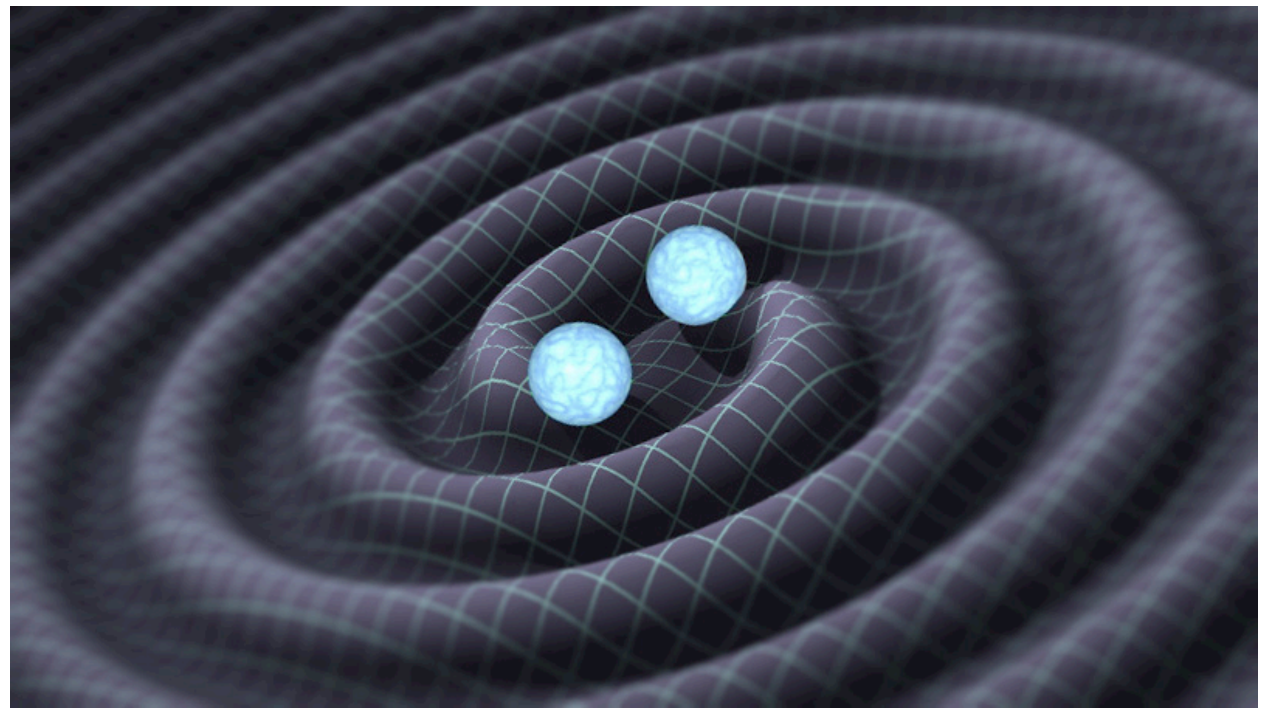
What does particle physics have to do with classical dynamics of astrophysical objects?

unbounded trajectory



gauge theories, QCD, electroweak
quantum field theory

bounded orbit



general relativity
classical physics

Black holes and neutron stars are point particles as far as long wavelength radiation is concerned.

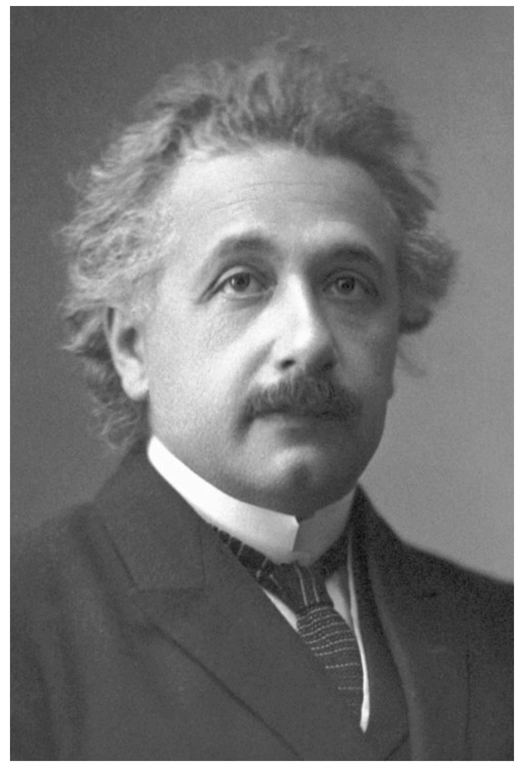
Iwasaki (1971); **Goldberger, Rothstein (2006)**; Vaydia, Foffa , Porto, Rothstein, Sturani; Kol; Bjerrum-Bohr, Donoghue, Holstein, Plante, Pierre Vanhove; Levi, Steinhoff; Vines etc

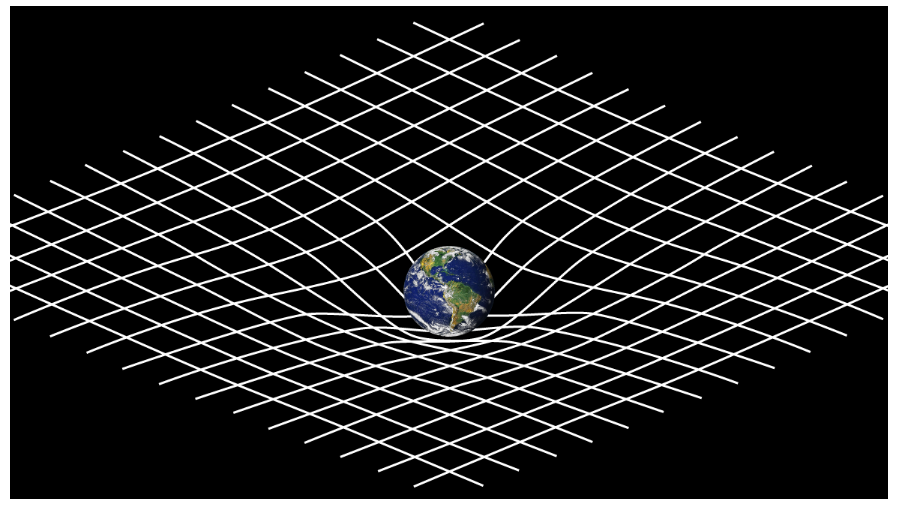
Will explain that scattering amplitudes are well suited for perturbative gravitational wave calculations.

Approach to General Relativity

The usual method to gravity is to solve Einstein's Field equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$





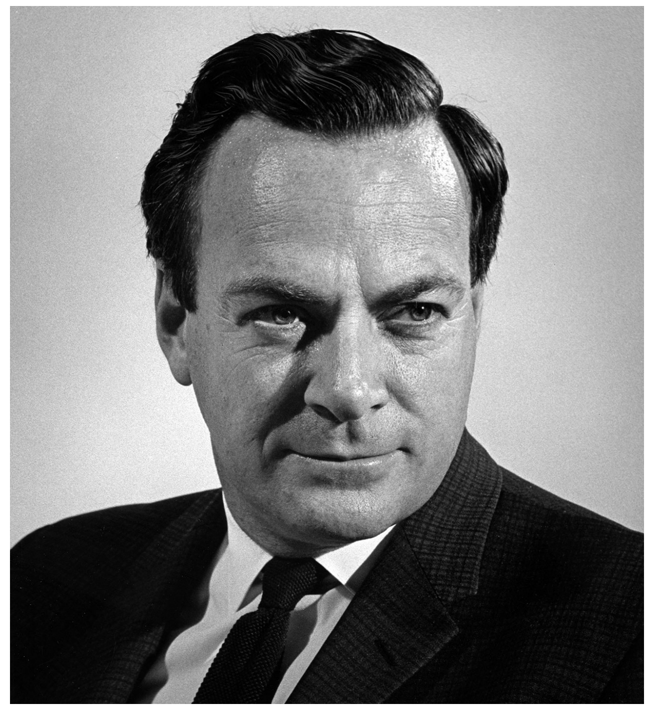
geometry

Amplitudes Approach to General Relativity

Our appoach does *not* start from usual Einstein Field equations.

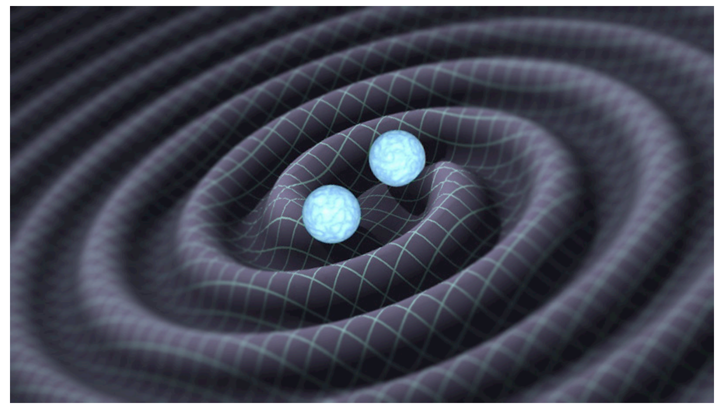
$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

geometry



Gravitons are spin 2 particles

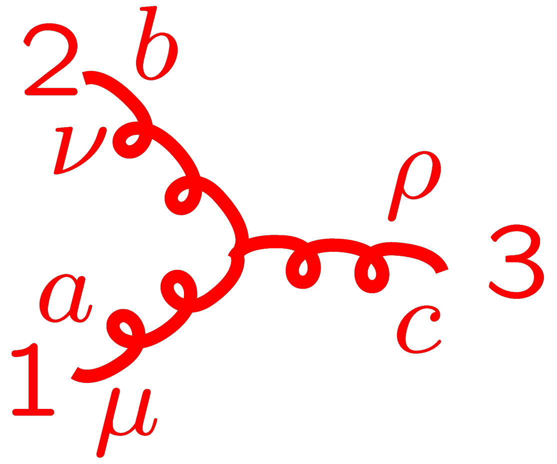
- **Not suited for all problems.**
- **Well suited for gravitational-wave physics from compact astrophysical objects.**



On-shell viewpoint has surprising simplicity.

On-shell three vertices contains all information: $k_i^2 = 0$

Yang-Mills gauge theory:

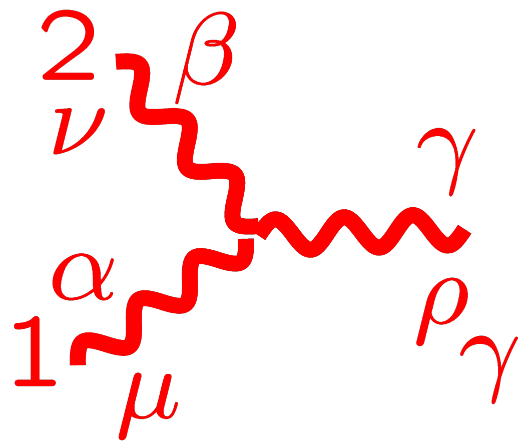


$$-gf^{abc}(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$

$$\sim gf^{abc}\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}$$

Only consistent vertices with correct dimensions

Einstein gravity:



$$i\kappa(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$
$$\times (\eta_{\alpha\beta}(k_1 - k_2)_\gamma + \text{cyclic})$$

$$\sim i\kappa\left(\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}\right)^2$$

"square" of Yang-Mills vertex.

$\kappa^2 = 32\pi G$

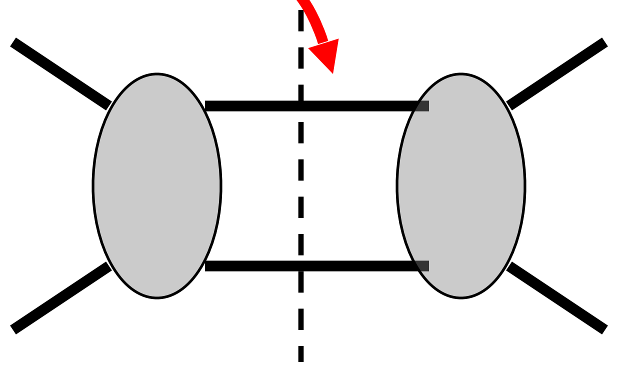
Using on-shell methods, BCFW recursion and unitarity method, we can build all tree and loop amplitudes in the theory.

From Tree to Loops: Generalized Unitarity Method

Use tree amplitudes to build higher order (loop) amplitudes.

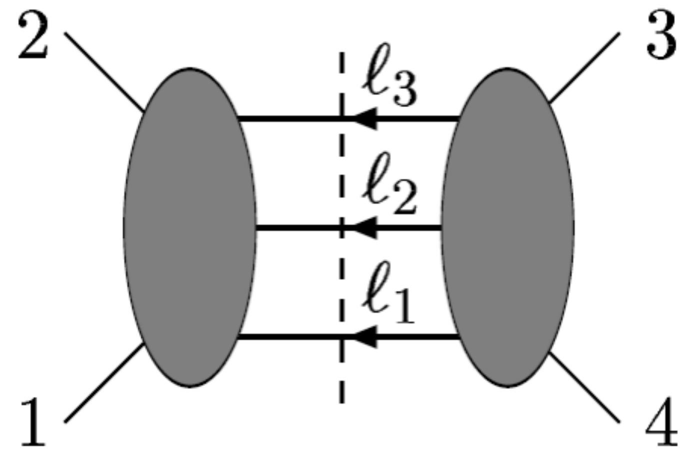
$E^2 = \vec{p}^{\,2} + m^2$ ← **on-shell**

Two-particle cut:



ZB, Dixon, Dunbar and Kosower (1994)

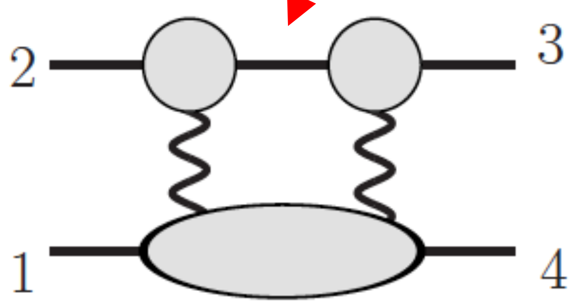
Three-particle cut:

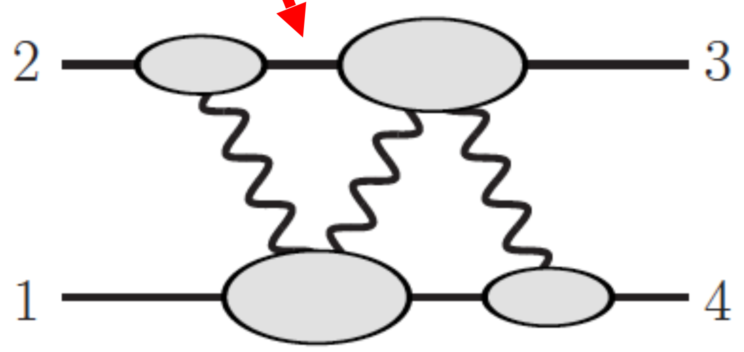


- **Systematic assembly of complete loop amplitudes from tree amplitudes.**
- **Works for any number of particles or loops.**

on-shell

Generalized unitarity as a practical tool for loops.





ZB, Dixon and Kosower;
ZB, Morgan;
Britto, Cachazo, Feng;
Ossala,Pittau,Papadopoulos;
Ellis, Kunszt, Melnikov;
Forde; Badger;
ZB, Carrasco, Johansson, Kosower
and many others

This is a very natural language for gravitational wave problems

Gravity as a Double copy of Gauge Theory

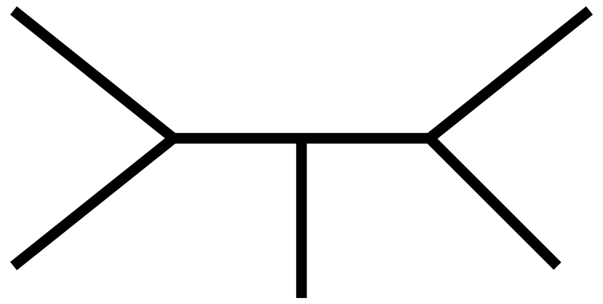
Kawai, Lewellen, Tye; ZB, Carrasco, Johansson

gauge theory (QCD): $\mathcal{A}_n^{\text{tree}} = i g^{n-2} \sum_i \frac{c_i\, n_i}{D_i}$

- c_i: **color factor**
- n_i: **kinematic numerator factor**
- D_i: **Feynman propagators**

$$c_k = c_i - c_j$$
$$n_k = n_i - n_j$$

$$\boxed{c_i \rightarrow n_i}$$



sum over diagrams with only 3 vertices

Einstein gravity: $\mathcal{M}_n^{\text{tree}} = i \kappa^{n-2} \sum_i \frac{n_i^2}{D_i}$

$$n_i \sim k_4 \cdot k_5\, k_2 \cdot \varepsilon_1\, \varepsilon_2 \cdot \varepsilon_3\, \varepsilon_4 \cdot \varepsilon_5 + \cdots$$

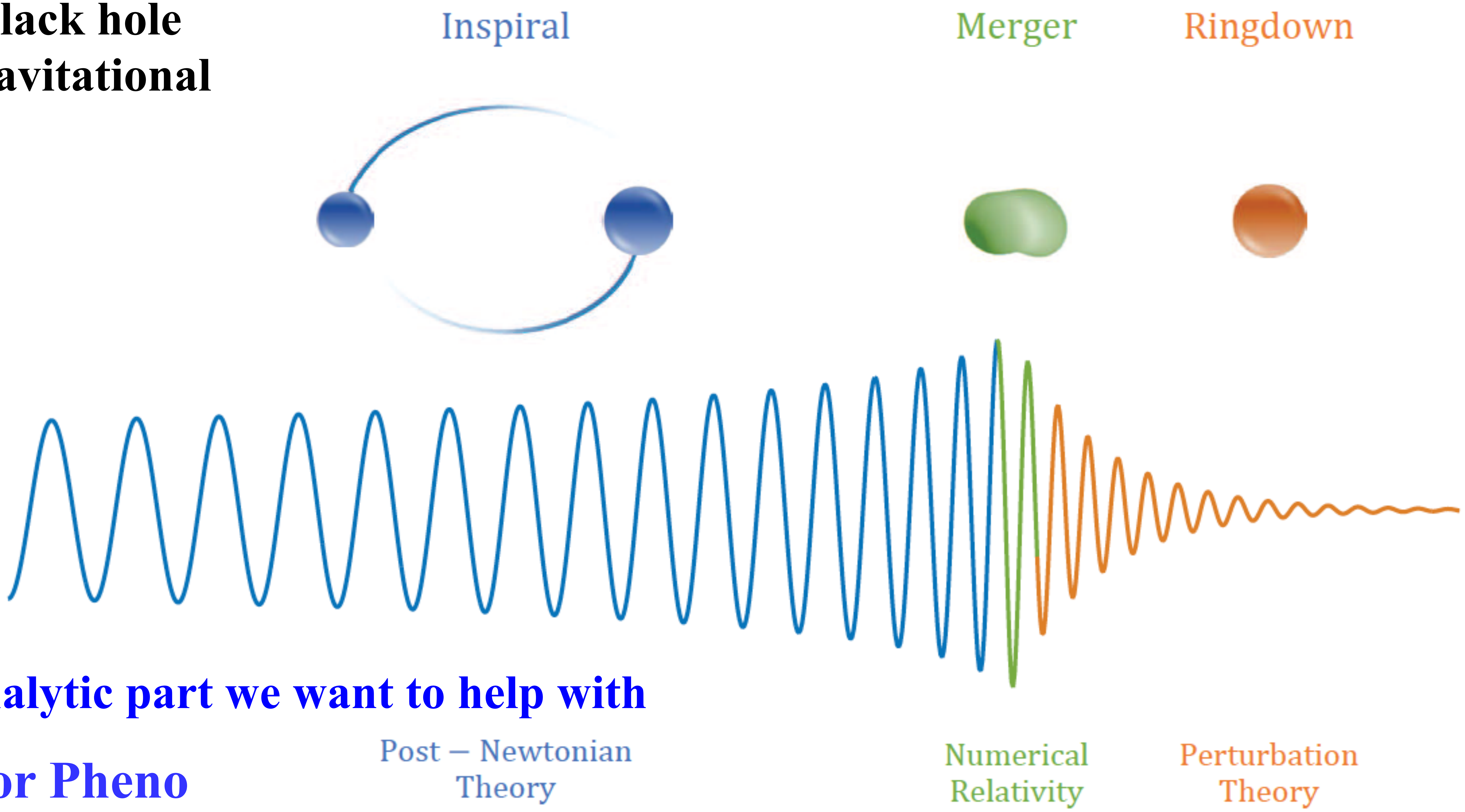
Gravity and gauge theory kinematic numerators are the same!

Same ideas conjectured to hold at loop level.

Goal: Higher Precision.

See Alessandra Buonanno's talk

Dynamics of black hole inspiral for gravitational waves.



analytic part we want to help with

PN + EOB or Pheno

Small errors accumulate. Need for high precision.

From Antelis and Moreno, arXiv:1610.03567

Basic Approaches

1. Post-Newtonian (**PN**): Expand in *G* and *v*
2. Post-Minkowskian (**PM**): Expand in *G*.
3. Self force (**GSW**): Expand in mass-ratio exact in *G*. (Semi numerical)
4. Numerical relativity (**NR**): Solve Einstein's equations numerically

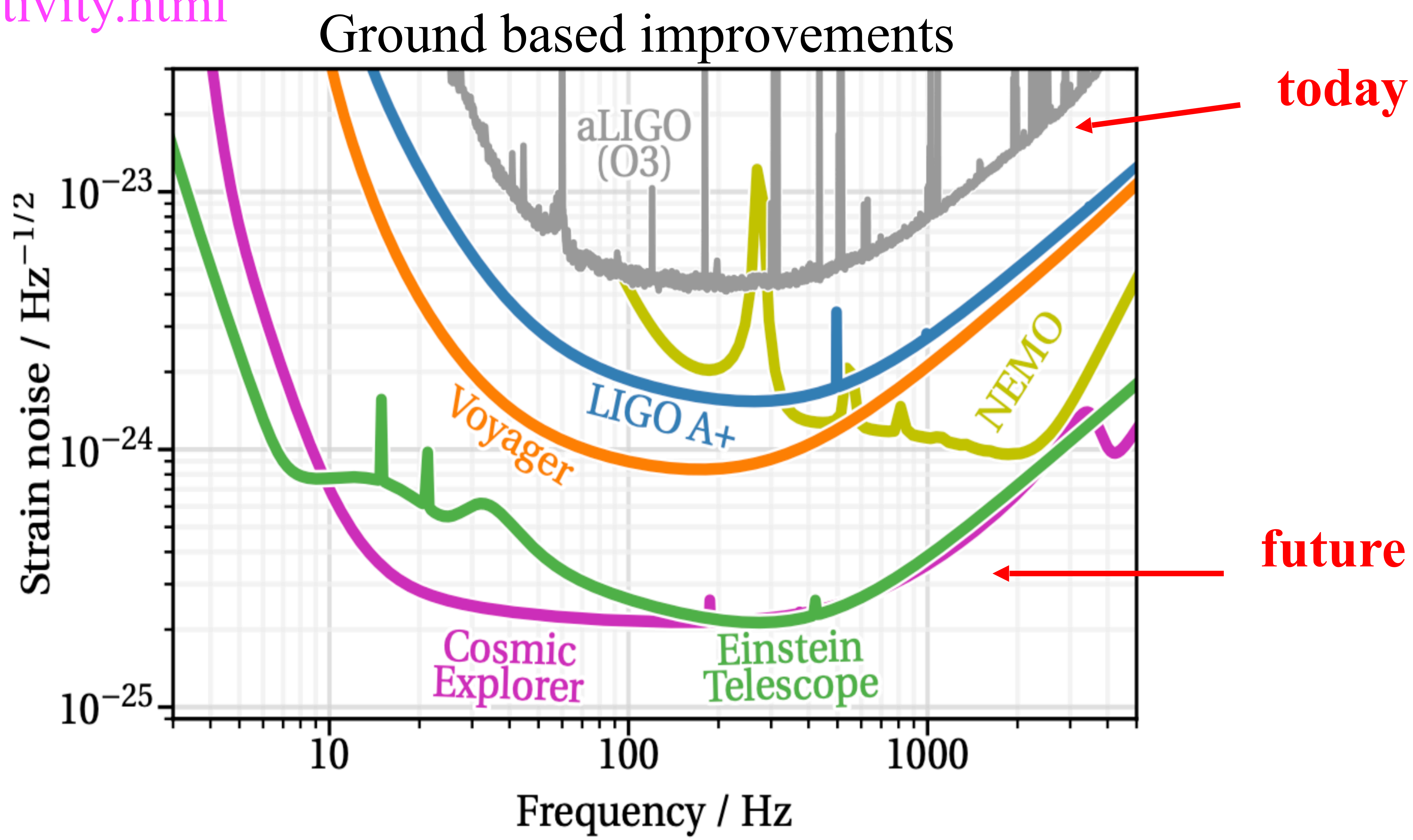
- PM approach fits naturally with scattering amplitudes.
- Waveform models import information from all approaches.

See Buonanno's talk

Future Detectors

https://cosmicexplorer.org/sensitivity.html

See Buonanno's talk



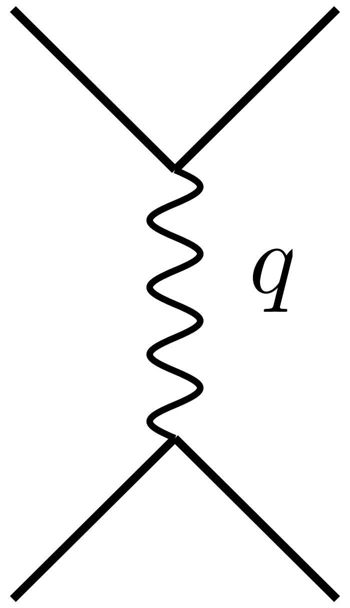
- **Depending on parameters, sensitivity improvements up to factor of 100.**
- **Highly nontrivial theoretical challenge to match upcoming experimental precision.**
- **Likely need 2 further perturbative orders**

2 Body Potentials and Amplitudes

Tree-level: Fourier transform gives classical potential.

$$V(r) \sim \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} A^{\text{tree}}(\mathbf{q})$$

$$H(\mathbf{p},\mathbf{r}) = \sqrt{\mathbf{p}^2 + m_1^2} + \sqrt{\mathbf{p}^2 + m_2^2} + V(\mathbf{p},\mathbf{r})$$



Newtonian potential follows follows from tree amplitude

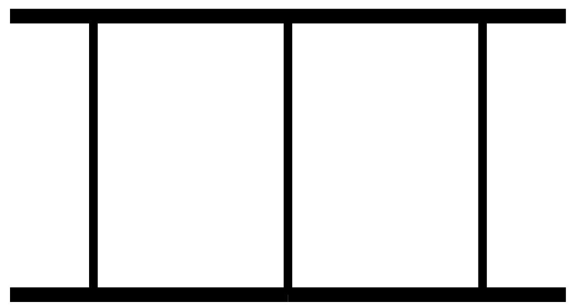
$$V(\mathbf{p},\mathbf{r}) = \sum_{i=1}^{\infty} c_i(\mathbf{p}^2)\left(\frac{G_N}{|\mathbf{r}|}\right)^i$$

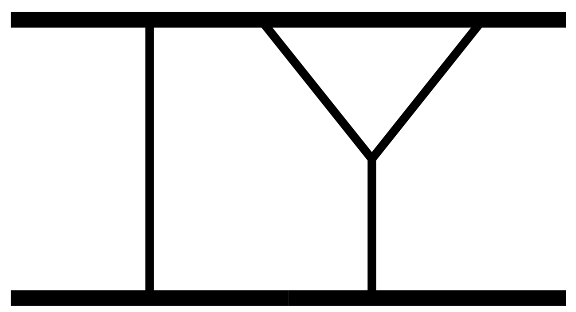
$$c_1(\boldsymbol{p}^2) = \frac{\nu^2 m^2}{\gamma^2 \xi}\left(1 - 2\sigma^2\right)$$

Beyond 1 loop less obvious:

- **Loops have classical pieces.**
- $1/\hbar^L$ **scaling of at** L **loop.**
- **Double counting and iteration.**

$$e^{iS_{\text{classical}}/\hbar}$$

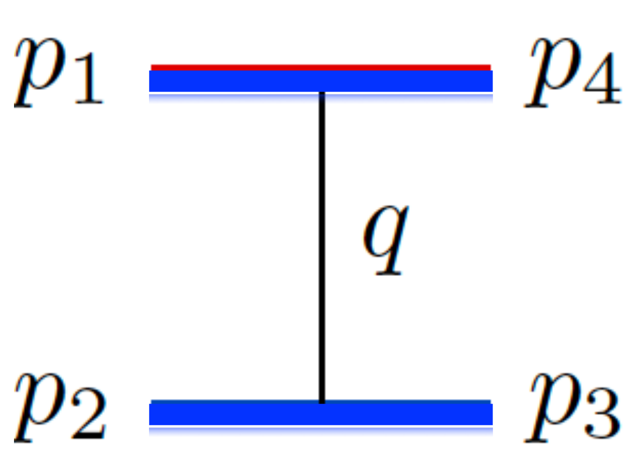




Piece of loops are classical: Our task is to efficiently extract these pieces.

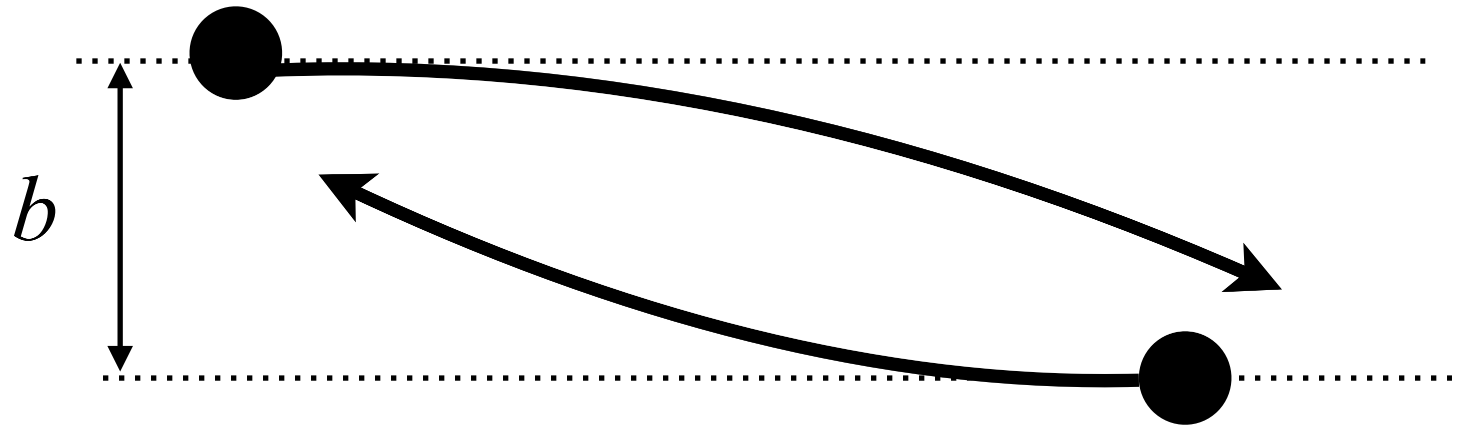
Classical Limit

Consider 2 to 2 scattering



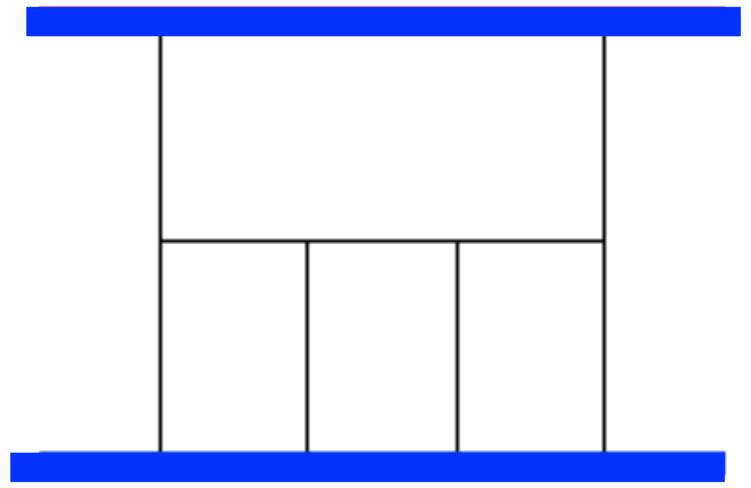
$$s = (p_1 + p_2)^2$$
$$t = (p_1 + p_4)^2$$

$$|q| \sim \frac{1}{|b|}$$



$$s, u, m_1^2, m_2^2 \sim J^2|t| \gg |t| = |q^2|$$

Large angular momentum limit



Classical contributions live in the soft graviton region

Useful to further subdivide into potential and radiation regions

Beneke and Smirnov

$$\text{potential:} \quad \ell \sim (v, \mathbf{1})|q|\,, \qquad \text{radiation:} \quad \ell \sim (v, \boldsymbol{v})|q|$$

Greatly simplifies the integrals. Eikonal matter propagators v characteristic velocity

Can also planarize the integrals.

Driesse, Jakobsen, Mogull, Plefka, Sauer, Usovitsch; ZB, Herrmann, Roiban, Ruf, Smirnov, Smirnov, Zeng;

Overview

Applying amplitudes methods to gravitational-wave physics is now well developed.

- **Pushing state of the art for high orders in G. Now pushing to G^5**

  ZB, Cheung, Roiban, Parra-Martinez, Ruf, Shen, Solon, Zeng; Di Vecchia, Heissenberg, Russo, Veneziano; Damgaard, Hansen, Plante, Vanhove; Dlapa, Kälin, Liu, Porto Ridgway, Shen; ZB, Herrmann, Roiban, Ruf, Smirnov, Smirnov, Zeng; Driesse, Jakobsen, Mogull, Plefka, Sauer, Usovitsch; Edison and Levi; etc

- **Waveforms** Cristofoli, Gonzo, Kosower, O'Connell; Herrmann, Parra-Martinez, Ruf, Zeng; Di Vecchia, Heissenberg, Russo, Veneziano; Herderschee, Roiban, Teng; Georgoudis, Heissenberg , Vazquez-Holm; Brandhuber, Brown, Chen, De Angelis, Gowdy , Travaglini; etc

- **Finite-size effects** Cheung and Solon; Haddad and Helset; Kälin, Liu, Porto; Cheung, Shah, Solon; ZB, Parra-Martinez, Roiban, Sawyer, Shen, etc

- **Spin** Vaidya; Geuvara, O'Connell, Vines; Chung, Huang, Kim, Lee; ZB, Luna, Roiban, Shen, Zeng; Kosmopoulos, Luna; Febres Cordero, Kraus, Lin, Ruf, Zeng; Aoude, Haddad, Helset; ZB, Kosmopoulos, Luna, Roiban, Teng; Kim, Steinhoff; Aoude and Ochirov; Ben-Shahar; Vine, Sheopner; Gatica; Jakobsen, Mogull, Plefka, Sauer Cangemi, Chiodaroli,Johansson,Ochirov, Pichini, Skvortsov; Luna,Moynihan, O'Connell, Ross; Bautista, Guevara,Kavanagh, Vines; Bautista, Bonelli, Iosa, Tanzini, Zhou; Buonanno, Mogull, Patil, Pompili; etc

- **Absorption** Goldberger and Rothstein; Aoude, Ochirov; Jones, Ruf; Chen, Hsieh, Huang, Kim; etc

Many great young people!

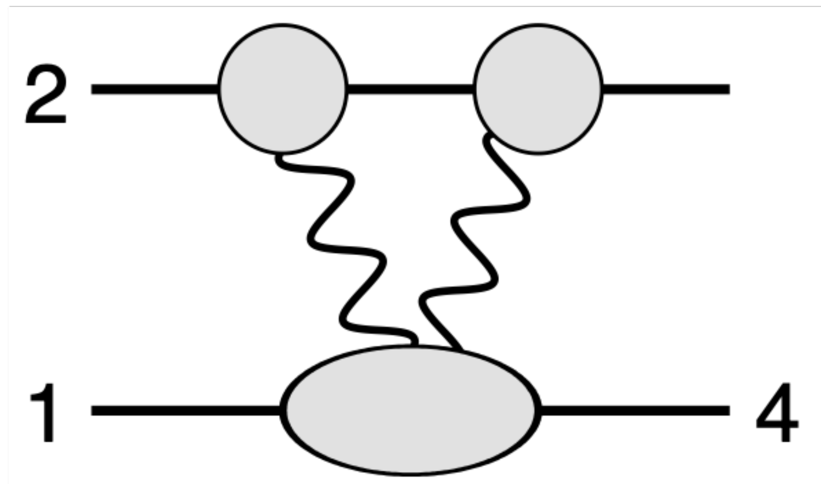
See also talks from Buonanno, Cangemi, De Angelis

Amplitudes Approach: Unitarity + Double Copy

- **Long-range force: Two matter lines must be separated by on-shell propagators.**
- **Classical potential: 1 matter line per loop is cut (on-shell).**

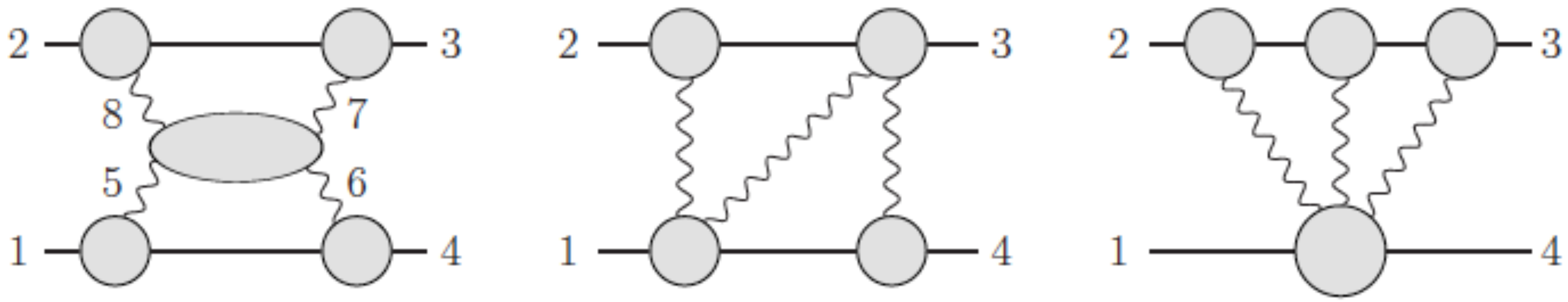
Neill and Rothstein ; Bjerrum-Bohr, Damgaard, Festuccia, Planté, Vanhove; Cheung, Rothstein, Solon

Only independent unitarity cut for O(G^2).



Treat exposed lines on-shell (long range).
Pieces we want are simple!

Independent generalized unitarity cuts for O(G^3).



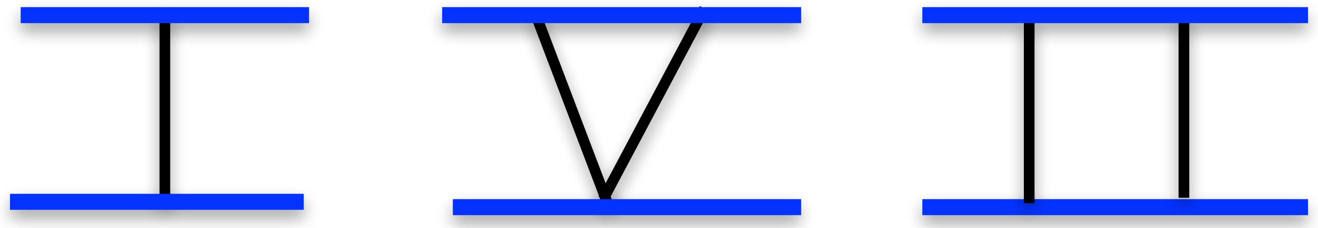
Amplitude tools fit perfectly with extracting classical pieces we want.

gravity **loops**

Structure of Higher Orders

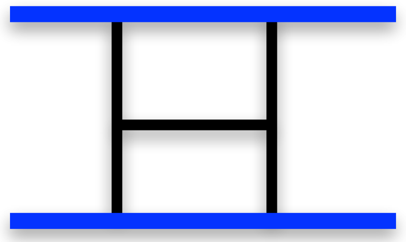
Moving up in orders of PM new effects and features encountered:

1PM and 2PM: Fixed by geodesic motion, 0SF.



$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{1}{1 - \frac{2M}{r}} dr^2 + r^2 d\theta^2 + r^2 \sin^2(\theta)\, d\phi^2$$

3PM: Interesting structure in high energy limit. 1SF, m_1/m_2

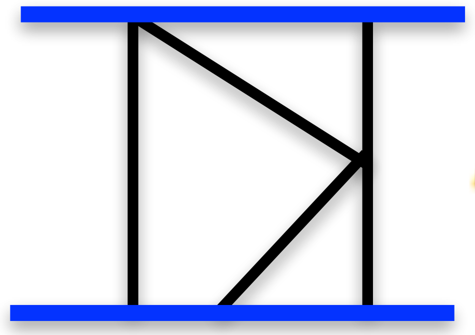


$$\log(E^2/m_1 m_2)$$

Poor high energy behavior cancels against real radiation

Di Vecchia, Heissenberg, Russo, Veneziano; Damour

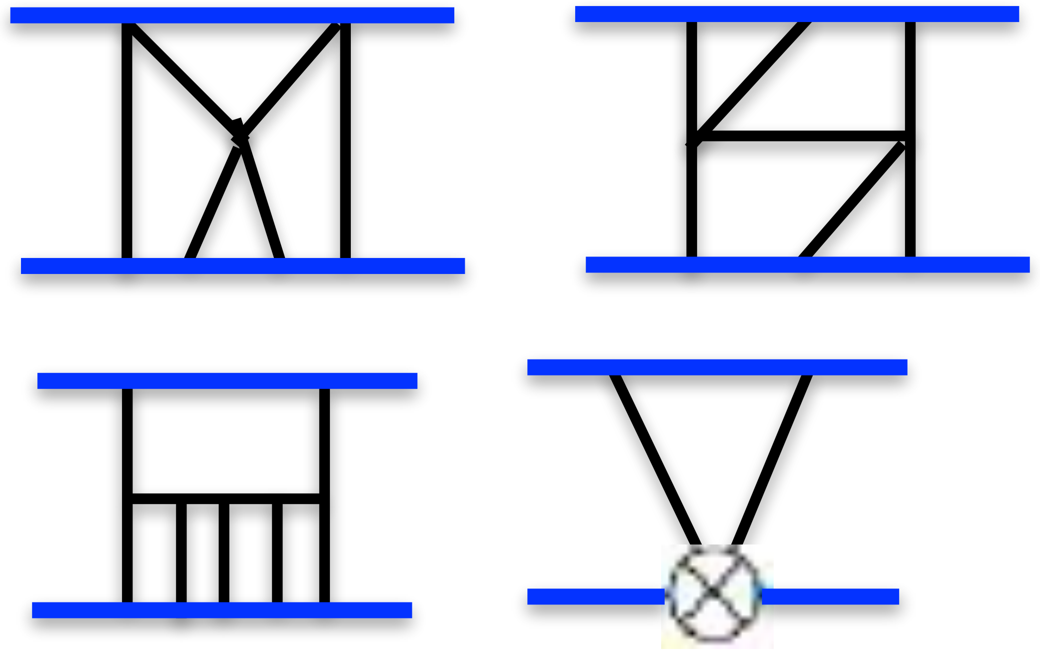
4PM: Tail effect, nontrivial analytic continuations, elliptic integrals, *non*-cancellation of poor high-energy behavior. Nonlocal in time effects.



$$\sim K^2\left(\frac{\sigma - 1}{\sigma + 1}\right)$$

5PM: 2SF, Calabi-Yau integrals.
Nontrivial to separate conservative and dissipative

6PM: Mixing with tidal operators, UV divergences.
Distinguish BHs from neutron stars.



Conservative Contribution 4PM O(G^4)

ZB, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng

O(G^4) amplitude — test particle — 1st self force — Iteration. No need to compute — Lower loop, already known, radial action

$$\mathcal{M}_4^{\rm cons} = G^4 M^7 \nu^2 |\boldsymbol{q}| \pi^2 \left[\mathcal{M}_4^{\rm p} + \nu\left(4\mathcal{M}_4^{\rm t}\log\left(\tfrac{p_\infty}{2}\right) + \mathcal{M}_4^{\pi^2} + \mathcal{M}_4^{\rm rem}\right)\right] + \int_\ell \frac{\tilde{I}^4_{r,1}}{Z_1 Z_2 Z_3} + \int_\ell \frac{\tilde{I}^2_{r,1}\tilde{I}_{r,2}}{Z_1 Z_2} + \int_\ell \frac{\tilde{I}_{r,1}\tilde{I}_{r,3}}{Z_1} + \int_\ell \frac{\tilde{I}^2_{r,2}}{Z_1}$$

tail effect

$$D = 4 - 2\epsilon$$

$$\mathcal{M}_4^{\rm t} = r_1 + r_2 \log\left(\tfrac{\sigma+1}{2}\right) + r_3 \frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\,, \qquad \mathcal{M}_4^{\rm p} = -\frac{35\left(1 - 18\sigma^2 + 33\sigma^4\right)}{8\left(\sigma^2-1\right)}$$

$$\mathcal{M}_4^{\pi^2} = r_4\pi^2 + r_5\,\mathrm{K}\left(\tfrac{\sigma-1}{\sigma+1}\right)\mathrm{E}\left(\tfrac{\sigma-1}{\sigma+1}\right) + r_6\,\mathrm{K}^2\left(\tfrac{\sigma-1}{\sigma+1}\right) + r_7\,\mathrm{E}^2\left(\tfrac{\sigma-1}{\sigma+1}\right)\,, \quad \longleftarrow \text{ elliptic}$$

$$\mathcal{M}_4^{\rm rem} = r_8 + r_9\log\left(\tfrac{\sigma+1}{2}\right) + r_{10}\frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}} + r_{11}\log(\sigma) + r_{12}\log^2\left(\tfrac{\sigma+1}{2}\right) + r_{13}\frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\log\left(\tfrac{\sigma+1}{2}\right) + r_{14}\frac{\operatorname{arccosh}^2(\sigma)}{\sigma^2-1}$$
$$+ r_{15}\,\mathrm{Li}_2\left(\tfrac{1-\sigma}{2}\right) + r_{16}\,\mathrm{Li}_2\left(\tfrac{1-\sigma}{1+\sigma}\right) + r_{17}\frac{1}{\sqrt{\sigma^2-1}}\left[\mathrm{Li}_2\left(-\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right) - \mathrm{Li}_2\left(\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right)\right].$$

For dissapative see: Manohar, Ridgeway, Shen; Dlapa, Kälin, Liu, Porto
Spin-orbit:
Jakobsen, Mogull, Plefka, Sauer, Xu

$$\nu = m_1 m_2/(m_1+m_2)^2 \qquad \sigma = p_1\cdot p_2/m_1 m_2,$$

r_{ij} rational coefficients

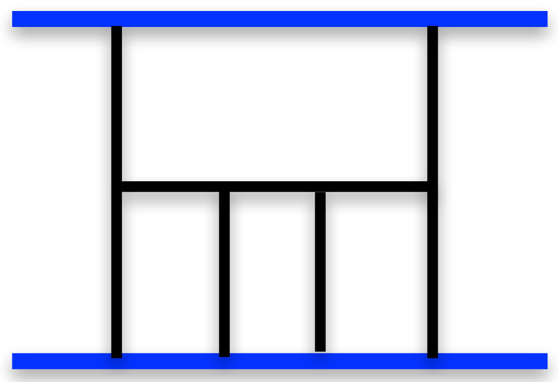
This is complete conservative contribution.

$$\mathcal{M}_4^{\rm radgrav,f} = \frac{12044}{75}p_\infty^2 + \frac{212077}{3675}p_\infty^4 + \frac{115917979}{793800}p_\infty^6 - \frac{9823091209}{76839840}p_\infty^8 + \frac{115240251793703}{1038874636800}p_\infty^{10} + \cdots$$

First 3 terms match 6PN results of Bini, Damour, Geralico.

- **Result for angle, including radiation effects completed.** Dlapa, Kälin, Liu, Porto
- **Potential subtlety remains with PN comparision.** Bluemlein, Maier, Marquard, Schafer; Foffa, Sturani. Luz Almeida, Muller, Foffa, Sturani
- **Analytic continuation to bound case not trivial: tail effect. Recent progress on local part.** Dlapa, Kälin, Liu, Porto; Bini and Damour

Towards 5PM, O(G^5)



Scattering Amplitudes/Worldline

Double copy
Generalized unitarity
Expansion in classical limit

Straightforward

Loop Integrand

Reduction to master integrals
DE's for master integrals
Solutions of DEs.

Hard

Currently working on integration

ZB, Herrmann, Roiban, Ruf, Smirnov, Smirnov, Zeng; Driesse, Jakobsen, Mogull, Plefka, Sauer, Usovitsch [Humbolt]

Integrated Amplitude

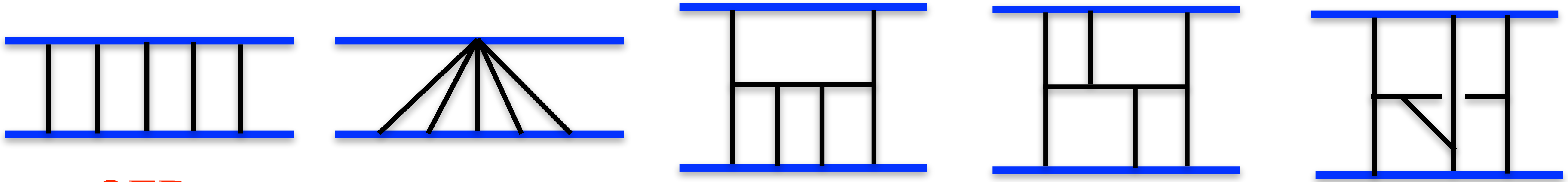
Eikonal, EFT matching computations
Amplitude action relation,
Pick your favorite formalism.

Straightforward

2-Body Hamiltonian or Observables

5PM problem nontrivial, so attack in stages.

Deal With Integration in Stages



QED 0SF 1SF 2SF 2SF

Stages:

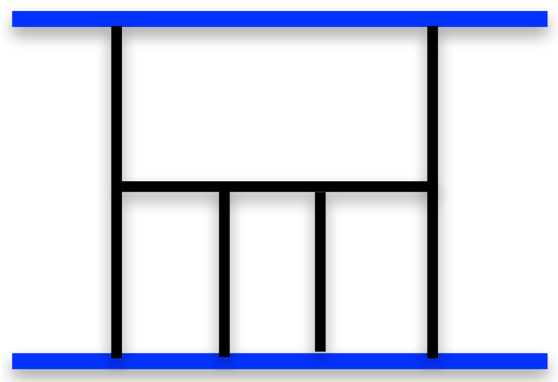
1. **QED warmup. Potential mode contributions.** **Done.**
   ZB, Herrmann, Roiban, Ruf, Smirnov, Smirnov, Zeng
2. **1 SF Conservative.** **Done.**
   Driesse, Jakobsen, Mogull, Plefka, Sauer, Usovitsch
3. **$\mathcal{N} = 8$ (lower tensor rank)** **1SF potential done, working on 2SF.**
   ZB, Herrmann, Roiban, Ruf, Smirnov, Smirnov, Zeng
4. **2 SF Conservative.** **Harder, but in reach.**
5. **Radiative effects.** **Similar**

$$M_{5\text{PM}} = M_{5\text{PM}}^{0\text{SF}} + \nu M_{5\text{PM}}^{1\text{SF}} + \nu^2 M_{\text{PM}}^{2\text{SF}} \qquad \nu = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

Learn from each stage to push forward the next one.

High-Loop Integration

In QCD/Amplitudes advanced technology for loop integrals which we import.



1. **IBP greatly simplified in classical limit.** Chetyrkin, Tkachov; Laporta; Henn; Henn and Smirnov Beneke and Smirnov; Parra-Martinez, Ruf, Zeng
2. **Choose master integrals to simplify the DEs and IBP.** A. Smirnov and V. Smirnov; Usovitsch
3. **Use finite prime fields and reconstruction for toughest IBPs.** Manteuffel, Schabinger; Peraro
4. **Set up a DEs for master integrals.** Kotikov, ZB, Dixon and Kosower; Gehrmann, Remiddi

IBP: $$0 = \int \prod_i^L \frac{d^D \ell_i}{(2\pi)^D} \frac{\partial}{\partial \ell_i^\mu} \frac{N^\mu(\ell_k, p_M)}{Z_1 \dots Z_n}$$

Solve linear relations for master integrals

Also discussed by Abreu, Trancredi

DEs: $$\partial_x \vec{I} = A(x,\epsilon)\vec{I}\,,$$

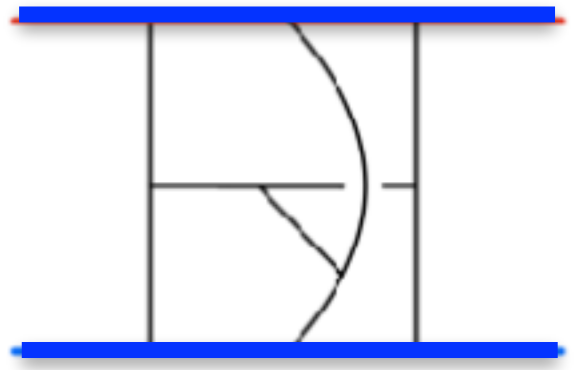
Solve DEs either as series or basis of functions.

Many tools available: We use upgraded FIRE, a private finite field IBP program, LiteRed, FiniteFlow.
Smirnov, Chuharev; Lee; Peraro

Another important tool is an upgraded version of KIRA
Maierhoefer, Usovitsch, Uwer;
Klappert, Lange, Maierhöfer, Usovitsch

Bottleneck: Integral Reduction

Primary bottleneck is integration by parts:



22 indices: 13 propagator and 9 irreducible scalar products (ISPs)

$$\int \frac{d^{4D}k\,[u_2\cdot k_1]^{a_{-14}}[u_2\cdot k_4]^{a_{-15}}[u_1\cdot k_2]^{a_{-16}}[u_1\cdot k_3]^{a_{-17}}[k_1\cdot q]^{a_{-18}}[k_2\cdot q]^{a_{-19}}[k_1\cdot k_2]^{a_{-20}}[k_1\cdot k_4]^{a_{-21}}[k_2\cdot k_3]^{a_{-22}}}{[-2u_2\cdot k_2]^{a_1}[-2u_2\cdot k_{123}]^{a_2}[2u_1\cdot k_{234}]^{a_3}[2u_1\cdot k_{1234}]^{a_4}[k_1^2]^{a_5}[k_2^2]^{a_6}[k_3^2]^{a_7}[k_{13}^2]^{a_8}[k_4^2]^{a_9}[k_{34}^2]^{a_{10}}[k_{234}^2]^{a_{11}}[(k_{123}-q)^2]^{a_{12}}[(k_{1234}-q)^2]^{a_{13}}}$$

Very similar to wordliine integrals. KMOC version probably identical to WLQFT

- **Can encounter up to 8 numerator powers and 4 doubled propagators.**
- **FIRE and KIRA need to be carefully tuned.**
- **Private code specialized for finite field numerical techniques.**

$$0 = \int \prod_i^L \frac{d^D\ell_i}{(2\pi)^D} \frac{\partial}{\partial \ell_i^\mu} \frac{N^\mu(\ell_k, p_i)}{D_1^{a_1}\cdots D_n^{a_n}}$$

ZB, Herrmann, Roiban, Ruf, Smirnov, Smirnov, Zeng; Driesse, Jakobsen, Mogull, Plefka, Sauer, Usovitsch

Some improvements:

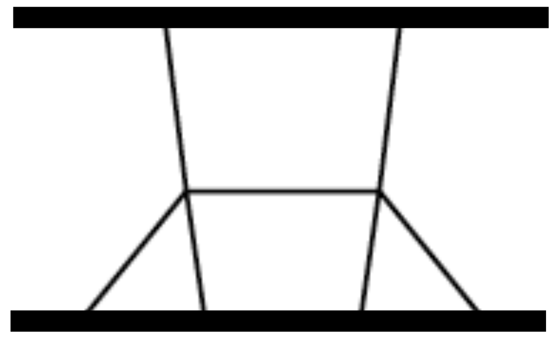
- **Identification of integrals with cancelled propagator across different diagram topologies.**
- **Improved modular arithmetic reconstruction alogorithm. Smoother use of MPI for parallelization**
- **Careful choice of seeds for IBP system.**
- **Use of parity to simplify the system of equations.**
- **Planarization: Special to classical limit. Nonplanar matter lines can be unwound.**

Iterated integrals

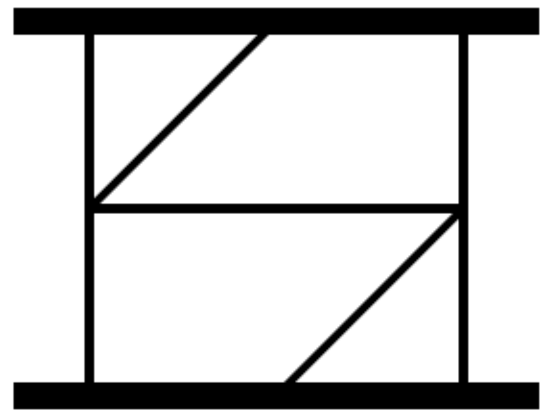
Elliptic and Calabi-Yau integrals appear in master integrals.

See Tancredi's talk for collider physics appearances

Frellesvig, Morales, Wilhelm; Klemm, Nega, Sauer, Plefka
ZB, Herrmann, Roiban, Ruf, Smirnov, Smirnov, Zeng.



At 1 SF, Elliptic integrals.



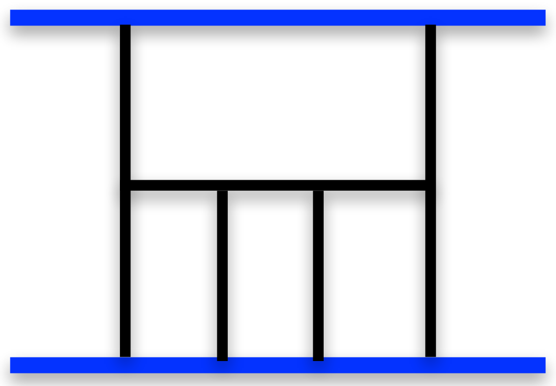
At 2 SF, Calabi-Yau 3-fold integrals.

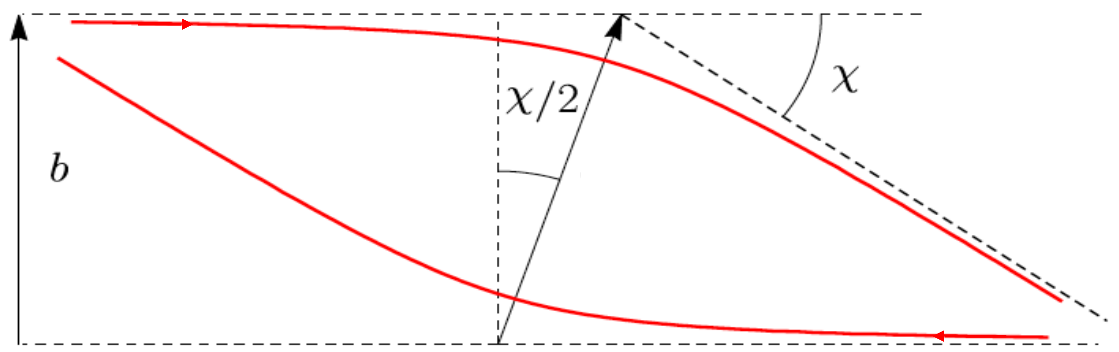
- **At 1SF no elliptic integrals in final result!**
- **Will this simplicity continue to 2 SF sector?**

Opportunity for those interested in the mathematics of Feynman integrals

5PM Scattering Angle $\mathcal{N} = 8$ Supergravity (1SF Potential)

ZB, Herrmann, Roiban, Ruf, Smirnov, Smirnov, Zeng





$$\tilde{I}_{r,5}^{\text{1SF,fin.}} = r_1 + r_2\,\mathrm{F}_0 + r_3\,\mathrm{F}_0^2 + r_4\,\mathrm{F}_1 + r_5\,\mathrm{F}_2\,.$$

$$\mathrm{F}_0 = \frac{2x}{1-x^2}\ln(x)\,,$$
$$\mathrm{F}_1 = \frac{2x}{1-x^2}\left[-\mathrm{Li}_2(1-x) - \mathrm{Li}_2(-x) - \ln(x)\ln(x+1) - \frac{1}{2}\zeta_2\right],$$
$$\mathrm{F}_2 = \frac{2x}{1-x^2}\left[-\mathrm{Li}_2(1-x) + \mathrm{Li}_2(-x) - \frac{1}{2}\ln^2(x) + \ln(x)\ln(x+1) + \frac{1}{2}\zeta_2\right]$$

Cyclotomic polylogs are natural functions to use, but here only up to dilogs. Ablinger, Bluemlein, Schneider.

Remarkably simple, elliptic integrals cancel.

$$r_1 = \frac{16c_\phi^3}{\sigma^2-1}\left[-\frac{\sigma\left(5\sigma^2-4\right)c_\phi^3}{5\left(\sigma^2-1\right)^3} - \frac{2c_\phi^2}{\sigma^2-1} + 8\right],$$
$$r_2 = 32c_\phi^2\left[-\frac{\sigma^2c_\phi^4}{\left(\sigma^2-1\right)^3} - \frac{4\sigma c_\phi^3}{\left(\sigma^2-1\right)^2} - \frac{9\left(2\sigma^2-1\right)c_\phi^2}{2\left(\sigma^2-1\right)^2} - \frac{8\sigma c_\phi}{\sigma^2-1} + 1\right],$$
$$r_3 = 16c_\phi\left[-\frac{\sigma^3c_\phi^5}{\left(\sigma^2-1\right)^3} - \frac{6\sigma^2c_\phi^4}{\left(\sigma^2-1\right)^2} + \frac{6\sigma\left(2-3\sigma^2\right)c_\phi^3}{\left(\sigma^2-1\right)^2} + \frac{\left(8-32\sigma^2\right)c_\phi^2}{\sigma^2-1} - \frac{92\sigma c_\phi}{3} - \frac{40}{3}\left(\sigma^2-1\right)\right],$$
$$r_4 = 32c_\phi\left[-\frac{\sigma c_\phi^3}{\left(\sigma^2-1\right)^2} - \frac{2c_\phi^2}{\sigma^2-1} - \frac{4\sigma c_\phi}{3\left(\sigma^2-1\right)} - \frac{8}{3}\right],$$
$$r_5 = 64c_\phi\left[\frac{\sigma^2c_\phi^3}{\left(\sigma^2-1\right)^2} + \frac{4\sigma c_\phi^2}{\sigma^2-1} + \frac{2\left(7\sigma^2-6\right)c_\phi}{3\left(\sigma^2-1\right)} + \frac{4\sigma}{3}\right].$$

See also very nice GR 1SF conservative results.

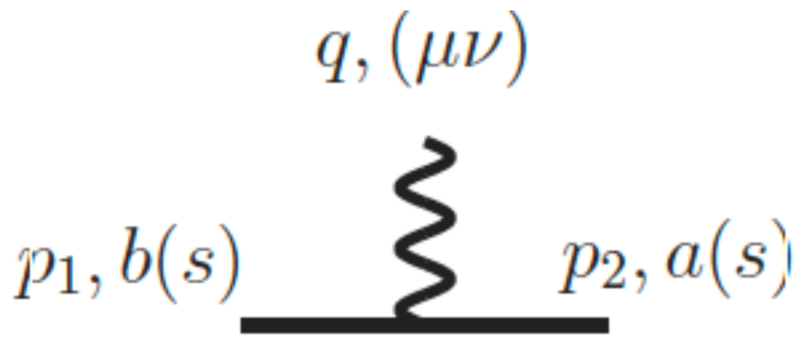
Driesse, Jakobsen, Mogull, Plefka, Sauer, Usovitsch

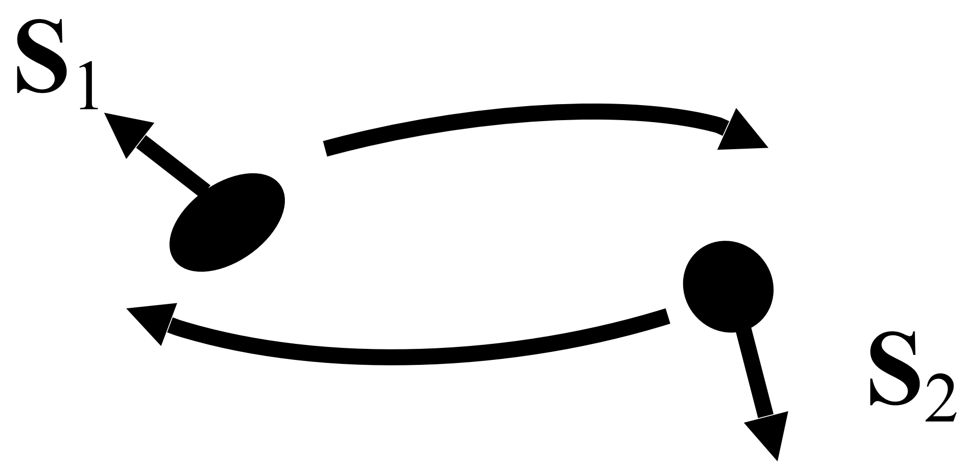
Scattering angle: $$\chi = -\frac{\partial I_r}{\partial J}$$

Very encouraging that results are so simple

Spinning Puzzles

Various puzzles for PM EFTs for spin:





Consider energy momentum tensor:

$$T^{\mu\nu}(p_1, q) = \frac{p_1^\mu p_1^\nu}{m} \sum_{n=0}^{\infty} \frac{C_{ES^{2n}}}{(2n)!} \left(\frac{q \cdot S(p_1)}{m}\right)^{2n} - \frac{i}{m} q_\rho p_1^{(\mu} S(p_1)^{\nu)\rho} \sum_{n=1}^{\infty} \frac{C_{BS^{2n+1}}}{(2n+1)!} \left(\frac{q \cdot S(p_1)}{m}\right)^{2n}.$$

See talk from Cangemi

Kerr black black hole: $C_{ES^{2n}} = 1, \quad C_{BS^{2n+1}} = 1$

Porto, Rothstein; Levi, Steinhoff; Vines

Can we determine all other BH Wilson coefficients and categorize all possible operators?

ZB, Kosmopoulos, Luna, Roiban, Teng; Aoude, Haddad, Helset; Mogull, Plefka, Steinhoff; Cangemi, Chiodaroli, Johansson, Ochirov, Pichini, Skvortsov

Starting at O(G^2) even basic questions become tricky.

— **Nontrivial analytic continuation at O($G^2 S^5$). How to separate conservative and dissipative?**

Bautista, Guevara, Kavanagh, Vines

— **How far can we push an eikonal interpretation at O(G^2) and beyond?**

ZB, Luna, Roiban, Shen, Zeng; Cristofoli, Gonzo, Moynihan, O'Connell, Ross, Sergola, White; Gatica; Luna, Moynihan, O'Connell, Ross

— **Mystery of extra Wilson coefficients O($G^2 S^2$). Spin magnitude change in conservative GR.**

$$S^{\mu\nu} = \frac{1}{m} \epsilon^{\mu\nu\rho\sigma} p_\rho S_\sigma + \frac{1}{m} p^{[\mu} K^{\nu]}$$

← K usually dropped via SSC

Apparently no effect on black holes

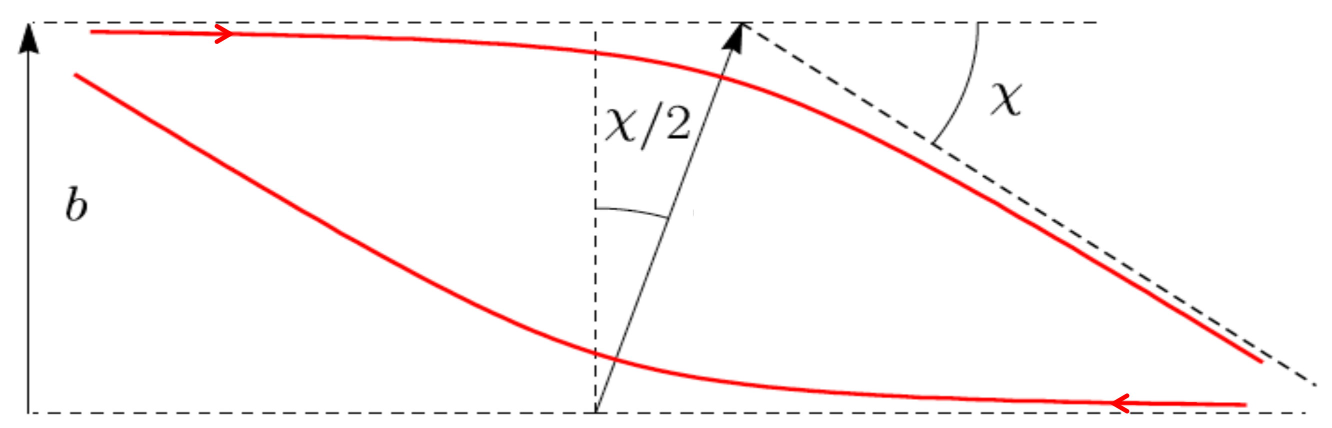
Alaverdian, ZB, Kosmopoulos, Luna, Roiban, Scheopner, Teng, Vines (also to appear)

d'Ambrosi, Kumar, van de Vis, van Holten (2015)

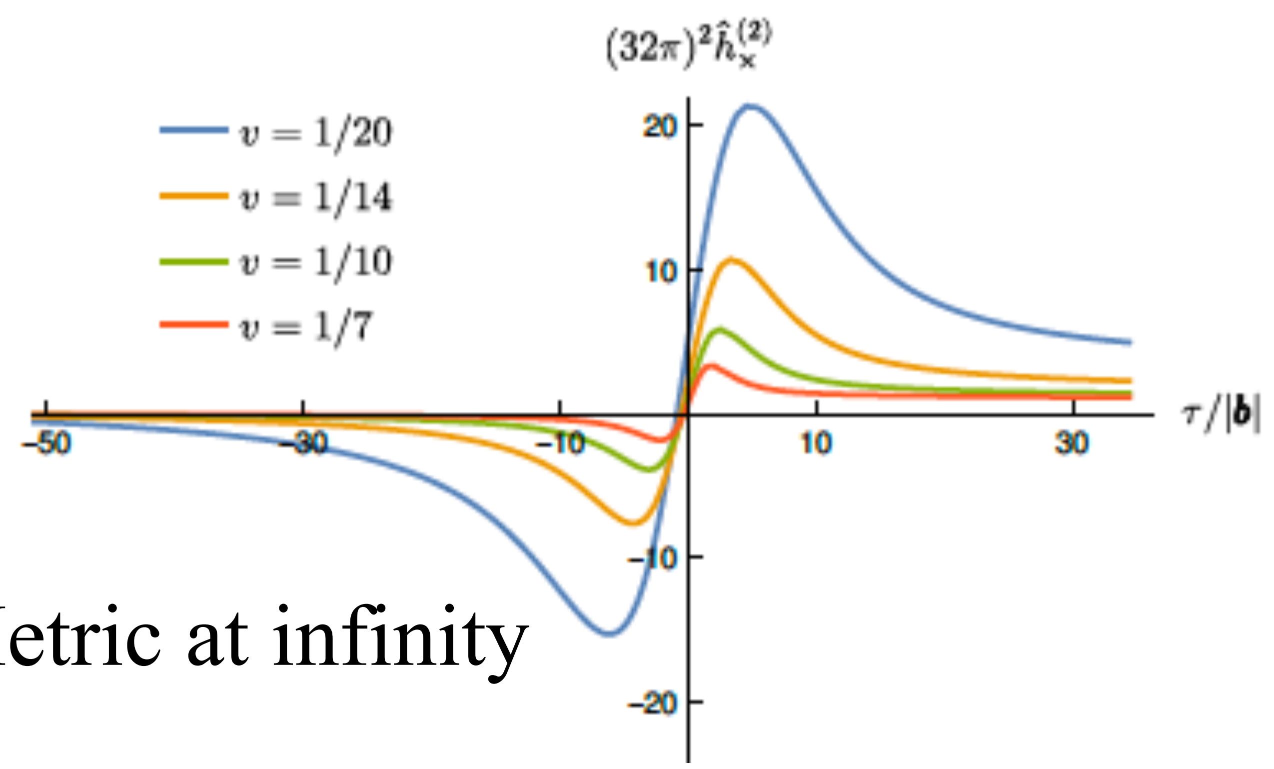
There are still many basic questions to answer

Waveform from scattering two black holes

$$g_{\mu\nu}\Big|_{|\boldsymbol{x}|\to\infty} = \eta_{\mu\nu} + \frac{\kappa^2 M}{8\pi|\boldsymbol{x}|}\left[\frac{\kappa^2 M}{\sqrt{-b^2}}\hat{h}^{(1)}_{\mu\nu} + \left(\frac{\kappa^2 M}{\sqrt{-b^2}}\right)^2 \hat{h}^{(2)}_{\mu\nu} + \dots\right]$$



NLO scattering waveform example



Metric at infinity

Herderschee, Roiban, Teng

Four different approaches at NLO:

1. **KMOC observable based:** Herderschee, Roiban, Teng
2. **Multipolar-Post- Minkowskian (MPM) formalism** Bini, Damour, Geralico
3. **Heavy mass effective field theory.** Brandhuber, Brown, Chen, De Angelis, Gowdy, Travaglini
4. **Eikonal approach** Alessandro Georgoudis, Carlo Heissenberg, Rodolfo Russo

Getting agreement not simple:

1. **Dim. reg. (*D* - 4)/(*D* - 4) finite terms.**
2. **Zero frequency gravitons contribute.**
3. **Nontrivial frame rotations to align results**

See De Angelis talk

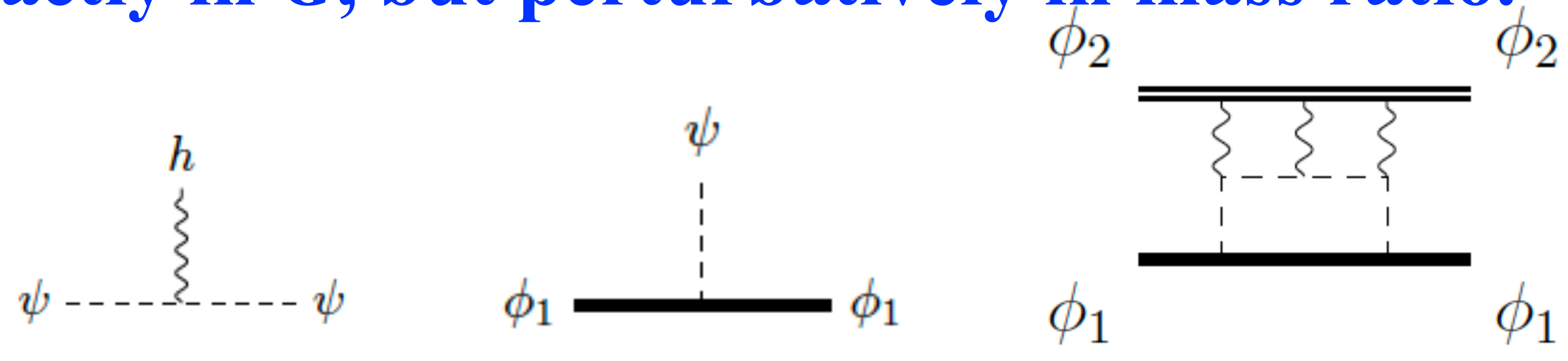
Bini, Damour, De Angelis, Geralico, Herderschee, Roiban, Teng

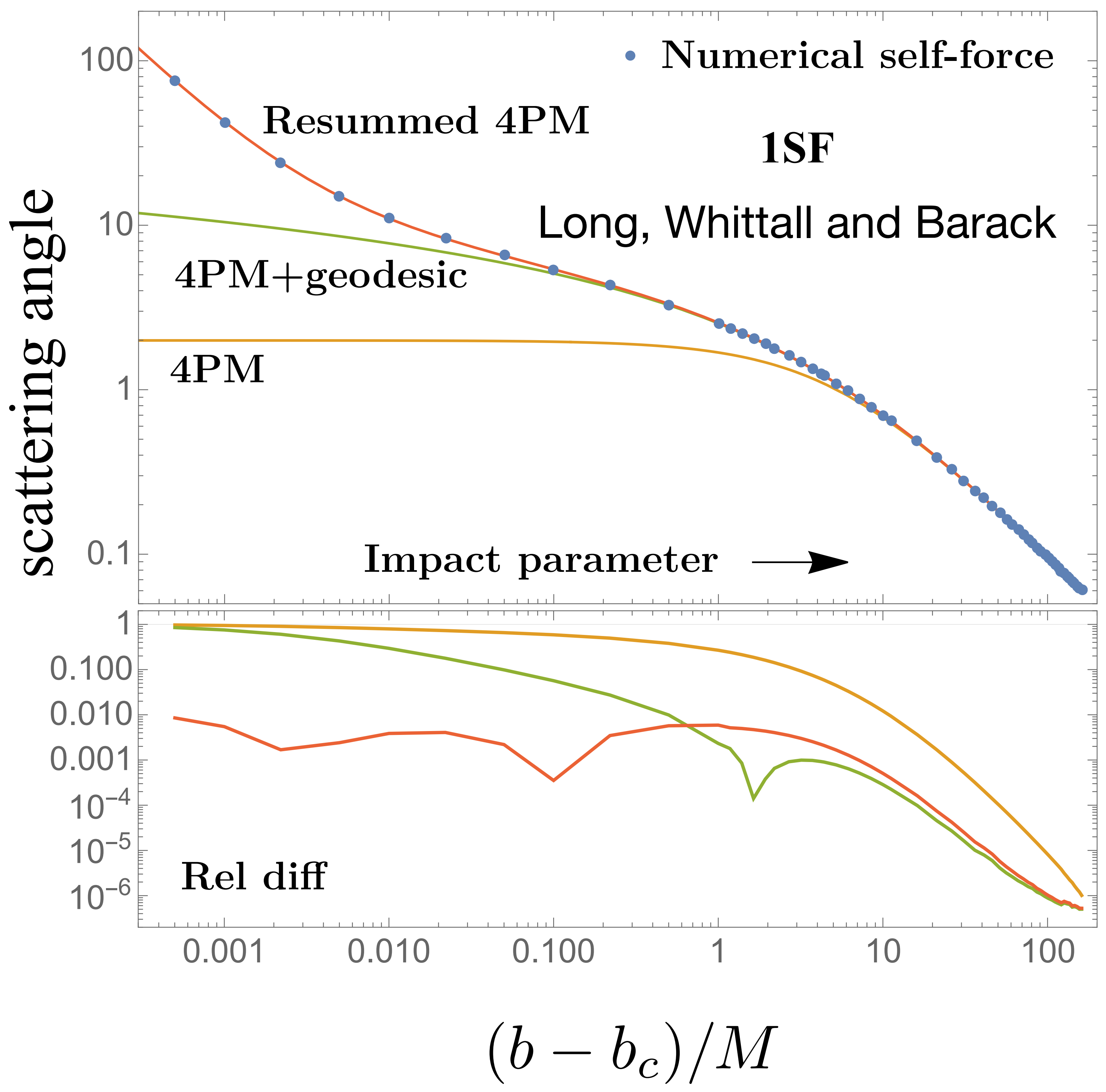
Great cooperation between amplitudes and more traditional approach.

Using Gravitational Self Force to Improve PM

GSF: Solve Einstein's equation semi-numerically exactly in *G*, but perturbatively in mass ratio.

Mino, Sasaki, Tanaka; Quinn, Wald; Poisson, Pound, Vega: Barack, Pound, etc





Scalar model, simplify GR. Charged scalar couples to 1 black hole.

Barack, ZB, Herrmann, Long, Parra-Martinez, Roiban, Ruf, Shen, Solon, Teng, Zeng; Jones and Ruf

Improve PM by importing detailed GSF information on log singularity at $b = bc$ further improves perturbative expansion.

Barack, Long, Whittall

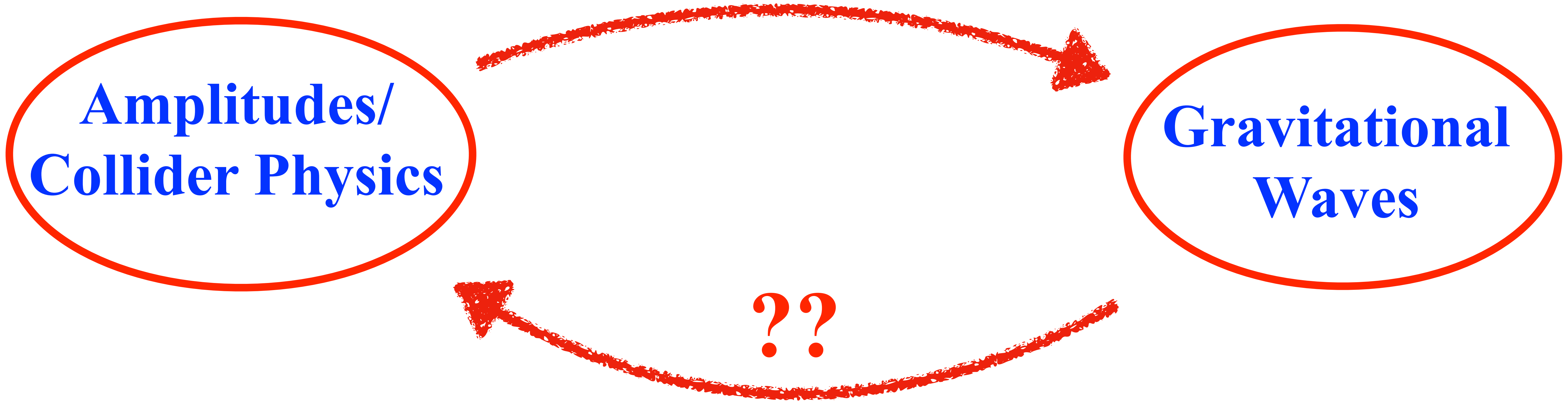
Precision of resummed PM in entire range impressive

See Buonanno's talk for more on PM resummations

Another example: use GSF ideas directly to reorganize PM in EFT context.
Why are we expanding Schwarszchild or Kerr?

Kosmopoulos and Solon; Cheung, Parra-Martinez, Rothstein, Shah, Wilson-Gerow

Can Gravitational-Wave Advances Help Amplitudes?



Idea flow from collider physics and amplitudes to gravitational waves:

- Use of EFTs
- Unitarity method
- Double copy
- Advanced loop integration methods
- Descriptions of spin

What can we import back to Amplitudes and/or Collider Physics?

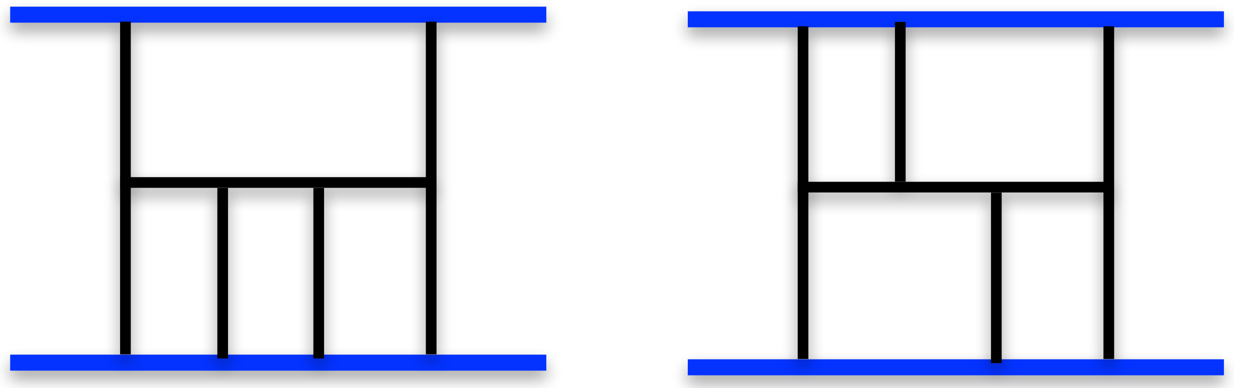
Can Gravitational Waves Help Amplitudes?

Integration:

ZB, Herrmann, Roiban, Ruf, Smirnov, Smirnov, Zeng (mostly FIRE based); Smirnov, Zeng
Driesse, Jakobsen, Mogull, Plefka, Sauer, Usovitsch (KIRA based)

5PM pushes the limits. Need to greatly improve IBP programs.

Already targeted and general improvements in FIRE, and KIRA and new private program.



Integrands:

ZB, Herrmann, Roiban, Ruf, Zeng (to appear)

Desire for clean integrands also pushed advances:

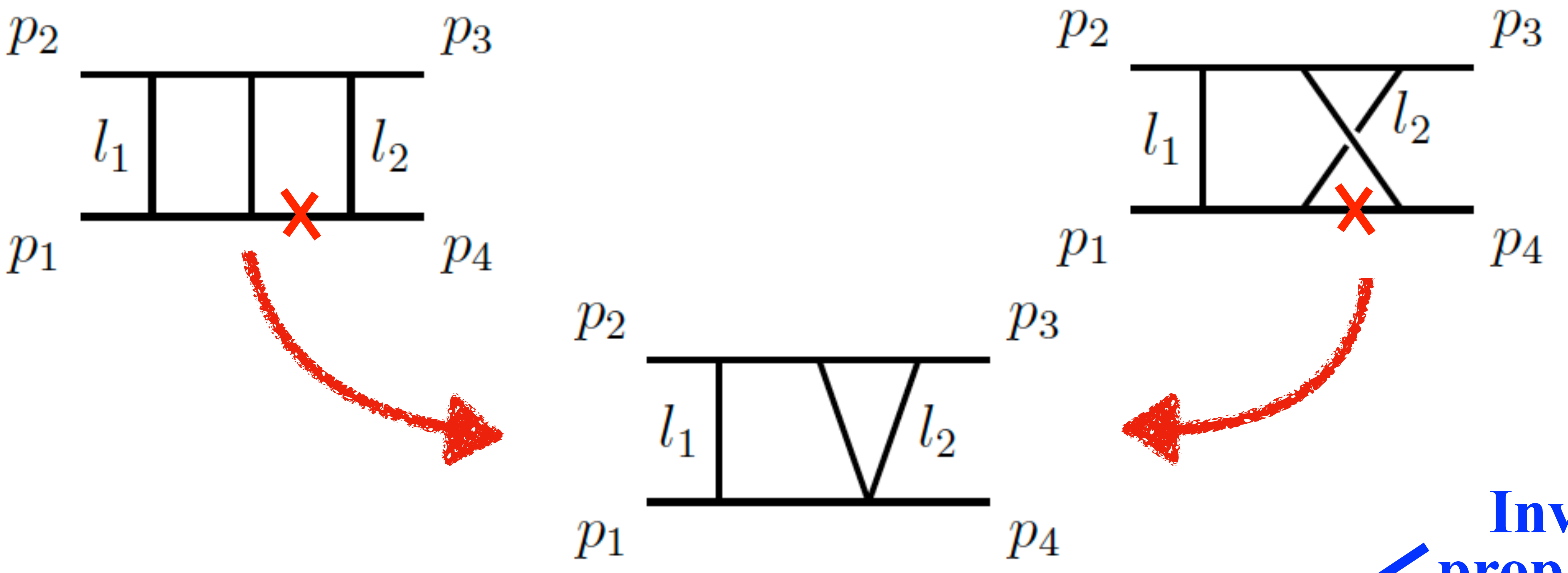
— An integrand basis, including nonplanar.

— Trivialize cut merging (terms in integrand aligned with terms in cuts)

— BCJ double copy to all loop orders.

An Integrand Basis

ZB, Herrmann, Roiban, Ruf, Zeng (to appear)



numerator terms can cancel propagators

diagram mix via contact terms

Start with the inverse propagator basis for each diagram:

Inverse propagator **ISP**

$$P_1 = l_1^2 - m_1^2, \qquad I_1 = p_4 \cdot l_1$$

$$\{P_1, P_2, P_3, P_4, P_5, P_6, P_7, I_1, I_2\}$$

Inverse propagator basis standard for IBP

Global integrand basis:

- **Explicitly cancel propagators**
- **Relabel duplicated diagrams to uniform labels and ISPs**
- **If there are external polarizations or spinors more thing to track but idea similar**

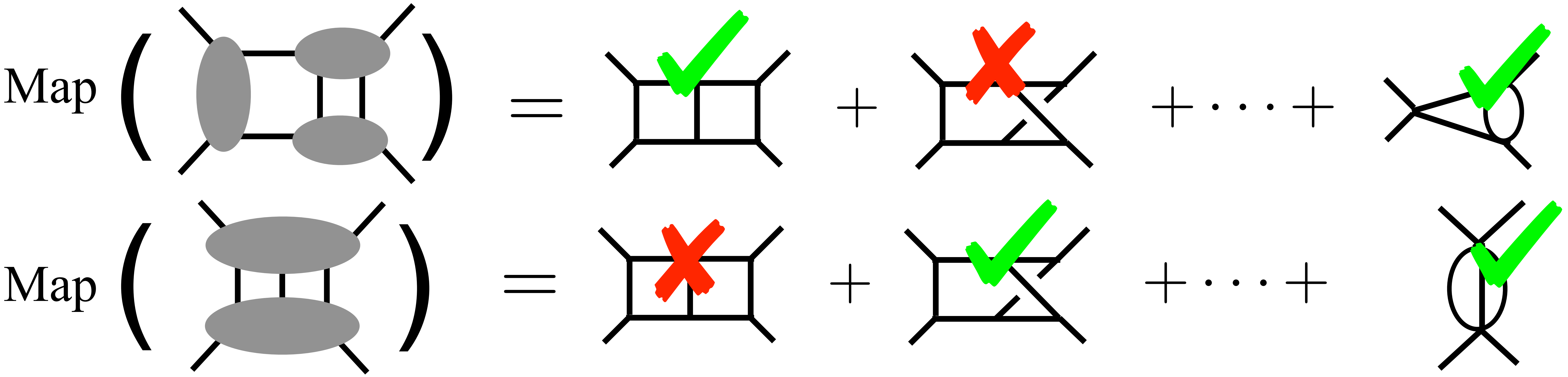
Key point: After mapping to unique labels and ISPs this is *global integrand basis**

What you can do with this?

* ignoring massless bubble on external legs issues

In Mapped Basis, Cut Terms are Integrand Terms

Inverse propagator basis: Cuts act trivially. Term present or not present.



Cancel all propagators that can be cancelled

Count terms once and only once (even in each cut)

mapped cut term = mapped integrand term.
cut merging trivialized. No linear algebra.

Related to earlier work e.g.

Inverse propagator basis is used for IBP — Chetyrkin, Tkachov; Laporta
Mapping between families widely used for IBPs
Multiloop cut merging is old. — ZB, Dixon, Kosower
Idea of finding a basis. — Bourjaily, Herrmann, Langer, Trnka; Bourjaily, Langer, Zhang

Also see talk from Figueiredo

Applications:

1. **Gauge invariance manifest. Any construction gives identical integrand***
2. **Meshes nicely with IBP. Stay on cuts and reconstruct at master integral level.**
3. **Bypasses difficulties with finding multiloop BCJ integrands. Double copy to all loop orders.**

Double Copy to All Loop Orders

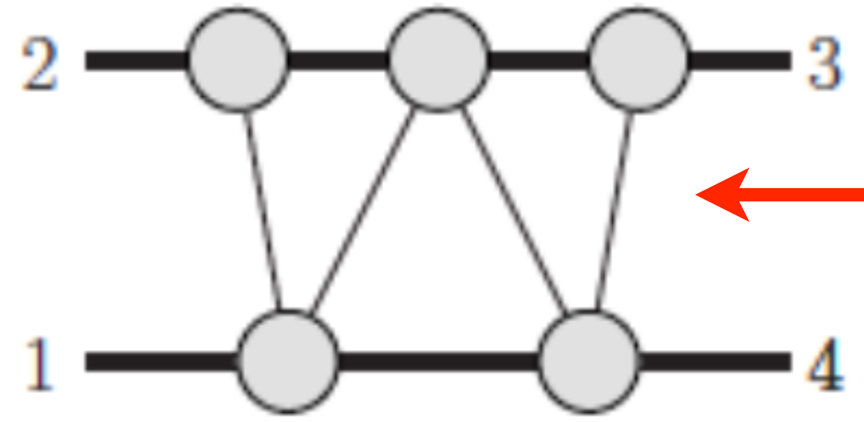
Sometimes difficult to find BCJ form of gauge-theory loop integrands.

— **2-loop 5-point pure YM (identical helicity) has solution but high power count.**

Mogull and O'Connell

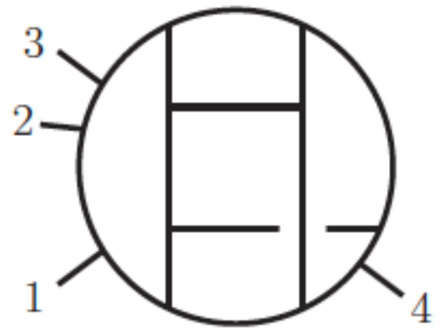
— **$N = 4$ sYM 5-loop 4-point patched double copy via correction formulas.**

ZB, Carrasco, Chen, Johannson, Roiban



Insert state projector.
e.g. remove dilaton

Double copy each tree



ZB, Herrmann, Roiban, Ruf, Zeng (to appear)

- **Use spanning set of cuts.**
- **Map to integrand basis: read off complete "unique" integrand.**

5PM Einstein gravity integrand constructed this way.

- **No need to bother finding multiloop BCJ integrand.**
- **Multiloop supergravity awaits! New studies of "enhanced UV cancellations"**

See talk from Kallosh

ZB, Chen, Edison, Gopalka, Jones, Herrmann, Roiban, Ruf (ongoing)

Conclusions

Challenges of gravitational waves push the limits of technical and conceptual issues.

1. **Over the past year major progress in various directions:**
   - — High orders, marching on 5PM, O(G^5)
   - — Spin, great progress but basic puzzles remain.
   - — Radiation and subtlety resolution via cooperation with GR.
   - — Cooperation with self force community.
   - — Absorption (didn't discuss)

2. **Feedback into amplitudes.**
   - — Improved integration programs.
   - — Simple Non-planar integrand bases. Trivialize cut merging.
   - — Double copy to all loop orders.

Judging from the pace of progress expect many new results in coming years.

Extra

Methods for Extracting Classical Physics.

There are now multiple alternative ways to extract classical physics.

- **EFT matching to 2 body Hamiltonian** — Cheng, Solon, Rothstein; ZB, Cheung, Roiban, Shen, Zeng
- **Map to EOB** — Bini, Damour, Geralico
- **Calculate physical observables** — Kosower, Maybee, O'Connell
- **Eikonal phase** — Amati, Ciafaloni, Veneziano; Di Vecchia, Heissenberg, Russo, Veneziano ADD SPIN
- **Amplitude radial-action relation** — ZB, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng
- **Exponential representation** — Damgaard, Plante, Vanhove; Bjerrum-Bohr, Plante, Vanhove
- **Heavy mass field theory** — Brandhuber, Chen, Travaglini, Wen Damgaard, Haddad, Helset
- **World line formalisms** — Goldberger, Rothstein; Levi, Steinhoff; Dlapa, Kälin, Liu, Porto; Jakobson, Mogul, Plefka, Steinhoff; Edison, Levi; etc

Some Results on Spin from Past Year

See the talk from Cangemi

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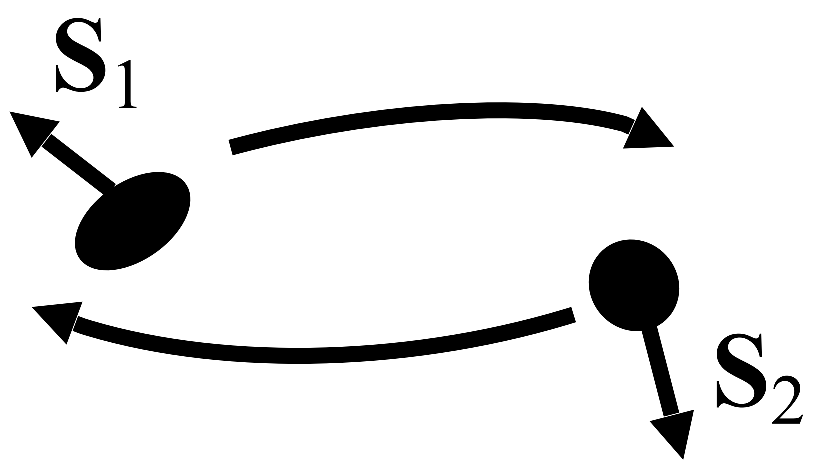


Very active subfield

- **Classical gravitational scattering amplitude at $O(G^2 S_1^\infty S_2^\infty)$** Aoude, Haddad, Helset
- **5 PN precision worldline EFT of spinning gravitating objects** Levi and Zin
- **Eikonal formulas for spin using KMOC** Gatica; Luna, Moynihan, O'Connell, Ross
- **WLQFT 4PM $O(G^4 S_1)$ including dissipation, impulse, spin kick and angle** Jakobsen, Mogull, Plefka, Sauer
- **WL QFT up to S^4, match to solutions of Teukolsky.** Ben-Shahar
- **Leading-order gravitational radiation to all spin orders** Aoude, Haddad Heissenberg, Helset
- **Fixing gravitational EFT couplings in the Kerr Solution.** Scheopner, Vines
- **Black hole absorption in presence of spin via on-shell approach** Jones, Ruf; Chen, Hsieh, Huang, Kim
- **First PM-informed spinning EOB waveform model: SEOBNR-PM.** Buonanno, Mogull, Patil, Pompili
- **Compton amplitude for Kerr BH and QFT and Teukosky equation** Bautista, Guevara, Kavanagh, Vines; Bautista, Bonelli, Iosa, Tanzini, Zhou; Cangemi, Chiodaroli, Johansson, Ochirov, Pichini, Skvortsov

Major advances in computations and understanding

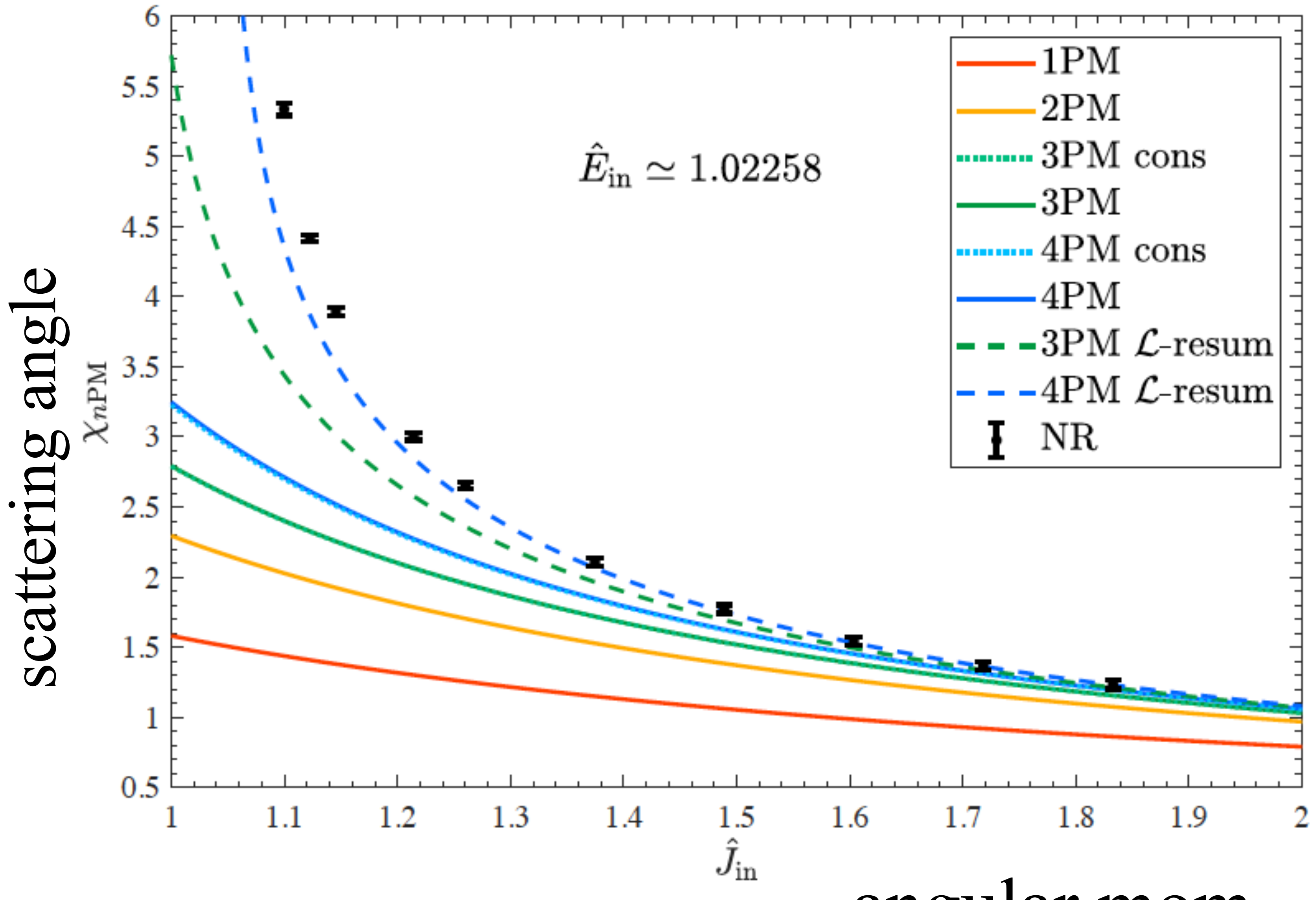
Transfer of Ideas from Amplitudes/Collider Physics to Gravitational Wave Problem

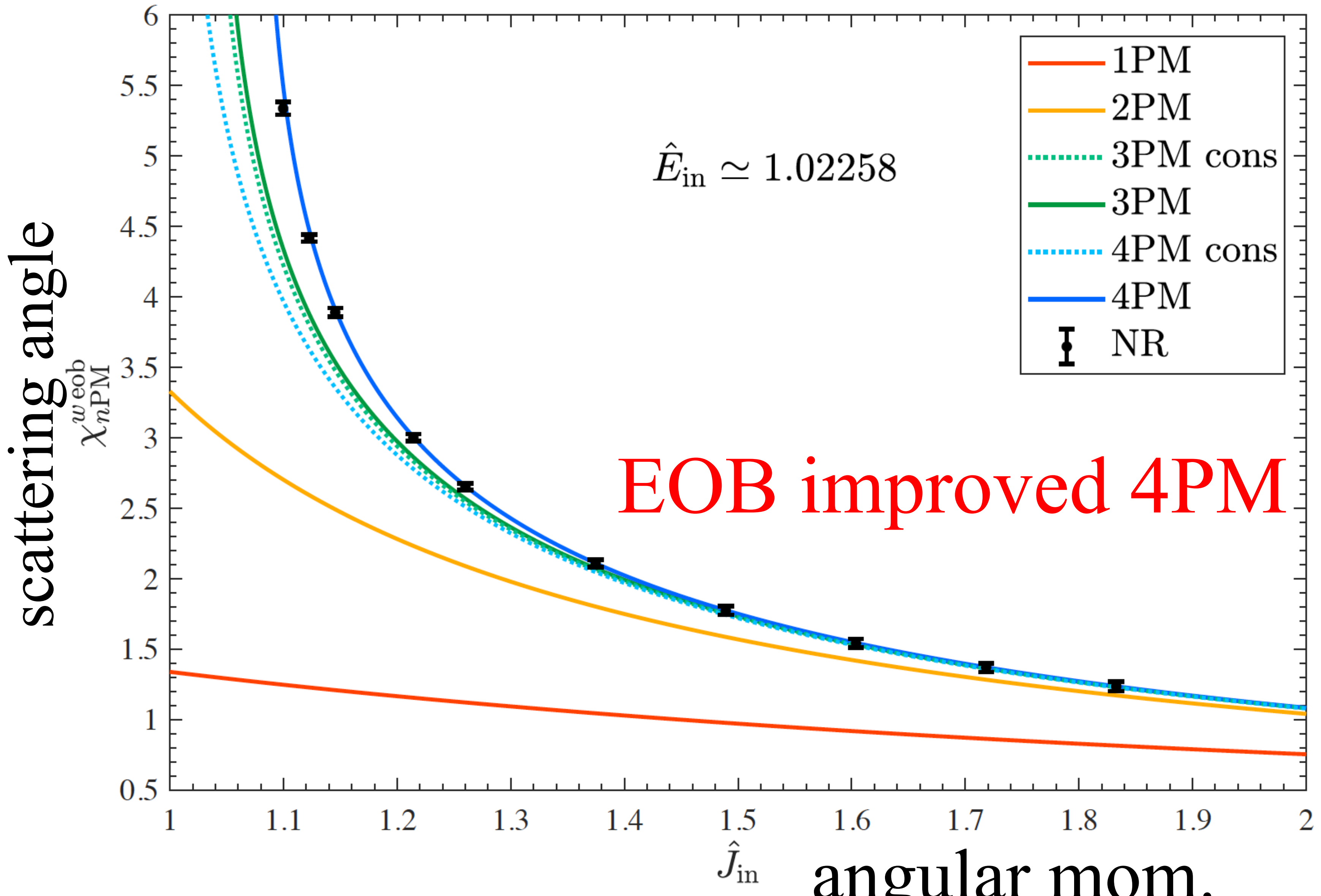
- QCD/HQEFT and EFT methods, separation of scales — Caswell, Lepage; Luke, Manohar, Rothstein; Golberger, Rothstein; Cheung, Rothstein, Solon
- Unitarity methods, recycling trees into loops — ZB, Dixon, Dunbar, Kosower
- Double copy: recycle gauge theory into gravity — Kawai, Lewellen, Tye; ZB, Carrasco, Johansson
- Integration by parts reduction to master integrals — Chetyrkin, Tkachov; Laporta
- Differential equations for master integrals — Kotikov; ZB, Dixon, Kosower; Gehrmann, Remiddi; Henn, Smirnov
- Method of Regions, very useful in classical limit! — Beneke, Smirnov
- Methods for evaluation of phase-space integrals. — Kosower, Page

Methods work well because classical limit is simpler

Comparison with Numerical Relativity

Khalil, Buonanno, Vines, Steinhoff; Damour and Rettegno





Plot uses:

4PM Conservative: ZB, Parra-Martinez, Roiban, Ruf. Shen, Solon, Zeng;
Damgaard, Hansen, Planté, Vanhove; Jakobsen, Gustav Mogull, Plefka, Sauer, Xu;
Bjerrum-Bohr, Plante,Vanhove.

4PM Dissipative: Manohar, Shen and Ridgeway; Dlapa, Kalen, Lui, Neef, Porto;
Damgaard, Hansen, Planté, Vanhove.

NR: Damour,. Guercilena, Hinder, Hopper, Nagar, Rezzolla;

- **Surprisingly good agreement with numerical relativity!**
- **Proves we are on a good track!**
- **Motivates continued work 5 PM order.**